Special Matrices

(1) Identity Matrix

- **Identity Matrix** is the matrix which is $n \times n$ square matrix where the diagonal consist of 1s and the other elements are all 0s.
- If any matrix is multiplied with the identity matrix, the result will be given matrix. The elements of the given matrix remain unchanged.
- It is also called as a *Unit Matrix* or *Elementary matrix*. It is represented as I_n or just by I, where n represents the size of the square matrix.
- For example.
- Given the I_n of matrices:
$$I_1 = 1$$
$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- ***Example:***

Let us check if the two matrices given below is an identity matrix by multiplying each other:
$$C = \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix}$$
$$D = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 1 & 0 \end{bmatrix}$$

```
format rat
C = [0 1 ; -2 1]
```

```
C =
       0              1
      -2              1
```

```
D = [1/2 -1/2 ; 1 0]
```

```
D =
       1/2           -1/2
       1              0
```

```
CD = C*D
```

```
CD =
       1              0
       0              1
```

```
DC = D*C
```

```
DC =
       1              0
       0              1
```

Solution:
$$CD = \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix} \times \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$DC = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(2) Diagonal Matrix

- A diagonal matrix is a square matrix where all entries outside the main diagonal (from top-left to bottom-right) are zero, though entries on the diagonal can be any value, including zero.
- It's a fundamental concept in linear algebra, simplifying calculations and representing transformations in simpler coordinate systems, with examples like the identity matrix (all 1s on diagonal) and zero matrix (all 0s) being special types.
- A 4x4 matrix representing a diagonal matrix:
$$\begin{bmatrix} a_{1,1} & 0 & 0 & 0 \\ 0 & a_{2,2} & 0 & 0 \\ 0 & 0 & a_{3,3} & 0 \\ 0 & 0 & 0 & a_{4,4} \end{bmatrix}$$

- ***Example:*** A 7x7 diagonal matrix generator (numbers are randomly generated):

```
size = 7;
Diagonal = zeros(size);
for c = 1:size
    Diagonal(c,c) = randi([-30,30]);
end
Diagonal
```

```
Diagonal =
      13              0              0              0              0              0              0
       0             25              0              0              0              0              0
       0              0             24              0              0              0              0
       0              0              0            -10              0              0              0
       0              0              0              0             12              0              0
       0              0              0              0              0            -18              0
       0              0              0              0              0              0            -29
```

(3) Upper-Triangular Matrix

- An ***upper triangular matrix*** is a square matrix where all entries below the main diagonal (from top-left to bottom-right) are zero, meaning $a_{ij} = 0$ for all row indices i greater than column indices $j(i > j)$.
- These matrices have non-zero elements only on or above the diagonal, forming a "triangle" of non-zeros in the upper portion.
- They are crucial in linear algebra for simplifying calculations like finding determinants (product of diagonal elements) and solving systems of equations through a process of **backward substitution**.
- A 4x4 matrix representing an upper-triangular matrix:
$$\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ 0 & a_{2,2} & a_{2,3} & a_{2,4} \\ 0 & 0 & a_{3,3} & a_{3,4} \\ 0 & 0 & 0 & a_{4,4} \end{bmatrix}$$

- ***Example:*** A 6x6 upper-triangle matrix generator (numbers are randomly generated):

```
size = 6;
LU_matrix = zeros(size);
rng = randi([-30,30],size);
for row = 1:size
    for col = 1:size
        if row <= col
            LU_Matrix(row,col) = rng(row,col);
        end
    end
end
```

```
LU_Matrix
```

```
LU_Matrix =
    15    22   -29    -2   -25     1
     0    19    -1   -27    19    29
     0     0   -20    11    19     9
     0     0     0   -28    14    18
     0     0     0     0   -21    -3
     0     0     0     0     0    -4
```

(4) Lower-Triangular Matrix

- A ***lower triangular matrix*** is a square matrix where all entries **above** the main diagonal (from top-left to bottom-right) are zero, meaning $L_{i,j} = 0$ for all row indices i smaller than column indices j.
- These matrices have non-zero elements only on or below the diagonal, forming a "triangle" of non-zeros in the lower portion.
- They are crucial in linear algebra for simplifying calculations like finding determinants (product of diagonal elements) and solving systems of equations through a process called **forward substitution**.
- $$\text{A 4x4 matrix representing a lower-triangular matrix:} \begin{bmatrix} a_{1,1} & 0 & 0 & 0 \\ a_{2,1} & a_{2,2} & 0 & 0 \\ a_{3,1} & a_{3,2} & a_{3,3} & 0 \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \end{bmatrix}$$

- ***Example:*** We'll use same 6x6 matrix (rng) from previous example to illustrate a lower-triangle matrix:

```
size = 6;
LU_Matrix = zeros(size);
for row = 1:size
    for col = 1:size
        if row >= col
            LU_Matrix(row,col) = rng(row,col);
        end
    end
end
LU_Matrix
```

```
LU_Matrix =
    15     0     0     0     0     0
     0    19     0     0     0     0
    -1     5   -20     0     0     0
    25   -19    29   -28     0     0
     7   -16    13   -26   -21     0
     7    24     0     1    10    -4
```