Mathmatical Exercise

Zichao Wei

This is **exercise** for Foundations of Mathmatical, WS24.

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1. Introduction

> **!** Practice makes perfect.
>
> We need many exercises to master the skill of mathmatical. However, it's not easy to find the exercise which just fit for you.
>
> So, I create this document to generate the exercise for you. You can use this document to generate the exercise for the Mathmatical course by GitHub Copilot or other LLM.

This is an LLM-generated exercise document for the Mathmatical course.

The syntax of this document is Quarto syntax, which is a markdown-based language. It is designed to be human-readable and easy to write, while also being powerful enough to support complex document structures.

You can check the syllabus of this course in the following link: Syllabus°.

Some prompts for LLM or Copilot to generate exercises are as follows:

- Use `$` to denote inline math, and `$$` to denote display math.
- Generate different types of questions related to the current topic. Such as multiple-choice questions, fill-in-the-blank questions, and short answer questions.
- Use random numbers to generate different questions.
- Use the `answer` environment to provide the answer to the questions. The answer should be in comment of html so only visible in the source code.
  - ‣ If the answer need to type in math, type the formula in advance and just let the user fill in the blank.
- Use the `solution` environment to provide the solution to the questions.
- Use the `hint` environment to provide hints to the questions.
- Use the `exercise` environment to provide the questions.

2. Vectors and Matrices

2.1. Vector Arithmetic

2.1.1. Addition and Subtraction

1. Let $\mathbf{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$. Compute $\mathbf{u} + \mathbf{v}$.

2. Let $\mathbf{u} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$. Compute $\mathbf{u} + \mathbf{v}$.

2.2. Matrix Arithmetic

1. Let $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$. Compute $A + B$.

2. Let $A = \begin{bmatrix} 4 & 2 \\ 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 8 & 2 \end{bmatrix}$. Compute $A - B$.

3. Let $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$. Compute $A \cdot B$.

4. Let $A = \begin{bmatrix} 2 & 5 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$. Compute $A \times B$.

5. Let $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$. Compute $A \cdot B$.

2.3. Gaussian Elimination

1. Solve the following system of equations using Gaussian elimination:

$$\begin{aligned} 2x + 3y &= 8 \\ 4x + 5y &= 18 \end{aligned} \tag{1}$$

2. Solve the following system of equations using Gaussian elimination:

$$\begin{aligned} 3x + 2y &= 11 \\ 5x + 3y &= 16 \end{aligned} \tag{2}$$

3. Solve the following system of equations using Gaussian elimination:

$$\begin{aligned} 2x + 3y + z &= 8 \\ 4x + 5y + z &= 18 \\ x + 2y + z &= 7 \end{aligned} \tag{3}$$

4. Solve the following system of equations using Gaussian elimination:

$$\begin{aligned} 2x + 3y + z &= 8 \\ 4x + 5y + z &= 18 \\ 3x + 4y + z &= 13 \end{aligned} \tag{4}$$

5. Solve the following system of equations using Gaussian elimination:

$$\begin{aligned} 2x + 3y + z &= 8 \\ 4x + 5y + z &= 18 \\ x + 2y + z &= 7 \end{aligned} \tag{5}$$

4. Choose the correct answer.

$$2x + 3y = 8$$
$$4x + 5y = 18$$
(6)

a) $x = 2, y = 2$

b) $x = 3, y = 2$

c) $x = 2, y = 3$

d) $x = 3, y = 3$

5 Choose the kind of solution for the following system of equations.

$$2x + 3y = 8$$
$$4x + 5y = 18$$
(7)

a) No solution

b) Unique solution

c) Infinite solutions

d) None of the above

Bibliography