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# Modeling of Active Aerostervoelastic Systems

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## **Abstract**

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# NOMENCLATURE

## Notation

[ ]	=	matrix
{ }	=	column vector

## Abbreviations

CZT	=	chirp Z-transform
DFT	=	discrete Fourier transform
FEM	=	finite element model
FRF	=	frequency response function
GLA	=	gust load alleviation
GVT	=	ground vibration testing
HIL	=	hardware-in-the-loop
LE	=	leading edge
MARGE	=	Model for Aeroelastic Response to Gust Excitation
ODE	=	ordinary differential equation
SISO	=	single-input single-output
TE	=	trailing edge

## Variables

$b$	=	reference semi-chord
$j$	=	imaginary unit
$k$	=	reduced frequency
$N_s$	=	# of elastic modes available
$n_s$	=	# of elastic modes used in modeling

$n_c$	=	# of rigid-body modes used in modeling
$n_{\text{lag}}$	=	# of lag terms used in modeling
$q_D$	=	dynamic pressure
$s$	=	Laplace variable
$U$	=	airspeed

### Subscripts

$s$	=	structural (flexible) component
$c$	=	control (rigid) component
$p$	=	plant
act	=	actuator
sens	=	sensor

## **ACKNOWLEDGMENTS**

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## Chapter 1

# INTRODUCTION

talk about aeronautics

talk about aerodynamics

talk about structural dynamics

talk about aeroelasticity

talk about control

talk about aeroservoelasticity

talk about MARGE

The goal of this project was to obtain a state-space model of MARGE in order to enable low-cost active aeroservoelastic control.

The remainder of this document is organized as follows: Lorem Ipsum.

## Chapter 2

### SYSTEM DESCRIPTION

The subject of this study's modeling effort is the Model for Aeroelastic Response to Gust Excitation (MARGE). MARGE is a flexible half-span wing-body-tail wind tunnel model which is capable of rigid-body rotation in the pitch axis. It was designed to allow rapid and accessible testing of gust alleviation control laws. Thus, it is of a simple and affordable construction. Details of the original design and construction of MARGE can be found in [3].

The structure of MARGE consists of flexible beams encapsulated by lightweight aerodynamic shells. The wing and tail spars are made of aluminum while the fuselage is made of steel. The aerodynamic shells are made of polylactic acid (PLA) and form a symmetrical NACA 0012 airfoil. There is a brass mass fixed at the wing tip to bring the structure's natural frequencies to the designed magnitude. The wing is joined to the fuselage at its root and the entire assembly rotates about a shaft which is suspended from the hanging sub-assembly by bearings. A diagram of the structural configuration of MARGE is shown in Fig. 2.1.

MARGE has three actuators: two servo-actuated control surfaces on the wing and one servo-actuated elevator on the tail. There are also two servo-activated gust vanes installed upstream of the test section in the 3x3 low-speed wind tunnel. The gust vanes move in unison to generate discrete or continuous gusts and are controlled as a single actuator.

MARGE has five sensors. There are two unidirectional accelerometers at the wingtip, one ahead of the wing spar and one aft of the wing spar. There is another unidirectional accelerometer at the tip of the tail. (All three accelerometers are aligned normal to the lifting surfaces.) There is a strain gauge at the wing root. Finally, there is a hall effect sensor inside the hanging sub-assembly by the model's rotating shaft. There is a magnet fixed to the shaft which allows the hall effect sensor to measure the rotation of the shaft. A diagram of the

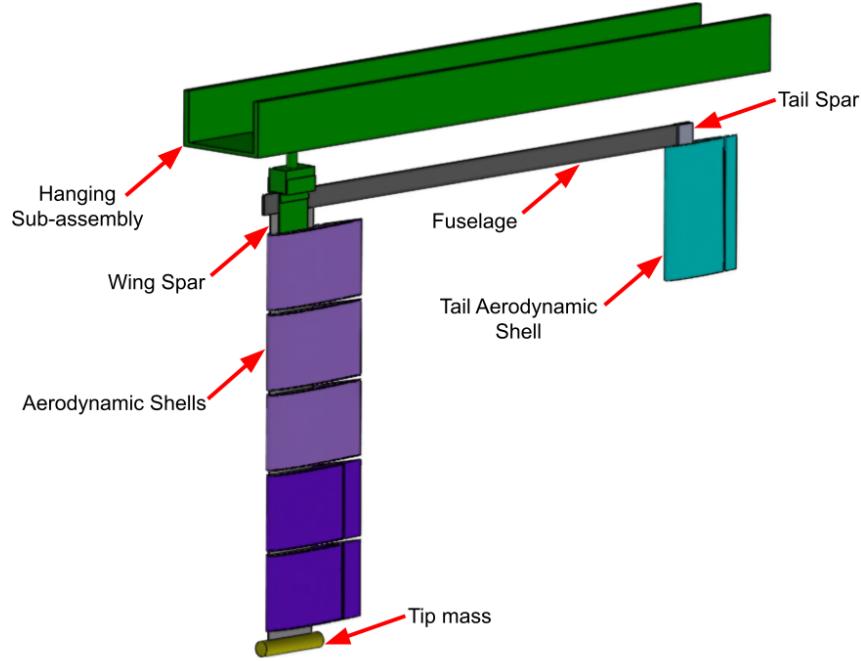


Figure 2.1: MARGE structural configuration

sensing and actuation configuration of MARGE is shown in Fig. 2.2.

Note that the sensor configuration is modified from the original design specified in [3]. The original design featured potentiometers on the control surfaces and a strain gauge on the fuselage. It also lacked the accelerometer on the tail. The potentiometers and the fuselage strain gauge have since been removed because they were found to be unnecessary. The accelerometer on the tail has since been added with the intent of capturing fuselage and tail flexible motions.

MARGE is designed to fit into the University of Washington's 3x3 low-speed wind tunnel. The 3x3 low-speed wind tunnel is an open-loop wind tunnel capable of speeds up to 60 m/s. The wind tunnel has flow straighteners, a 9:1 contraction, gust vanes, and a 3 ft. by 3 ft. by 8 ft. test section. Further details about the 3x3 low-speed wind tunnel can be found in [2]. When installed, MARGE hangs vertically from the ceiling of the test section. A diagram

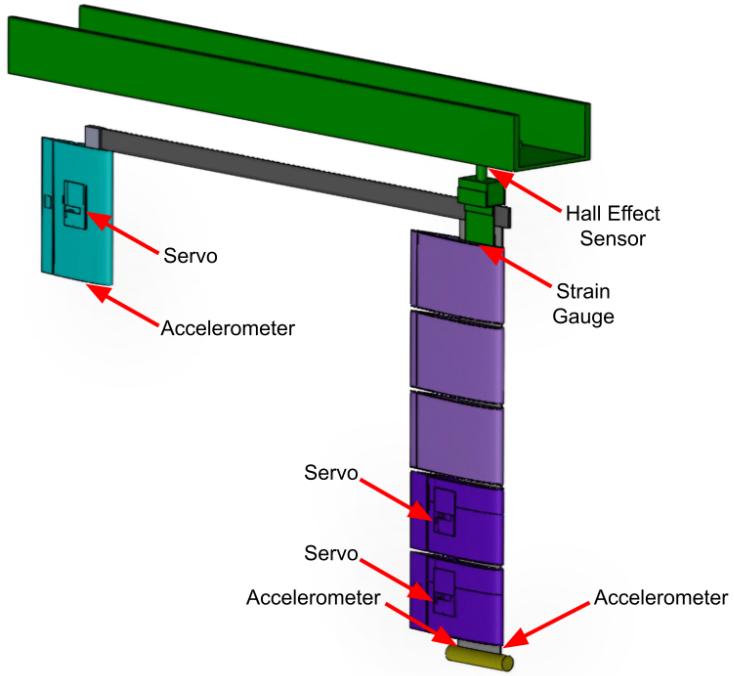


Figure 2.2: MARGE sensing and actuation configuration

indicating the key features of the 3x3 low-speed wind tunnel is shown in Fig. 2.3.

The physical interface to MARGE's acutation and sensing is a National Instruments DAQ coupled with a National Instruments terminal block. The exceptions to this are the gust vanes, which are controlled through HTTP on the local network. Data is transmitted and received to and from these interfaces through Simulink Real-Time.

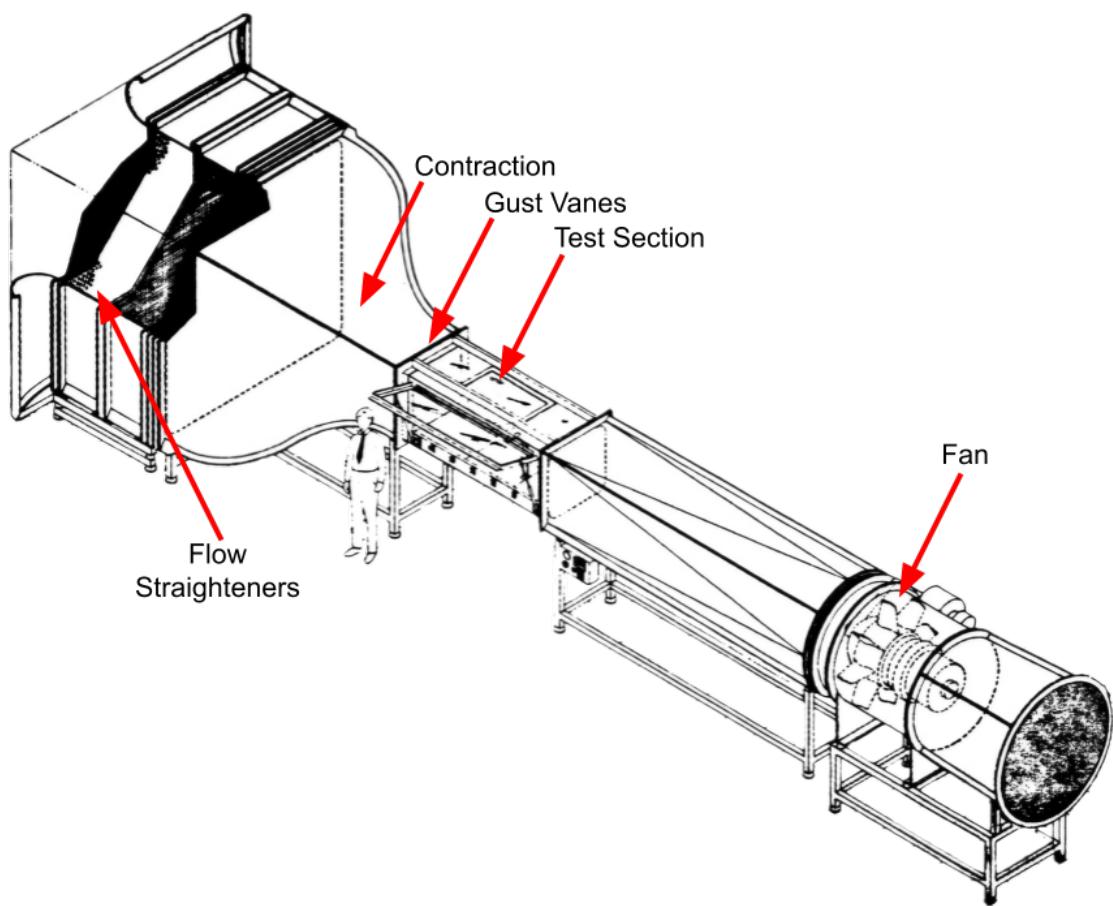


Figure 2.3: Univeristy of Washington 3x3 low-speed wind tunnel

## Chapter 3

### DYNAMIC SYSTEM MODELING

The goal of this section is to derive a preliminary state-space model for the MARGE in the form

