Exercise 1:

Foundations of Mathematical, WS24

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This is **exercise** 1 for Foundations of Mathematical, WS24. Generated on 2024-11-25 with 10 problems per section.

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1. Problems

1.1. Vector Arithmetic

1.1.1. Addition

Find the sum of the following vectors $\mathbf{u}$ and $\mathbf{v}$

1. $\mathbf{u} = \begin{bmatrix} -1 \\ -7 \\ -7 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 6 \\ -6 \\ -9 \end{bmatrix}$

2. $\mathbf{u} = \begin{bmatrix} -9 \\ 8 \\ -9 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -5 \\ -4 \\ 6 \end{bmatrix}$

3. $\mathbf{u} = \begin{bmatrix} -9 \\ 6 \\ -7 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -7 \\ -2 \\ -7 \end{bmatrix}$

4. $\mathbf{u} = \begin{bmatrix} -6 \\ -8 \\ -7 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -8 \\ 9 \\ -5 \end{bmatrix}$

5. $\mathbf{u} = \begin{bmatrix} 0 \\ 4 \\ 9 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 7 \\ -2 \\ 0 \end{bmatrix}$

6. $\mathbf{u} = \begin{bmatrix} -3 \\ 2 \\ -7 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix}$

7. $\mathbf{u} = \begin{bmatrix} -8 \\ -9 \\ 8 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 7 \\ 3 \\ -7 \end{bmatrix}$

8. $\mathbf{u} = \begin{bmatrix} -9 \\ 10 \\ 10 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -3 \\ 10 \\ -10 \end{bmatrix}$

9. $\mathbf{u} = \begin{bmatrix} -4 \\ -7 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 4 \\ -7 \\ -5 \end{bmatrix}$

10. $\mathbf{u} = \begin{bmatrix} -3 \\ 3 \\ 5 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -8 \\ -2 \\ -4 \end{bmatrix}$

1.1.2. Subtraction

Find the difference of the following vectors $\mathbf{u}$ and $\mathbf{v}$

1. $\mathbf{u} = \begin{bmatrix} 7 \\ -2 \\ -3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -4 \\ 1 \\ 7 \end{bmatrix}$

2. $\mathbf{u} = \begin{bmatrix} 2 \\ 3 \\ 8 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 6 \\ 8 \\ 6 \end{bmatrix}$

3. $\mathbf{u} = \begin{bmatrix} -4 \\ -1 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -7 \\ -10 \\ 0 \end{bmatrix}$

4. $\mathbf{u} = \begin{bmatrix} -8 \\ 9 \\ 6 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -1 \\ 7 \\ -6 \end{bmatrix}$

5. $\mathbf{u} = \begin{bmatrix} -7 \\ -1 \\ 8 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

6. $\mathbf{u} = \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -4 \\ -1 \\ 0 \end{bmatrix}$

7. $\mathbf{u} = \begin{bmatrix} 7 \\ 9 \\ 0 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -9 \\ -4 \\ -3 \end{bmatrix}$

8. $\mathbf{u} = \begin{bmatrix} -9 \\ -1 \\ -8 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -4 \\ -10 \\ 8 \end{bmatrix}$

9. $\mathbf{u} = \begin{bmatrix} -1 \\ 2 \\ 9 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -1 \\ -4 \\ -4 \end{bmatrix}$

10. $\mathbf{u} = \begin{bmatrix} -8 \\ -4 \\ 10 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -2 \\ 8 \\ -9 \end{bmatrix}$

1.1.3. Scalar Multiplication

Find the scalar product of the following vector $\mathbf{u}$ and scalar k

1. $\mathbf{u} = \begin{bmatrix} 5 \\ -10 \\ -6 \end{bmatrix}$ 9v.

2. $\mathbf{u} = \begin{bmatrix} -9 \\ -6 \\ -3 \end{bmatrix}$ −9v.

3. $\mathbf{u} = \begin{bmatrix} 9 \\ 8 \\ 1 \end{bmatrix}$ −7v.

4. $\mathbf{u} = \begin{bmatrix} 4 \\ -9 \\ 3 \end{bmatrix}$ −6v.

