

Exercise 4:

Foundations of Mathematical, WS24

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This is **exercise** 4 for Foundations of Mathematical, WS24. Generated on 2024-12-16 with 10 problems per section.

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1. Problems

1.1. Vector Arithmetic

1.1.1. Addition

Find the sum of the following vectors \mathbf{u} and \mathbf{v}

1. $\mathbf{u} = \begin{bmatrix} -2 \\ -2 \\ -3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 5 \\ 2 \\ -7 \end{bmatrix}$
2. $\mathbf{u} = \begin{bmatrix} -2 \\ -2 \\ 10 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 0 \\ -1 \\ -8 \end{bmatrix}$
3. $\mathbf{u} = \begin{bmatrix} 8 \\ -5 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -1 \\ -7 \\ 9 \end{bmatrix}$
4. $\mathbf{u} = \begin{bmatrix} -8 \\ -2 \\ -4 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 2 \\ -10 \\ -8 \end{bmatrix}$
5. $\mathbf{u} = \begin{bmatrix} -8 \\ 4 \\ 8 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -3 \\ -4 \\ 9 \end{bmatrix}$
6. $\mathbf{u} = \begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -1 \\ -6 \\ 0 \end{bmatrix}$
7. $\mathbf{u} = \begin{bmatrix} 10 \\ 1 \\ 5 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 5 \\ 7 \\ -1 \end{bmatrix}$
8. $\mathbf{u} = \begin{bmatrix} 4 \\ -9 \\ 1 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix}$
9. $\mathbf{u} = \begin{bmatrix} 2 \\ -8 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -8 \\ -6 \\ 9 \end{bmatrix}$
10. $\mathbf{u} = \begin{bmatrix} -3 \\ -5 \\ 10 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 10 \\ 10 \\ -1 \end{bmatrix}$

1.1.2. Subtraction

Find the difference of the following vectors \mathbf{u} and \mathbf{v}

1. $\mathbf{u} = \begin{bmatrix} 5 \\ -8 \\ -3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -10 \\ -1 \\ -9 \end{bmatrix}$
2. $\mathbf{u} = \begin{bmatrix} 2 \\ 2 \\ -9 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 5 \\ 2 \\ -3 \end{bmatrix}$
3. $\mathbf{u} = \begin{bmatrix} 0 \\ 4 \\ -6 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 3 \\ 5 \\ 4 \end{bmatrix}$
4. $\mathbf{u} = \begin{bmatrix} -7 \\ 9 \\ -9 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix}$
5. $\mathbf{u} = \begin{bmatrix} 7 \\ -6 \\ -9 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 9 \\ -4 \\ -5 \end{bmatrix}$

6. $\mathbf{u} = \begin{bmatrix} 5 \\ 3 \\ 3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -1 \\ 6 \\ -5 \end{bmatrix}$
7. $\mathbf{u} = \begin{bmatrix} -5 \\ -9 \\ 1 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 5 \\ 4 \\ -4 \end{bmatrix}$
8. $\mathbf{u} = \begin{bmatrix} 4 \\ 9 \\ 9 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -9 \\ -5 \\ -7 \end{bmatrix}$
9. $\mathbf{u} = \begin{bmatrix} 7 \\ 9 \\ 4 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 4 \\ -1 \\ -1 \end{bmatrix}$
10. $\mathbf{u} = \begin{bmatrix} 1 \\ 10 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 10 \\ 3 \\ 8 \end{bmatrix}$

1.1.3. Scalar Multiplication

Find the scalar product of the following vector \mathbf{u} and scalar k

1. $\mathbf{u} = \begin{bmatrix} 0 \\ 0 \\ -6 \end{bmatrix} \quad 9\mathbf{v}.$
2. $\mathbf{u} = \begin{bmatrix} 8 \\ -6 \\ -8 \end{bmatrix} \quad -9\mathbf{v}.$
3. $\mathbf{u} = \begin{bmatrix} 6 \\ 10 \\ -8 \end{bmatrix} \quad -8\mathbf{v}.$
4. $\mathbf{u} = \begin{bmatrix} 6 \\ -6 \\ -8 \end{bmatrix} \quad -2\mathbf{v}.$
5. $\mathbf{u} = \begin{bmatrix} 5 \\ 5 \\ -3 \end{bmatrix} \quad -3\mathbf{v}.$
6. $\mathbf{u} = \begin{bmatrix} 9 \\ -5 \\ -6 \end{bmatrix} \quad 5\mathbf{v}.$
7. $\mathbf{u} = \begin{bmatrix} 8 \\ -6 \\ 9 \end{bmatrix} \quad 4\mathbf{v}.$
8. $\mathbf{u} = \begin{bmatrix} 10 \\ -6 \\ 5 \end{bmatrix} \quad -7\mathbf{v}.$
9. $\mathbf{u} = \begin{bmatrix} -3 \\ 6 \\ 6 \end{bmatrix} \quad -6\mathbf{v}.$
10. $\mathbf{u} = \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix} \quad 6\mathbf{v}.$

