Exercise 6:

Foundations of Mathematical, WS24

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This is **exercise** 6 for Foundations of Mathematical, WS24. Generated on 2024-12-30 with 10 problems per section.

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1. Problems

1.1. Vector Arithmetic

1.1.1. Addition

Find the sum of the following vectors $\mathbf{u}$ and $\mathbf{v}$

1. $\mathbf{u} = \begin{bmatrix} 7 \\ 9 \\ -1 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -1 \\ -9 \\ -3 \end{bmatrix}$
2. $\mathbf{u} = \begin{bmatrix} 10 \\ 0 \\ 10 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 7 \\ -4 \\ -2 \end{bmatrix}$
3. $\mathbf{u} = \begin{bmatrix} -8 \\ 5 \\ -4 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -3 \\ -2 \\ 7 \end{bmatrix}$
4. $\mathbf{u} = \begin{bmatrix} -5 \\ 0 \\ -10 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -9 \\ 4 \\ -9 \end{bmatrix}$
5. $\mathbf{u} = \begin{bmatrix} -9 \\ -1 \\ -3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -7 \\ -4 \\ 6 \end{bmatrix}$
6. $\mathbf{u} = \begin{bmatrix} 1 \\ -3 \\ -6 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -9 \\ 3 \\ 10 \end{bmatrix}$
7. $\mathbf{u} = \begin{bmatrix} -3 \\ 1 \\ -3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 7 \\ -2 \\ -6 \end{bmatrix}$
8. $\mathbf{u} = \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 7 \\ 3 \\ -7 \end{bmatrix}$
9. $\mathbf{u} = \begin{bmatrix} 7 \\ 7 \\ 0 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 7 \\ 7 \\ 5 \end{bmatrix}$
10. $\mathbf{u} = \begin{bmatrix} 10 \\ 7 \\ -10 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 10 \\ -2 \\ 10 \end{bmatrix}$

1.1.2. Subtraction

Find the difference of the following vectors $\mathbf{u}$ and $\mathbf{v}$

1. $\mathbf{u} = \begin{bmatrix} -6 \\ 0 \\ 5 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 8 \\ 4 \\ 3 \end{bmatrix}$
2. $\mathbf{u} = \begin{bmatrix} 6 \\ 10 \\ -9 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 9 \\ -9 \\ 7 \end{bmatrix}$
3. $\mathbf{u} = \begin{bmatrix} 8 \\ -5 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -6 \\ 9 \\ -6 \end{bmatrix}$
4. $\mathbf{u} = \begin{bmatrix} -8 \\ -4 \\ 9 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -3 \\ 8 \\ 4 \end{bmatrix}$
5. $\mathbf{u} = \begin{bmatrix} 4 \\ 7 \\ 5 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 4 \\ 10 \\ 4 \end{bmatrix}$

6. $\mathbf{u} = \begin{bmatrix} 2 \\ 1 \\ -9 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 3 \\ -9 \\ 6 \end{bmatrix}$
7. $\mathbf{u} = \begin{bmatrix} -2 \\ 6 \\ 0 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 3 \\ -3 \\ -7 \end{bmatrix}$
8. $\mathbf{u} = \begin{bmatrix} -2 \\ 10 \\ -5 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -5 \\ -8 \\ -9 \end{bmatrix}$
9. $\mathbf{u} = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -6 \\ -8 \\ -7 \end{bmatrix}$
10. $\mathbf{u} = \begin{bmatrix} -7 \\ 10 \\ 9 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 4 \\ 1 \\ 7 \end{bmatrix}$

1.1.3. Scalar Multiplication

Find the scalar product of the following vector $\mathbf{u}$ and scalar k

1. $\mathbf{u} = \begin{bmatrix} 2 \\ -1 \\ -8 \end{bmatrix} -5\mathbf{v}.$
2. $\mathbf{u} = \begin{bmatrix} 6 \\ 9 \\ 8 \end{bmatrix} 7\mathbf{v}.$
3. $\mathbf{u} = \begin{bmatrix} -5 \\ -2 \\ 7 \end{bmatrix} -3\mathbf{v}.$
4. $\mathbf{u} = \begin{bmatrix} -7 \\ -10 \\ -8 \end{bmatrix} 7\mathbf{v}.$
5. $\mathbf{u} = \begin{bmatrix} -8 \\ 0 \\ -4 \end{bmatrix} -6\mathbf{v}.$
6. $\mathbf{u} = \begin{bmatrix} 9 \\ -10 \\ 2 \end{bmatrix} 6\mathbf{v}.$
7. $\mathbf{u} = \begin{bmatrix} -8 \\ -5 \\ -7 \end{bmatrix} 3\mathbf{v}.$
8. $\mathbf{u} = \begin{bmatrix} -5 \\ 3 \\ 7 \end{bmatrix} 3\mathbf{v}.$
9. $\mathbf{u} = \begin{bmatrix} 3 \\ -5 \\ 10 \end{bmatrix} 8\mathbf{v}.$
10. $\mathbf{u} = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix} -3\mathbf{v}.$

