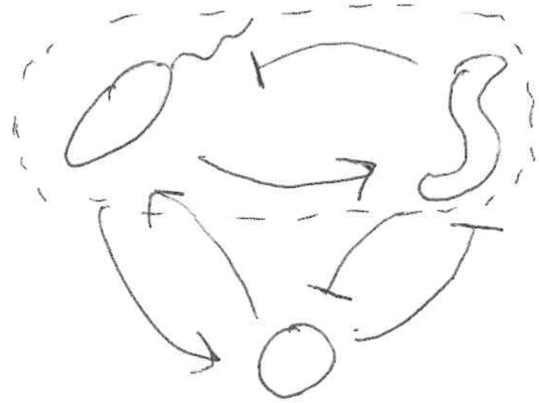
L11: Generalized Lotka-Volterra



"Predator/Prey"

Generalized Lotka-Volterra (GLV) models species interactions w/ arbitrary number of species and types of interactions.

$$\frac{dx_i}{dt} = x_i\Big(\underbrace{\mu_i}_{\text{Individual growth rates}} + \sum_{j=1}^{n} \underbrace{\alpha_{ij} x_j}_{\text{species interactions}}\Big)$$

What parameters give the special case of predator prey dynamics?

$n = 2$, $\mu = ?$, $\alpha = ?$

$$\frac{dx_1}{dt} = x_1\left(a + (-b)\,x_2\right)$$

$$\mu = \begin{array}{cc} i & \begin{bmatrix} a \\ -c \end{bmatrix} \end{array}$$

$$\frac{dx_2}{dt} = x_2\left((-c) + d\,x_1\right)$$

$$\alpha = \begin{array}{cc} & j \\ i & \begin{bmatrix} 0 & -b \\ d & 0 \end{bmatrix} \end{array}$$

How many parameters does this model have as a function of n? $n^2 + n$, $n = 12$ $(12^2 + 12) = 156$

Self Interactions: α_{ii}

Monoculture Growth:

if $\alpha_{ii} = 0$ (Lotka-Volterra Predator Prey)

$$\frac{dx}{dt} = \mu x \quad \text{exponential growth}$$

if $\alpha_{ii} < 0$ (Assumed in Venturelli et al. 2018)

$$\frac{dx}{dt} = x(\mu - |\alpha| x)$$

Steady state:

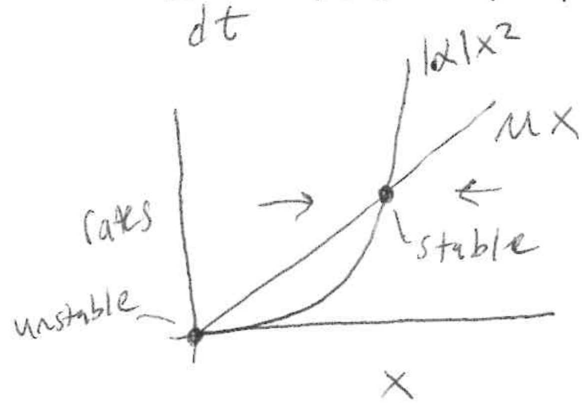
$$0 = x(\mu - |\alpha| x)$$

Steady-state monospecies abundance

$$x = 0, \quad x = \frac{\mu}{|\alpha|} = \frac{-\mu}{\alpha}$$

Stability:

$$\frac{dx}{dt} = \mu x - |\alpha| x^2$$



Logistic Equation:

$$\frac{dx}{dt} = r x\left(1 - \frac{x}{K}\right)$$

r = growth rate, K = carrying capacity

$$r = \mu$$

$$K = \frac{\mu}{|\alpha|}$$