5. $\mathbf{u} = \begin{bmatrix} 8 \\ -6 \\ -6 \end{bmatrix}$ −10v.

6. $\mathbf{u} = \begin{bmatrix} -7 \\ 2 \\ -4 \end{bmatrix}$ −9v.

7. $\mathbf{u} = \begin{bmatrix} 5 \\ 9 \\ 8 \end{bmatrix}$ −6v.

8. $\mathbf{u} = \begin{bmatrix} 8 \\ -7 \\ 4 \end{bmatrix}$ 10v.

9. $\mathbf{u} = \begin{bmatrix} 1 \\ -9 \\ 4 \end{bmatrix}$ 1v.

10. $\mathbf{u} = \begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix}$ 2v.

1.2. Matrix Arithmetic

1.2.1. Addition

Find the sum of the following matrices A and B

1. $A = \begin{bmatrix} 2 & 9 & 2 \\ -10 & 1 & 1 \\ -4 & 4 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -4 & 6 \\ -1 & 6 & 5 \\ 1 & 3 & 0 \end{bmatrix}$

2.
$$A = \begin{bmatrix} -3 & 7 & 9 \\ 9 & -3 & -8 \\ 8 & -1 & -8 \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 & -4 & 6 \\ -1 & 2 & -9 \\ -2 & 4 & -10 \end{bmatrix}$$

3.
$$A = \begin{bmatrix} -1 & 4 & 2 \\ 0 & -3 & -10 \\ -10 & -1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} -5 & -8 & -4 \\ -10 & -8 & 6 \\ -1 & 9 & 1 \end{bmatrix}$$

4.
$$A = \begin{bmatrix} -4 & 5 & 1 \\ 2 & -3 & 0 \\ -1 & -8 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -10 & 0 & -9 \\ 0 & 7 & 6 \\ -2 & -5 & 8 \end{bmatrix}$$

5.
$$A = \begin{bmatrix} -2 & 5 & -8 \\ 6 & -2 & -2 \\ -8 & -1 & -6 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & -9 & 5 \\ 5 & 5 & -6 \\ -7 & -7 & 2 \end{bmatrix}$$

6.
$$A = \begin{bmatrix} 4 & 4 & -7 \\ 2 & 8 & -8 \\ 4 & 5 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 4 & 0 \\ -2 & -1 & 9 \\ 8 & -3 & -4 \end{bmatrix}$$

7.
$$A = \begin{bmatrix} -2 & 8 & 3 \\ -6 & 6 & 9 \\ 3 & -5 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} -9 & 2 & -3 \\ 5 & 6 & -8 \\ -6 & -8 & 0 \end{bmatrix}$$

8.
$$A = \begin{bmatrix} -2 & -3 & 3 \\ 7 & -9 & -4 \\ -4 & -6 & 7 \end{bmatrix} \text{ and } B = \begin{bmatrix} -9 & 8 & -1 \\ 4 & -8 & -7 \\ 4 & -2 & 8 \end{bmatrix}$$

9.
$$A = \begin{bmatrix} -8 & -2 & -3 \\ 6 & -4 & -10 \\ 3 & -3 & -4 \end{bmatrix} \text{ and } B = \begin{bmatrix} -7 & -10 & 2 \\ -6 & 5 & 6 \\ 7 & 6 & -8 \end{bmatrix}$$

10.
$$A = \begin{bmatrix} 4 & 0 & -5 \\ 2 & 8 & -9 \\ 1 & 1 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} -4 & 4 & 0 \\ -7 & 2 & 2 \\ 8 & -10 & -9 \end{bmatrix}$$

1.2.2. Subtraction

Find the difference of the following matrices A and B

1.
$$A = \begin{bmatrix} -10 & -10 & 6 \\ 3 & -9 & 1 \\ -6 & 4 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 1 & 6 \\ -3 & -6 & -5 \\ -5 & -6 & 9 \end{bmatrix}$$

2.
$$A = \begin{bmatrix} 6 & 5 & -3 \\ -5 & 0 & 0 \\ 7 & 4 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & -1 & -8 \\ 0 & 8 & -10 \\ 6 & -3 & -8 \end{bmatrix}$$