1.2. Matrix Arithmetic

1.2.1. Addition

Find the sum of the following matrices A and B

1. $A = \begin{bmatrix} 5 & 6 & 2 \\ -1 & 9 & 3 \\ 5 & 9 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & 8 & -8 \\ 5 & 9 & 4 \\ -2 & -1 & 8 \end{bmatrix}$

2. $A = \begin{bmatrix} 3 & -10 & -5 \\ -4 & -7 & 9 \\ 4 & 9 & -10 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 4 & 9 \\ 9 & 7 & -10 \\ 1 & -8 & 3 \end{bmatrix}$
3. $A = \begin{bmatrix} -7 & 2 & 8 \\ -3 & 8 & 3 \\ 3 & 0 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & -6 & 2 \\ 0 & 3 & -6 \\ 8 & -1 & 4 \end{bmatrix}$
4. $A = \begin{bmatrix} -3 & -6 & 1 \\ 3 & 0 & -7 \\ 1 & 5 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} -7 & 9 & 9 \\ -1 & -5 & -8 \\ 2 & -8 & -4 \end{bmatrix}$
5. $A = \begin{bmatrix} 2 & -1 & -6 \\ 3 & 6 & -7 \\ 9 & -8 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -3 & 1 \\ -4 & 3 & -4 \\ -9 & 2 & 5 \end{bmatrix}$
6. $A = \begin{bmatrix} 7 & 9 & -1 \\ 1 & -8 & 7 \\ 2 & -8 & -9 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & -9 & -1 \\ 8 & -8 & 3 \\ 8 & 9 & 1 \end{bmatrix}$
7. $A = \begin{bmatrix} -4 & -5 & 2 \\ 9 & -3 & -10 \\ 9 & -6 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} -6 & 7 & -9 \\ -2 & -2 & -7 \\ 6 & -4 & 5 \end{bmatrix}$
8. $A = \begin{bmatrix} 5 & 6 & 6 \\ -9 & -9 & 7 \\ -3 & -9 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 4 & -10 \\ -7 & -6 & 0 \\ -6 & 8 & 6 \end{bmatrix}$
9. $A = \begin{bmatrix} -7 & 3 & -8 \\ -7 & -8 & -2 \\ 9 & -5 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -7 & -9 & 8 \\ -7 & 8 & 0 \\ 3 & 5 & 0 \end{bmatrix}$
10. $A = \begin{bmatrix} -4 & -5 & -3 \\ -9 & 6 & -7 \\ 0 & 4 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} -8 & -10 & 7 \\ 6 & -10 & 2 \\ -5 & 9 & 2 \end{bmatrix}$

1.2.2. Subtraction

Find the difference of the following matrices A and B

1. $A = \begin{bmatrix} 4 & 9 & 1 \\ -6 & -6 & 1 \\ 3 & 6 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 6 & -9 \\ 6 & 9 & 7 \\ 0 & 7 & 7 \end{bmatrix}$
2. $A = \begin{bmatrix} 4 & -10 & 2 \\ 2 & 4 & 3 \\ 1 & 7 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 9 \\ -9 & 0 & 0 \\ -3 & -10 & -10 \end{bmatrix}$
3. $A = \begin{bmatrix} -8 & -6 & 8 \\ 1 & 4 & -5 \\ -8 & 1 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & 5 & -2 \\ -9 & -3 & 4 \\ 2 & 4 & -6 \end{bmatrix}$
4. $A = \begin{bmatrix} 2 & 4 & 4 \\ -6 & -2 & -2 \\ 1 & 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -6 & 5 & 9 \\ -5 & 8 & -4 \\ 6 & 4 & -6 \end{bmatrix}$
5. $A = \begin{bmatrix} 1 & -8 & 6 \\ 0 & 2 & -6 \\ -4 & 3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 2 \\ 6 & -10 & 2 \\ -3 & 3 & 0 \end{bmatrix}$
6. $A = \begin{bmatrix} 6 & 7 & -3 \\ -4 & -5 & -10 \\ 6 & 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 5 & -7 \\ 8 & -4 & 0 \\ -10 & -8 & 0 \end{bmatrix}$
7. $A = \begin{bmatrix} 9 & -4 & 3 \\ -1 & 3 & 7 \\ 2 & 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 8 & -10 \\ 8 & -9 & 2 \\ 5 & -3 & -4 \end{bmatrix}$
8. $A = \begin{bmatrix} -6 & 7 & 3 \\ -3 & 6 & -7 \\ -4 & -1 & -8 \end{bmatrix}$ and $B = \begin{bmatrix} -10 & 3 & -3 \\ -3 & -4 & -8 \\ 7 & -2 & -6 \end{bmatrix}$