$$\begin{aligned}s\{x\} &= [A]\{x\} + [B]\{u\} \\ \{y\} &= [C]\{x\} + [D]\{u\}\end{aligned}\tag{3.1}$$

based on first principles.

#### **3.1 Plant Dynamics**

This section will derive the plant dynamics for MARGE in the form

$$s\{x\} = [A_p]\{x_p\} + [B_p]\{u_p\}\tag{3.2}$$

##### *3.1.1 Structural Dynamics*

MARGE's structure is modeled as a linear, second-order system with damping and external forcing. The equations of motion for such a system are

$$s^2[M]\{q(s)\} + s[C]\{q(s)\} + [K]\{q(s)\} = \{f(s)\}\tag{3.3}$$

where  $\{q\}$  is the structural dynamic state expressed in generalized modal coordinates,  $\{f(s)\}$  is the generalized external forcing, and  $[M]$ ,  $[C]$ , and  $[K]$  are generalized mass, damping, and stiffness matrices respectively. Note that the modal coordinates include both elastic motions (of the structure) denoted by the subscript  $s$  and rigid-body motions (of the structure and the control surfaces) denoted by the subscript  $c$ :

$$\{q\} = \begin{Bmatrix} q_s \\ q_c \end{Bmatrix}\tag{3.4}$$

The forcing for the aeroservoelastic wing can be decomposed into the aerodynamic forcing due to the state and the forcing from actuator hinge moments:

$$s^2[M]\{q(s)\} + s[C]\{q(s)\} + [K]\{q(s)\} = q_D[A(s)]\{q(s)\} + \begin{Bmatrix} \{0\} \\ \{H_c\} \end{Bmatrix} \quad (3.5)$$

where  $[A(s)]$  is the aerodynamic influence matrix for the state and  $\{H_c\}$  are the hinge moments from the actuators.

The whole system can be further decomposed into structural and control components:

$$\begin{aligned} & \left( s^2 \begin{bmatrix} [M_{ss}] & [M_{sc}] \\ [M_{cs}] & [M_{cc}] \end{bmatrix} + s \begin{bmatrix} [C_{ss}] & [C_{sc}] \\ [C_{cs}] & [C_{cc}] \end{bmatrix} + \begin{bmatrix} [K_{ss}] & [K_{sc}] \\ [K_{cs}] & [K_{cc}] \end{bmatrix} \right) \begin{Bmatrix} \{q_s(s)\} \\ \{q_c(s)\} \end{Bmatrix} \\ &= q_D \begin{bmatrix} [A_{ss}(s)] & [A_{sc}(s)] \\ [A_{cs}(s)] & [A_{cc}(s)] \end{bmatrix} \begin{Bmatrix} \{q_s(s)\} \\ \{q_c(s)\} \end{Bmatrix} + \begin{Bmatrix} \{0\} \\ \{H_c\} \end{Bmatrix} \end{aligned} \quad (3.6)$$

The control modes are those corresponding to rigid-body motions of control surfaces. The structural modes are all other modes, including flexible-body modes and rigid-body pitching of the entire model.

It is assumed that the dynamics of the control modes are completely determined by the control inputs and the inputs are not directly affected by the control modes, i.e. the actuators are irreversible controls. Then, interest is only in the dynamics of the structural modes:

$$\begin{aligned} & \left( s^2 \begin{bmatrix} [M_{ss}] & [M_{sc}] \end{bmatrix} + s \begin{bmatrix} [C_{ss}] & [C_{sc}] \end{bmatrix} + \begin{bmatrix} [K_{ss}] & [K_{sc}] \end{bmatrix} \right) \begin{Bmatrix} \{q_s(s)\} \\ \{q_c(s)\} \end{Bmatrix} \\ &= q_D \begin{bmatrix} [A_{ss}(s)] & [A_{sc}(s)] \end{bmatrix} \begin{Bmatrix} \{q_s(s)\} \\ \{q_c(s)\} \end{Bmatrix} \end{aligned} \quad (3.7)$$

Note that since the control modes are rigid-body modes, they have no stiffness and the

equations of motion further simplify to

$$\begin{aligned} \left( s^2 \begin{bmatrix} [M_{ss}] & [M_{sc}] \end{bmatrix} + s \begin{bmatrix} [C_{ss}] & [C_{sc}] \end{bmatrix} + \begin{bmatrix} [K_{ss}] & [0] \end{bmatrix} \right) \begin{Bmatrix} \{q_s(s)\} \\ \{q_c(s)\} \end{Bmatrix} \\ = q_D \begin{bmatrix} [A_{ss}(s)] & [A_{sc}(s)] \end{bmatrix} \begin{Bmatrix} \{q_s(s)\} \\ \{q_c(s)\} \end{Bmatrix} \quad (3.8) \end{aligned}$$

### 3.1.2 The Roger Approximation

The aerodynamic influence matrix  $[A]$  is a function of the Laplace variable  $s$ . For a thin airfoil oscillating in a potential flow, Theodorsen's solution applies [7]. This solution is nonlinear and unable to be represented in a linear system. In order to obtain the linear state-space system,  $[A]$  must be approximated as an analytic function of  $s$ .

The Roger approximation [5] is a method of generating a rational function approximation of the aerodynamic influence matrix in the form

$$[A(s)] \approx [P_0] + s[P_1] + s^2[P_2] + \sum_{n=1}^{N_{\text{lag}}} \frac{s}{s + \beta_n} [P_{n+2}] \quad (3.9)$$

where  $[P]$  are the unknown real-valued matrices that are fit to tabulated known  $[A(s)]$  matrices (obtained from an unsteady aerodynamic solver such as that found in NASTRAN). Aside from the zeroth, first, and second order terms, there are  $N_{\text{lag}}$  additional “lag term” approximating functions which are defined by their respective constants  $\beta$ .

Begin with an approximation only along the frequency domain:

$$[A(jk)] \approx [\bar{P}_0] + jk[\bar{P}_1] + (jk)^2[\bar{P}_2] + \sum_{n=1}^{N_{\text{lag}}} \frac{jk}{jk + \bar{\beta}_n} [\bar{P}_{n+2}] \quad (3.10)$$

where  $k$  is the reduced frequency

$$k = \frac{\omega \cdot b}{U} \quad (3.11)$$

Given the tabulated set of aerodynamic influence matrices  $[A]$  computed across  $N$  reduced frequencies, each entry  $p$  of the matrices  $[\bar{P}]$  in the above approximation can be fit to the

corresponding entries  $a$  of the aerodynamic influence matrices  $[A]$ . The steady-state component of the approximation  $[\bar{P}_0]$  is fixed to be equal to the steady-state aerodynamic influence matrix (with zero reduced frequency):

$$[\bar{P}_0] = [A(k=0)] \quad (3.12)$$

The remaining  $[\bar{P}]$  matrices are determined using a linear regression:

$$\begin{bmatrix} 0 & -k_1^2 & \frac{k_1^2}{k_1^2 + \bar{\beta}_1^2} & \cdots & \frac{k_1^2}{k_1^2 + \bar{\beta}_{N_{\text{lag}}}^2} \\ k_1 & 0 & \frac{k_1 \bar{\beta}_1}{k_1^2 + \bar{\beta}_1^2} & \cdots & \frac{k_1 \bar{\beta}_1}{k_1^2 + \bar{\beta}_{N_{\text{lag}}}^2} \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & -k_N^2 & \frac{k_N^2}{k_N^2 + \bar{\beta}_1^2} & \cdots & \frac{k_N^2}{k_N^2 + \bar{\beta}_{N_{\text{lag}}}^2} \\ k_N & 0 & \frac{k_N \bar{\beta}_1}{k_N^2 + \bar{\beta}_1^2} & \cdots & \frac{k_N \bar{\beta}_1}{k_N^2 + \bar{\beta}_{N_{\text{lag}}}^2} \end{bmatrix} \begin{Bmatrix} \bar{p}_1 \\ \vdots \\ \bar{p}_{N_{\text{lag}}+2} \end{Bmatrix} = \begin{Bmatrix} \text{Re}(a(jk_1)) - a(0) \\ \text{Im}(a(jk_1)) \\ \vdots \\ \text{Re}(a(jk_N)) - a(0) \\ \text{Im}(a(jk_N)) \end{Bmatrix} \quad (3.13)$$

The linear regression in Eq. 3.13 can then be solved using the method of least-squares to obtain the fitted  $[\bar{P}]$  matrices.