1.2. Matrix Arithmetic

1.2.1. Addition

Find the sum of the following matrices A and B

1. $A = \begin{bmatrix} 1 & 7 & 8 \\ -2 & -2 & 3 \\ -4 & 7 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & -4 & -6 \\ -7 & 2 & 7 \\ -4 & 6 & -8 \end{bmatrix}$

2. $A = \begin{bmatrix} -8 & -4 & 6 \\ -2 & -2 & -7 \\ 1 & -5 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} -7 & 2 & -8 \\ -9 & -3 & -1 \\ -1 & 2 & -8 \end{bmatrix}$
3. $A = \begin{bmatrix} 9 & 8 & -7 \\ 0 & -9 & -10 \\ -4 & -1 & -9 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & -5 & 7 \\ 8 & 0 & -6 \\ -10 & 3 & -8 \end{bmatrix}$
4. $A = \begin{bmatrix} 9 & -6 & -1 \\ 4 & -5 & -4 \\ 2 & -9 & -8 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 & -9 \\ 7 & -5 & 5 \\ 7 & 6 & 7 \end{bmatrix}$
5. $A = \begin{bmatrix} 2 & 6 & -10 \\ 0 & 8 & -6 \\ -4 & 8 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -2 & -9 \\ 1 & 6 & 4 \\ -8 & 9 & 1 \end{bmatrix}$
6. $A = \begin{bmatrix} 0 & -1 & 6 \\ -3 & -6 & 2 \\ -5 & 9 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & -4 \\ -2 & -4 & -2 \\ -2 & -1 & -6 \end{bmatrix}$
7. $A = \begin{bmatrix} 6 & 6 & 2 \\ -4 & 3 & 8 \\ 0 & 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -3 & 3 \\ 2 & 1 & -3 \\ -5 & -10 & -2 \end{bmatrix}$
8. $A = \begin{bmatrix} -7 & 9 & 9 \\ 4 & -2 & 4 \\ -10 & -9 & -8 \end{bmatrix}$ and $B = \begin{bmatrix} -6 & -6 & -7 \\ 7 & -7 & 4 \\ -4 & -10 & -7 \end{bmatrix}$
9. $A = \begin{bmatrix} -2 & 2 & 5 \\ -7 & -5 & -2 \\ 5 & 2 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} -7 & -3 & -9 \\ -10 & -8 & -4 \\ 3 & -2 & 5 \end{bmatrix}$
10. $A = \begin{bmatrix} 8 & -1 & 1 \\ 6 & -8 & 9 \\ -1 & 4 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 6 & -4 \\ -2 & 3 & 7 \\ 2 & -4 & -3 \end{bmatrix}$

1.2.2. Subtraction

Find the difference of the following matrices A and B

1. $A = \begin{bmatrix} 3 & -4 & 5 \\ 2 & -5 & -10 \\ -9 & -6 & -9 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 2 & -9 \\ -4 & 8 & -8 \\ 9 & -3 & -9 \end{bmatrix}$
2. $A = \begin{bmatrix} -1 & 7 & -3 \\ 9 & -7 & -1 \\ -9 & -10 & -9 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -8 & 0 \\ 8 & 7 & -6 \\ -1 & 9 & 6 \end{bmatrix}$
3. $A = \begin{bmatrix} 0 & -6 & 6 \\ 9 & -6 & -3 \\ -2 & -2 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 8 & -10 \\ 9 & 2 & 1 \\ -4 & 9 & -4 \end{bmatrix}$
4. $A = \begin{bmatrix} 3 & 8 & 5 \\ 3 & -9 & -8 \\ 0 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & -9 & -9 \\ 6 & 1 & 7 \\ 1 & -6 & -9 \end{bmatrix}$
5. $A = \begin{bmatrix} -6 & 6 & -4 \\ -6 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -7 & -3 \\ -2 & 8 & -10 \\ -4 & 1 & 7 \end{bmatrix}$
6. $A = \begin{bmatrix} -8 & -4 & -4 \\ -10 & 8 & 3 \\ 9 & -1 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 8 & 2 \\ -2 & -1 & -4 \\ -3 & -8 & -2 \end{bmatrix}$
7. $A = \begin{bmatrix} 4 & -6 & -9 \\ -8 & 8 & 7 \\ 6 & 7 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 8 & 6 \\ -1 & 2 & 6 \\ 5 & -10 & 0 \end{bmatrix}$
8. $A = \begin{bmatrix} -5 & 5 & -1 \\ 8 & -8 & -8 \\ 3 & -5 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -8 & 6 \\ 5 & -7 & 5 \\ -6 & 1 & -10 \end{bmatrix}$

