Exercise 8:

Foundations of Mathematical, WS24

Zichao Wei

This is **exercise** 8 for Foundations of Mathematical, WS24. Generated on 2025-01-13 with 10 problems per section.

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1. Problems

1.1. Vector Arithmetic

1.1.1. Addition

Find the sum of the following vectors $\mathbf{u}$ and $\mathbf{v}$

1. $\mathbf{u} = \begin{bmatrix} -9 \\ -4 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 4 \\ -9 \\ -9 \end{bmatrix}$

2. $\mathbf{u} = \begin{bmatrix} 9 \\ 7 \\ 0 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -1 \\ 0 \\ 9 \end{bmatrix}$

3. $\mathbf{u} = \begin{bmatrix} -7 \\ 1 \\ -10 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -3 \\ -1 \\ -7 \end{bmatrix}$

4. $\mathbf{u} = \begin{bmatrix} 8 \\ -5 \\ 4 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -10 \\ 7 \\ -3 \end{bmatrix}$

5. $\mathbf{u} = \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 8 \\ 1 \\ -5 \end{bmatrix}$

6. $\mathbf{u} = \begin{bmatrix} -2 \\ 4 \\ 4 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 0 \\ -7 \\ 0 \end{bmatrix}$

7. $\mathbf{u} = \begin{bmatrix} 7 \\ -1 \\ 5 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -3 \\ 8 \\ -8 \end{bmatrix}$

8. $\mathbf{u} = \begin{bmatrix} -4 \\ 6 \\ -7 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -2 \\ -5 \\ 3 \end{bmatrix}$

9. $\mathbf{u} = \begin{bmatrix} 6 \\ 9 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 4 \\ 6 \\ 1 \end{bmatrix}$

10. $\mathbf{u} = \begin{bmatrix} 0 \\ -3 \\ 8 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 10 \\ -4 \\ -4 \end{bmatrix}$

1.1.2. Subtraction

Find the difference of the following vectors $\mathbf{u}$ and $\mathbf{v}$

1. $\mathbf{u} = \begin{bmatrix} 2 \\ -9 \\ -4 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}$

2. $\mathbf{u} = \begin{bmatrix} -7 \\ -2 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

3. $\mathbf{u} = \begin{bmatrix} 3 \\ 7 \\ 9 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -10 \\ -4 \\ 2 \end{bmatrix}$

4. $\mathbf{u} = \begin{bmatrix} 2 \\ 8 \\ 10 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix}$

5. $\mathbf{u} = \begin{bmatrix} 0 \\ -9 \\ -1 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -4 \\ 4 \\ -7 \end{bmatrix}$

6. $\mathbf{u} = \begin{bmatrix} 10 \\ 6 \\ -10 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -4 \\ -4 \\ -10 \end{bmatrix}$

7. $\mathbf{u} = \begin{bmatrix} -6 \\ 8 \\ -4 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -3 \\ -10 \\ 0 \end{bmatrix}$

8. $\mathbf{u} = \begin{bmatrix} -4 \\ 6 \\ -6 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -6 \\ -4 \\ 3 \end{bmatrix}$

9. $\mathbf{u} = \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -5 \\ -6 \\ -5 \end{bmatrix}$

10. $\mathbf{u} = \begin{bmatrix} -6 \\ 0 \\ -4 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 6 \\ 3 \\ -3 \end{bmatrix}$