3.
$$A = \begin{bmatrix} -1 & -5 & 6 \\ 5 & -10 & -7 \\ -8 & -4 & -2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 8 & -7 & -9 \\ -8 & -3 & -3 \\ 5 & 8 & -8 \end{bmatrix}$$

4.
$$A = \begin{bmatrix} -1 & -5 & 7 \\ -4 & -5 & -8 \\ -5 & 9 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 & -4 & -5 \\ 2 & -9 & 3 \\ 4 & 1 & 4 \end{bmatrix}$$

5.
$$A = \begin{bmatrix} -8 & 2 & 2 \\ -6 & 3 & -4 \\ -4 & -10 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 & 2 & -8 \\ 0 & -1 & -1 \\ 5 & 3 & -4 \end{bmatrix}$$

6.
$$A = \begin{bmatrix} -4 & -10 & 8 \\ 3 & -4 & -4 \\ -10 & -8 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 & 3 & 0 \\ -10 & -7 & 5 \\ 7 & 3 & 9 \end{bmatrix}$$

7.
$$A = \begin{bmatrix} 1 & -1 & -5 \\ 7 & 2 & 6 \\ -4 & 6 & -3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 & -4 & 9 \\ -9 & -10 & 5 \\ -10 & -2 & 9 \end{bmatrix}$$

8.
$$A = \begin{bmatrix} 4 & -2 & -4 \\ -2 & 2 & -2 \\ 7 & 7 & -4 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & -7 & -2 \\ 0 & -3 & 0 \\ -7 & -7 & 3 \end{bmatrix}$$

9. $A = \begin{bmatrix} -4 & -5 & -3 \\ 8 & 2 & 7 \\ -7 & 9 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & -4 & 7 \\ 3 & -1 & 2 \\ -7 & 8 & -10 \end{bmatrix}$

10. $A = \begin{bmatrix} -5 & -9 & 2 \\ -5 & 9 & 4 \\ -7 & -5 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 7 & -6 \\ -8 & -1 & 0 \\ -3 & -7 & -2 \end{bmatrix}$

1.2.3. Multiplication

Find the product of the following matrices A and B

1. $A = \begin{bmatrix} 7 & 7 & -7 \\ 8 & -10 & 2 \\ 6 & 8 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 4 & -6 \\ 3 & -2 & 1 \\ -8 & -3 & 6 \end{bmatrix}$

2. $A = \begin{bmatrix} -7 & 4 & 9 \\ 1 & -3 & 3 \\ -10 & -4 & -7 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & -6 \\ -2 & 9 & 0 \\ 3 & 8 & 1 \end{bmatrix}$

3. $A = \begin{bmatrix} -2 & -4 & 1 \\ 0 & -7 & 2 \\ -6 & 1 & -8 \end{bmatrix}$ and $B = \begin{bmatrix} -9 & 9 & 0 \\ 4 & -8 & -9 \\ 4 & 1 & 1 \end{bmatrix}$

4. $A = \begin{bmatrix} 8 & 0 & 1 \\ 6 & 2 & -5 \\ -2 & 6 & -9 \end{bmatrix}$ and $B = \begin{bmatrix} -7 & -9 & -7 \\ 8 & 8 & 8 \\ 5 & 6 & 2 \end{bmatrix}$

5. $A = \begin{bmatrix} 8 & -6 & -3 \\ 5 & 1 & 3 \\ -3 & 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & -4 & -10 \\ 0 & -7 & -3 \\ -2 & -1 & -7 \end{bmatrix}$

6. $A = \begin{bmatrix} 7 & 9 & -2 \\ 8 & 7 & 2 \\ -10 & -5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & -5 & -10 \\ 6 & -1 & -2 \\ -7 & 2 & 8 \end{bmatrix}$

7. $A = \begin{bmatrix} 7 & 3 & -7 \\ 0 & 4 & 8 \\ -4 & 7 & -7 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & -9 & 6 \\ -7 & -10 & 2 \\ -3 & -4 & 3 \end{bmatrix}$

8. $A = \begin{bmatrix} 7 & -8 & 3 \\ -10 & 0 & -6 \\ -10 & 8 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -2 & 0 \\ -2 & -6 & 7 \\ 2 & 2 & 7 \end{bmatrix}$

