Mathematics Date - 05/03/22

Q1. Graph A graph "G" consists set of vector,

$$V = \{v_1, v_2, \ldots, v_n\}$$

called vertices or points or nodes and other set

$$E = \{e_1, e_2, \ldots, e_n\}.$$

the set $V(G)$ is called vertex set of G and $E(G)$ is called edge set.

usually the graph is denoted by

$G(V, E)$.

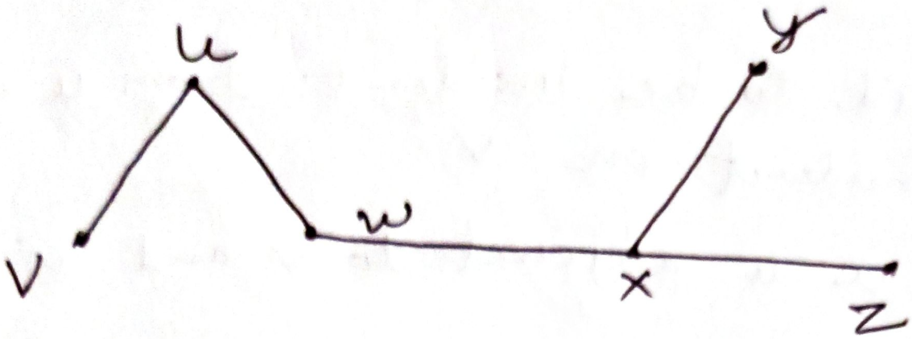
if $\boxed{e = uv}$ is an edge of graph G then we say that u & v are adjacent in G.

for example

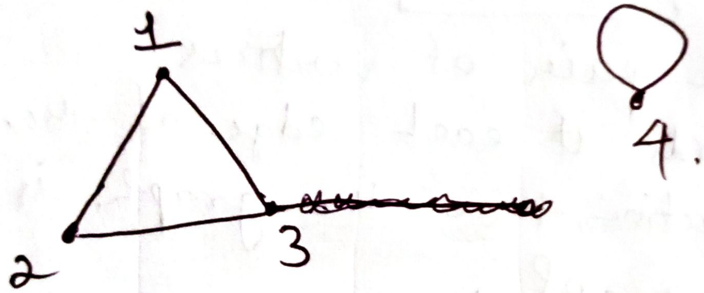
$$V(G) = \{u, v, w, x, y, z\}. \text{ and}$$

$$E(G) = \{uv, uw, wx, xy, xz\}.$$

we can draw the graph by considering the elements.

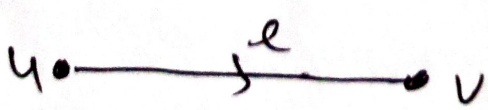


Example Draw a graph with vertices $V = \{1, 2, 3, 4\}$ and $E = \{(1,2), (1,3), (3,2), (4,4)\}$.



Directed Graph

A directed graph 'G' consists of a set V of vertices & set E of edges such that $\boxed{e \in E}$ is associated with an ordered pair of vertices in another words if each edge of the graph 'G' has a direction then the graph is called directed Graph.



Here u is called intial vertex of e.
and v is called terminal vertex of e.