9. $A = \begin{bmatrix} -1 & 2 & 7 \\ 8 & 7 & 4 \\ -9 & 2 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 5 & 3 \\ -3 & -7 & -8 \\ 3 & -8 & -10 \end{bmatrix}$

10. $A = \begin{bmatrix} 6 & -10 & 4 \\ 7 & 2 & 0 \\ -8 & -1 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -8 & -9 \\ -8 & -7 & -3 \\ -8 & 9 & -10 \end{bmatrix}$

1.2.3. Multiplication

Find the product of the following matrices A and B

1. $A = \begin{bmatrix} -10 & -4 & -1 \\ -4 & -3 & 1 \\ -7 & 6 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & -8 & -7 \\ 8 & 7 & 8 \\ -9 & -10 & -6 \end{bmatrix}$

2. $A = \begin{bmatrix} -6 & 5 & 9 \\ 4 & 7 & -10 \\ -8 & 5 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & -3 & -4 \\ -7 & -3 & 6 \\ -6 & 7 & 1 \end{bmatrix}$

3. $A = \begin{bmatrix} -1 & -8 & 4 \\ -3 & 6 & 6 \\ 4 & -8 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 6 & -9 \\ -8 & 9 & 3 \\ -5 & -4 & 2 \end{bmatrix}$

4. $A = \begin{bmatrix} -5 & -9 & -8 \\ 8 & 1 & -3 \\ 8 & 0 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 9 & 1 \\ 9 & -4 & 6 \\ -2 & 8 & 4 \end{bmatrix}$

5. $A = \begin{bmatrix} 6 & 0 & -6 \\ -9 & 7 & 2 \\ 6 & -2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 3 & -7 \\ -1 & 7 & -2 \\ 5 & 9 & 8 \end{bmatrix}$

6. $A = \begin{bmatrix} 2 & -4 & -2 \\ 7 & 9 & -9 \\ 3 & -2 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 7 & -8 \\ 4 & -1 & 6 \\ 1 & 6 & -10 \end{bmatrix}$

7. $A = \begin{bmatrix} -3 & -7 & -1 \\ 7 & 3 & 4 \\ -10 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -4 & -6 \\ -1 & -1 & 8 \\ 5 & -8 & -7 \end{bmatrix}$

8. $A = \begin{bmatrix} -2 & 3 & 3 \\ 2 & -8 & 2 \\ 8 & 0 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} -6 & 1 & 7 \\ -9 & 0 & 2 \\ 3 & 5 & 3 \end{bmatrix}$

9. $A = \begin{bmatrix} -4 & 3 & 0 \\ 2 & 0 & -9 \\ -6 & 6 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & -7 & 1 \\ 8 & 9 & -9 \\ 0 & -8 & 5 \end{bmatrix}$

10. $A = \begin{bmatrix} 8 & -6 & 7 \\ -5 & -10 & -2 \\ -1 & -7 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -3 & 5 \\ 3 & -3 & 8 \\ 8 & -9 & 9 \end{bmatrix}$

1.3. Matrix Properties

1.3.1. Properties

For each matrix A, find:

a) rank(A)

b) nullity(A)

c) det(A)

d) A^{-1} (if exists)

e) basis of ker(A)

1. $A = \begin{bmatrix} 1 & 4 & 4 \\ 0 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & -1 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$
3. $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 2 \\ 1 & -1 & 3 \end{bmatrix}$
4. $A = \begin{bmatrix} 0 & 2 & -3 \\ 0 & 1 & -2 \\ -1 & 2 & -1 \end{bmatrix}$
5. $A = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 2 & -5 \\ 0 & 0 & 1 \end{bmatrix}$
6. $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -2 \\ -2 & -4 & -2 \end{bmatrix}$
7. $A = \begin{bmatrix} 1 & -2 & -3 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$
8. $A = \begin{bmatrix} 5 & -12 & -12 \\ 0 & 1 & 1 \\ 2 & -4 & -4 \end{bmatrix}$
9. $A = \begin{bmatrix} 5 & 7 & 5 \\ 2 & 3 & 2 \\ 0 & 0 & 1 \end{bmatrix}$
10. $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & -2 \\ -1 & -2 & 4 \end{bmatrix}$