Replacing the reduced frequency with the equivalent angular frequency yields

$$[A(j\omega)] \approx [P_0] + j\omega[P_1] - \omega^2[P_2] + \sum_{n=1}^{N_{\text{lag}}} \frac{j\omega}{j\omega + \beta_n} [P_{n+2}] \quad (3.14)$$

where

$$\begin{aligned} [P_0] &= [\bar{P}_0] \\ [P_1] &= [\bar{P}_1] \frac{b}{U} \\ [P_2] &= [\bar{P}_2] \left( \frac{b}{U} \right)^2 \\ [P_3] &= [\bar{P}_3] \\ &\vdots \\ [P_{N_{\text{lag}}+2}] &= [\bar{P}_{N_{\text{lag}}+2}] \\ \beta_1 &= \bar{\beta}_1 \frac{U}{b} \\ &\vdots \\ \beta_{N_{\text{lag}}} &= \bar{\beta}_{N_{\text{lag}}} \frac{U}{b} \end{aligned} \quad (3.15)$$

This is then extended from the imaginary axis to the full Laplace domain through analytic continuation, leading to the final form of the Roger approximation for aerodynamic influence matrices:

$$[A(s)] \approx [P_0] + s[P_1] + s^2[P_2] + \sum_{n=1}^{N_{\text{lag}}} \frac{s}{s + \beta_n} [P_{n+2}] \quad (3.16)$$

It should be noted that the Roger approximation is only valid over the tabulated reduced frequency range. Thus, any result from the model is only valid if the airspeed that the model is analyzed at is sufficiently above

$$U_{\min} = \frac{\omega_{\max} \cdot b}{k_{\max}} \quad (3.17)$$

where  $\omega_{\max}$  is the maximum frequency of interest in the analysis. Thus, modeling which utilizes the Roger Approximation cannot be used for purely structural dynamic analysis (where  $U = 0$ ).

### 3.1.3 Coupled Aeroelastic Dynamics

This section describes how the structural dynamic equations of motion can be reduced to the form in Eq. 3.2.

Replacing the aerodynamic influence matrices in Eq. 3.8 with the equivalent Roger approximation and collecting like terms yields

$$\begin{aligned} & \left( s^2[M_{ss}] + s[C_{ss}] + [K_{ss}] - q_D \left( [P_{ss,0}] + s[P_{ss,1}] + s^2[P_{ss,2}] + \sum_{n=1}^{N_{\text{lag}}} \frac{s}{s + \beta_n} [P_{ss,n+2}] \right) \right) \{q_s(s)\} \\ &= - \left( s^2[M_{sc}] + s[C_{sc}] - q_D \left( [P_{sc,0}] + s[P_{sc,1}] + s^2[P_{sc,2}] + \sum_{n=1}^{N_{\text{lag}}} \frac{s}{s + \beta_n} [P_{sc,n+2}] \right) \right) \{q_c(s)\} \end{aligned} \quad (3.18)$$

Define the following coupled aeroelastic mass, damping, and stiffness matrices:

$$\begin{aligned}\left[\bar{\bar{M}}\right] &= [M] - q_D[P_2] \\ \left[\bar{\bar{C}}\right] &= [C] - q_D[P_1] \\ \left[\bar{\bar{K}}\right] &= [K] - q_D[P_0]\end{aligned}\quad (3.19)$$

Equation 3.18 then simplifies to

$$\begin{aligned}&\left(s^2 \left[\bar{\bar{M}}_{ss}\right] + s \left[\bar{\bar{C}}_{ss}\right] + \left[\bar{\bar{K}}_{ss}\right]\right) \{q_s(s)\} - q_D \left(\sum_{n=1}^{N_{\text{lag}}} \frac{s}{s + \beta_n} [P_{ss,n+2}]\right) \{q_s(s)\} \\ &= - \left(s^2 \left[\bar{\bar{M}}_{sc}\right] + s \left[\bar{\bar{C}}_{sc}\right] + \left[\bar{\bar{K}}_{sc}\right]\right) \{q_c(s)\} + q_D \left(\sum_{n=1}^{N_{\text{lag}}} \frac{s}{s + \beta_n} [P_{sc,n+2}]\right) \{q_c(s)\}\end{aligned}\quad (3.20)$$

Rearranging the lag terms,

$$\begin{aligned}&\left(s^2 \left[\bar{\bar{M}}_{ss}\right] + s \left[\bar{\bar{C}}_{ss}\right] + \left[\bar{\bar{K}}_{ss}\right]\right) \{q_s(s)\} \\ &- q_D \begin{bmatrix} \sum_{n=1}^{N_{\text{lag}}} \frac{s}{s + \beta_n} [P_{ss,n+2}] & \sum_{n=1}^{N_{\text{lag}}} \frac{s}{s + \beta_n} [P_{sc,n+2}] \end{bmatrix} \begin{Bmatrix} q_s(s) \\ q_c(s) \end{Bmatrix} \\ &= - \left(s^2 \left[\bar{\bar{M}}_{sc}\right] + s \left[\bar{\bar{C}}_{sc}\right] + \left[\bar{\bar{K}}_{sc}\right]\right) \{q_c(s)\}\end{aligned}\quad (3.21)$$

Define the following aerodynamic ‘‘lag states’’ as

$$\begin{aligned}\{r_1\} &= \begin{bmatrix} [P_{ss,3}] & [P_{sc,3}] \end{bmatrix} \frac{s}{s + \beta_1} \\ &\vdots \\ \{r_{N_{\text{lag}}}\} &= \begin{bmatrix} [P_{ss,N_{\text{lag}}+2}] & [P_{sc,N_{\text{lag}}+2}] \end{bmatrix} \frac{s}{s + \beta_{N_{\text{lag}}}}\end{aligned}\quad (3.22)$$

Rearranging this yields an expression for the first derivative of the lag states:

$$\begin{aligned}s\{r_1\} &= s[P_{ss,3}]\{q_s(s)\} + s[P_{sc,3}]\{q_c(s)\} - \beta_1\{r_1\} \\ &\vdots \\ s\{r_{N_{\text{lag}}}\} &= s[P_{ss,N_{\text{lag}}+2}]\{q_s\} + s[P_{sc,N_{\text{lag}}+2}]\{q_c\} - \beta_{N_{\text{lag}}}\{r_{N_{\text{lag}}}\}\end{aligned}\quad (3.23)$$

Combining these lag state dynamic equations into a single matrix equation yields

$$s\{r\} = s[B_{rs}]\{q_s\} + s[B_{rc}]\{q_c\} + [A_r]\{r\} \quad (3.24)$$

where  $\{r\}$ ,  $[B_{rs}]$ ,  $[B_{rc}]$ , and  $[A_r]$  are defined as

$$\begin{aligned} \{r\} &= \begin{Bmatrix} \{r_1\} \\ \vdots \\ \{r_{N_{\text{lag}}}\} \end{Bmatrix} \\ [B_{rs}] &= \begin{bmatrix} [P_{ss,3}] \\ \vdots \\ [P_{ss,N_{\text{lag}}+2}] \end{bmatrix} \\ [B_{rc}] &= \begin{bmatrix} [P_{sc,3}] \\ \vdots \\ [P_{sc,N_{\text{lag}}+2}] \end{bmatrix} \\ [A_r] &= \begin{bmatrix} -\beta_1[I] & & \\ & \ddots & \\ & & -\beta_{N_{\text{lag}}}[I] \end{bmatrix} \end{aligned} \quad (3.25)$$

With the lag states defined, Eq.3.21 can now be rewritten as

$$\begin{aligned} \left( s^2 \left[ \bar{\bar{M}}_{ss} \right] + s \left[ \bar{\bar{C}}_{ss} \right] + \left[ \bar{\bar{K}}_{ss} \right] \right) \{q_s(s)\} - q_D \sum_{n=1}^{N_{\text{lag}}} \{r_n\} \\ = - \left( s^2 \left[ \bar{\bar{M}}_{sc} \right] + s \left[ \bar{\bar{C}}_{sc} \right] + \left[ \bar{\bar{K}}_{sc} \right] \right) \{q_c(s)\} \end{aligned} \quad (3.26)$$

The sum of lag states can be simplified introducing the matrix  $[I_r]$  such that

$$\{r_1\} + \cdots + \{r_{N_{\text{lag}}}\} = \begin{bmatrix} [I] & \dots & [I] \end{bmatrix} \begin{Bmatrix} \{r_1\} \\ \vdots \\ \{r_{N_{\text{lag}}}\} \end{Bmatrix} = [I_r]\{r\} \quad (3.27)$$

This further compacts Eq. 3.26, which becomes

$$\begin{aligned} & \left( s^2 \left[ \bar{\bar{M}}_{ss} \right] + s \left[ \bar{\bar{C}}_{ss} \right] + \left[ \bar{\bar{K}}_{ss} \right] \right) \{q_s(s)\} - q_D[I_r]\{r\} \\ & = - \left( s^2 \left[ \bar{\bar{M}}_{sc} \right] + s \left[ \bar{\bar{C}}_{sc} \right] + \left[ \bar{\bar{K}}_{sc} \right] \right) \{q_c(s)\} \end{aligned} \quad (3.28)$$

Define new structural dynamic state vectors as

$$\{x_1\} = \{q\} \quad (3.29)$$

$$\{x_2\} = s\{q\} \quad (3.30)$$

Using these, Eq. 3.28 can be rewritten as a first-order linear ODE for  $\{x_2\}$ :

$$\begin{aligned} s\{x_2\} = & - \left[ \bar{\bar{M}}_{ss} \right]^{-1} \left[ \bar{\bar{C}}_{ss} \right] \{x_2\} - \left[ \bar{\bar{M}}_{ss} \right]^{-1} \left[ \bar{\bar{K}}_{ss} \right] \{x_1\} + q_D \left[ \bar{\bar{M}}_{ss} \right]^{-1} [I_r]\{r\} \\ & - \left[ \bar{\bar{M}}_{ss} \right]^{-1} \left( s^2 \left[ \bar{\bar{M}}_{sc} \right] + s \left[ \bar{\bar{C}}_{sc} \right] + \left[ \bar{\bar{K}}_{sc} \right] \right) \{q_c(s)\} \end{aligned} \quad (3.31)$$