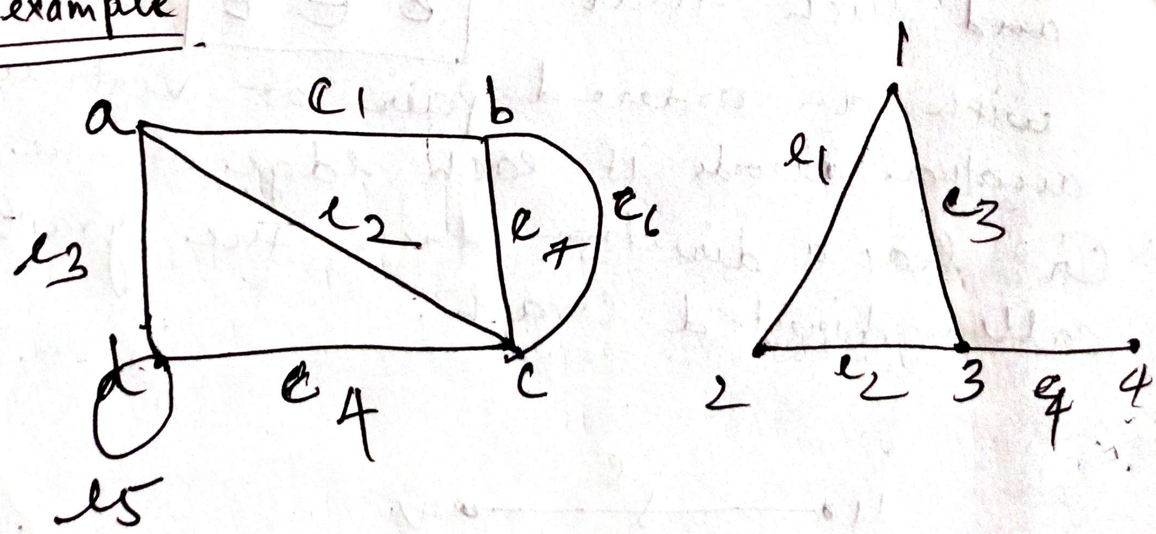
e is said to be incident from u and to be incident on v.

Here u is adjacent to v and vice-versa.

Undirected Graph

An undirected Graph 'G' consists of set V of vertices & a set E of edge such that each edge $\boxed{e \in E}$ is associated with an unordered pair of vertices.
in other words if each edge of the graph G has no direction then the graph is called undirected graph.
Here, we can refer an edge joining the vertex pair i and j as either (i,j) or (j,i).

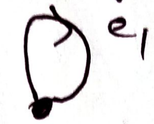
for example



loop (self loop) : An edge of a graph that joins a vertex/node to its self is called loop or self loop.

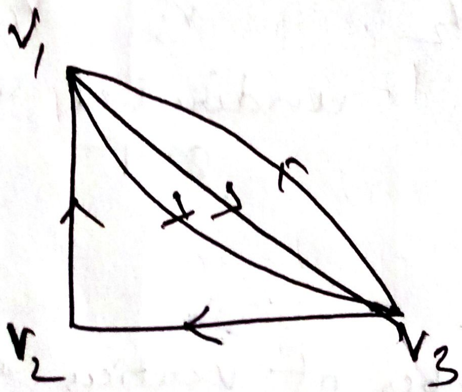
for example.

A loop is an edge for vertices (v_i, v_j)

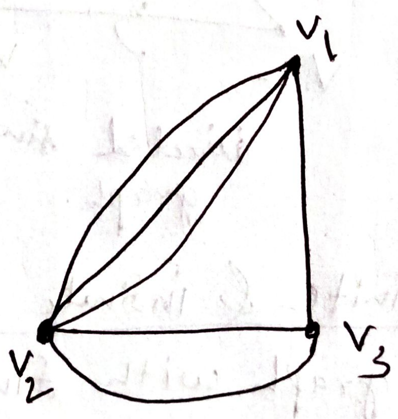


in other words, An edge is said to be a loop if its initial & terminal vertices are same.

Multi graph : In a multi graph no loops are allowed but more than one edge can join two vertices this edges are called multiple edges or parallel edges and the graph is called multigraph.