9. $A = \begin{bmatrix} -2 & -3 & -7 \\ -1 & -8 & 4 \\ -5 & -5 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} -7 & 7 & 0 \\ -2 & -10 & 7 \\ 1 & -7 & -6 \end{bmatrix}$

10. $A = \begin{bmatrix} -1 & -5 & -9 \\ 3 & 0 & 1 \\ 3 & -8 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 9 & -9 \\ -2 & 2 & -2 \\ 7 & -8 & -7 \end{bmatrix}$

1.3. Matrix Properties

1.3.1. Properties

For each matrix A, find:

a) rank(A)

b) nullity(A)

c) det(A)

d) A^{-1} (if exists)

e) basis of ker(A)

1. $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & -2 \\ -1 & 2 & 7 \end{bmatrix}$

3. $A = \begin{bmatrix} -3 & -6 & 0 \\ -1 & -1 & -1 \\ 2 & 4 & 0 \end{bmatrix}$

4. $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 4 & 0 & -3 \end{bmatrix}$

5. $A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 3 & 12 \\ 0 & 0 & 1 \end{bmatrix}$

6. $A = \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

7. $A = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

8. $A = \begin{bmatrix} 1 & -2 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$

9. $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

10. $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 5 \\ 0 & 5 & 13 \end{bmatrix}$

1.3.2. RREF

Find the Reduced Row Echelon Form of the following matrix A

1. $A = \begin{bmatrix} 1 & 1 & 0 \\ -2 & -1 & -1 \\ 2 & 1 & 2 \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$

4. $A = \begin{bmatrix} 1 & 2 & 2 \\ -1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

5. $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

6. $A = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

7. $A = \begin{bmatrix} 2 & 3 & 0 \\ -1 & -1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$

8. $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -2 & 1 & 0 \end{bmatrix}$

9. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$

10. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

2. Solutions

2.1. Vector Arithmetic

2.1.1. Addition

$$\begin{bmatrix} 5 \\ -13 \\ -16 \end{bmatrix} \begin{bmatrix} -14 \\ 4 \\ -3 \end{bmatrix} \begin{bmatrix} -16 \\ 4 \\ -14 \end{bmatrix} \begin{bmatrix} -14 \\ 1 \\ -12 \end{bmatrix} \begin{bmatrix} 7 \\ 2 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 2 \\ -12 \end{bmatrix} \begin{bmatrix} -1 \\ -6 \\ 1 \end{bmatrix} \begin{bmatrix} -12 \\ 20 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ -14 \\ -3 \end{bmatrix} \begin{bmatrix} -11 \\ 1 \\ 1 \end{bmatrix}$$

2.1.2. Subtraction

$$\begin{bmatrix} 11 \\ -3 \\ -10 \end{bmatrix} \begin{bmatrix} -4 \\ -5 \\ 2 \end{bmatrix} \begin{bmatrix} 3 \\ 9 \\ 2 \end{bmatrix} \begin{bmatrix} -7 \\ 2 \\ 12 \end{bmatrix} \begin{bmatrix} -5 \\ -1 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 6 \\ 5 \\ 8 \end{bmatrix} \begin{bmatrix} 16 \\ 13 \\ 3 \end{bmatrix} \begin{bmatrix} -5 \\ 9 \\ -16 \end{bmatrix} \begin{bmatrix} 0 \\ 6 \\ 13 \end{bmatrix} \begin{bmatrix} -6 \\ -12 \\ 19 \end{bmatrix}$$

2.1.3. Scalar Multiplication

1: $\begin{bmatrix} 45 \\ -90 \\ -54 \end{bmatrix}$ 2: $\begin{bmatrix} 81 \\ 54 \\ 27 \end{bmatrix}$ 3: $\begin{bmatrix} -63 \\ -56 \\ -7 \end{bmatrix}$ 4: $\begin{bmatrix} -24 \\ 54 \\ -18 \end{bmatrix}$ 5: $\begin{bmatrix} -80 \\ 60 \\ 60 \end{bmatrix}$