$$9. A = \begin{bmatrix} -7 & 4 & 5 \\ -10 & 1 & 9 \\ 9 & 1 & 7 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 7 & 6 \\ 5 & 2 & -8 \\ 7 & 1 & 5 \end{bmatrix}$$

$$10. A = \begin{bmatrix} -8 & -2 & -2 \\ 1 & 3 & -8 \\ 8 & 9 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 8 & -6 \\ 1 & -4 & -10 \\ 9 & 1 & -2 \end{bmatrix}$$

1.2.3. Multiplication

Find the product of the following matrices A and B

$$1. A = \begin{bmatrix} 7 & 7 & -6 \\ -10 & -6 & 4 \\ 7 & -6 & -8 \end{bmatrix} \text{ and } B = \begin{bmatrix} -10 & 5 & -6 \\ 9 & -9 & -8 \\ 5 & 5 & -1 \end{bmatrix}$$

$$2. A = \begin{bmatrix} -2 & -3 & 7 \\ 0 & 1 & 9 \\ -9 & 6 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 1 & -3 \\ 6 & -5 & -7 \\ -10 & 8 & -8 \end{bmatrix}$$

$$3. A = \begin{bmatrix} 1 & 9 & 0 \\ 2 & 5 & -9 \\ -8 & 2 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & -7 & -9 \\ -8 & -8 & -2 \\ 3 & 1 & -10 \end{bmatrix}$$

$$4. A = \begin{bmatrix} 7 & 8 & 2 \\ 3 & 0 & -6 \\ -3 & -4 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 & -7 & 4 \\ 3 & -8 & 7 \\ -3 & 9 & 0 \end{bmatrix}$$

$$5. A = \begin{bmatrix} -6 & -6 & -5 \\ -9 & -7 & -8 \\ -2 & 7 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 & 4 & -2 \\ 5 & -3 & 9 \\ -9 & 3 & -2 \end{bmatrix}$$

$$6. A = \begin{bmatrix} -1 & -6 & 8 \\ -1 & -10 & -10 \\ -2 & 1 & -9 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 & -4 & 9 \\ -8 & -1 & -5 \\ 2 & -6 & -6 \end{bmatrix}$$

$$7. A = \begin{bmatrix} -2 & 1 & -7 \\ -3 & -2 & -10 \\ -8 & -1 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & -2 & 4 \\ -1 & -8 & -6 \\ 5 & -8 & -5 \end{bmatrix}$$

$$8. A = \begin{bmatrix} 7 & 4 & 6 \\ 6 & 8 & -3 \\ 9 & -7 & -2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & -10 & 6 \\ -6 & -2 & -2 \\ 5 & -9 & 9 \end{bmatrix}$$

$$9. A = \begin{bmatrix} -1 & 9 & 1 \\ -9 & 6 & -10 \\ -1 & -8 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} -7 & 3 & 1 \\ 3 & 2 & 9 \\ -6 & 0 & 7 \end{bmatrix}$$

$$10. A = \begin{bmatrix} 9 & 7 & -1 \\ -6 & -2 & -2 \\ 7 & 1 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -5 & 2 & 7 \\ -8 & -9 & -4 \\ -4 & -1 & -2 \end{bmatrix}$$

1.3. Matrix Properties

1.3.1. Properties

For each matrix A , find:

- $\text{rank}(A)$
- $\text{nullity}(A)$
- $\det(A)$
- A^{-1} (if exists)
- basis of $\ker(A)$

$$1. A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix}$$

2. $A = \begin{bmatrix} -1 & -1 & 8 \\ 0 & 1 & -2 \\ -1 & 0 & 5 \end{bmatrix}$
3. $A = \begin{bmatrix} 1 & 3 & -1 \\ -1 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
4. $A = \begin{bmatrix} -5 & 11 & -10 \\ -6 & 13 & -11 \\ 2 & -4 & 3 \end{bmatrix}$
5. $A = \begin{bmatrix} 1 & 0 & -1 \\ -2 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$
6. $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
7. $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 5 & -4 \\ 3 & 6 & -3 \end{bmatrix}$
8. $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 3 & -4 \\ -1 & -2 & 3 \end{bmatrix}$
9. $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -2 \\ -1 & -2 & 3 \end{bmatrix}$
10. $A = \begin{bmatrix} 3 & 6 & 6 \\ 0 & 1 & -1 \\ 1 & 4 & 0 \end{bmatrix}$