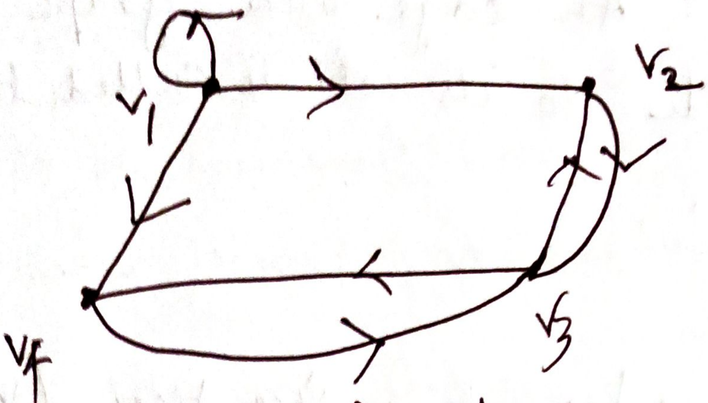


Directed Multigraph

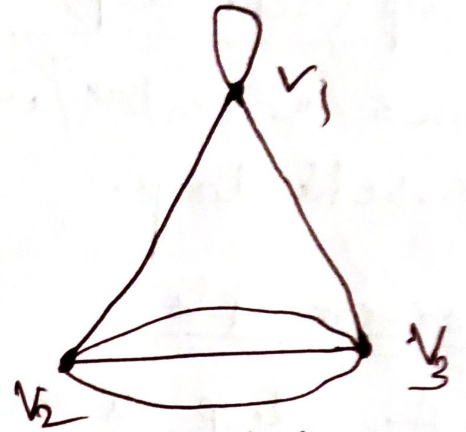


undirected Multigraph.

Pseudo Graph : A graph in which loops & multiple edges are allowed is called a sudo graph.



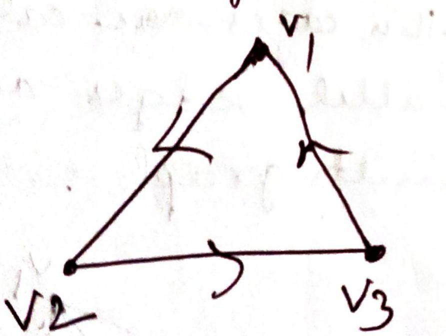
Directed Pseudo graph.



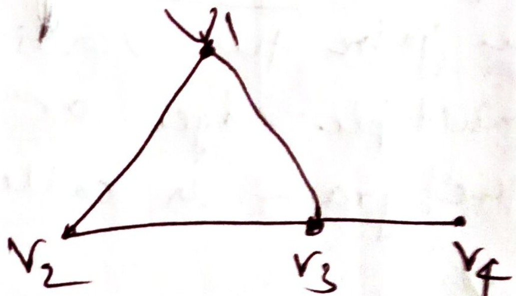
undirected Pseudo graph

Simple graph: A graph which has neither loops nor multiple edges is called simple graph.

i.e each edge connects two distinct vertices & no two edges connects two vertices.



directed simple graph.



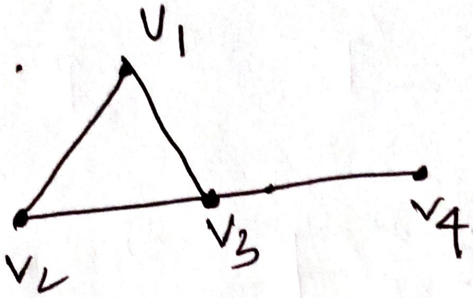
undirected simple graph

finite & infinite graph

A graph with finite number of vertices as well as finite number of edges is called finite graph. Otherwise it is called infinite graph.

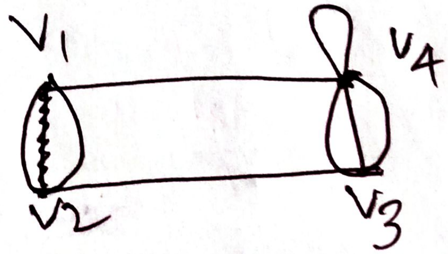
Degree of a vertex:

the number of edges incident on a vertex v_i with self loops counted twice is called the degree of vertex is denoted by $deg(v_i)$ or $d(v_i)$.



vertex(vi)	deg(vi)
v1	2
v2	2
v3	3
v4	1

total deg of graph is 8.



vertex(vi)	deg(vi)
V1	3
V2	3
V3	4
V4	6
	16.