6: $\begin{bmatrix} 63 \\ -18 \\ 36 \end{bmatrix}$ 7: $\begin{bmatrix} -30 \\ -54 \\ -48 \end{bmatrix}$ 8: $\begin{bmatrix} 80 \\ -70 \\ 40 \end{bmatrix}$ 9: $\begin{bmatrix} 1 \\ -9 \\ 4 \end{bmatrix}$ 10: $\begin{bmatrix} 4 \\ 4 \\ -4 \end{bmatrix}$

2.2. Matrix Arithmetic

2.2.1. Addition

1: $\begin{bmatrix} 6 & 5 & 8 \\ -11 & 7 & 6 \\ -3 & 7 & 7 \end{bmatrix}$ 2: $\begin{bmatrix} 1 & 3 & 15 \\ 8 & -1 & -17 \\ 6 & 3 & -18 \end{bmatrix}$ 3: $\begin{bmatrix} -6 & -4 & -2 \\ -10 & -11 & -4 \\ -11 & 8 & 3 \end{bmatrix}$ 4: $\begin{bmatrix} -14 & 5 & -8 \\ 2 & 4 & 6 \\ -3 & -13 & 9 \end{bmatrix}$ 5: $\begin{bmatrix} -4 & -4 & -3 \\ 11 & 3 & -8 \\ -15 & -8 & -4 \end{bmatrix}$

6: $\begin{bmatrix} 4 & 8 & -7 \\ 0 & 7 & 1 \\ 12 & 2 & -1 \end{bmatrix}$ 7: $\begin{bmatrix} -11 & 10 & 0 \\ -1 & 12 & 1 \\ -3 & -13 & 4 \end{bmatrix}$ 8: $\begin{bmatrix} -11 & 5 & 2 \\ 11 & -17 & -11 \\ 0 & -8 & 15 \end{bmatrix}$ 9: $\begin{bmatrix} -15 & -12 & -1 \\ 0 & 1 & -4 \\ 10 & 3 & -12 \end{bmatrix}$ 10: $\begin{bmatrix} 0 & 4 & -5 \\ -5 & 10 & -7 \\ 9 & -9 & -6 \end{bmatrix}$

2.2.2. Subtraction

1: $\begin{bmatrix} -9 & -11 & 0 \\ 6 & -3 & 6 \\ -1 & 10 & -8 \end{bmatrix}$ 2: $\begin{bmatrix} 8 & 6 & 5 \\ -5 & -8 & 10 \\ 1 & 7 & 10 \end{bmatrix}$ 3: $\begin{bmatrix} -9 & 2 & 15 \\ 13 & -7 & -4 \\ -13 & -12 & 6 \end{bmatrix}$ 4: $\begin{bmatrix} -5 & -1 & 12 \\ -6 & 4 & -11 \\ -9 & 8 & -4 \end{bmatrix}$ 5: $\begin{bmatrix} -15 & 0 & 10 \\ -6 & 4 & -3 \\ -9 & -13 & 5 \end{bmatrix}$

6: $\begin{bmatrix} -9 & -13 & 8 \\ 13 & 3 & -9 \\ -17 & -11 & -14 \end{bmatrix}$ 7: $\begin{bmatrix} -6 & 3 & -14 \\ 16 & 12 & 1 \\ 6 & 8 & -12 \end{bmatrix}$ 8: $\begin{bmatrix} 5 & 5 & -2 \\ -2 & 5 & -2 \\ 14 & 14 & -7 \end{bmatrix}$ 9: $\begin{bmatrix} -13 & -1 & -10 \\ 5 & 3 & 5 \\ 0 & 1 & 9 \end{bmatrix}$ 10: $\begin{bmatrix} -14 & -16 & 8 \\ 3 & 10 & 4 \\ -4 & 2 & 9 \end{bmatrix}$

2.2.3. Multiplication

1: $\begin{bmatrix} 140 & 35 & -77 \\ 26 & 46 & -46 \\ 94 & 14 & -40 \end{bmatrix}$ 2: $\begin{bmatrix} 12 & 101 & 51 \\ 16 & -2 & -3 \\ -23 & -102 & 53 \end{bmatrix}$ 3: $\begin{bmatrix} 6 & 15 & 37 \\ -20 & 58 & 65 \\ 26 & -70 & -17 \end{bmatrix}$ 4: $\begin{bmatrix} -51 & -66 & -54 \\ -51 & -68 & -36 \\ 17 & 12 & 44 \end{bmatrix}$ 5: $\begin{bmatrix} 78 & 13 & -41 \\ 39 & -30 & -74 \\ -25 & -1 & 31 \end{bmatrix}$