1.3.2. RREF

Find the Reduced Row Echelon Form of the following matrix A

1. $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$
2. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
3. $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & -2 \\ 0 & 1 & -1 \end{bmatrix}$
4. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & -2 & 5 \end{bmatrix}$
5. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
6. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 0 \end{bmatrix}$
7. $A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$
8. $A = \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 0 \end{bmatrix}$

$$9. \quad A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$10. \quad A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

2. Solutions

2.1. Vector Arithmetic

2.1.1. Addition

$$\begin{bmatrix} 3 \\ 0 \\ -10 \end{bmatrix} + \begin{bmatrix} -2 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ -12 \\ 11 \end{bmatrix} + \begin{bmatrix} -6 \\ -12 \\ -12 \end{bmatrix} = \begin{bmatrix} -11 \\ 0 \\ 17 \end{bmatrix}$$
$$\begin{bmatrix} -2 \\ -4 \\ -2 \end{bmatrix} + \begin{bmatrix} 15 \\ 8 \\ 4 \end{bmatrix} = \begin{bmatrix} 7 \\ -13 \\ 2 \end{bmatrix} + \begin{bmatrix} -6 \\ -14 \\ 11 \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \\ 9 \end{bmatrix}$$

2.1.2. Subtraction

$$\begin{bmatrix} 15 \\ -7 \\ 6 \end{bmatrix} - \begin{bmatrix} -3 \\ 0 \\ -6 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ -10 \end{bmatrix} - \begin{bmatrix} -9 \\ 6 \\ -4 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ -4 \end{bmatrix}$$
$$\begin{bmatrix} 6 \\ -3 \\ 8 \end{bmatrix} - \begin{bmatrix} -10 \\ -13 \\ 5 \end{bmatrix} = \begin{bmatrix} 13 \\ 14 \\ 16 \end{bmatrix} - \begin{bmatrix} 3 \\ 10 \\ 5 \end{bmatrix} = \begin{bmatrix} -9 \\ 7 \\ -6 \end{bmatrix}$$

2.1.3. Scalar Multiplication

$$1: \begin{bmatrix} 0 \\ 0 \\ -54 \end{bmatrix} \quad 2: \begin{bmatrix} -72 \\ 54 \\ 72 \end{bmatrix} \quad 3: \begin{bmatrix} -48 \\ -80 \\ 64 \end{bmatrix} \quad 4: \begin{bmatrix} -12 \\ 12 \\ 16 \end{bmatrix} \quad 5: \begin{bmatrix} -15 \\ -15 \\ 9 \end{bmatrix}$$
$$6: \begin{bmatrix} 45 \\ -25 \\ -30 \end{bmatrix} \quad 7: \begin{bmatrix} 32 \\ -24 \\ 36 \end{bmatrix} \quad 8: \begin{bmatrix} -70 \\ 42 \\ -35 \end{bmatrix} \quad 9: \begin{bmatrix} 18 \\ -36 \\ -36 \end{bmatrix} \quad 10: \begin{bmatrix} -6 \\ 18 \\ -12 \end{bmatrix}$$

2.2. Matrix Arithmetic

2.2.1. Addition

$$1: \begin{bmatrix} -2 & 3 & 2 \\ -9 & 0 & 10 \\ -8 & 13 & -13 \end{bmatrix} \quad 2: \begin{bmatrix} -15 & -2 & -2 \\ -11 & -5 & -8 \\ 0 & -3 & -13 \end{bmatrix} \quad 3: \begin{bmatrix} 16 & 3 & 0 \\ 8 & -9 & -16 \\ -14 & 2 & -17 \end{bmatrix} \quad 4: \begin{bmatrix} 7 & -3 & -10 \\ 11 & -10 & 1 \\ 9 & -3 & -1 \end{bmatrix} \quad 5: \begin{bmatrix} 2 & 4 & -19 \\ 1 & 14 & -2 \\ -12 & 17 & 2 \end{bmatrix}$$
$$6: \begin{bmatrix} 2 & -2 & 2 \\ -5 & -10 & 0 \\ -7 & 8 & -9 \end{bmatrix} \quad 7: \begin{bmatrix} 9 & 3 & 5 \\ -2 & 4 & 5 \\ -5 & -8 & 0 \end{bmatrix} \quad 8: \begin{bmatrix} -13 & 3 & 2 \\ 11 & -9 & 8 \\ -14 & -19 & -15 \end{bmatrix} \quad 9: \begin{bmatrix} -9 & -1 & -4 \\ -17 & -13 & -6 \\ 8 & 0 & -1 \end{bmatrix} \quad 10: \begin{bmatrix} 12 & 5 & -3 \\ 4 & -5 & 16 \\ 1 & 0 & 6 \end{bmatrix}$$