1.1.3. Scalar Multiplication

Find the scalar product of the following vector $\mathbf{u}$ and scalar k

1. $\mathbf{u} = \begin{bmatrix} -3 \\ 2 \\ -3 \end{bmatrix} -9\mathbf{v}.$

2. $\mathbf{u} = \begin{bmatrix} 8 \\ -4 \\ 2 \end{bmatrix} -10\mathbf{v}.$

3. $\mathbf{u} = \begin{bmatrix} 8 \\ -6 \\ -8 \end{bmatrix} -8\mathbf{v}.$

4. $\mathbf{u} = \begin{bmatrix} -9 \\ -2 \\ -5 \end{bmatrix} 10\mathbf{v}.$

5. $\mathbf{u} = \begin{bmatrix} -9 \\ -1 \\ 7 \end{bmatrix} -10\mathbf{v}.$

6. $\mathbf{u} = \begin{bmatrix} 2 \\ 5 \\ 5 \end{bmatrix} 8\mathbf{v}.$

7. $\mathbf{u} = \begin{bmatrix} -6 \\ -5 \\ 2 \end{bmatrix} 9\mathbf{v}.$

8. $\mathbf{u} = \begin{bmatrix} -9 \\ 10 \\ -10 \end{bmatrix} -3\mathbf{v}.$

9. $\mathbf{u} = \begin{bmatrix} 0 \\ -5 \\ 4 \end{bmatrix} 8\mathbf{v}.$

10. $\mathbf{u} = \begin{bmatrix} 4 \\ 6 \\ -6 \end{bmatrix} -2\mathbf{v}.$

1.2. Matrix Arithmetic

1.2.1. Addition

Find the sum of the following matrices A and B

1. $A = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 9 & 1 \\ -10 & -3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -6 & -2 \\ 9 & -4 & 7 \\ -7 & 3 & -7 \end{bmatrix}$

2. $A = \begin{bmatrix} -7 & 3 & -9 \\ 0 & -3 & 7 \\ -6 & 6 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & -6 & -8 \\ -4 & -7 & -3 \\ 6 & 9 & -2 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & -8 & -1 \\ -6 & 9 & 8 \\ -8 & 6 & -9 \end{bmatrix}$ and $B = \begin{bmatrix} -8 & 8 & -4 \\ 4 & -2 & -6 \\ 6 & 4 & -2 \end{bmatrix}$

4. $A = \begin{bmatrix} 3 & -10 & -9 \\ 9 & 7 & -8 \\ -5 & -1 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} -6 & 2 & -2 \\ -3 & 7 & 1 \\ 9 & 3 & 4 \end{bmatrix}$

5. $A = \begin{bmatrix} -3 & 5 & 8 \\ -5 & -7 & -3 \\ 1 & -10 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -4 & -8 \\ 4 & -1 & 0 \\ -7 & 7 & 4 \end{bmatrix}$

6. $A = \begin{bmatrix} -1 & 1 & 5 \\ -6 & 5 & 6 \\ -2 & -9 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & -5 & 7 \\ -5 & 9 & -2 \\ -5 & -6 & -7 \end{bmatrix}$

7. $A = \begin{bmatrix} -5 & 6 & 0 \\ -5 & 1 & -7 \\ -7 & -1 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & -6 & -4 \\ 8 & 7 & 0 \\ 0 & -6 & -5 \end{bmatrix}$

8. $A = \begin{bmatrix} -5 & 6 & -8 \\ 7 & 1 & -7 \\ 0 & 3 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -8 & 7 & -10 \\ 0 & 5 & -5 \\ -7 & 0 & 1 \end{bmatrix}$

9. $A = \begin{bmatrix} -4 & -5 & -9 \\ 1 & 3 & -3 \\ 1 & -8 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 2 & -8 \\ 0 & -7 & 6 \\ 4 & -7 & -2 \end{bmatrix}$

10. $A = \begin{bmatrix} 2 & -10 & 5 \\ 7 & -5 & -8 \\ 4 & -5 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & -1 & -6 \\ 2 & 4 & 0 \\ -6 & -8 & 5 \end{bmatrix}$

1.2.2. Subtraction

Find the difference of the following matrices A and B

1. $A = \begin{bmatrix} 5 & -4 & 1 \\ -9 & 0 & 3 \\ -9 & 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -10 & 9 \\ -10 & 3 & 9 \\ 1 & 8 & -6 \end{bmatrix}$

2. $A = \begin{bmatrix} 5 & 0 & 0 \\ 7 & 3 & 0 \\ 8 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -8 & 5 & -7 \\ 7 & 0 & 7 \\ -3 & 0 & -6 \end{bmatrix}$

3. $A = \begin{bmatrix} 9 & -1 & -7 \\ 2 & 6 & 8 \\ 5 & -7 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & 8 & 7 \\ -7 & -10 & -9 \\ -2 & 9 & -4 \end{bmatrix}$

4. $A = \begin{bmatrix} -8 & -6 & 2 \\ -3 & 9 & -2 \\ -4 & -6 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} -10 & 5 & -9 \\ -3 & 0 & 2 \\ -4 & 8 & 5 \end{bmatrix}$

5. $A = \begin{bmatrix} -5 & 8 & -6 \\ -2 & 5 & 8 \\ 4 & -9 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -9 & 2 \\ -7 & -8 & -6 \\ 8 & 5 & -7 \end{bmatrix}$