6: $\begin{bmatrix} 131 & -48 & -104 \\ 100 & -43 & -78 \\ -141 & 61 & 134 \end{bmatrix}$ 7: $\begin{bmatrix} 56 & -65 & 27 \\ -52 & -72 & 32 \\ -60 & -6 & -31 \end{bmatrix}$ 8: $\begin{bmatrix} 15 & 40 & -35 \\ -2 & 8 & -42 \\ -14 & -36 & 28 \end{bmatrix}$ 9: $\begin{bmatrix} 13 & 65 & 21 \\ 27 & 45 & -80 \\ 53 & -41 & -83 \end{bmatrix}$ 10: $\begin{bmatrix} -59 & 53 & 82 \\ 25 & 19 & -34 \\ 6 & 43 & 17 \end{bmatrix}$

2.3. Matrix Properties

2.3.1. Properties

Solution

Row Operations:

Step 1: $r_1 := r_1 - r_2$ $\begin{bmatrix} 1 & 0 & 0 & | & 1 & -1 & 0 \\ 0 & 1 & 0 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix}$

Results:

a) $\text{rank}(A) = 3$

b) $\text{nullity}(A) = 0$

c) $\det(A) = 0$

d) $A^{-1} = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

e) $\ker(A) = \{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_2 := r_2 - (2)r_1$ $\begin{bmatrix} 1 & 0 & -2 & | & 1 & 0 & 0 \\ 0 & 1 & 2 & | & -2 & 1 & 0 \\ -1 & 2 & 7 & | & 0 & 0 & 1 \end{bmatrix}$

Step 2: $r_3 := r_3 - (-1)r_1$ $\begin{bmatrix} 1 & 0 & -2 & | & 1 & 0 & 0 \\ 0 & 1 & 2 & | & -2 & 1 & 0 \\ 0 & 2 & 5 & | & 1 & 0 & 1 \end{bmatrix}$

Step 3: $r_3 := r_3 - (2)r_2$ $\begin{bmatrix} 1 & 0 & -2 & | & 1 & 0 & 0 \\ 0 & 1 & 2 & | & -2 & 1 & 0 \\ 0 & 0 & 1 & | & 5 & -2 & 1 \end{bmatrix}$

Step 4: $r_1 := r_1 - (-2)r_3$ $\begin{bmatrix} 1 & 0 & 0 & | & 11 & -4 & 2 \\ 0 & 1 & 2 & | & -2 & 1 & 0 \\ 0 & 0 & 1 & | & 5 & -2 & 1 \end{bmatrix}$

Step 5: $r_2 := r_2 - (2)r_3$ $\begin{bmatrix} 1 & 0 & 0 & | & 11 & -4 & 2 \\ 0 & 1 & 0 & | & -12 & 5 & -2 \\ 0 & 0 & 1 & | & 5 & -2 & 1 \end{bmatrix}$

Results:

a) $\text{rank}(A) = 3$

b) $\text{nullity}(A) = 0$

c) $\det(A) = 0$

d) $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 5 & -2 & 1 \end{bmatrix}$

e) $\ker(A) = \{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_1 := -1/3r_1$ $\begin{bmatrix} 1 & 2 & 0 & | & -1/3 & 0 & 0 \\ -1 & -1 & -1 & | & 0 & 1 & 0 \\ 2 & 4 & 0 & | & 0 & 0 & 1 \end{bmatrix}$

Step 2: $r_2 := r_2 - (-1)r_1$ $\begin{bmatrix} 1 & 2 & 0 & | & -1/3 & 0 & 0 \\ 0 & 1 & -1 & | & -1/3 & 1 & 0 \\ 2 & 4 & 0 & | & 0 & 0 & 1 \end{bmatrix}$