2.2.2. Subtraction

$$1: \begin{bmatrix} -3 & -6 & 14 \\ 6 & -13 & -2 \\ -18 & -3 & 0 \end{bmatrix} \quad 2: \begin{bmatrix} -5 & 15 & -3 \\ 1 & -14 & 5 \\ -8 & -19 & -15 \end{bmatrix} \quad 3: \begin{bmatrix} 3 & -14 & 16 \\ 0 & -8 & -4 \\ 2 & -11 & 13 \end{bmatrix} \quad 4: \begin{bmatrix} -5 & 17 & 14 \\ -3 & -10 & -15 \\ -1 & 7 & 14 \end{bmatrix} \quad 5: \begin{bmatrix} -5 & 13 & -1 \\ -4 & -6 & 11 \\ 6 & -1 & -4 \end{bmatrix}$$
$$6: \begin{bmatrix} -13 & -12 & -6 \\ -8 & 9 & 7 \\ 12 & 7 & -1 \end{bmatrix} \quad 7: \begin{bmatrix} 4 & -14 & -15 \\ -7 & 6 & 1 \\ 1 & 17 & 6 \end{bmatrix} \quad 8: \begin{bmatrix} -1 & 13 & -7 \\ 3 & -1 & -13 \\ 9 & -6 & 4 \end{bmatrix} \quad 9: \begin{bmatrix} -6 & -3 & -1 \\ -15 & -1 & 17 \\ 2 & 0 & 2 \end{bmatrix} \quad 10: \begin{bmatrix} -9 & -10 & 4 \\ 0 & 7 & 2 \\ -1 & 8 & 2 \end{bmatrix}$$

2.2.3. Multiplication

$$1: \begin{bmatrix} -37 & -58 & -92 \\ 66 & 24 & 104 \\ -164 & 49 & 14 \end{bmatrix} \quad 2: \begin{bmatrix} -82 & 69 & -29 \\ -84 & 67 & -79 \\ 23 & -7 & -47 \end{bmatrix} \quad 3: \begin{bmatrix} -72 & -79 & -27 \\ -67 & -63 & 62 \\ -10 & 42 & 48 \end{bmatrix} \quad 4: \begin{bmatrix} 67 & -95 & 84 \\ 39 & -75 & 12 \\ -30 & 44 & -40 \end{bmatrix} \quad 5: \begin{bmatrix} -9 & -21 & -32 \\ 1 & -39 & -29 \\ 72 & -44 & 77 \end{bmatrix}$$
$$6: \begin{bmatrix} 59 & -38 & -27 \\ 55 & 74 & 101 \\ -36 & 61 & 31 \end{bmatrix} \quad 7: \begin{bmatrix} -42 & 52 & 21 \\ -57 & 102 & 50 \\ -48 & 64 & -1 \end{bmatrix} \quad 8: \begin{bmatrix} 6 & -132 & 88 \\ -63 & -49 & -7 \\ 32 & -58 & 50 \end{bmatrix} \quad 9: \begin{bmatrix} 28 & 15 & 87 \\ 141 & -15 & -25 \\ -47 & -19 & -38 \end{bmatrix} \quad 10: \begin{bmatrix} -97 & -44 & 37 \\ 54 & 8 & -30 \\ -39 & 6 & 47 \end{bmatrix}$$

2.3. Matrix Properties

2.3.1. Properties

Solution

Row Operations:

$$\text{Step 1: } r_3 := r_3 - r_1 \begin{bmatrix} 1 & 2 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & -1 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & -1 & 0 & 1 \end{bmatrix}$$

$$\text{Step 2: } r_1 := r_1 - (2)r_2 \begin{bmatrix} 1 & 0 & 2 & | & 1 & -2 & 0 \\ 0 & 1 & -1 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & -1 & 0 & 1 \end{bmatrix}$$

$$\text{Step 3: } r_1 := r_1 - (2)r_3 \begin{bmatrix} 1 & 0 & 0 & | & 3 & -2 & -2 \\ 0 & 1 & -1 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & -1 & 0 & 1 \end{bmatrix}$$

$$\text{Step 4: } r_2 := r_2 - (-1)r_3 \begin{bmatrix} 1 & 0 & 0 & | & 3 & -2 & -2 \\ 0 & 1 & 0 & | & -1 & 1 & 1 \\ 0 & 0 & 1 & | & -1 & 0 & 1 \end{bmatrix}$$