6. $A = \begin{bmatrix} 4 & 6 & 0 \\ -1 & 8 & 2 \\ 1 & -4 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -6 & 8 \\ -4 & 3 & 9 \\ -7 & -4 & -5 \end{bmatrix}$

7. $A = \begin{bmatrix} -7 & 0 & 9 \\ 6 & -8 & 4 \\ 2 & -6 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 9 & 5 \\ 3 & -7 & 0 \\ 8 & -3 & -3 \end{bmatrix}$

8. $A = \begin{bmatrix} 0 & 2 & -9 \\ 0 & 4 & -4 \\ -1 & -7 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & -4 & 1 \\ 3 & -10 & 6 \\ 3 & 0 & 0 \end{bmatrix}$

9. $A = \begin{bmatrix} -7 & -2 & -5 \\ 0 & -8 & -10 \\ 6 & -7 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 5 \\ -4 & 9 & -9 \\ 5 & -8 & -6 \end{bmatrix}$

10. $A = \begin{bmatrix} 5 & 5 & -5 \\ -5 & 4 & -4 \\ 8 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -8 & 6 & -8 \\ 8 & 3 & 6 \\ -7 & -10 & 7 \end{bmatrix}$

1.2.3. Multiplication

Find the product of the following matrices A and B

1. $A = \begin{bmatrix} 5 & 1 & -5 \\ -8 & -7 & -2 \\ -3 & 7 & -10 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 5 & 7 \\ -6 & -1 & 1 \\ 6 & 9 & -3 \end{bmatrix}$

2. $A = \begin{bmatrix} -2 & 3 & -5 \\ 4 & 1 & 4 \\ -10 & -9 & -7 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 4 & 9 \\ -7 & -1 & -6 \\ 4 & -6 & -1 \end{bmatrix}$

3. $A = \begin{bmatrix} 9 & -10 & 6 \\ -1 & 3 & 5 \\ -4 & -4 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -10 & -7 \\ 4 & 0 & 1 \\ -7 & -7 & -2 \end{bmatrix}$

4. $A = \begin{bmatrix} -1 & -9 & -6 \\ 1 & -9 & 5 \\ -9 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & 4 \\ -7 & 5 & -5 \\ 8 & 2 & -3 \end{bmatrix}$

5. $A = \begin{bmatrix} -7 & -7 & -3 \\ 3 & -4 & -5 \\ -2 & 2 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -9 & -2 \\ 9 & -5 & 4 \\ -4 & -2 & -2 \end{bmatrix}$

6. $A = \begin{bmatrix} 1 & 7 & -8 \\ -3 & 2 & 8 \\ -4 & 7 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} -10 & -3 & 4 \\ 7 & -2 & 5 \\ -10 & -3 & -4 \end{bmatrix}$

7. $A = \begin{bmatrix} 8 & 9 & 9 \\ 9 & 6 & -2 \\ 7 & -8 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -4 & -6 \\ -1 & 0 & -9 \\ -4 & 8 & -2 \end{bmatrix}$

8. $A = \begin{bmatrix} -5 & -9 & 7 \\ 3 & -8 & 6 \\ -8 & 3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -3 & -9 \\ -9 & -3 & -9 \\ -4 & 2 & -2 \end{bmatrix}$

9. $A = \begin{bmatrix} -1 & -9 & -8 \\ 9 & -2 & 4 \\ -6 & -7 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -10 & -7 \\ 3 & -6 & -6 \\ 0 & 1 & -10 \end{bmatrix}$

10. $A = \begin{bmatrix} 2 & 3 & -4 \\ 3 & 5 & 5 \\ 6 & 2 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 2 & -7 \\ -2 & 2 & 6 \\ 7 & 7 & -4 \end{bmatrix}$

1.3. Matrix Properties

1.3.1. Properties

For each matrix A, find:

a) rank(A)

b) nullity(A)

c) det(A)

d) A^{-1} (if exists)

e) basis of ker(A)

1. $A = \begin{bmatrix} 1 & -1 & 4 \\ -1 & 3 & -3 \\ 1 & -2 & 4 \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & 2 & -5 \\ 1 & 3 & -7 \\ 1 & 2 & -5 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & -3 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4. $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & -1 \\ -4 & -4 & 9 \end{bmatrix}$

5. $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & -1 \\ 1 & 1 & -1 \end{bmatrix}$

6. $A = \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & -3 \\ 2 & -6 & 5 \end{bmatrix}$