Rearranging,

$$\begin{aligned} s\{x_2\} = & - \left[ \bar{\bar{M}}_{ss} \right]^{-1} \left[ \bar{\bar{C}}_{ss} \right] \{x_2\} - \left[ \bar{\bar{M}}_{ss} \right]^{-1} \left[ \bar{\bar{K}}_{ss} \right] \{x_1\} \\ & + q_D \left[ \bar{\bar{M}}_{ss} \right]^{-1} [I_r]\{r\} - \left[ \bar{\bar{M}}_{ss} \right]^{-1} \left[ \left[ \bar{\bar{K}}_{sc} \right] \quad \left[ \bar{\bar{C}}_{sc} \right] \quad \left[ \bar{\bar{M}}_{sc} \right] \right] \begin{Bmatrix} \{q_c(s)\} \\ s\{q_c(s)\} \\ s^2\{q_c(s)\} \end{Bmatrix} \end{aligned} \quad (3.32)$$

Define the following simplifying matrices:

$$[T_{21}] = - \left[ \bar{\bar{M}}_{ss} \right]^{-1} \left[ \bar{\bar{K}}_{ss} \right] \quad (3.33)$$

$$[T_{22}] = - \left[ \bar{\bar{M}}_{ss} \right]^{-1} \left[ \bar{\bar{C}}_{ss} \right] \quad (3.34)$$

$$[T_{2r}] = q_D \left[ \bar{\bar{M}}_{ss} \right]^{-1} [I_r] \quad (3.35)$$

$$[T_{2c}] = - \left[ \bar{\bar{M}}_{ss} \right]^{-1} \left[ \left[ \bar{\bar{K}}_{sc} \right] \quad \left[ \bar{\bar{C}}_{sc} \right] \quad \left[ \bar{\bar{M}}_{sc} \right] \right] \quad (3.36)$$

Equation 3.32 then simplifies to

$$s\{x_2\} = [T_{21}]\{x_1\} + [T_{22}]\{x_2\} + [T_{2r}]\{r\} + [T_{2c}]\{q_c\} \quad (3.37)$$

Now there exist first-order linear ODEs for the structural dynamic states (Eq. 3.32) and the aerodynamic lag states (Eq. 3.24) so the dynamics of the full system can be written in state-space form:

$$s \begin{Bmatrix} \{x_1\} \\ \{x_2\} \\ \{r\} \end{Bmatrix} = \begin{bmatrix} [0] & [I] & [0] \\ [T_{21}] & [T_{22}] & [T_{2r}] \\ [0] & [B_{rs}] & [A_r] \end{bmatrix} \begin{Bmatrix} \{x_1\} \\ \{x_2\} \\ \{r\} \end{Bmatrix} + \begin{bmatrix} [0] & [0] & [0] \\ [0] & [T_{2c}] & [0] \\ [0] & [B_{rc}] & [0] \end{bmatrix} \begin{Bmatrix} \{q_c\} \\ s\{q_c\} \\ s^2\{q_c\} \end{Bmatrix} \quad (3.38)$$

Define the state vector of the MARGE plant to be

$$\{x_p\} = \begin{Bmatrix} \{x_1\} \\ \{x_2\} \\ \{r\} \end{Bmatrix} \quad (3.39)$$

and the input vector of the MARGE plant to be

$$\{u_p\} = \begin{Bmatrix} \{q_c\} \\ s\{q_c\} \\ s^2\{q_c\} \end{Bmatrix} \quad (3.40)$$

Then, Eq. 3.38 can be written in the form that is desired:

$$s\{x_p\} = [A_p]\{x_p\} + [B_p]\{u_p\} \quad (3.41)$$

where  $[A_p]$  and  $[B_p]$  are the block matrices in Eq. 3.38.

### 3.2 Plant Output Modeling

This section will derive the output equations for MARGE in the form

$$\{y_p\} = [C_p]\{x_p\} + [D_p]\{u_p\} \quad (3.42)$$

The output of MARGE consists of three acceleration, one strain, and one pitch measurements. The plant output model to be presented here captures all of these as well as an

optional pitch rate output. Thus,

$$\{u_p\} = \begin{Bmatrix} \text{acceleration 1} \\ \text{acceleration 2} \\ \text{acceleration 3} \\ \text{strain} \\ \text{rotation} \\ \text{rotation rate} \end{Bmatrix} \quad (3.43)$$

These outputs are the inputs to the sensors described in the next section.

The structural dynamic generalized state  $\{q\}$  has modal coordinates which correspond to  $n_s$  mode shapes of the structure  $\{\phi_1\}, \dots, \{\phi_{n_s}\}$ . These modes shapes are in physical coordinates and are known from the structural dynamic solution of the finite-element model. The motion of the system in physical coordinates  $\{z(t)\}$  is

$$\{z(t)\} = [\{\phi_1\} \dots \{\phi_{n_s}\}] \{q\} \quad (3.44)$$

For the  $i$ th degree of freedom (DOF) of the system, this is

$$z_i(t) = [\phi^i] \{q(t)\} \quad (3.45)$$

where  $[\phi^i]$  contains the contribution to that DOF's motion from each mode:

$$[\phi^i] = [\phi_1^i \dots \phi_{n_s}^i] \quad (3.46)$$

The motion of the system can be expressed in terms of the full plant state vector defined in Eq. 3.39 as

$$\{z_i(t)\} = [\phi^i] [T_{\text{disp}}] \{x_p(t)\} \quad (3.47)$$

where  $[T_{\text{disp}}]$  is a matrix which extracts the  $\{x_1\}$  component of  $\{x_p\}$ :

$$[T_{\text{disp}}] = \begin{bmatrix} [I]_{n_s \times n_s} & [0]_{n_s \times n_s} & [0]_{n_s \times n_{\text{lag}}} \end{bmatrix} \quad (3.48)$$

Here,  $n_s$  is the number of elastic “structural” modes used,  $n_c$  is the number of rigid-body “control” modes used, and  $n_{\text{lag}}$  is the number of lag terms used in the model.

Similarly, the rates of the system’s motion at DOF  $i$  can be expressed using the  $\{x_2\}$  component of the state vector  $\{x_p\}$ :

$$\frac{d\{z_i(t)\}}{dt} = [\phi^i] [T_{\text{vel}}]\{x_p(t)\} \quad (3.49)$$

where  $[T_{\text{vel}}]$  is a matrix which extracts the  $\{x_2\}$  component of  $\{x_p\}$ :

$$[T_{\text{vel}}] = \begin{bmatrix} [0]_{n_s \times n_s} & [I]_{n_s \times n_s} & [0]_{n_s \times n_{\text{lag}}} \end{bmatrix} \quad (3.50)$$

### 3.2.1 Acceleration Output

The system’s accelerations at DOF  $i$  can then be expressed as

$$\frac{d^2\{z_i(t)\}}{dt^2} = [\phi^i] [T_{\text{vel}}] \frac{d\{x_p(t)\}}{dt} \quad (3.51)$$

The derivative of  $\{x_p(t)\}$  is contained in Eq. 3.38. Using that expression here yields

$$\frac{d^2\{z_i(t)\}}{dt^2} = [\phi^i] [T_{\text{vel}}] ([A_p]\{x_p\} + [B_p]\{u_p(t)\}) \quad (3.52)$$

For the three accelerometer locations (DOFs) on MARGE with corresponding modal motions  $\{\phi_1\}$ ,  $\{\phi_2\}$ , and  $\{\phi_3\}$ , the accelerations experienced by the sensors are

$$\begin{Bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{Bmatrix} = \begin{Bmatrix} [\phi^1] [T_{\text{vel}}][A_p] \\ [\phi^2] [T_{\text{vel}}][A_p] \\ [\phi^3] [T_{\text{vel}}][A_p] \end{Bmatrix} \{x_p(t)\} + \begin{Bmatrix} [\phi^1] [T_{\text{vel}}][B_p] \\ [\phi^2] [T_{\text{vel}}][B_p] \\ [\phi^3] [T_{\text{vel}}][B_p] \end{Bmatrix} \{u_p(t)\} \quad (3.53)$$

### 3.2.2 Pitch Output

For the pitch output which is a measure of position instead of acceleration, Eq. 3.47 is used instead:

$$u_5(t) = [\phi^5] [T_{\text{disp}}]\{x_p(t)\} \quad (3.54)$$

The corresponding pitch rate output is similarly given by 3.49

$$u_6(t) = [\phi^5] [T_{\text{vel}}] \{x_p(t)\} \quad (3.55)$$

Note that the pitch rate output uses the same DOF in the mode shape ( $\phi^5$ ) as the pitch output.

### 3.2.3 Strain Output

The structural dynamic generalized state  $\{q\}$  has modal coordinates which also correspond to the load (force or moment) at each DOF in each of  $n_s$  mode shapes of the structure  $\{\psi_1\}, \dots, \{\psi_{n_s}\}$ . (This is also known from the finite element model.) The loading state of the system in physical coordinates  $\{L(t)\}$  is

$$\{L(t)\} = [\{\psi_1\} \dots \{\psi_{n_s}\}] \{q(t)\} \quad (3.56)$$

For the  $i$ th DOF of the system,

$$L_i(t) = [\psi^i] \{q(t)\} \quad (3.57)$$

where

$$\psi^i = [\psi_1^i \dots \psi_{n_s}^i] \{q(t)\} \quad (3.58)$$

In a similar fashion as in Eq. 3.47, this DOF's load can be expressed in terms of the system state:

$$\psi^i = [\psi_1^i \dots \psi_{n_s}^i] [T_{\text{disp}}] \{x_p(t)\} \quad (3.59)$$

Given this load, the strain due to bending is determined using the following relation from Euler-Bernoulli linear beam theory:

$$\varepsilon_y = -\frac{M_x \cdot z}{I_{zz} \cdot E} \quad (3.60)$$

where  $\varepsilon_y$  is the strain,  $M_x$  is the bending moment (from the relevant DOF of  $\{L\}$ ),  $z$  is the  $z$ -distance of the sensor from the elastic axis of the beam,  $I_{zz}$  is the  $z$ -axis area moment of inertia, and  $E$  is the modulus of elasticity. Although the axial load in the  $y$ -direction would also add to the strain output, its effect is negligible compared to that of bending. This is especially true because the axial  $y$ -stress at rest due to the weight of the model is tared out at before tests. Thus, its effect has been omitted in modeling.