Results:

a) $\text{rank}(A) = 3$

b) $\text{nullity}(A) = 0$

c) $\det(A) = 0$

d) $A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

e) $\ker(A) = \{\mathbf{0}\}$

Solution

Row Operations:

$$\text{Step 1: } r_1 := -1r_1 \begin{bmatrix} 1 & 1 & -8 & | & -1 & 0 & 0 \\ 0 & 1 & -2 & | & 0 & 1 & 0 \\ -1 & 0 & 5 & | & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Step 2: } r_3 := r_3 - (-1)r_1 \begin{bmatrix} 1 & 1 & -8 & | & -1 & 0 & 0 \\ 0 & 1 & -2 & | & 0 & 1 & 0 \\ 0 & 1 & -3 & | & -1 & 0 & 1 \end{bmatrix}$$

$$\text{Step 3: } r_1 := r_1 - r_2 \begin{bmatrix} 1 & 0 & -6 & | & -1 & -1 & 0 \\ 0 & 1 & -2 & | & 0 & 1 & 0 \\ 0 & 1 & -3 & | & -1 & 0 & 1 \end{bmatrix}$$

$$\text{Step 4: } r_3 := r_3 - r_2 \begin{bmatrix} 1 & 0 & -6 & | & -1 & -1 & 0 \\ 0 & 1 & -2 & | & 0 & 1 & 0 \\ 0 & 0 & -1 & | & -1 & -1 & 1 \end{bmatrix}$$

$$\text{Step 5: } r_3 := -1r_3 \begin{bmatrix} 1 & 0 & -6 & | & -1 & -1 & 0 \\ 0 & 1 & -2 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & 1 & 1 & -1 \end{bmatrix}$$

$$\text{Step 6: } r_1 := r_1 - (-6)r_3 \begin{bmatrix} 1 & 0 & 0 & | & 5 & 5 & -6 \\ 0 & 1 & -2 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & 1 & 1 & -1 \end{bmatrix}$$

$$\text{Step 7: } r_2 := r_2 - (-2)r_3 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & 5 & -6 \\ 0 & 1 & 0 & 2 & 3 & -2 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{array} \right]$$

Results:

a) $\text{rank}(A) = 3$

b) $\text{nullity}(A) = 0$

c) $\det(A) = 0$

d) $A^{-1} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix}$

e) $\ker(A) = \{\mathbf{0}\}$

Solution

Row Operations:

$$\text{Step 1: } r_2 := r_2 - (-1)r_1 \quad \left[\begin{array}{ccc|ccc} 1 & 3 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right]$$

$$\text{Step 2: } r_1 := r_1 - (3)r_2 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & -2 & -3 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right]$$

Results:

a) $\text{rank}(A) = 2$

b) $\text{nullity}(A) = 1$

c) $\det(A) = 0$

d) $A^{-1} = \text{does not exist}$

e) $\ker(A) = \text{span} \left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

Solution

Row Operations:

$$\text{Step 1: } r_1 := -1/5r_1 \quad \left[\begin{array}{ccc|ccc} 1 & -11/5 & 2 & -1/5 & 0 & 0 \\ -6 & 13 & -11 & 0 & 1 & 0 \\ 2 & -4 & 3 & 0 & 0 & 1 \end{array} \right]$$

$$\text{Step 2: } r_2 := r_2 - (-6)r_1 \quad \left[\begin{array}{ccc|ccc} 1 & -11/5 & 2 & -1/5 & 0 & 0 \\ 0 & -1/5 & 1 & -6/5 & 1 & 0 \\ 2 & -4 & 3 & 0 & 0 & 1 \end{array} \right]$$

$$\text{Step 3: } r_3 := r_3 - (2)r_1 \quad \left[\begin{array}{ccc|ccc} 1 & -11/5 & 2 & -1/5 & 0 & 0 \\ 0 & -1/5 & 1 & -6/5 & 1 & 0 \\ 0 & 2/5 & -1 & 2/5 & 0 & 1 \end{array} \right]$$

$$\text{Step 4: } r_2 := -5r_2 \quad \left[\begin{array}{ccc|ccc} 1 & -11/5 & 2 & -1/5 & 0 & 0 \\ 0 & 1 & -5 & 6 & -5 & 0 \\ 0 & 2/5 & -1 & 2/5 & 0 & 1 \end{array} \right]$$

$$\text{Step 5: } r_1 := r_1 - (-11/5)r_2 \begin{bmatrix} 1 & 0 & -9 & | & 13 & -11 & 0 \\ 0 & 1 & -5 & | & 6 & -5 & 0 \\ 0 & 2/5 & -1 & | & 2/5 & 0 & 1 \end{bmatrix}$$