7. $A = \begin{bmatrix} 1 & -4 & 4 \\ -1 & 5 & -6 \\ 0 & 0 & 0 \end{bmatrix}$

8. $A = \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -1 \\ 0 & -2 & 3 \end{bmatrix}$

9. $A = \begin{bmatrix} -1 & -2 & 0 \\ -2 & -3 & 0 \\ 2 & 4 & -1 \end{bmatrix}$

10. $A = \begin{bmatrix} 1 & 3 & -8 \\ 0 & 1 & -2 \\ 2 & 8 & -20 \end{bmatrix}$

1.3.2. RREF

Find the Reduced Row Echelon Form of the following matrix A

1. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & -2 & -1 \\ 1 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ -1 & -1 & -2 \end{bmatrix}$

4. $A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

5. $A = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 2 \\ 1 & -1 & 1 \end{bmatrix}$

6. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 2 & 0 \end{bmatrix}$

7. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

8. $A = \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

9. $A = \begin{bmatrix} 1 & -2 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

10. $A = \begin{bmatrix} 5 & 2 & -2 \\ 0 & 1 & -2 \\ 2 & 1 & -1 \end{bmatrix}$

2. Solutions

2.1. Vector Arithmetic

2.1.1. Addition

$\begin{bmatrix} -5 \\ -13 \\ -7 \end{bmatrix}$ $\begin{bmatrix} 8 \\ 7 \\ 9 \end{bmatrix}$ $\begin{bmatrix} -10 \\ 0 \\ -17 \end{bmatrix}$ $\begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 8 \\ 4 \\ -2 \end{bmatrix}$

$\begin{bmatrix} -2 \\ -3 \\ 4 \end{bmatrix}$ $\begin{bmatrix} 4 \\ 7 \\ -3 \end{bmatrix}$ $\begin{bmatrix} -6 \\ 1 \\ -4 \end{bmatrix}$ $\begin{bmatrix} 10 \\ 15 \\ 3 \end{bmatrix}$ $\begin{bmatrix} 10 \\ -7 \\ 4 \end{bmatrix}$

2.1.2. Subtraction

$\begin{bmatrix} 0 \\ -6 \\ -8 \end{bmatrix}$ $\begin{bmatrix} -9 \\ 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 13 \\ 11 \\ 7 \end{bmatrix}$ $\begin{bmatrix} -3 \\ 5 \\ 12 \end{bmatrix}$ $\begin{bmatrix} 4 \\ -13 \\ 6 \end{bmatrix}$

$\begin{bmatrix} 14 \\ 10 \\ 0 \end{bmatrix}$ $\begin{bmatrix} -3 \\ 18 \\ -4 \end{bmatrix}$ $\begin{bmatrix} 2 \\ 10 \\ -9 \end{bmatrix}$ $\begin{bmatrix} 11 \\ 8 \\ 8 \end{bmatrix}$ $\begin{bmatrix} -12 \\ -3 \\ -1 \end{bmatrix}$

2.1.3. Scalar Multiplication

1: $\begin{bmatrix} 27 \\ -18 \\ 27 \end{bmatrix}$ 2: $\begin{bmatrix} -80 \\ 40 \\ -20 \end{bmatrix}$ 3: $\begin{bmatrix} -64 \\ 48 \\ 64 \end{bmatrix}$ 4: $\begin{bmatrix} -90 \\ -20 \\ -50 \end{bmatrix}$ 5: $\begin{bmatrix} 90 \\ 10 \\ -70 \end{bmatrix}$

6: $\begin{bmatrix} 16 \\ 40 \\ 40 \end{bmatrix}$ 7: $\begin{bmatrix} -54 \\ -45 \\ 18 \end{bmatrix}$ 8: $\begin{bmatrix} 27 \\ -30 \\ 30 \end{bmatrix}$ 9: $\begin{bmatrix} 0 \\ -40 \\ 32 \end{bmatrix}$ 10: $\begin{bmatrix} -8 \\ -12 \\ 12 \end{bmatrix}$

2.2. Matrix Arithmetic

2.2.1. Addition

1: $\begin{bmatrix} -2 & -3 & -1 \\ 8 & 5 & 8 \\ -17 & 0 & -5 \end{bmatrix}$ 2: $\begin{bmatrix} -12 & -3 & -17 \\ -4 & -10 & 4 \\ 0 & 15 & 6 \end{bmatrix}$ 3: $\begin{bmatrix} -7 & 0 & -5 \\ -2 & 7 & 2 \\ -2 & 10 & -11 \end{bmatrix}$ 4: $\begin{bmatrix} -3 & -8 & -11 \\ 6 & 14 & -7 \\ 4 & 2 & 10 \end{bmatrix}$ 5: $\begin{bmatrix} -3 & 1 & 0 \\ -1 & -8 & -3 \\ -6 & -3 & 12 \end{bmatrix}$