Step 3: $r_3 := r_3 - (2)r_1$ $\begin{bmatrix} 1 & 2 & 0 & | & -1/3 & 0 & 0 \\ 0 & 1 & -1 & | & -1/3 & 1 & 0 \\ 0 & 0 & 0 & | & 2/3 & 0 & 1 \end{bmatrix}$

Step 4: $r_1 := r_1 - (2)r_2$ $\begin{bmatrix} 1 & 0 & 2 & | & 1/3 & -2 & 0 \\ 0 & 1 & -1 & | & -1/3 & 1 & 0 \\ 0 & 0 & 0 & | & 2/3 & 0 & 1 \end{bmatrix}$

Results:

a) rank(A) = 2

b) nullity(A) = 1

c) det(A) = 0

d) A^{-1} = does not exist

e) ker(A) = span $\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

Solution

Row Operations:

Step 1: $r_2 := r_2 - (2)r_1$ $\begin{bmatrix} 1 & 0 & -1 & | & 1 & 0 & 0 \\ 0 & 1 & 2 & | & -2 & 1 & 0 \\ 4 & 0 & -3 & | & 0 & 0 & 1 \end{bmatrix}$

Step 2: $r_3 := r_3 - (4)r_1$ $\begin{bmatrix} 1 & 0 & -1 & | & 1 & 0 & 0 \\ 0 & 1 & 2 & | & -2 & 1 & 0 \\ 0 & 0 & 1 & | & -4 & 0 & 1 \end{bmatrix}$

Step 3: $r_1 := r_1 - (-1)r_3$ $\begin{bmatrix} 1 & 0 & 0 & | & -3 & 0 & 1 \\ 0 & 1 & 2 & | & -2 & 1 & 0 \\ 0 & 0 & 1 & | & -4 & 0 & 1 \end{bmatrix}$

Step 4: $r_2 := r_2 - (2)r_3$ $\begin{bmatrix} 1 & 0 & 0 & | & -3 & 0 & 1 \\ 0 & 1 & 0 & | & 6 & 1 & -2 \\ 0 & 0 & 1 & | & -4 & 0 & 1 \end{bmatrix}$

Results:

a) rank(A) = 3

b) nullity(A) = 0

c) det(A) = 0

d) $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$

e) $\ker(A) = \{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_2 := r_2 - (2)r_1$ $\begin{bmatrix} 1 & 1 & 3 & | & 1 & 0 & 0 \\ 0 & 1 & 6 & | & -2 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix}$

Step 2: $r_1 := r_1 - r_2$ $\begin{bmatrix} 1 & 0 & -3 & | & 3 & -1 & 0 \\ 0 & 1 & 6 & | & -2 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix}$

Step 3: $r_1 := r_1 - (-3)r_3$ $\begin{bmatrix} 1 & 0 & 0 & | & 3 & -1 & 3 \\ 0 & 1 & 6 & | & -2 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix}$

Step 4: $r_2 := r_2 - (6)r_3$ $\begin{bmatrix} 1 & 0 & 0 & | & 3 & -1 & 3 \\ 0 & 1 & 0 & | & -2 & 1 & -6 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix}$

Results:

a) $\text{rank}(A) = 3$

b) $\text{nullity}(A) = 0$

c) $\det(A) = 0$

d) $A^{-1} = \begin{bmatrix} 5 & -2 & 10 \\ -2 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$

e) $\ker(A) = \{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_1 := r_1 - r_2$ $\begin{bmatrix} 1 & 0 & -2 & | & 1 & -1 & 0 \\ 0 & 1 & 0 & | & 0 & 1 & 0 \\ 0 & -1 & 1 & | & 0 & 0 & 1 \end{bmatrix}$

Step 2: $r_3 := r_3 - (-1)r_2$ $\begin{bmatrix} 1 & 0 & -2 & | & 1 & -1 & 0 \\ 0 & 1 & 0 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 1 & 1 \end{bmatrix}$

Step 3: $r_1 := r_1 - (-2)r_3$ $\begin{bmatrix} 1 & 0 & 0 & | & 1 & 1 & 2 \\ 0 & 1 & 0 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 1 & 1 \end{bmatrix}$

Results:

a) $\text{rank}(A) = 3$

b) $\text{nullity}(A) = 0$

c) $\det(A) = 0$

d) $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$

e) $\ker(A) = \{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_1 := r_1 - r_2$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