$$\text{Step 6: } r_3 := r_3 - (2/5)r_2 \begin{bmatrix} 1 & 0 & -9 & | & 13 & -11 & 0 \\ 0 & 1 & -5 & | & 6 & -5 & 0 \\ 0 & 0 & 1 & | & -2 & 2 & 1 \end{bmatrix}$$

$$\text{Step 7: } r_1 := r_1 - (-9)r_3 \begin{bmatrix} 1 & 0 & 0 & | & -5 & 7 & 9 \\ 0 & 1 & -5 & | & 6 & -5 & 0 \\ 0 & 0 & 1 & | & -2 & 2 & 1 \end{bmatrix}$$

$$\text{Step 8: } r_2 := r_2 - (-5)r_3 \begin{bmatrix} 1 & 0 & 0 & | & -5 & 7 & 9 \\ 0 & 1 & 0 & | & -4 & 5 & 5 \\ 0 & 0 & 1 & | & -2 & 2 & 1 \end{bmatrix}$$

Results:

a) $\text{rank}(A) = 3$

b) $\text{nullity}(A) = 0$

c) $\det(A) = 0$

d) $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 3 \\ -2 & 2 & 1 \end{bmatrix}$

e) $\ker(A) = \{\mathbf{0}\}$

Solution

Row Operations:

$$\text{Step 1: } r_2 := r_2 - (-2)r_1 \begin{bmatrix} 1 & 0 & -1 & | & 1 & 0 & 0 \\ 0 & 1 & 1 & | & 2 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Step 2: } r_1 := r_1 - (-1)r_3 \begin{bmatrix} 1 & 0 & 0 & | & 1 & 0 & 1 \\ 0 & 1 & 1 & | & 2 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Step 3: } r_2 := r_2 - r_3 \begin{bmatrix} 1 & 0 & 0 & | & 1 & 0 & 1 \\ 0 & 1 & 0 & | & 2 & 1 & -1 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix}$$

Results:

a) $\text{rank}(A) = 3$

b) $\text{nullity}(A) = 0$

c) $\det(A) = 0$

d) $A^{-1} = \begin{bmatrix} 5 & 2 & -2 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

e) $\ker(A) = \{\mathbf{0}\}$

Solution

Row Operations:

$$\text{Step 1: } r_1 := r_1 - (-1)r_2 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$\text{Step 2: } r_1 := r_1 - r_3 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

Results:

a) $\text{rank}(A) = 3$

b) $\text{nullity}(A) = 0$

c) $\det(A) = 0$

d) $A^{-1} = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

e) $\ker(A) = \{\mathbf{0}\}$

Solution

Row Operations:

$$\text{Step 1: } r_2 := r_2 - (2)r_1 \quad \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 3 & 6 & -3 & 0 & 0 & 1 \end{array} \right]$$

$$\text{Step 2: } r_3 := r_3 - (3)r_1 \quad \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 1 \end{array} \right]$$

$$\text{Step 3: } r_1 := r_1 - (2)r_2 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 5 & -2 & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 1 \end{array} \right]$$

Results:

a) $\text{rank}(A) = 2$

b) $\text{nullity}(A) = 1$

c) $\det(A) = 0$

d) $A^{-1} = \text{does not exist}$

e) $\ker(A) = \text{span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\}$

Solution

Row Operations:

$$\text{Step 1: } r_2 := r_2 - r_1 \quad \left[\begin{array}{ccc|ccc} 1 & 2 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ -1 & -2 & 3 & 0 & 0 & 1 \end{array} \right]$$

$$\text{Step 2: } r_3 := r_3 - (-1)r_1 \quad \left[\begin{array}{ccc|ccc} 1 & 2 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array} \right]$$

$$\begin{aligned} \text{Step 3: } r_1 &:= r_1 - (2)r_2 \quad \begin{bmatrix} 1 & 0 & 2 & | & 3 & -2 & 0 \\ 0 & 1 & -2 & | & -1 & 1 & 0 \\ 0 & 0 & 1 & | & 1 & 0 & 1 \end{bmatrix} \\ \text{Step 4: } r_1 &:= r_1 - (2)r_3 \quad \begin{bmatrix} 1 & 0 & 0 & | & 1 & -2 & -2 \\ 0 & 1 & -2 & | & -1 & 1 & 0 \\ 0 & 0 & 1 & | & 1 & 0 & 1 \end{bmatrix} \\ \text{Step 5: } r_2 &:= r_2 - (-2)r_3 \quad \begin{bmatrix} 1 & 0 & 0 & | & 1 & -2 & -2 \\ 0 & 1 & 0 & | & 1 & 1 & 2 \\ 0 & 0 & 1 & | & 1 & 0 & 1 \end{bmatrix} \end{aligned}$$