The strain gauge on MARGE is mounted on the lower surface of the rectangular cross-sectioned wing spar. Thus,  $z$  is equal to negative one half of the spar thickness  $t$ :

$$\varepsilon_y = -\frac{M_x \cdot (-t/2)}{I_{zz} \cdot E} \quad (3.61)$$

Combining Eqs. 3.59 and yields the final expression for MARGE's strain output,  $u_4$ :

$$u_4 = \frac{t/2}{I_{zz} \cdot E} \begin{bmatrix} \psi_1^4 & \dots & \psi_{n_s}^4 \end{bmatrix} [T_{\text{disp}}] \{x_p(t)\} \quad (3.62)$$

Thus, the full expression for the plant output of MARGE is

$$\{y_p\} = \begin{bmatrix} [\phi^1] [T_{\text{vel}}][A_p] \\ [\phi^2] [T_{\text{vel}}][A_p] \\ [\phi^3] [T_{\text{vel}}][A_p] \\ \frac{t/2}{I_{zz} \cdot E} \begin{bmatrix} \psi_1^4 & \dots & \psi_{n_s}^4 \end{bmatrix} [T_{\text{disp}}] \\ [\phi^5] [T_{\text{disp}}] \\ [\phi^5] [T_{\text{vel}}] \end{bmatrix} \{x_p\} + \begin{bmatrix} [\phi^1] [T_{\text{vel}}][B_p] \\ [\phi^2] [T_{\text{vel}}][B_p] \\ [\phi^3] [T_{\text{vel}}][B_p] \\ 0 \\ 0 \\ 0 \end{bmatrix} \{u_p\} \quad (3.63)$$

Thus, the plant output of MARGE has been derived the form that is desired

$$\{y_p\} = [C_p] \{x_p\} + [D_p] \{u_p\} \quad (3.64)$$

where  $[C_p]$  and  $[D_p]$  are the block matrices in Eq. 3.63.

### 3.3 Actuation and Sensing Dynamics

When developing a model that is used for control design, the dynamics of the actuators and sensors must be accounted for. The output of the control law will be fed not into the

plant, but the imperfect actuators. The input of the control law will come not directly from the plant, but from the imperfect sensors. Thus, the goal of this section is to determine dynamic models for the actuators and the sensors in the forms

$$\begin{aligned}\frac{d}{dt}\{x_{\text{act}}\} &= [A_{\text{act}}]\{x_{\text{act}}\} + [B_{\text{act}}]\{u_{\text{act}}\} \\ \{y_{\text{act}}\} &= [C_{\text{act}}]\{x_{\text{act}}\} + [D_{\text{act}}]\{u_{\text{act}}\}\end{aligned}\quad (3.65)$$

and

$$\begin{aligned}\frac{d}{dt}\{x_{\text{sens}}\} &= [A_{\text{sens}}]\{x_{\text{sens}}\} + [B_{\text{sens}}]\{u_{\text{sens}}\} \\ \{y_{\text{sens}}\} &= [C_{\text{sens}}]\{x_{\text{sens}}\} + [D_{\text{sens}}]\{u_{\text{sens}}\}\end{aligned}\quad (3.66)$$

The servo-actuated control surfaces on the wing and tail are known (from [3]) to have dynamics according to the following transfer function<sup>1</sup>:

$$G(s) = \frac{1461}{s^2 + 62.2s + 1461} \quad (3.67)$$

This transfer function was then converted to the following equivalent state-space representation:

$$\begin{aligned}s\{x\} &= \begin{bmatrix} -62.2 & -1461 \\ 1 & 0 \end{bmatrix} \{x\} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 0 & 1461 \end{bmatrix} \{x\}\end{aligned}\quad (3.68)$$

where  $u$  is the input,  $y$  is the output, and  $\{x\}$  is the internal state of the actuator.

The wind tunnel gust vanes were measured to have “perfect” dynamics except for a pure time delay of 0.34 seconds. This pure delay was approximated as a second-order transfer function using a Padé approximant:

$$G(s) = \frac{s^2 - 176.47s + 10381}{s^2 + 176.47s + 10381} \quad (3.69)$$

The Padé approximant matches the pure delay’s response well in the frequency range of interest (<20 Hz); the step response and phase shift behavior of a pure delay and the Padé approximant are compared in Fig. 3.3.

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<sup>1</sup>The numerator of this transfer function differs from that defined in [3] because the actuator is calibrated to have unity DC gain before use.

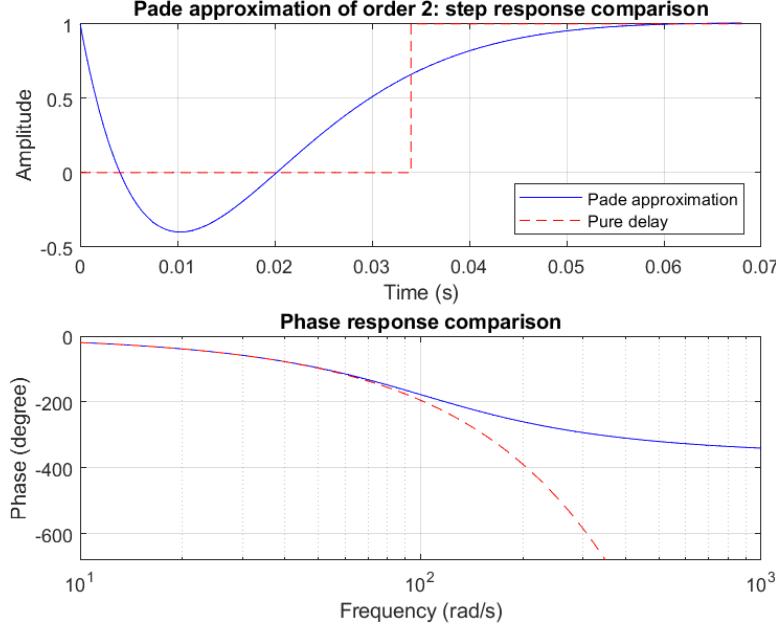


Figure 3.1: Padé approximant of the pure-delay response of the wind tunnel gust vanes

This transfer function was then converted to the following equivalent state-space representation:

$$\begin{aligned} s\{x\} &= \begin{bmatrix} -176.47 & -10381 \\ 1 & 0 \end{bmatrix} \{x\} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \\ y &= \begin{bmatrix} -352.94 & 0 \end{bmatrix} \{x\} + \begin{bmatrix} 1 \end{bmatrix} u \end{aligned} \quad (3.70)$$

where  $u$  is the input,  $y$  is the output, and  $\{x\}$  is the internal state of the actuator.

The state-space models for the four actuators (three servo-actuated control surfaces and one pair of wind tunnel gust vanes) are combined to form one combined state-space model for all actuators with input, output, and state

$$\begin{aligned} \{u_{\text{act}}\} &= \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} & \{y_{\text{act}}\} &= \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} & \{x_{\text{act}}\} &= \begin{bmatrix} \{x\}_1 \\ \{x\}_2 \\ \{x\}_3 \\ \{x\}_4 \end{bmatrix} \end{aligned} \quad (3.71)$$

respectively. The combined actuation state-space model is then

$$\begin{aligned} \{x_{\text{act}}\} &= \begin{bmatrix} [A_1] & & & \\ & [A_2] & & \\ & & [A_3] & \\ & & & [A_4] \end{bmatrix} \{x_{\text{act}}\} + \begin{bmatrix} [B_1] & & & \\ & [B_2] & & \\ & & [B_3] & \\ & & & [B_4] \end{bmatrix} \{u_{\text{act}}\} \\ \{y_{\text{act}}\} &= \begin{bmatrix} [C_1] & & & \\ & [C_2] & & \\ & & [C_3] & \\ & & & [C_4] \end{bmatrix} \{x_{\text{act}}\} + \begin{bmatrix} [D_1] & & & \\ & [D_2] & & \\ & & [D_3] & \\ & & & [D_4] \end{bmatrix} \{u_{\text{act}}\} \end{aligned} \quad (3.72)$$

where the  $[A]$ ,  $[B]$ ,  $[C]$ , and  $[D]$  system matrices for the two types of actuators are defined above in Eq. 3.68 and 3.70. This then forms the actuator block shown in Fig. 3.4.

A similar process would be appropriate for a set of imperfect sensors. However, the high-rate sensors used in MARGE have approximately no dynamics in the frequency range of interest. Thus, the sensor response was approximated as

$$\{y_{\text{sens}}\} = [I]\{u_{\text{sens}}\} \quad (3.73)$$

In other words, the output of the sensor was taken as the output of the plant. This then forms the sensor block shown in Fig. 3.4.

### 3.4 Coupled Aerervoelastic Modeling

The imperfect actuators and sensors can be accounted for in modeling by combining the actuator, plant, and sensor models into an integrated system model that can then be used for control design; see Fig. 3.4 for a block diagram of this system of systems. In the system illustration in Fig. 3.4, the actuators take in the control command and output control forces; the plant takes in control forces and outputs motions; the sensors take in motions and output measured motions; and a control law can then use this measured motion to create a control command for MARGE.

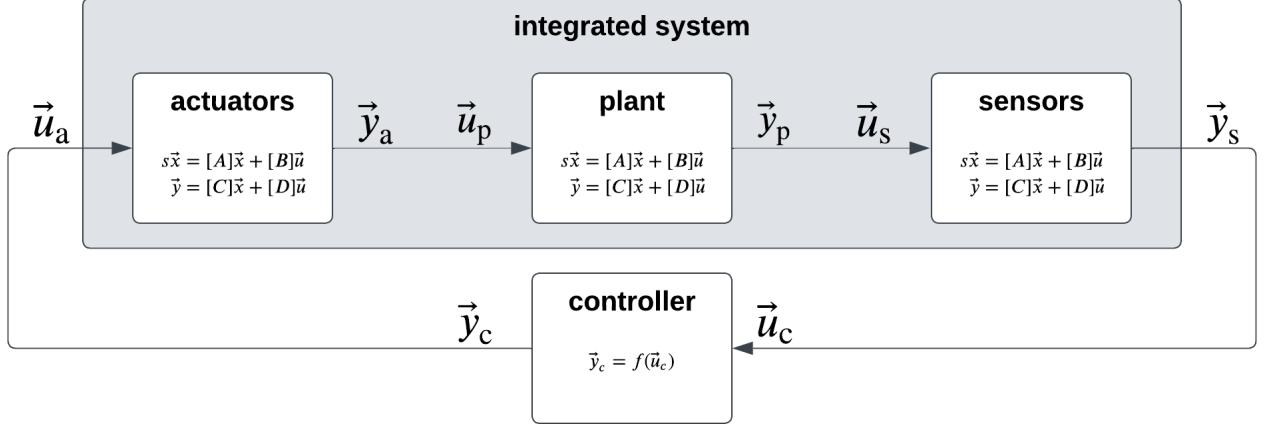


Figure 3.2: Integrated model of actuation, plant, and sensing in a control loop

First, the actuator dynamics can be coupled to the plant dynamics. In state-space form, this is

$$s \begin{Bmatrix} \{x_{act}\} \\ \{x_p\} \end{Bmatrix} = \begin{bmatrix} [A_{act}] & [0] \\ [B_p][C_{act}] & [A_p] \end{bmatrix} \begin{Bmatrix} \{x_{act}\} \\ \{x_p\} \end{Bmatrix} + \begin{bmatrix} [B_{act}] \\ [B_p][D_{act}] \end{bmatrix} \{u_{act}\} \quad (3.74)$$

where the entries in the block matrices containing  $[B_p]$  are used to convert  $\{x_{act}\}$  and  $\{u_{act}\}$  into the plant's input,  $\{u_p\} = \{y_{act}\}$ .