Results:

a) $\operatorname{rank}(A) = 2$

b) $\operatorname{nullity}(A) = 1$

c) $\det(A) = 0$

d) $A^{-1} =$ does not exist

e) $\ker(A) = \operatorname{span}\left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \right\}$

Solution

Row Operations:

Step 1: $r_1 := r_1 - (-2)r_2$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & -6 & 1 & 2 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

Results:

a) $\operatorname{rank}(A) = 2$

b) $\operatorname{nullity}(A) = 1$

c) $\det(A) = 0$

d) $A^{-1} =$ does not exist

e) $\ker(A) = \operatorname{span}\left\{ \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \right\}$

Solution

Row Operations:

Results:

a) $\operatorname{rank}(A) = 2$

b) $\operatorname{nullity}(A) = 1$

c) $\det(A) = 0$

d) $A^{-1} =$ does not exist

e) $\ker(A) = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right\}$

Solution

Row Operations:

Step 1: $r_2 := 1/2r_2$
$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 5/2 & 0 & 1/2 & 0 \\ 0 & 5 & 13 & 0 & 0 & 1 \end{array}\right]$$

Step 2: $r_1 := r_1 - (-1)r_2$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 5/2 & 1 & 1/2 & 0 \\ 0 & 1 & 5/2 & 0 & 1/2 & 0 \\ 0 & 5 & 13 & 0 & 0 & 1 \end{array}\right]$$

Step 3: $r_3 := r_3 - (5)r_2$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 5/2 & 1 & 1/2 & 0 \\ 0 & 1 & 5/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 0 & -5/2 & 1 \end{array}\right]$$

Step 4: $r_3 := 2r_3$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 5/2 & 1 & 1/2 & 0 \\ 0 & 1 & 5/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & -5 & 2 \end{array}\right]$$

Step 5: $r_1 := r_1 - (5/2)r_3$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 13 & -5 \\ 0 & 1 & 5/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & -5 & 2 \end{array}\right]$$

Step 6: $r_2 := r_2 - (5/2)r_3$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 13 & -5 \\ 0 & 1 & 0 & 0 & 13 & -5 \\ 0 & 0 & 1 & 0 & -5 & 2 \end{array}\right]$$

Results:

a) $\text{rank}(A) = 3$

b) $\text{nullity}(A) = 0$

c) $\det(A) = 0$

d)
$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & -4 & 1 \end{bmatrix}$$

e) $\ker(A) = \{\mathbf{0}\}$

2.3.2. RREF

Solution

Elementary Row Operations:

(1) $r_3 := r_3 - r_2$
$$\begin{bmatrix} 1 & 1 & 0 \\ -2 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

(2) $r_2 := r_2 - r_3$
$$\begin{bmatrix} 1 & 1 & 0 \\ -2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(3) $r_2 := r_2 - (2)r_1$
$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_2 := r_2 - r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(2) $r_2 := r_2 - r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_2 := r_2 + (-2)r_3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

(2) $r_3 := r_3 - r_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(3) $r_1 := r_1 - r_3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_2 := r_2 - r_1$

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

(2) $r_1 := r_1 - (2)r_3$

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

(3) $r_2 := r_2 - (2)r_3$

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_2 := r_2 - r_1$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(2) $r_1 := r_1 - r_3$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_1 := r_1 - (2)r_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

(2) $r_3 := r_3 - r_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_3 := r_3 + (-1)r_2$

$$\begin{bmatrix} 2 & 3 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(2) $r_1 := r_1 - r_2$

$$\begin{bmatrix} 1 & 2 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(3) $r_2 := r_2 - r_1$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_3 := r_3 + (-2)r_2$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -2 & -1 & 0 \end{bmatrix}$$

(2) $r_3 := r_3 + (-1)r_2$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -2 & -2 & 0 \end{bmatrix}$$

(3) $r_3 := r_3 - (2)r_1$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_3 := r_3 + (-2)r_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

(2) $r_3 := r_3 + (-2)r_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_1 := r_1 - r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(2) $r_2 := r_2 + (-2)r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(3) $r_2 := r_2 - (2)r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$