Results:

- a) $\text{rank}(A) = 3$
- b) $\text{nullity}(A) = 0$
- c) $\det(A) = 0$
- d) $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$
- e) $\ker(A) = \{\mathbf{0}\}$

Solution

Row Operations:

$$\begin{aligned} \text{Step 1: } r_3 &:= r_3 - (-1)r_1 \quad \begin{bmatrix} 1 & 0 & 2 & | & 1 & 0 & 0 \\ 0 & 1 & -2 & | & 0 & 1 & 0 \\ 0 & -2 & 5 & | & 1 & 0 & 1 \end{bmatrix} \\ \text{Step 2: } r_3 &:= r_3 - (-2)r_2 \quad \begin{bmatrix} 1 & 0 & 2 & | & 1 & 0 & 0 \\ 0 & 1 & -2 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & 1 & 2 & 1 \end{bmatrix} \\ \text{Step 3: } r_1 &:= r_1 - (2)r_3 \quad \begin{bmatrix} 1 & 0 & 0 & | & -1 & -4 & -2 \\ 0 & 1 & -2 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & 1 & 2 & 1 \end{bmatrix} \\ \text{Step 4: } r_2 &:= r_2 - (-2)r_3 \quad \begin{bmatrix} 1 & 0 & 0 & | & -1 & -4 & -2 \\ 0 & 1 & 0 & | & 2 & 5 & 2 \\ 0 & 0 & 1 & | & 1 & 2 & 1 \end{bmatrix} \end{aligned}$$

Results:

- a) $\text{rank}(A) = 3$
- b) $\text{nullity}(A) = 0$
- c) $\det(A) = 0$
- d) $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$
- e) $\ker(A) = \{\mathbf{0}\}$

Solution

Row Operations:

$$\text{Step 1: } r_1 := 1/3r_1 \quad \left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1/3 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 1 & 4 & 0 & 0 & 0 & 1 \end{array} \right]$$

$$\text{Step 2: } r_3 := r_3 - r_1 \quad \left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1/3 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 2 & -2 & -1/3 & 0 & 1 \end{array} \right]$$

$$\text{Step 3: } r_1 := r_1 - (2)r_2 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 1/3 & -2 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 2 & -2 & -1/3 & 0 & 1 \end{array} \right]$$

$$\text{Step 4: } r_3 := r_3 - (2)r_2 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 1/3 & -2 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1/3 & -2 & 1 \end{array} \right]$$

Results:

a) $\text{rank}(A) = 2$

b) $\text{nullity}(A) = 1$

c) $\det(A) = 0$

d) A^{-1} does not exist

e) $\ker(A) = \text{span} \left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \right\}$

2.3.2. RREF

Solution

Elementary Row Operations:

(1) $r_1 := r_1 + (-1)r_2$

$$\left[\begin{array}{ccc} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{array} \right]$$

(2) $r_2 := r_2 + (-1)r_1$

$$\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{array} \right]$$

(3) $r_3 := r_3 + (-1)r_1$

$$\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

Result:

$$\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

Solution

Elementary Row Operations:

(1) $r_3 := r_3 + (-1)r_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(2) \ r_1 := r_1 + (-2)r_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution

Elementary Row Operations:

$$(1) \ r_2 := r_2 - r_1$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 1 & -1 \end{bmatrix}$$

$$(2) \ r_3 := r_3 + (-1)r_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(3) \ r_2 := r_2 - r_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

Elementary Row Operations:

$$(1) \ r_3 := r_3 - (2)r_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(2) \ r_2 := r_2 - (2)r_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

Elementary Row Operations:

$$(1) \ r_3 := r_3 - r_1$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

$$(2) \ r_3 := r_3 + (-2)r_1$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution**Elementary Row Operations:**

$$(1) \ r_3 := r_3 - (2)r_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(2) \ r_2 := r_2 - r_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution**Elementary Row Operations:**

$$(1) \ r_2 := r_2 - (2)r_3$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(2) \ r_2 := r_2 - r_3$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

Elementary Row Operations:

$$(1) \ r_1 := r_1 + (-1)r_2$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 0 \end{bmatrix}$$

$$(2) \ r_3 := r_3 - (2)r_2$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(3) \ r_2 := r_2 - r_3$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution**Elementary Row Operations:**

$$(1) \ r_2 := r_2 + (-1)r_1$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(2) \ r_1 := r_1 + (-2)r_3$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution**Elementary Row Operations:**

$$(1) \ r_3 := r_3 + (-2)r_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 0 \end{bmatrix}$$

$$(2) \ r_3 := r_3 - (2)r_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$