6: $\begin{bmatrix} 6 & -4 & 12 \\ -11 & 14 & 4 \\ -7 & -15 & 0 \end{bmatrix}$ 7: $\begin{bmatrix} -10 & 0 & -4 \\ 3 & 8 & -7 \\ -7 & -7 & -9 \end{bmatrix}$ 8: $\begin{bmatrix} -13 & 13 & -18 \\ 7 & 6 & -12 \\ -7 & 3 & 4 \end{bmatrix}$ 9: $\begin{bmatrix} 3 & -3 & -17 \\ 1 & -4 & 3 \\ 5 & -15 & 0 \end{bmatrix}$ 10: $\begin{bmatrix} -1 & -11 & -1 \\ 9 & -1 & -8 \\ -2 & -13 & 3 \end{bmatrix}$

2.2.2. Subtraction

1: $\begin{bmatrix} 7 & 6 & -8 \\ 1 & -3 & -6 \\ -10 & -5 & 5 \end{bmatrix}$ 2: $\begin{bmatrix} 13 & -5 & 7 \\ 0 & 3 & -7 \\ 11 & 1 & 11 \end{bmatrix}$ 3: $\begin{bmatrix} 14 & -9 & -14 \\ 9 & 16 & 17 \\ 7 & -16 & 2 \end{bmatrix}$ 4: $\begin{bmatrix} 2 & -11 & 11 \\ 0 & 9 & -4 \\ 0 & -14 & 3 \end{bmatrix}$ 5: $\begin{bmatrix} -8 & 17 & -8 \\ 5 & 13 & 14 \\ -4 & -14 & 4 \end{bmatrix}$

6: $\begin{bmatrix} 5 & 12 & -8 \\ 3 & 5 & -7 \\ 8 & 0 & 1 \end{bmatrix}$ 7: $\begin{bmatrix} -12 & -9 & 4 \\ 3 & -1 & 4 \\ -6 & -3 & -2 \end{bmatrix}$ 8: $\begin{bmatrix} 3 & 6 & -10 \\ -3 & 14 & -10 \\ -4 & -7 & 6 \end{bmatrix}$ 9: $\begin{bmatrix} -6 & -4 & -10 \\ 4 & -17 & -1 \\ 1 & 1 & 15 \end{bmatrix}$ 10: $\begin{bmatrix} 13 & -1 & 3 \\ -13 & 1 & -10 \\ 15 & 12 & -4 \end{bmatrix}$

2.2.3. Multiplication

1: $\begin{bmatrix} -11 & -21 & 51 \\ -10 & -51 & -57 \\ -117 & -112 & 16 \end{bmatrix}$ 2: $\begin{bmatrix} -59 & 19 & -31 \\ 45 & -9 & 26 \\ -55 & 11 & -29 \end{bmatrix}$ 3: $\begin{bmatrix} -73 & -132 & -85 \\ -24 & -25 & 0 \\ -76 & -16 & 8 \end{bmatrix}$ 4: $\begin{bmatrix} 12 & -59 & 59 \\ 106 & -33 & 34 \\ -4 & -19 & -37 \end{bmatrix}$ 5: $\begin{bmatrix} -72 & 104 & -8 \\ -7 & 3 & -12 \\ 20 & 12 & 16 \end{bmatrix}$

6: $\begin{bmatrix} 119 & 7 & 71 \\ -36 & -19 & -34 \\ 29 & -20 & -5 \end{bmatrix}$ 7: $\begin{bmatrix} -77 & 40 & -147 \\ -34 & -52 & -104 \\ -40 & 12 & 20 \end{bmatrix}$ 8: $\begin{bmatrix} 58 & 56 & 112 \\ 45 & 27 & 33 \\ -43 & 27 & 33 \end{bmatrix}$ 9: $\begin{bmatrix} -29 & 56 & 141 \\ 12 & -74 & -91 \\ -33 & 110 & 4 \end{bmatrix}$ 10: $\begin{bmatrix} -24 & -18 & 20 \\ 40 & 51 & -11 \\ 12 & 2 & -22 \end{bmatrix}$