The actuator can similarly be coupled to the plant in the output equations:

$$\{y_p\} = \begin{bmatrix} [D_p][C_{act}] & [C_p] \end{bmatrix} \begin{Bmatrix} \{x_{act}\} \\ \{x_p\} \end{Bmatrix} + [D_p][D_{act}]\{u_{act}\} \quad (3.75)$$

where similarly, the terms involving  $[D_p]$  are used to convert  $\{x_{act}\}$  and  $\{u_{act}\}$  into the plant's input,  $\{u_p\} = \{y_{act}\}$ .

The block matrices in Eq. 3.74 and 3.75 form the coupled actuator-plant system in Fig. 3.4. Since the sensors are modeled as perfect, this is also equivalent to the coupled actuator-plant-sensor system. Thus, Eq. 3.74 and 3.75 define the state-full, coupled space model for MARGE in the form required as defined in Eq. 3.1.

## Chapter 4

### FINITE ELEMENT MODELING

An aeroelastic finite element model (FEM) of MARGE was previously constructed using NASTRAN. The model includes MARGE, the wind tunnel test section walls, and the gust vanes.

The wing structure and tail structure were each modeled as a single chain of Euler-Bernoulli beam elements along their respective spar. The wind tunnel walls were modeled as extremely rigid panels whose contribution to the structural dynamics of the FEM is negligible. The area moments of inertia and material properties of the beam elements in the finite element model are reported in Table 4.1.

Table 4.1: Properties of Beam Finite Elements

	$E$ , GPa	$G$ , GPa	$I_1$ , mm $^4$	$I_2$ , mm $^4$	$J$ , mm $^4$
wing spar	68.9	0.0125	25.41	58530	58560
tail spar	68.9	25.9	1829	13010	14840
fuselage	200	76.0	74.52	4476	4550
rigid	$10^4$	$10^4$	25.41	58530	58560

Note that the values in Table 4.1 (and also Table 4) are the final values after corrections. The values before corrections and correction process are discussed in the following chapter.

The aerodynamic loads on the NASTRAN model are based on the doublet-lattice model (DLM) for aerodynamics. This linear aerodynamic model assumes incompressible, inviscid, irrotational flow around thin lifting surfaces. The loads on the aerodynamic panels were transferred to the structural nodes via a spline interpolation.

The structural dynamic solution to the FEM includes natural frequencies and mode shapes. These are summarized in Table 4.

Table 4.2: NASTRAN Modal Properties

#	$\omega_n$	Description
1	0	pitching
2	1.444	wing bending 1
3	10.487	wing bending 2
4	16.638	fuselage bending 1
5	19.200	wing twisting 1
6	21.948	fuselage in-plane bending 1
7	32.311	wing bending 3
8	60.852	fuselage bending 2
9	66.011	wing bending 4
10	69.298	wing in-plane bending 1
11	113.674	wing bending 5
12	120.642	fuselage bending 3
13	153.716	fuselage in-plane bending 2
14	160.909	fuselage bending 4
15	175.941	wing bending 6

## Chapter 5

# GROUND VIBRATION TESTING

Ground vibration testing (GVT) was performed to validate the preliminary finite-element model of MARGE. The frequency responses of accelerometers to an impulse input were generated from the experimental data. Using these, the natural frequencies and the damping ratios of dynamic modes of the system were determined.

Two sets of data were collected. The first set of data was collected with MARGE as designed, including the rigid-body pitching mode. This data was used to determine the damping ratios of the modes. The second set of data was collected with the root of the MARGE wing clamped to eliminate the rotational rigid-body mode. This was done to enable data acquisition of flexible-body modes without exciting and losing energy to the rigid-body mode. This data was used to tune the finite-element model and to determine the damping ratios of the wing bending modes.

### **5.1 Experiment**

This section describes the GVT setup and procedure.

#### *5.1.1 Test Setup*

The equipment used for the test include:

- PCB Piezotronics ICP Impact Hammer Model 086C03
- PCB Piezotronics ICP Accelerometer Model 352C22
- National Instruments DAQ Model NI-9234

The impact hammer and accelerometers were connected to the DAQ system which was connected to a personal computer via Ethernet. The computer interfaced with the DAQ system using the Data Acquisition Toolbox for MATLAB. Data was acquired at a rate of at least 6400 Hz.

The accelerometers were mounted in locations such that all of the flexible natural modes of interest were observable. This was done by placing accelerometers near anti-nodal points of the natural modes as predicted by the preliminary finite-element model. The accelerometer locations (and corresponding FEM node IDs) for the two sets of testing are shown in Fig. 5.1.1. The impact hammer hits were also placed at these same locations on the structure.

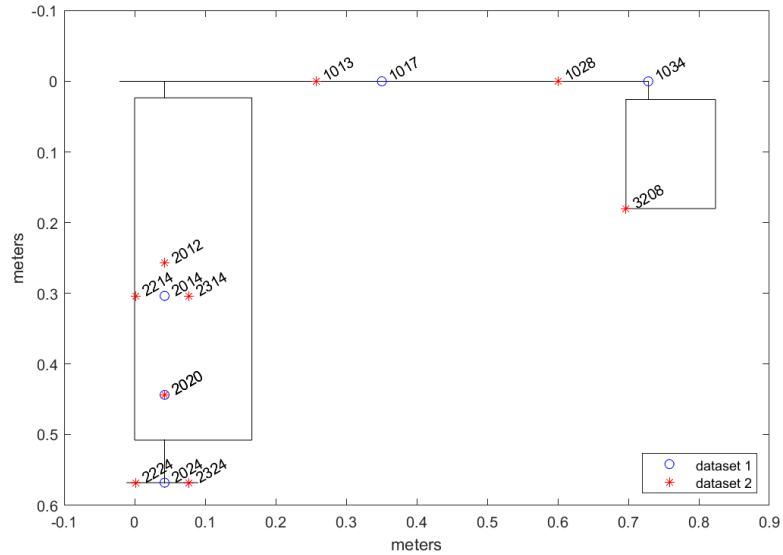


Figure 5.1: Accelerometer placement in ground vibration testing of MARGE

For the second dataset, pairs of accelerometers on the wing located a short distance apart chordwise were also treated as a fictional third accelerometer by taking the difference of their signals. This was done to simulate a sensor observing only torsional modes of the wing.

### 5.1.2 Test Procedure

Tests were performed by executing a MATLAB script which would prompt the user to name and record data. After data acquisition for a run the script automatically searched for the moment of impact generated a plot of the full time-series as well as a magnified plot of the time-series data around the impact. This magnified view was used to inspect for double-hits from the impact hammer. The script also automatically rejected data which was of insufficient length or which had too weak or strong of an impact from the hammer.

After accelerometers were mounted to MARGE and the DAQ was connected to the host computer, the following procedure was used for each run of GVT:

1. Start recording data
2. Hit the target point with the impact hammer
3. After at least five seconds has elapsed, stop recording data
4. Inspect the automatically-generated plot of the time-series data for irregularities, especially double-hits of the impact hammer or bumps of the accelerometers
5. Name and save the data

## 5.2 Generating Frequency Response Functions

Each GVT test point (input-output combination) was post-processed to generate the frequency response functions (FRFs) of the accelerometers to the impacts. This section describes the steps in this process.

### 5.2.1 Time-Domain Post-Processing

The time-series data was truncated to start just before the impulse input and end after  $t$  seconds where  $t$  was chosen to be at least four times the period of the lowest frequency of

interest. (In most cases,  $t = 4$  seconds.) This ensured that irrelevant segments of the signal were eliminated while keeping still enough data to perform the frequency-domain analysis.

Each test point was recorded as three (in the second dataset) or five (in the first dataset) separate impacts. After truncation, the signals from these impacts were concatenated to form one continuous time-domain signal. The mean of this combined signal was then subtracted from it so that there would be no steady-state component before proceeding to compute the frequency response.

### 5.2.2 Computing Frequency Response Functions

The frequency response functions for each of these concatenated SISO signal pairs were computed using the method described in [8]. This section summarizes this method as it was implemented for the GVT data.

First, the signal was buffered into overlapping Hann windows and transformed using a chirp z-transform (CZT). The CZT has an advantage over the similar Discrete Fourier Transform (DFT) in that it has the ability to allocate the full frequency-domain resolution to the bandwidth of interest. The purpose of first buffering the signal is to reduce the effect of noise at the expense of frequency resolution.

