Corrections to *A Numerical Study of Vortices and Turbulence in Quantum Fluids*

In the corrected thesis the following changes have occurred:

- Corrected spelling, grammar, and small errors throughout.
- Corrected throughout so that $\rho = \mu/g = N/D^3$ always takes uniform values and $n(\mathbf{r}, t) = |\psi(\mathbf{r}, t)|^2$ is always spatially dependent, for consistency.
- Page 5 - Changed volume from $V$ to $D^3$, for consistency.
- Page 6 - Added that quantised circulation is predicted by describing the system in terms of a wavefunction.
- Page 6 - Added two sentences detailing the experimental observation of rotons and the $^4$He dispersion relation.
- Page 8 - Added that s-wave scattering dominates over p-wave and d-wave scattering in the limit of low energy collisions.
- Page 8 - Expanded description of attractive BEC instability to collapse.
- Page 9 - Described how a helium II or atomic superfluid can be forced to rotate.
- Page 11 - Rephrased "fluids" to "interfacial fluid waves" for accuracy, and removed the word "shallow" to avoid misleading readers, as shallow water waves are sometimes described by the *defocusing* NLS.
- Page 12 - Expanded the discussion of classical and quantum turbulence.
- Page 13 - Added "the collision of two vortex lines such that there is a change of topology" as a description of reconnection.
- Page 18 - Rewrote this paragraph so as to describe the meaning of the symbols for the chemical potential ($\mu$) and particle mass ($m$), and introduce the definition of the terms $V_{\text{obj}}$ and $V_{\text{trap}}$.
- Page 18 - Replaced "$T \approx T_\lambda/2$" with the more accurate "providing the temperature of the system is small compared to the critical temperature for Bose-Einstein condensation".
- Page 18 - Swapped Sections 2.3 and 2.4 so that I refer to equations only after they've appeared.
- Page 18 - Removed $\exp(-i\mu t/\hbar)$ from $\psi = \psi_0(r)$ when deriving the time independent GPE, as the $-\mu$ term is already part of the GPE.
- Page 19 - Explained the reasoning behind Equation 2.7.
- Page 21 - Added a sentence explaining that the density vanishes at the centre of a vortex.
- Page 22 - Introduced the formula for the speed of sound $c$ for a homogeneous condensate here, also referencing Appendix B.4 for details.
- Page 23 - Rewrote several sentences in Section 2.9 so as to rephrase the description of decaying imaginary time solutions.
- Page 27 - Removed the word "coherent" as $\psi$ is already in the mean field formulation.
- Page 27 - Corrected reference to Equation 2.2.
- Page 28 - Moved Figure 2.2 and associated text so that the meaning of $\gamma$ is introduced first.
- Page 31 - Corrected the vortex core solution by adding $\sqrt{\rho}$.

- Page 32 - Rephrased sentence on the "asymptotic behaviour" of $f(r)$, instead quoting the values of $f(0)$ and $f(r)$ as $r \to \infty$, also citing [188] in reference to the the accuracy of the Padé approximation.
- Page 33 - Rephrased the $1/r$ dependence so that it references Equation 2.37 rather than a figure, and instead introduced Figure 2.7 (b) elsewhere.
- Page 33 - Rephrased the sentence introducing "strongly repulsive" interactions to define what "strong" means (the interaction term is large compared to the size of the kinetic energy term).
- Page 35 - Better explained the distribution of $|a_k|$, and explained that the number density, $N/V$, and kinetic energy density, $E_{\text{kin}}/V$, uniquely determines the dynamics and equilibrium temperature.
- Page 40 - Rewrote a paragraph in Section 3.1.2 to better describe the process of substituting and rearranging with $\xi$ to find the healing length in terms of $l_r$.
- Page 43 - Added a sentence defining the vortex field as the field of values $\Delta\theta(C_{[i,j]})$ for every $[i, j]$.
- Page 43 - Rephrased "each connected component" as "each set of connected grid points where $\Delta\theta(C_{[i,j]}) \neq 0$.
- Page 44 - Added a sentence explaining at what scale the low-pass filter should act.
- Page 50 - Added a sentence explaining that the parameters chosen to generate random points are arbitrary, chosen just for demonstration.
- Page 51 - Added a paragraph explaining that $\mathbf{R}_C$ and $R_C$ vary over time, and so time based averaging should also be used.
- Page 54 - Explained our choice of using the filtering technique over other methods for quasi-condensate visualisation.
- Page 61 - Expanded the explanation of why the finite boundary has a negligible effect of vortex shedding.
- Page 62 - Added "where $c$ is the speed of sound." to Figure 4.2.
- Page 64 - Added that the reason the vortex pair separation decreases is due to the velocity field of surrounding nucleated vortices.
- Page 65 - Removed direct reference to Figure 4.6, so that Figure 4.5 is referenced in the text first.
- Page 66 - Moved paragraph explaining the Eulerian limit closer to the first mention of the Eulerian limit.
- Page 66 - Corrected the derivation of $v_{c2}$ in section 4.3.4 so that quantities are expressed in dimensional form.
- Page 66 - Defined $v_{c1}$ and $v_{c2}$ in the text.
- Page 73 - Extended the description of the "superfluid Reynolds number" and compared the findings of Reeves *et. al.* to our simulations in Table 4.1.
- Page 75 - Rephrased typical condensate sizing to "only one or two orders of magnitude larger than the vortex core size".
- Page 76 - Corrected $Nal_z^3/l_r^3 \ll 1$ to $Nal_z^3/l_r^4 \ll 1$.
- Page 76 - Explained why Kelvin waves are suppressed in $z$.