2.3. Matrix Properties

2.3.1. Properties

Solution

Row Operations:

Step 1: $r_2 := r_2 - (-1)r_1$
$$\left[\begin{array}{ccc|ccc} 1 & -1 & 4 & 1 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 \\ 1 & -2 & 4 & 0 & 0 & 1 \end{array}\right]$$

Step 2: $r_3 := r_3 - r_1$
$$\left[\begin{array}{ccc|ccc} 1 & -1 & 4 & 1 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \end{array}\right]$$

Step 3: $r_2 := 1/2 r_2$
$$\left[\begin{array}{ccc|ccc} 1 & -1 & 4 & 1 & 0 & 0 \\ 0 & 1 & 1/2 & 1/2 & 1/2 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \end{array}\right]$$

Step 4: $r_1 := r_1 - (-1)r_2$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 9/2 & 3/2 & 1/2 & 0 \\ 0 & 1 & 1/2 & 1/2 & 1/2 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \end{array}\right]$$

Step 5: $r_3 := r_3 - (-1)r_2$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 9/2 & 3/2 & 1/2 & 0 \\ 0 & 1 & 1/2 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1/2 & -1/2 & 1/2 & 1 \end{array}\right]$$

Step 6: $r_3 := 2r_3$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 9/2 & 3/2 & 1/2 & 0 \\ 0 & 1 & 1/2 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{array}\right]$$

Step 7: $r_1 := r_1 - (9/2)r_3$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 6 & -4 & -9 \\ 0 & 1 & 1/2 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{array}\right]$$

Step 8: $r_2 := r_2 - (1/2)r_3$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 6 & -4 & -9 \\ 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{array}\right]$$

Results:

a) rank(A) = 3

b) nullity(A) = 0

c) det(A) = 0

d) $A^{-1} = \begin{bmatrix} 3 & -2 & -4 \\ 0 & 1 & 1 \\ -1 & 1 & 2 \end{bmatrix}$

e) ker(A) = $\{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_2 := r_2 - r_1$
$$\left[\begin{array}{ccc|ccc} 1 & 2 & -5 & 1 & 0 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 1 & 2 & -5 & 0 & 0 & 1 \end{array}\right]$$

Step 2: $r_3 := r_3 - r_1$
$$\left[\begin{array}{ccc|ccc} 1 & 2 & -5 & 1 & 0 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{array}\right]$$

Step 3: $r_1 := r_1 - (2)r_2$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 3 & -2 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{array}\right]$$

Results:

a) rank(A) = 2

b) nullity(A) = 1

c) det(A) = 1440

d) A^{-1} = does not exist

e) ker(A) = span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\}$

Solution

Row Operations:

Step 1: $r_1 := r_1 - (-3)r_2$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 6 & 1 & 3 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

Step 2: $r_1 := r_1 - (6)r_3$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & -6 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

Results:

a) rank(A) = 3

b) nullity(A) = 0

c) det(A) = 0

d) $A^{-1} = \begin{bmatrix} 1 & 3 & -7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

e) ker(A) = $\{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_3 := r_3 - (-4)r_1$
$$\left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 4 & -3 & 4 & 0 & 1 \end{array}\right]$$

Step 2: $r_1 := r_1 - (2)r_2$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 4 & -3 & 4 & 0 & 1 \end{array}\right]$$

Step 3: $r_3 := r_3 - (4)r_2$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 4 & -4 & 1 \end{array}\right]$$

Step 4: $r_1 := r_1 - (-1)r_3$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & -6 & 1 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 4 & -4 & 1 \end{array}\right]$$

Step 5: $r_2 := r_2 - (-1)r_3$ $\begin{bmatrix} 1 & 0 & 0 & | & 5 & -6 & 1 \\ 0 & 1 & 0 & | & 4 & -3 & 1 \\ 0 & 0 & 1 & | & 4 & -4 & 1 \end{bmatrix}$

Results:

a) $\operatorname{rank}(A) = 3$

b) $\operatorname{nullity}(A) = 0$

c) $\det(A) = 0$

d) $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 4 & -4 & 1 \end{bmatrix}$

e) $\ker(A) = \{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_2 := r_2 - (2)r_1$ $\begin{bmatrix} 1 & 1 & -1 & | & 1 & 0 & 0 \\ 0 & 1 & 1 & | & -2 & 1 & 0 \\ 1 & 1 & -1 & | & 0 & 0 & 1 \end{bmatrix}$