The products of the CZT are the power spectra of the signals. For any given accelerometer power spectrum  $S_y(\omega)$  and impact hammer power spectrum  $S_x(\omega)$ , the cross-spectrum correlation estimates can be computed as

$$G_{xy}(\omega) = S_x^* \cdot S_y(\omega) \quad (5.1)$$

$$G_{yx}(\omega) = S_y^* \cdot S_x(\omega) \quad (5.2)$$

and the autospectrum correlation estimates can be computed as

$$G_{xx}(\omega) = |S_x|^2 \quad (5.3)$$

$$G_{yy}(\omega) = |S_y|^2 \quad (5.4)$$

Three possible ways to estimate the FRF from the above are the  $H_1$  FRF, the  $H_2$  FRF, and the  $H_v$  FRF:

$$FRF_{H_1}(\omega) = \frac{G_{xy}(\omega)}{G_{xx}(\omega)} \quad (5.5)$$

$$FRF_{H_2}(\omega) = \frac{G_{yy}(\omega)}{G_{yx}(\omega)} \quad (5.6)$$

$$FRF_{H_v}(\omega) = \sqrt{FRF_{H_1}(\omega) \cdot FRF_{H_2}(\omega)} \quad (5.7)$$

The  $H_1$  estimate tends to under-estimate the FRF when there is noise in the input, while the  $H_2$  estimate tends to over-estimate the FRF when there is noise at the output [8]. The  $H_v$  estimate of the FRF is thus used in this study as a conservative choice which makes no assumption of the source or nature of noise in the system. (Subsequent references to the FRF can be assumed to refer to the  $H_v$  FRF.)

The coherence can also be computed as

$$\text{coh}(\omega) = \frac{|G_{xy}|^2}{|G_{xx}||G_{yy}|} \quad (5.8)$$

The frequency response functions and coherence computed for the GVT data are shown in Appendix ??.

### 5.3 Determining Modal Properties

Once the FRFs were obtained, the frequencies and damping ratios of the natural modes were computed from the FRFs. This section describes this computation.

#### 5.3.1 Computing Natural Frequencies

First, natural frequencies visible as peaks in the data were noted. These often were visible across multiple FRFs, confirming that they were not artifacts from the noise of a single experimental trial.

These experimental natural frequencies were compared to those predicted by the NASTRAN finite-element model; if they matched well, it was assumed that the experimental

natural frequency corresponded to the mode shape generated by the NASTRAN model. This could be further validated by observing the antinodal points of the relevant NASTRAN mode shape and checking that the FRFs in which the natural frequency peaks are visible correspond to sensors placed near those antinodal points.

In some cases, there were clear natural modes visible in the experimental data that were not predicted by the NASTRAN finite-element model. It was inferred that two of these natural modes appeared in that these were torsional modes of the wing, because the FRFs they appeared most prominently in were from the aforementioned “fictional” accelerometers which had manipulated signals to enhance the response to torsional modes. These torsional modes were not predicted by the preliminary NASTRAN finite-element model; this was corrected in the subsequent FEM tuning process.

Each experimental natural frequency  $\omega_n$  was then measured in an automated way: first, all FRFs with a local maximum magnitude at  $\omega_n$  which was at least twice the magnitude of its surroundings was identified. The median of all of these measured natural frequencies, each from a different FRF, was then taken to be the true experimental natural frequency for that natural mode.

### 5.3.2 Computing Damping Ratios

The damping ratio was also measured in a similar automated way. The damping ratio was computed for each identifiable natural frequency in each FRF using the half-power method:

$$\zeta = \frac{\omega_2 - \omega_1}{2\omega_n} \quad (5.9)$$

where

$$FRF(\omega_1) = FRF(\omega_2) = \frac{1}{2}FRF(\omega_n) \quad (5.10)$$

$$\omega_1 < \omega_2$$

The median of all of these damping ratios, each from a different FRF, was then taken to be the true damping ratio for that natural mode. The experimentally obtained natural frequencies and corresponding damping ratios can be found in Table 5.3.2.

Table 5.1: Experimental Natural Modes

$\omega_n$	$\zeta$	Description
0		pitching
1.422	0.030	wing bending 1
10.142	0.046	wing bending 2
18.094	0.113	wing twisting 1
19.893	0.033	fuselate in-plane bending 1
19.897	0.031	fuselage bending 1
32.545	0.019	wing bending 3
51.706	0.084	wing torsion 2
60.482	0.035	wing bending 4
74.521	0.023	fuselage bending 2

#### 5.4 Finite Element Model Correction

The finite-element model was adjusted to better match the experimental GVT data as well as static testing data from a separate study. This was done by adjusting the bending and torsional stiffness of the various materials until the natural frequencies best matched that of the experiment. The uncorrected and corrected natural frequencies of the FEM are compared to the experimental natural frequencies in Table 5.4. Note that damping ratios are not involved here, as the FEM does not model damping. The damping ratios are used only in the modeling in Chapter 3.

Table 5.2: Natural Frequencies of Uncorrected FEM, Corrected FEM, and Experiment

Uncorrected FEM	Corrected FEM	Experiment	Description
0	0		pitching
1.454401	1.44397	1.422	wing bending 1
10.51099	10.48667	10.142	wing bending 2
-	19.1997	18.094	wing twisting 1
23.13519	21.94804	19.893	fuselage in-plane bending 1
17.81394	16.63828	19.897	fuselage bending 1
32.33148	32.31077	32.545	wing bending 3
-	-	51.706	wing twisting 2
66.02579	66.01089	60.482	wing bending 4
69.7988	69.29826		wing in-plane bending 1
62.68383	60.85218	74.521	fuselage bending 2
113.6852	113.6736		wing bending 5
120.8188	120.6415		fuselage bending 3
161.7024	153.716		fuselage in-plane bending 2
175.9587	175.9407		wing bending 6
159.36	160.9085		fuselage bending 4
225.7709	-		fuselage bending 5

## Chapter 6

### WIND TUNNEL TESTING AND MODEL TUNING

This chapter describes the wind-tunnel testing of MARGE and the tuning of the aerovielastic model to match the experimental data.

#### **6.1 Data Acquisition and Analysis**

MARGE's response was tested at the University of Washington's 3x3 low-speed wind tunnel. It was tested at six flight conditions,  $q_D = \{60, 100, 163, 207, 281, 343\}$  Pa. At each flight condition, the response to each of the four inputs was tested three times. For the gust vanes, a discrete 4-degree doublet gust was generated with a frequency of 1.45 Hz (approximately equivalent to the first wing bending natural frequency). For the ailerons, a 5-degree frequency sweep from 1 Hz to 2 Hz was performed. For the elevator, a 2-degree frequency sweep from 1 Hz to 2 Hz was performed. This frequency band was chosen as it encompassed the first wing bending mode and also stayed within the bandwidth of the actuators.

The exception to the above is at  $q_d = 343$  Pa. At this dynamic pressure, the aileron sweeps were reduced in magnitude to 3.5 degrees and the elevator sweep was reduced in magnitude to 1 degree. This was done in order to reduce the risk of damage to the model due to violent responses at high speeds. <verify with john>

All of these tests were controlled and recorded using Simulink Real-Time. The testing yielded time-series data of input commands and sensor readings for each test.

### 6.1.1 Data Postprocessing

The time-domain data was post-processed in a similar way as was done for the GVT data in Section 5.2. Each run's data was truncated to start when the input began and end five seconds after the input ended. The data was re-centered to have zero mean and then the three runs of each test were concatenated.

This single concatenated time-domain signal was then buffered into overlapping Hann windows and transformed using the CZT. The FRFs were then computing using Equation 5.7.

### Accelerometer Data Postprocessing

issues will be further discussed in the next chapter <insert>

bandwidth out [1,2] <insert>

bandpass in [0.8,25] <insert>

re-center <insert>

## 6.2 Model Tuning

The state-space model generated in Chapter 3 was adjusted such that its frequency response fit the experimental frequency response. This was done by adjusting certain parameters in the model that each represented some uncertainty in the model's physical characteristics. This adjustment was first done by hand, and then repeated in a automated process through optimization.

### 6.2.1 Tuning Parameters

The state-space model derived in Chapter 3 assumes linear structural dynamics, linear aerodynamics, and thin airfoils. It also requires perfect knowledge of mode shapes. In reality, MARGE does not conform to any of these assumptions perfectly. Thus, tuning parameters

were implemented in the state-space model generation script which adjusted the structural and aerodynamic modeling to compensate.

The first tuning parameter is the first wing bending natural frequency  $\omega_{n,2}$ . This quantity has a low uncertainty, but was treated as a tuning parameter for purposes of investigation.

The next tuning parameters are the damping ratios  $\zeta$  for the modes. These have higher uncertainty since viscous damping is not the most physically accurate model for structural damping. Furthermore, aerodynamic damping of motions is not accounted for in the model. These can be compensated for by adjusting the damping ratios with this tuning parameter.

The next tuning parameters are the effectiveness  $\tau$  of the controls. Each control surface will not be as effective as predicted by the linear aerodynamic model due to airfoil thickness, viscous effects (such as flow separation), roughness, and gaps in construction. These can be compensated for by adjusting the control effectiveness of each control surface. These tuning parameters scale the forces predicted for the control surfaces by the linear aerodynamic model.

The final tuning parameters are scaling factors  $\tau_{P_{ss}}$  for each entry of  $[P_{ss}]$ . (Refer to Eq. 3.18 for the context in which  $[P_{ss}]$  is used in modeling.) These scale the lift of the lifting surfaces (not including the control surfaces) to account for airfoil thickness, viscous effects, roughness, and gaps in construction.

A summary of tuning parameters and their default values (which apply no correction) is shown in Table 6.2.1. The comparison of this initial un-tuned model's FRFs and the experimental FRFs can be seen in Fig. 6.2.1.