Isolated vertex :

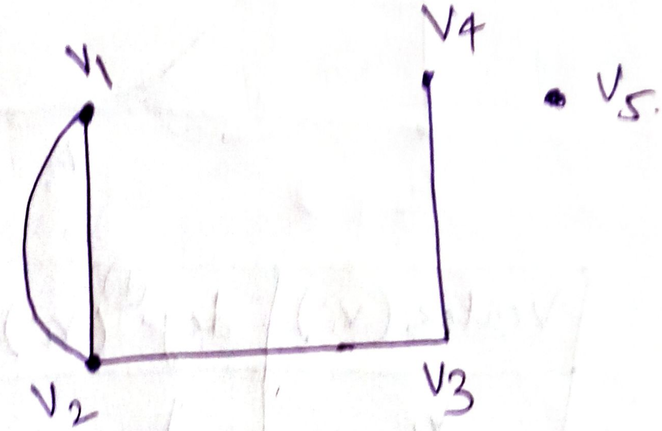
A vertex having no vertical edge is called Isolated vertex.

In order words isolated vertices are those with zero degree.

Pendent vertex :—

A vertex of degree one is called pendent vertex.

for example :

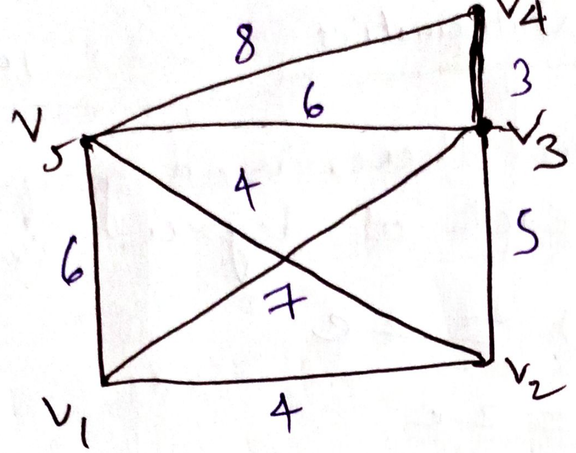


Here V_4 is Pendent vertex.

& V_5 is Isolated vertex.

Indegree and Out degree

In a graph 'G' the out degree of A vertex is denoted by $deg^{(+)}(V_1)$. is the number of edges begining at V_1 and Indegree of V_1 is denoted by $deg^{(-)}(V_1)$ is the no. of edges ending at V_1.



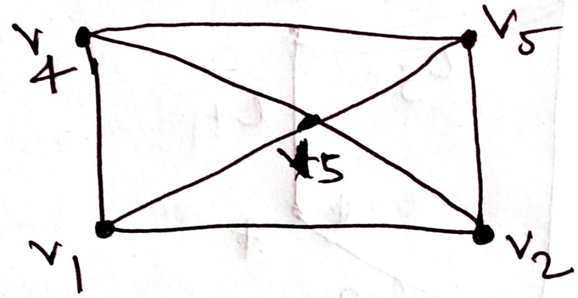
Remarks :

(1) - if 'G' = (V, E) be an undirected graph. ~~let G = (~~ with V, vertices & e, edges then $\sum_{v \in V} deg(v) = 2e$

this theorem is known as hand shaking theorem.

(2) - In an undirected graph the total no. of odd degree vertices is always even.

Q. verify the handshaking theorem for the following.



sol^n

Vertex	deg(v)
V_1	3
V_2	3
V_3	3
V_4	3
V_5	4

$\boxed{\text{Here, } e = 4}$

total degree = 16

and A/c to handshaking theorem.

$\sum deg(v) = 2 \times e$

$16 = 16$ Hence verify

Mathematics 10/04/23

Q How many edges are there in a graph with ten vertices each of degree five.

Sol^n

$$\sum \deg(V) = 2e$$

$$10 \times 5 = 2e$$

$$\boxed{e = 25}$$ Ans

Q Draw a graph where $V = \{1, 2, 3, 4,\}$

& $E = \{e_1, e_2, e_3, e_4, e_5\}$.

let us have,

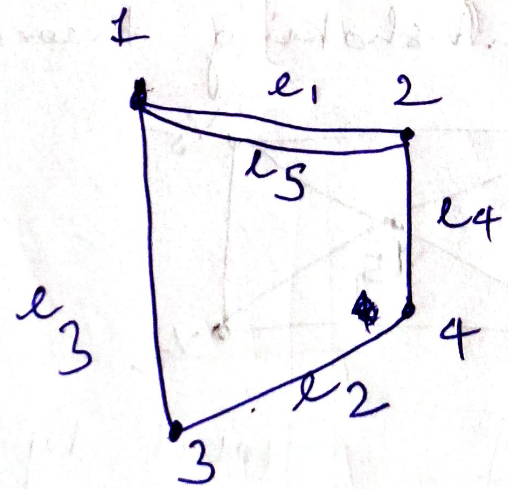
$e_1 = e_5 = (1, 2)$.