Step 2: $r_3 := r_3 - r_1$ $\begin{bmatrix} 1 & 1 & -1 & | & 1 & 0 & 0 \\ 0 & 1 & 1 & | & -2 & 1 & 0 \\ 0 & 0 & 0 & | & -1 & 0 & 1 \end{bmatrix}$

Step 3: $r_1 := r_1 - r_2$ $\begin{bmatrix} 1 & 0 & -2 & | & 3 & -1 & 0 \\ 0 & 1 & 1 & | & -2 & 1 & 0 \\ 0 & 0 & 0 & | & -1 & 0 & 1 \end{bmatrix}$

Results:

a) $\operatorname{rank}(A) = 2$

b) $\operatorname{nullity}(A) = 1$

c) $\det(A) = 0$

d) $A^{-1} = $ does not exist

e) $\ker(A) = \operatorname{span} \left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \right\}$

Solution

Row Operations:

Step 1: $r_3 := r_3 - (2)r_1$ $\begin{bmatrix} 1 & -3 & 2 & | & 1 & 0 & 0 \\ 0 & 1 & -3 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & -2 & 0 & 1 \end{bmatrix}$

Step 2: $r_1 := r_1 - (-3)r_2$ $\begin{bmatrix} 1 & 0 & -7 & | & 1 & 3 & 0 \\ 0 & 1 & -3 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & -2 & 0 & 1 \end{bmatrix}$

Step 3: $r_1 := r_1 - (-7)r_3$ $\begin{bmatrix} 1 & 0 & 0 & | & -13 & 3 & 7 \\ 0 & 1 & -3 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & -2 & 0 & 1 \end{bmatrix}$

Step 4: $r_2 := r_2 - (-3)r_3$ $\begin{bmatrix} 1 & 0 & 0 & | & -13 & 3 & 7 \\ 0 & 1 & 0 & | & -6 & 1 & 3 \\ 0 & 0 & 1 & | & -2 & 0 & 1 \end{bmatrix}$

Results:

a) rank(A) = 3

b) nullity(A) = 0

c) det(A) = 0

d) $A^{-1} = \begin{bmatrix} -1 & 1 & 1 \\ -2 & 1 & 1 \\ -2 & 0 & 1 \end{bmatrix}$

e) ker(A) = $\{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_2 := r_2 - (-1)r_1$ $\begin{bmatrix} 1 & -4 & 4 & | & 1 & 0 & 0 \\ 0 & 1 & -2 & | & 1 & 1 & 0 \\ 0 & 0 & 0 & | & 0 & 0 & 1 \end{bmatrix}$

Step 2: $r_1 := r_1 - (-4)r_2$ $\begin{bmatrix} 1 & 0 & -4 & | & 5 & 4 & 0 \\ 0 & 1 & -2 & | & 1 & 1 & 0 \\ 0 & 0 & 0 & | & 0 & 0 & 1 \end{bmatrix}$

Results:

a) rank(A) = 2

b) nullity(A) = 1

c) det(A) = 0

d) A^{-1} = does not exist

e) ker(A) = span $\left\{ \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \right\}$

Solution

Row Operations:

Step 1: $r_1 := r_1 - (2)r_2$ $\begin{bmatrix} 1 & 0 & 0 & | & 1 & -2 & 0 \\ 0 & 1 & -1 & | & 0 & 1 & 0 \\ 0 & -2 & 3 & | & 0 & 0 & 1 \end{bmatrix}$

Step 2: $r_3 := r_3 - (-2)r_2$ $\begin{bmatrix} 1 & 0 & 0 & | & 1 & -2 & 0 \\ 0 & 1 & -1 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 2 & 1 \end{bmatrix}$

Step 3: $r_2 := r_2 - (-1)r_3$ $\begin{bmatrix} 1 & 0 & 0 & | & 1 & -2 & 0 \\ 0 & 1 & 0 & | & 0 & 3 & 1 \\ 0 & 0 & 1 & | & 0 & 2 & 1 \end{bmatrix}$

Results:

a) rank(A) = 3

b) nullity(A) = 0

c) det(A) = 0

d) $A^{-1} = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$

e) ker(A) = $\{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_1 := -1r_1$ $\begin{bmatrix} 1 & 2 & 0 & | & -1 & 0 & 0 \\ -2 & -3 & 0 & | & 0 & 1 & 0 \\ 2 & 4 & -1 & | & 0 & 0 & 1 \end{bmatrix}$