- Page 77 - Explained that $V_0$ and $d$ are defined “so as to match the experimental parameters”.
- Page 78 - Added a sentence explaining the effect of changing $\gamma$.
- Page 82 - Described which line is which for $N_d(t)$ and $N_a(t)$ in the caption of Figure 5.5.
- Page 84 - Explained why $N_d$ is not of a decaying form, citing [259].
- Page 85 - Detailed the 4-body loss argument.
- Page 87 - Expanded the discussion of the crescent shaped density depletion, adding the observations of references [267, 261, 262].
- Page 89 - Quantified the reduction in sound production from using an elliptical obstacle by adding Figure 5.9, a plot of the compressible kinetic energy over time.
- Page 91 - Replaced “semi-classical” with “quasi-classical” for consistency.
- Page 92 - Rephrased the reference to the simulations of Frisch *et. al.* for style.
- Page 93 - Added the sentence “By using a strongly non-equilibrium initial condition, with random phase at all points in space, we model an infinitely fast quenched system.”
- Page 94 - Removed erroneous averaging from the energy density $\langle E \rangle / D^3 \rightarrow E/D^3$, as $E$ is already an integrated quantity.
- Page 95 - Expanded the discussion on weak turbulence predictions, leadsing to R-J distribution.
- Page 95 - Described the ‘shoulder’ in the integral distribution, indicating a macroscopic fraction of the particles in low-k modes, facilitating the choice of $k_c$.
- Page 95 - Added a second filtering level, $k_c = 40(2\pi/D)$, expanding discussion of quasi-condensate filtering throughout the chapter.
- Page 96 - Added definition of the homogeneous density of the quasi-condensate, $\rho_0$.
- Page 98 - Corrected $\sqrt{\rho}$ to $\sqrt{\rho\xi^3}$ in Equation 6.3.
- Page 98 - Explained that $k_\ell$ is allowed to vary over time and $k_\xi$ is defined using the value of $\xi$ at equilibrium.
- Page 99 - Removed the $k^{-1}$ scaling prediction from the text and Figure 6.4.
- Page 99 - Added a plot to Figure 6.4 showing the unfiltered incompressible kinetic energy.
- Page 99 - Added an inset to Figure 6.4 showing the incompressible kinetic energy filtered by $k_c = 40(2\pi/D)$.
- Page 101 - Updated Table 6.2 so as to also show the power law fits for $k_c = 40(2\pi/D)$.
- Page 101 - Removed the phrase “cooler simulations” for style.
- Page 102 - Added the plot of $\ell(t)$ to Figure 6.6 (b), so as to show an assertion made in the text.
- Page 102 - Added the sentence “In all cases, at the time of $t = 5000/\tau$ there remains only a single vortex ring in the system”.
- Page 103 - Explained in words the effect of $k_c$ on $f$ and $I$.
- Page 108 - Added Figure 6.10, a schematic picture of the polarisation $R$.
- Page 109 - Compared our results to that of Nore *et. al.* and Kobayashi *et. al.*.
- Page 113 - Described ways in which we expect vortex kinetic energy loss to sound emission.

- Page 113 - Expanded the description of the HVBK equations.
- Page 114 - Clarified the $z$ boundary conditions.
- Page 115 - Clarified that $v_c \approx 0.36$ in the Eulerian limit (obstacle diameter $d \gg \xi$).
- Page 116 - Moved Table 7.1 closer to its first mention in text.
- Page 117 - Corrected the statement as surface roughness is not required for secondary vortex nucleation, but instead vastly reduces the velocity requirements.
- Page 117 - Rephrased the description of why the largest mountains are the main producer of vorticity so as to correctly refer to local curvature.
- Page 118 - Changed all occurrences of values for $\beta$ from e.g 50% to 0.5.
- Page 120 - Clarified the formation of a 2D boundary layer through the addition of Figure 7.8.
- Page 124 - Referenced Figure 7.12 in the text.
- Page 125 - Repeated the definition of the local velocity here.
- Page 126 - Rephrased "which in superflow at absolute zero are absent" to "which in superflow at absolute zero operate through a fundamentally different mechanism".
- Page 135 - Added a section in the appendix, "Critical temperature and condensate fraction for a non-interacting Bose gas".
- Page 140 - Explained the origin of the normalisation term $N!/n_1!...n_\infty!$.
- Page 141 - Added a definition of the functions $\phi_i(\mathbf{r})$.
- Page 142 - Added a description of why the derivation is only valid when particle interactions are weak.
- Page 149 - Expanded description of $\xi$ to show $\xi = \hbar/\sqrt{mgn}$ and $\xi = \hbar/\sqrt{mg\rho}$ are the same when considering a homogeneous system, as $n(\mathrm{r}, t) = \rho$ everywhere.
- Page 149 - Added detailed definition of the local and homogeneous speed of sound, $c_{\mathrm{local}}$ and $c$.
- Corrected and added several references.