Functional Programming

Monad Transformers

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Reminder: Monad

Definition of a Monad – Previous lecture

- abstract datatype for instructions that produce values
- built-in combination >>=
- abstracts over different interpretations (computations)

Monad definition

The type class Monad

```
1 class Monad m where
2   (>>=) :: m a -> (a -> m b) -> m b
3   return :: a -> m a
4   fail :: String -> m a
```

with the following laws:

- `return x >>= f == f x`
- `m >>= return == m`
- `(m >>= f) >>= g == m >>= (\x -> f x >>= g)`

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  - If `f` and `g` are applicatives, what about `Comp f g`?
  - If `f` and `g` are monads, what about `Comp f g`?
- We sometimes want to use multiple functors, applicatives, monads at once!

Why combine functors, applicatives, monads

Lecture 12: Monadic interpreters.
Interpreters can have many features:

- Failure (**`Maybe`**).
- Keeping some state (`State`).
- Reading from the environment (`Reader`).
- ...

To implement an interpreter, we need to combine all these monads!

Let's start by combining functors!

To show that `Comp f g` is a functor ...

- Implement `fmap` (i.e., give an instance of the **`Functor`** class)
- Show that the functor laws hold
  - The identity function gets mapped to the identity function.
  - Functor composition commutes with function composition.

Let's combine applicatives!

To show that `Comp f g` is an applicative ...

- Implement `pure` and `(<*>)` (i.e., give an instance of the `Applicative` class)
- Show that the applicative laws hold ...

Let's combine Monads! – State alone

The State monad

```
1 data ST s a = ST (s -> (a, s))
2 runST (ST sas) = sas
3
4 instance Monad (ST s) where
5   return a = ST (\s -> (a, s))
6   m >>= f = ST (\s ->
7                 let (a, s') = runST m s in
8                 runST (f a) s')
```

Let's combine Monads! - Maybe+State

Consider `Comp (ST s) Maybe`

- Corresponds to `s -> (Maybe a, s)`, a stateful computation that may fail
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Consider `Comp Maybe (ST s)`

- Corresponds to `Maybe (s -> (a, s))`
- `return a = return_Maybe (return_ST a)`
- But there's not way to write the bind function:

```
1 Nothing >>= f = Nothing
2 Just sta >>= f = ???
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Lessons

- Monads do not compose, in general
- Monad composition is not commutative!

Let's combine Monads! – Maybe+State

A different construction: The MaybeState monad

- Purpose: propagate state and signaling of errors
- Attention: the state is lost

```
1 data MaybeState s a = MST { runMST :: s -> Maybe (a, s) }
2
3 ....
4
5 instance Monad (MST s) where
6   return a = MST (\s -> return (a, s))
7   ms >>= f = MST (\s -> runMST ms s >>= \(a, s') -> runMST (f a) s')
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```

- Interestingly, the implementation does not depend on **Maybe** at all!
- We don't have to write this definition again for other combinations!

Alternative solution: Monad transformers

We've seen a particular instance of a *monad transformer*:

```
1 class MonadTrans t where
2   lift :: Monad m => m a -> t m a
```

A monad transformer `t` takes a monad `m` and yields a new monad `(t m)`.
Function `lift` moves a computation from the underlying monad to the new monad.

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Intermezzo

- What's the kind of `t` in `MonadTrans`?
- Answer: `t :: (* -> *) -> (* -> *)`

The MaybeT monad transformer

Definition

```haskell
newtype MaybeT m a = MaybeT { runMaybeT :: m (Maybe a) }

instance (Monad m) => Monad (MaybeT m) where
  return = MaybeT . return . Just
  mmx >>= f = MaybeT $ do
    mx <- runMaybeT mmx
    case mx of
      Nothing -> return Nothing
      Just x -> runMaybeT (f x)

instance MonadTrans MaybeT where
  lift mx = MaybeT $ mx >>= (return . Just)
```

A simple use of MaybeT

We can recover the "normal" monad by applying to `Identity`.

```
1 type MaybeLike = MaybeT Identity
```

The `StateT` monad transformer

Definition

```
1  newtype StateT s m a = StateT { runStateT :: s -> m (a,s) }
2
3  instance (Monad m) => Monad (StateT m) where
4    return a = StateT $ \s -> return (a, s)
5    m >>= f = StateT $ \s -> do
6       (a, s') <- runStateT m s
7       runStateT (f a) s'
8
9  instance MonadTrans StateT where
10   lift ma = StateT $ \s -> do { a <- ma ; return (a, s) }
```

Let's combine Monads with transformers!

Demo!

The ReaderT monad transformer

Definition

```haskell
newtype ReaderT r m a = ReaderT { runReaderT :: r -> m a }

ask :: (Monad m) => ReaderT r m r
ask = ReaderT return

instance Monad m => Monad (ReaderT r m) where
   return = lift . return
   m >>= k = ReaderT $ \r -> do
             a <- runReaderT m r
             runReaderT (k a) r

instance MonadTrans (ReaderT r) where
   lift m = ReaderT (const m)
```

Back to interpreters

Earlier we had a monadic interpreter for:

```
1 data Term = Con Integer
2           | Bin Term Op Term
3             deriving (Eq, Show)
4
5 data Op = Add | Sub | Mul | Div
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Different interpreters with various features:

- Failure ($\Rightarrow$ exception/Maybe monad)
- Counting instructions ($\Rightarrow$ state monad)
- Traces ($\Rightarrow$ writer monad)

Key points

- Monads do not always compose:
  if `m1` and `m2` are monads, there is no general definition that makes `Comp m1 m2` a monad

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- Order is important:
  `StateT s` **`Maybe`** $\neq$ `MaybeT (ST s)`
- You should not overdo it.
- It's all in the `mtl` library.