Step 2: $r_2 := r_2 - (-2)r_1$ $\begin{bmatrix} 1 & 2 & 0 & | & -1 & 0 & 0 \\ 0 & 1 & 0 & | & -2 & 1 & 0 \\ 2 & 4 & -1 & | & 0 & 0 & 1 \end{bmatrix}$

Step 3: $r_3 := r_3 - (2)r_1$ $\begin{bmatrix} 1 & 2 & 0 & | & -1 & 0 & 0 \\ 0 & 1 & 0 & | & -2 & 1 & 0 \\ 0 & 0 & -1 & | & 2 & 0 & 1 \end{bmatrix}$

Step 4: $r_1 := r_1 - (2)r_2$ $\begin{bmatrix} 1 & 0 & 0 & | & 3 & -2 & 0 \\ 0 & 1 & 0 & | & -2 & 1 & 0 \\ 0 & 0 & -1 & | & 2 & 0 & 1 \end{bmatrix}$

Step 5: $r_3 := -1r_3$ $\begin{bmatrix} 1 & 0 & 0 & | & 3 & -2 & 0 \\ 0 & 1 & 0 & | & -2 & 1 & 0 \\ 0 & 0 & 1 & | & -2 & 0 & -1 \end{bmatrix}$

Results:

a) rank(A) = 3

b) nullity(A) = 0

c) det(A) = 8

d) $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ -2 & 0 & -1 \end{bmatrix}$

e) ker(A) = $\{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_3 := r_3 - (2)r_1$ $\begin{bmatrix} 1 & 3 & -8 & | & 1 & 0 & 0 \\ 0 & 1 & -2 & | & 0 & 1 & 0 \\ 0 & 2 & -4 & | & -2 & 0 & 1 \end{bmatrix}$

Step 2: $r_1 := r_1 - (3)r_2$ $\begin{bmatrix} 1 & 0 & -2 & | & 1 & -3 & 0 \\ 0 & 1 & -2 & | & 0 & 1 & 0 \\ 0 & 2 & -4 & | & -2 & 0 & 1 \end{bmatrix}$

Step 3: $r_3 := r_3 - (2)r_2$ $\begin{bmatrix} 1 & 0 & -2 & | & 1 & -3 & 0 \\ 0 & 1 & -2 & | & 0 & 1 & 0 \\ 0 & 0 & 0 & | & -2 & -2 & 1 \end{bmatrix}$

Results:

a) $\operatorname{rank}(A) = 2$

b) $\operatorname{nullity}(A) = 1$

c) $\det(A) = 0$

d) $A^{-1} = \text{does not exist}$

e) $\ker(A) = \operatorname{span}\left\{ \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \right\}$

2.3.2. RREF

Solution

Elementary Row Operations:

(1) $r_3 := r_3 - r_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

(2) $r_2 := r_2 - r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_2 := r_2 + (-1)r_1$

$$\begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

(2) $r_2 := r_2 - r_3$

$$\begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(3) $r_1 := r_1 - r_3$

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_3 := r_3 - r_1$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(2) $r_1 := r_1 + (-1)r_2$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(3) $r_2 := r_2 + (-2)r_3$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_2 := r_2 - r_1$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(2) $r_1 := r_1 + (-1)r_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(3) $r_1 := r_1 + (-2)r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_2 := r_2 - (2)r_1$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

(2) $r_3 := r_3 + (-1)r_1$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_1 := r_1 - r_2$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 2 & 2 & 0 \end{bmatrix}$$

(2) $r_3 := r_3 + (-2)r_1$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(3) $r_1 := r_1 + (-1)r_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_2 := r_2 - (2)r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

(2) $r_2 := r_2 - r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

(3) $r_2 := r_2 + (-2)r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_1 := r_1 - (2)r_2$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(2) $r_2 := r_2 + (-1)r_3$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(3) $r_1 := r_1 - r_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_1 := r_1 + (-2)r_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

(2) $r_2 := r_2 - (2)r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

(3) $r_3 := r_3 + (-1)r_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_1 := r_1 + (-2)r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 2 & 1 & -1 \end{bmatrix}$$

(2) $r_3 := r_3 + (-1)r_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 2 & 0 & 1 \end{bmatrix}$$

(3) $r_3 := r_3 + (-2)r_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$