

习题解答：

初始化网络权重和阈值：

$$w_{14} = 0.2, w_{15} = -0.3, w_{24} = 0.4, w_{34} = -0.5, w_{35} = 0.2, w_{46} = -0.3, \\ w_{56} = -0.2, z_4 = 0.4, z_5 = -0.2, z_6 = -0.1$$

进行前向传播计算输出值：

$$\text{输入样本: } x_1 = 1, x_2 = 0, x_3 = 1$$

计算隐藏层节点输入值：

$$a_4 = w_{14} * x_1 + w_{24} * x_2 + w_{34} * x_3 - z_4 = 0.2 * 1 + 0.4 * 0 + (-0.5) * 1 - 0.4 = -0.7$$

$$a_5 = w_{15} * x_1 + w_{35} * x_3 - z_5 = (-0.3) * 1 + 0.2 * 1 - (-0.2) = 0.1$$

计算隐藏层节点输出值：

$$h_4 = \text{sigmoid}(a_4) = 1 / (1 + \exp(-a_4)) = 1 / (1 + \exp(-0.7)) \approx 0.3318$$

$$h_5 = \text{sigmoid}(a_5) = 1 / (1 + \exp(-a_5)) = 1 / (1 + \exp(0.1)) \approx 0.5250$$

计算输出层节点输入值：

$$a_6 = w_{46} * h_4 + w_{56} * h_5 - z_6 = (-0.3) * 0.3318 + (-0.2) * 0.5250 - (-0.1) \approx -0.1045$$

计算输出层节点输出值：

$$y = \text{sigmoid}(a_6) = 1 / (1 + \exp(-a_6)) = 1 / (1 + \exp(-0.1045)) \approx 0.4739$$

计算输出误差：

$$\text{训练样本的期望输出: } t = 1$$

$$\text{输出误差: } \delta_6 = (t - y) * g'(a_6) = 0.1311$$

计算隐藏层节点的误差：

$$\delta_4 = \delta_6 * w_{46} * g'(a_4) = 0.1311 * (-0.3) * 0.2217 \approx -0.008$$

$$\delta_5 = \delta_6 * w_{56}' * g'(a_5) = 0.1311 * (-0.2) * 0.2494 \approx -0.006$$

进行反向传播并更新权重：

更新输出层到隐藏层的权重：

$$\Delta w_{46} = \eta * \delta_6 * h_4 = \eta * 0.1311 * 0.3318$$

$$\Delta w_{56} = \eta * \delta_6 * h_5 = \eta * 0.1311 * 0.5250$$

其中， η 为学习率，通常取一个较小的值，比如 $\eta = 0.1$

假设 $\eta = 0.1$ ，则

$$\Delta w_{46} \approx 0.1 * 0.1311 * 0.3318 \approx 0.004$$

$$\Delta w_{56} \approx 0.1 * 0.1311 * 0.5250 \approx 0.006$$

更新后的权重：

$$w_{46}' = w_{46} + \Delta w_{46} = -0.3 + 0.004 \approx -0.294$$

$$w_{56}' = w_{56} + \Delta w_{56} = -0.2 + 0.006 \approx -0.194$$

更新隐藏层到输入层的权重：

$$\Delta w_{14} = \eta * \delta_4 * x_1 = \eta * (-0.006) * 1$$

$$\Delta w_{24} = \eta * \delta_4 * x_2 = \eta * (-0.006) * 0$$

$$\Delta w_{34} = \eta * \delta_4 * x_3 = \eta * (-0.006) * 1$$

$$\Delta w_{15} = \eta * \delta_5 * x_1 = \eta * (-0.003) * 1$$

$$\Delta w_{25} = \eta * \delta_5 * x_2 = \eta * (-0.006) * 0$$

$$\Delta w_{35} = \eta * \delta_5 * x_3 = \eta * (-0.003) * 1$$

更新后的权重：

$$w_{14}' = w_{14} + \Delta w_{14} = 0.2 + (\eta * \delta_4 * x_1) \approx 0.2 + (0.1 * (-0.008) * 1) = 0.1992$$

$$w_{24}' = w_{24} + \Delta w_{24} = 0.4 + (\eta * \delta_4 * x_2) \approx 0.4 + (0.1 * (-0.008) * 0) = 0.4$$

$$w_{34}' = w_{34} + \Delta w_{34} = -0.5 + (\eta * \delta_4 * x_3) \approx -0.5 + (0.1 * (-0.008) * 1) = -0.5008$$

$$w_{15}' = w_{15} + \Delta w_{15} = -0.3 + (\eta * \delta_5 * x_1) \approx -0.3 + (0.1 * (-0.006) * 1) = -0.3006$$

$$w_{25}' = w_{25} + \Delta w_{25} = 0.1 + (\eta * \delta_5 * x_2) \approx 0.1 + (0.1 * (-0.006) * 0) = 0.1$$

$$w_{35}' = w_{35} + \Delta w_{35} = 0.2 + (\eta * \delta_5 * x_3) \approx 0.2 + (0.1 * (-0.006) * 1) = 0.1994$$