$e_2 = (4, 3)$

$e_4 = (2, 4)$

and $e_3 = (1, 3)$

Sol^n



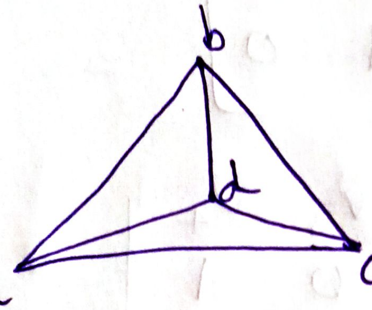
* Representation of a graph

1) Representation of a undirected graph.

(*) Adjacency Matrix Representation.

Let any graph 'G' = (V, E) where $V = \{v_1, v_2, \}$ then Adjacency matrix of graph is an $n \times n$ matrix. $A = \{a_{ij}\}$ and defined by

$$a_{ij} = \begin{cases} 1, & \text{if } (v_i, v_j) \in E \text{ i.e. } v_i \text{ is adjacent to } v_j \\ 0, & \text{if there is no edge between } v_i \,\&\, v_j. \end{cases}$$

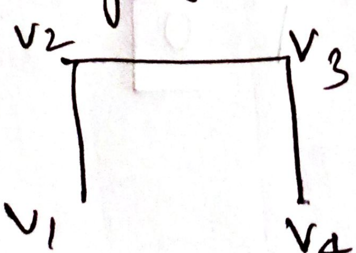


Here graph 'G' consists four vertices therefore the Adjacency matrix will be of 4×4.

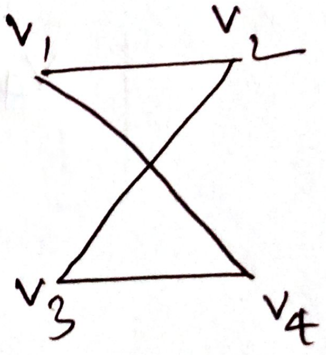
$$A = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 1 & 1 \\ b & 1 & 0 & 1 & 1 \\ c & 1 & 1 & 0 & 1 \\ d & 1 & 1 & 1 & 0 \end{array}$$

Example define the adjacency matrix for the following graph.

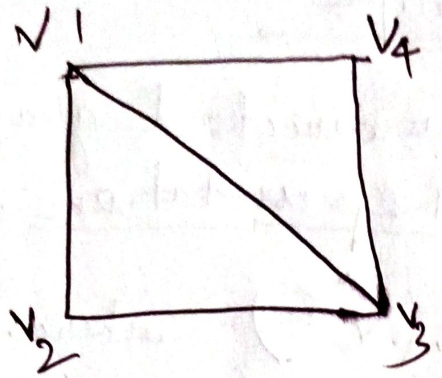
1)



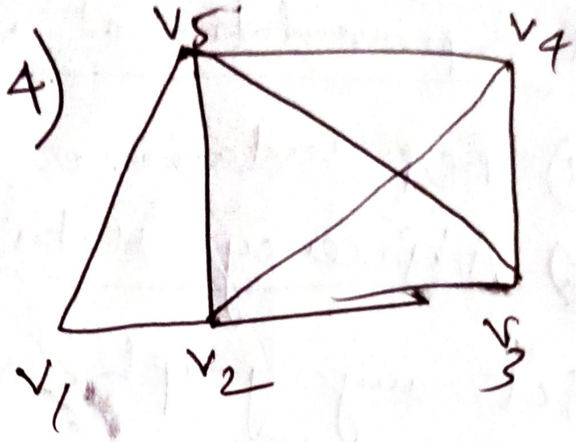
2)



3)



4)



sol^n

(1).

