turbulence with oscillating obstacles, including grids [145], wires [146–148], spheres [149] and others [150–154]. Methods without the use of oscillating structures have been also been devised, such as the use of heat flux [155] or container spin-down [156], to generate turbulence . A mutual friction [18] couples the normal fluid and superfluid components, and at finite temperature leads to a dissipation of the resulting turbulent vortex tangle. In the limit of very low temperatures the mutual friction tends to zero. It is expected that in this limit Kelvin wave excitations [143, 144] and vortex reconnections leads to dissipation via sound waves, but the entire mechanism is not yet fully understood [157].

1.5.2 Turbulence in BECs

Weakly interacting atomic BECs present a key improvement over helium II superfluidity for the study of quantum turbulence. Although the spatial extent of BECs support much fewer vortices than for superfluid helium, the structure of individual vortices and their turbulent dynamics can be experimentally resolved much easier. This is due to the relatively large vortex core size, on the scale of a micron, available to atomic BECs and the fact that the state of the entire atomic gas can be readily visualised through imaging techniques. Expansion imaging has provided a technical leap in this area, allowing individual vortex cores and reconnection events to be directly observed [85, 98, 101]. Improved real time and non-destructive imaging of condensates is now possible [65] by out-coupling of a small representative fraction of the gas for each image, allowing for observation of quantum vortex trajectories. A further recent improvement in this area [158] has allowed for the imaging of vortex polarity as well as its location in space. These technical leaps in imaging have made atomic condensates an extremely attractive medium for exploring quantum vortex turbulence.

The range of length scales available to classical turbulence and superfluid helium turbulence far outshines those currently available to atomic BECs, and much theory of classical turbulence is defined by the distribution of kinetic energy over the large number of length scales available. Nevertheless, numerical studies show quantum turbulence can distribute the kinetic energy in an atomic condensate in agreement with Kolmogorov's $k^{-5/3}$ law [159–161].

1.6 Thesis overview

An outline of the structure of this thesis and a brief description of each chapter is given in this section. The thesis is split into three main parts. Part I introduces the formalism, models and numerical tools that we go on to use in part II to model quantum fluids and generate numerical results. Part III is a collection of appendices. The following publications partially feature the results shown in the thesis, and collaborative contributions are highlighted here.

- Quantum analogues of classical wakes in Bose-Einstein condensates
  G. W. Stagg, N. G. Parker and C. F. Barenghi, J Phys B: At. Mol. Opt. Phys. **47** 095304 (2014)
- Generation and decay of two-dimensional quantum turbulence in a trapped Bose-Einstein condensate
  G. W. Stagg, A. J. Allen, N. G. Parker and C. F. Barenghi, Phys. Rev. A **91**, 013612 (2015)
- Classical-like wakes past elliptical obstacles in atomic Bose-Einstein condensates
  G. W. Stagg, A. J. Allen, C. F. Barenghi, N. G. Parker, J. Phys.: Conf. Ser. **594** 012044 (2015)
- Critical velocity of a finite-temperature Bose gas
  G. W. Stagg, R. W. Pattinson, C. F. Barenghi, N. G. Parker, Phys. Rev. A **93**, 023640 (2016)
- A superfluid boundary layer
  G. W. Stagg, N. G. Parker, C. F. Barenghi, arXiv:1603.01165 (2016)

Part I - Introduction and Theory

Chapter 1 introduces the concept of Bose-Einstein condensation, superfluidity, and the nature of quantum turbulence in weakly interacting atomic condensates and superfluid liquid helium.

Chapter 2 describes the theoretical concepts and mean-field methodology that allows us to accurately model a dilute, weakly interacting atomic Bose gas. We describe the Gross-Pitaevskii equation (GPE), a non-linear Schrödinger equation used to model condensates at zero temperature, and go on to extend the GPE to take into account finite temperature effects through phenomenological damping or the classical-field method. We detail some of the consequences of the model, such as quantised circulation, and show various initial conditions and simple solutions of the GPE.

Chapter 3 describes the theory and implementation of various numerical procedures we use to generate the simulations and results shown in part II. Our numerical time-stepping GPE solver is described, followed by an extensive description of our vortex locating and tracking routines. Finally a method is described for the filtering of the thermal part of the field when using the classical-field method.

Part II - Numerical Studies

Part II consists of a selection of the numerical simulations we have performed, interpretations of the results and any applications to real experimental systems.

Chapter 4 extends recent studies of moving obstacles in superfluids, a system mimicking a well known problem in classical viscous flows. We present numerical simulations of classical-like wakes, consisting of vortex clusters, generated by the presence of elliptical obstacles.

Chapter 5 was motivated by the recent experimental work of Kwon *et al.* [101]. We model their experimental set up in which a trapped condensate is translated past an obstacle, and study the decay of the resulting 2D quantum turbulence and vortex annihilation events. Communications with Y. Shin and A. Cidrim were helpful in the interpretation of the numerical simulations shown in this chapter.

Chapter 6 was motivated by further work of Kwon *et al.* [162], highlighting the need to extend the study of the critical velocity for vortex nucleation to finite temperatures. We use classical field methods to simulate finite temperature homogeneous Bose gases from a strongly non-equilibrium initial distribution, and classify the nature of the resulting turbulent vortex tangle. We introduce a cylindrical obstacle into equilibrium states for various temperatures, measure the critical velocity, and characterise the nature of the resulting vortex lines. The previous numerical work of A. J. Youd was helpful in the development of our classical-field code, used in this chapter.

Chapter 7 is inspired by recent experimental studies in helium II, generating turbulence with oscillating obstacles such as wires, spheres and grids [145–154]. We simulate a quantum fluid in the presence of a real rough boundary obtained via atomic force microscopy [163], with data kindly provided by C. R. Lawson. We find surprising evidence of the formation of a thin 'superfluid boundary layer' consisting of vortex loops and rings.

In Chapter 8 we briefly review the main conclusions of our work and discuss opportunities for further research in the area.

Part III - Appendix

Part III is a collection of appendices. Appendix A consists of various detailed derivations related to the theoretical modelling of an atomic BEC. Appendix B contains definitions of various quantities we refer to throughout the thesis. Appendix C is a collection of algorithms that we have used.