Table 6.1: Default Values of MARGE Tuning Parameters

Name	Default Value	Description
$\omega_{n,2}$	1.4544	first wing bending mode frequency
$\zeta_1$	0	rigid pitching damping ratio
$\zeta_2$	0.028	first wing bending damping ratio
$\tau_{ail1}$	1	aileron 1 effectiveness
$\tau_{ail2}$	1	aileron 2 effectiveness
$\tau_{elev}$	1	elevator effectiveness
$\tau_{gust}$	1	gust vane effectiveness
$[\tau_{P_{ss}}]$	all ones	lift scaling

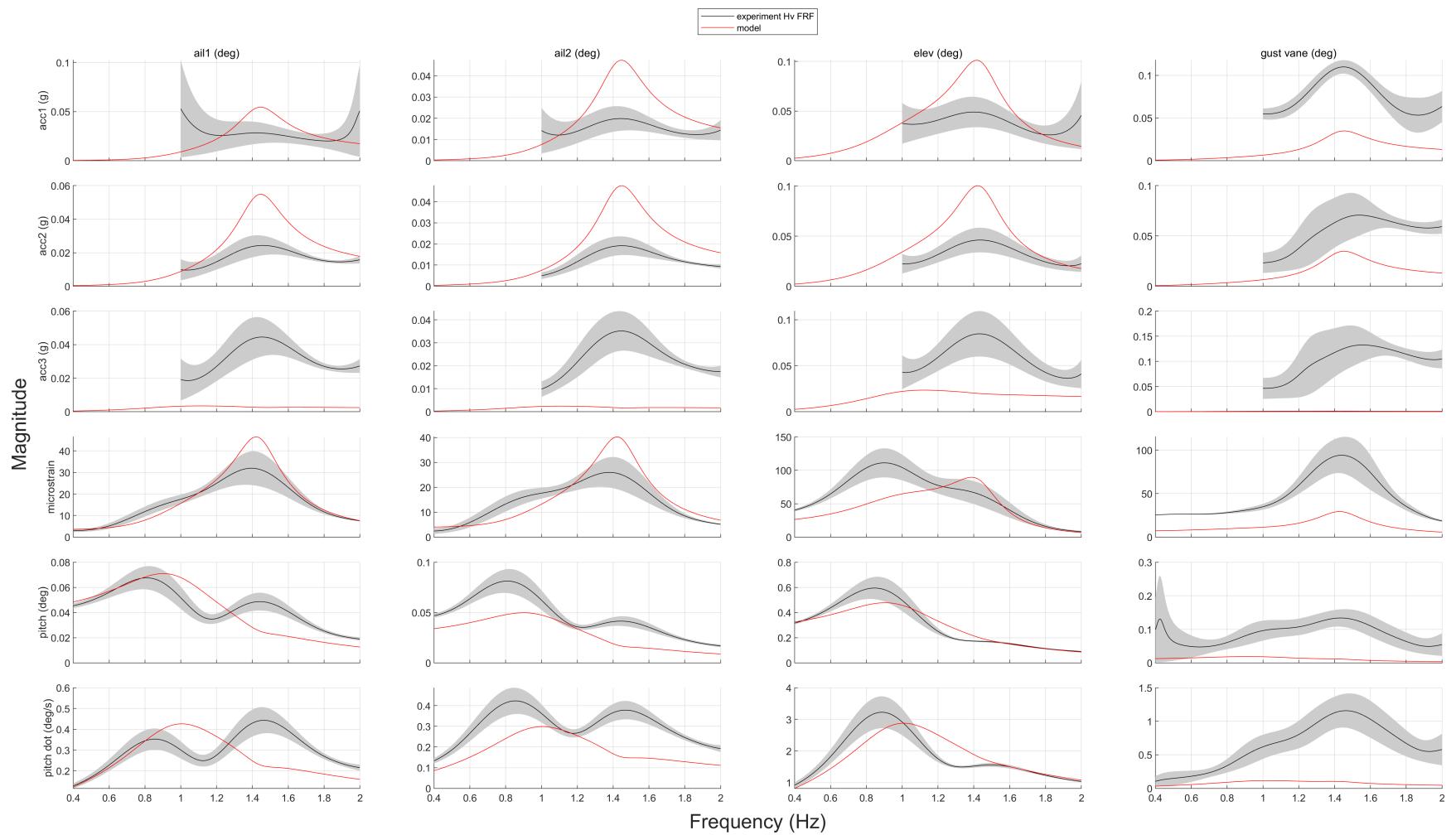


Figure 6.1: A comparison of experimental FRFs and untuned model FRFs. The grey region is enclosed by the  $H_1$  FRF below and the  $H_2$  FRF above of the experimental data.

### 6.2.2 Manual Model Tuning

The first step in tuning was experimenting with manual changes to the tuning parameters with the goal of closing the gap between the model's FRF and the experimental FRF. This was done by thinking intuitively about the modeling uncertainties which are most likely effecting the known discrepancies in the FRFs. For example, it was known that linear aerodynamics would overpredict the control surface effectiveness by up to 50% even in reasonably small ( $< 10^\circ$ ) deflections [10, 4]. Thus, the control effectiveness of the ailerons and elevator were lowered until the FRFs with these controls as inputs better matched the experimental FRFs in magnitude.

The resultant parameters and FRF comparison of the manually tuned model are shown below in Table 6.2.2 and Fig. 6.2.2. The poor fit in Fig 6.2.2 indicates that the simple and “intuitive” manual tuning is not sufficient to completely match the experiment.

Table 6.2: Manually Tuned Values of MARGE Tuning Parameters

Name	Default Value	Adjusted Value
$\omega_{n,2}$	1.4544	N/A
$\zeta_1$	0	60
$\zeta_2$	0.028	N/A
$\tau_{ail1}$	1	0.6
$\tau_{ail2}$	1	0.7
$\tau_{elev}$	1	0.6
$\tau_{gust}$	1	N/A
$[\tau_{P_{ss}}]$	all ones	$\begin{bmatrix} 0.9 & 1 \\ 0.5 & 1.5 \end{bmatrix}$

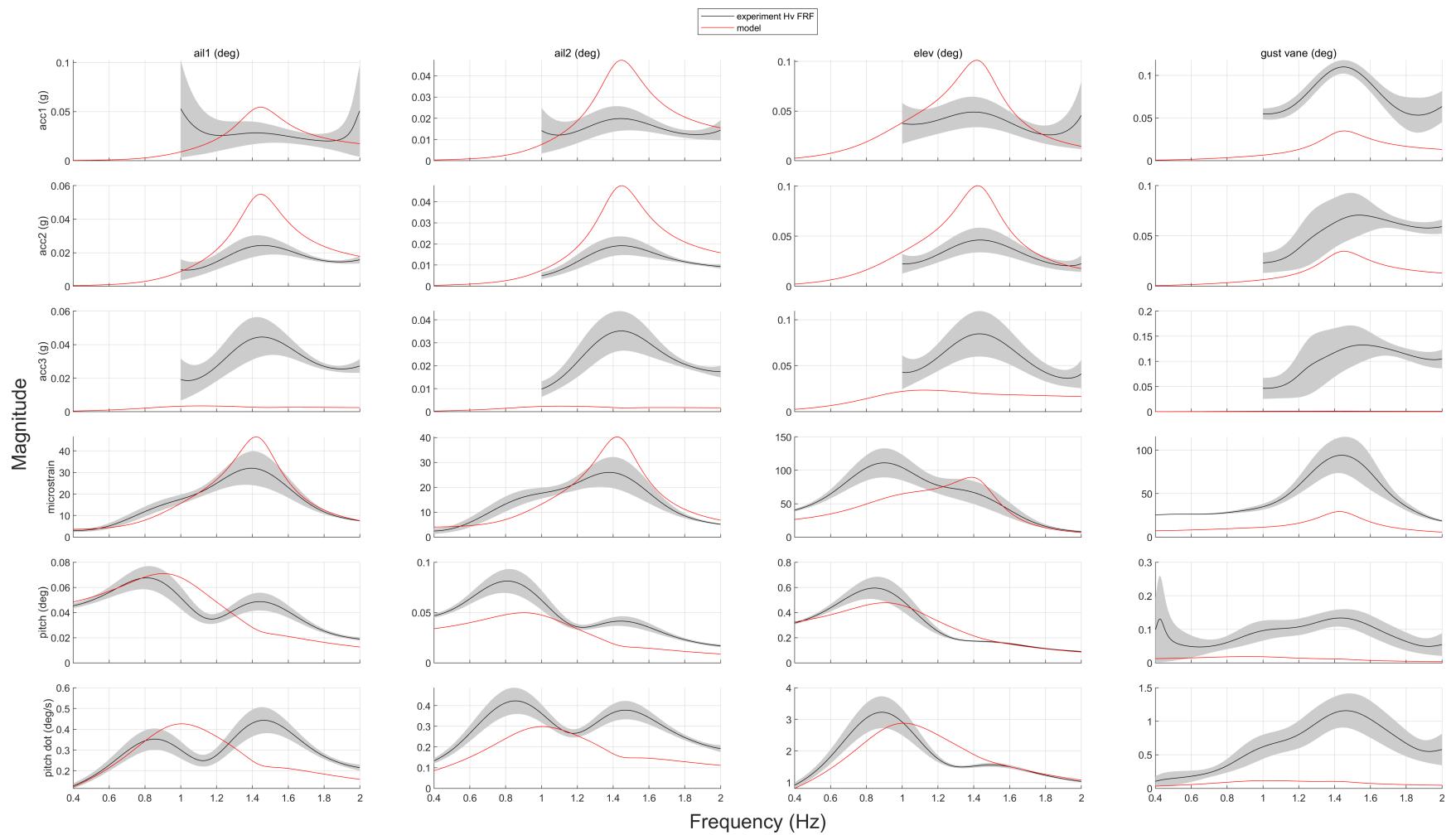


Figure 6.2: A comparison of experimental FRFs and manually-tuned model FRFs. The grey region is enclosed by the  $H_1$  FRF below and the  $H_2$  FRF above of the experimental data.

### 6.2.3 Model Optimization

obj

$$F(\{x\}) = \sum_{i=1}^{N_{\text{in}}} \sum_{j=1}^{N_{\text{out}}} f_{i,j}(\{x\}) \quad (6.1)$$

$$f_{ij}(\{x\}) = \epsilon_{ij \text{ mag}} \sqrt{\lambda} + \epsilon_{ij \text{ phase}} \frac{1}{\sqrt{\lambda}} \quad (6.2)$$

$$\epsilon_{ij \text{ mag}} = \sum_{\omega} \frac{|FRF_{\text{exp}}(\omega)| - |FRF_{\text{model}}(\omega)|}{|FRF_{\text{exp}}(\omega)|} \quad (6.3)$$

$$\epsilon_{ij \text{ phase}} = \sum_{\omega} (\angle FRF_{\text{exp}}(\omega) - \angle FRF_{\text{model}}(\omega)) \quad (6.4)$$

## Chapter 7

### RESULTS AND DISCUSSION

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### 7.1 *Hardware Limitations*

## Chapter 8

# CONCLUSION

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## Appendix A

### GROUND VIBRATION TEST FREQUENCY RESPONSE PLOTS

The plots each indicate the acceleration response at a location to an impulse input at a location. The locations are labeled by node number and direction. The locations of the respective nodes can be found in Figure 5.1.1.