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

(2)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

(3)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

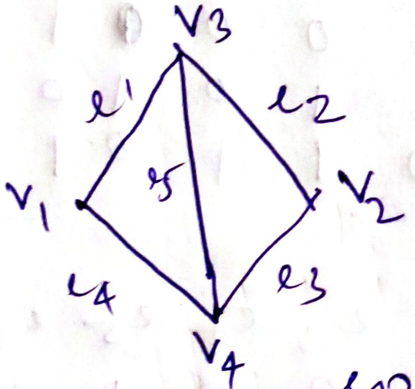
4)

$$
\begin{array}{c|ccccc}
 & V_1 & V_2 & V_3 & V_4 & V_5 \\
\hline
V_1 & 0 & 1 & 0 & 0 & 1 \\
V_2 & 1 & 0 & 1 & 1 & 1 \\
V_3 & 0 & 1 & 0 & 1 & 1 \\
V_4 & 0 & 1 & 1 & 0 & 1 \\
V_5 & 1 & 1 & 1 & 1 & 0
\end{array}
$$

* Incident Matrix Representation

Let $G = (V, E)$, $V = \{v_1, v_2, \ldots v_n\}$ and
$E = \{e_1, e_2, \ldots e_m\}$
then the incident matrix is $n \times m$.

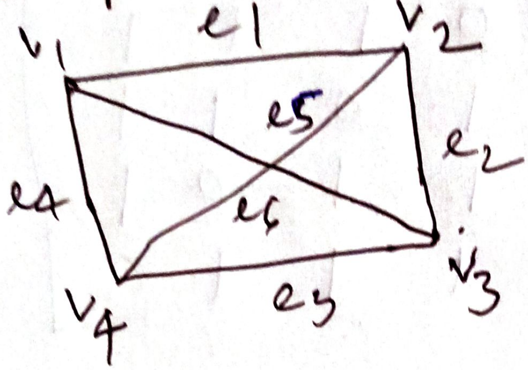
$C = \{a_{ij}\}$ by $a_{ij} = \begin{cases} 1, & \text{if the vertex } v_i \text{ incident by edge } e_j \\ 0, & \text{if otherwise.} \end{cases}$



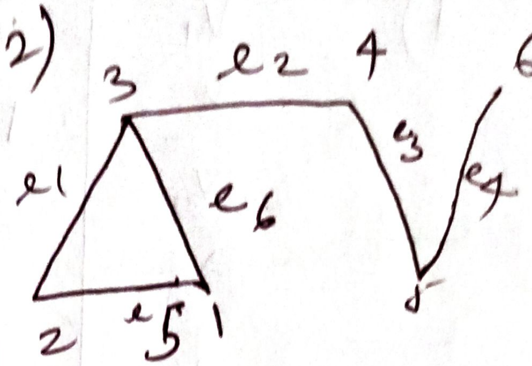
there number vertices are $n = 4$.
and number of edges are $m = 5$.
so, the incident matrix is of $\underline{4 \times 5}$

$$
C = \begin{array}{c|ccccc}
 & e_1 & e_2 & e_3 & e_4 & e_5 \\
\hline
v_1 & 1 & 0 & 0 & 1 & 0 \\
v_2 & 0 & 1 & 1 & 0 & 0 \\
v_3 & 1 & 1 & 0 & 0 & 1 \\
v_4 & 0 & 0 & 1 & 1 & 1
\end{array}
$$

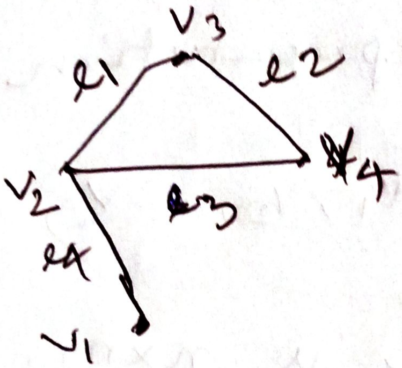
1) Develop



2)



3)



Sol^n

1) $$C = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

2) $$C = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

3)

$$C = \begin{array}{c c} & \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$