1.3.2. RREF

Find the Reduced Row Echelon Form of the following matrix A

1. $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -2 & 0 & 3 \end{bmatrix}$
2. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$
3. $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$
4. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$
5. $A = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 1 \\ 2 & -3 & 0 \end{bmatrix}$
6. $A = \begin{bmatrix} 1 & 0 & -2 \\ -1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$
7. $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
8. $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 2 & 5 & 1 \end{bmatrix}$

9. $A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ -2 & 0 & 5 \end{bmatrix}$

10. $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

2. Solutions

2.1. Vector Arithmetic

2.1.1. Addition

$\begin{bmatrix} 6 \\ 0 \\ -4 \end{bmatrix}$ $\begin{bmatrix} 17 \\ -4 \\ 8 \end{bmatrix}$ $\begin{bmatrix} -11 \\ 3 \\ 3 \end{bmatrix}$ $\begin{bmatrix} -14 \\ 4 \\ -19 \end{bmatrix}$ $\begin{bmatrix} -16 \\ -5 \\ 3 \end{bmatrix}$

$\begin{bmatrix} -8 \\ 0 \\ 4 \end{bmatrix}$ $\begin{bmatrix} 4 \\ -1 \\ -9 \end{bmatrix}$ $\begin{bmatrix} 12 \\ 7 \\ -4 \end{bmatrix}$ $\begin{bmatrix} 14 \\ 14 \\ 5 \end{bmatrix}$ $\begin{bmatrix} 20 \\ 5 \\ 0 \end{bmatrix}$

2.1.2. Subtraction

$\begin{bmatrix} -14 \\ -4 \\ 2 \end{bmatrix}$ $\begin{bmatrix} -3 \\ 19 \\ -16 \end{bmatrix}$ $\begin{bmatrix} 14 \\ -14 \\ 8 \end{bmatrix}$ $\begin{bmatrix} -5 \\ -12 \\ 5 \end{bmatrix}$ $\begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix}$

$\begin{bmatrix} -1 \\ 10 \\ -15 \end{bmatrix}$ $\begin{bmatrix} -5 \\ 9 \\ 7 \end{bmatrix}$ $\begin{bmatrix} 3 \\ 18 \\ 4 \end{bmatrix}$ $\begin{bmatrix} 8 \\ 14 \\ 11 \end{bmatrix}$ $\begin{bmatrix} -11 \\ 9 \\ 2 \end{bmatrix}$

2.1.3. Scalar Multiplication

1: $\begin{bmatrix} -10 \\ 5 \\ 40 \end{bmatrix}$ 2: $\begin{bmatrix} 42 \\ 63 \\ 56 \end{bmatrix}$ 3: $\begin{bmatrix} 15 \\ 6 \\ -21 \end{bmatrix}$ 4: $\begin{bmatrix} -49 \\ -70 \\ -56 \end{bmatrix}$ 5: $\begin{bmatrix} 48 \\ 0 \\ 24 \end{bmatrix}$

6: $\begin{bmatrix} 54 \\ -60 \\ 12 \end{bmatrix}$ 7: $\begin{bmatrix} -24 \\ -15 \\ -21 \end{bmatrix}$ 8: $\begin{bmatrix} -15 \\ 9 \\ 21 \end{bmatrix}$ 9: $\begin{bmatrix} 24 \\ -40 \\ 80 \end{bmatrix}$ 10: $\begin{bmatrix} -3 \\ -12 \\ 6 \end{bmatrix}$

2.2. Matrix Arithmetic

2.2.1. Addition

1: $\begin{bmatrix} 0 & 14 & -6 \\ 4 & 18 & 7 \\ 3 & 8 & 13 \end{bmatrix}$ 2: $\begin{bmatrix} 9 & -6 & 4 \\ 5 & 0 & -1 \\ 5 & 1 & -7 \end{bmatrix}$ 3: $\begin{bmatrix} 2 & -4 & 10 \\ -3 & 11 & -3 \\ 11 & -1 & 10 \end{bmatrix}$ 4: $\begin{bmatrix} -10 & 3 & 10 \\ 2 & -5 & -15 \\ 3 & -3 & -8 \end{bmatrix}$ 5: $\begin{bmatrix} 0 & -4 & -5 \\ -1 & 9 & -11 \\ 0 & -6 & 1 \end{bmatrix}$

6: $\begin{bmatrix} 13 & 0 & -2 \\ 9 & -16 & 10 \\ 10 & 1 & -8 \end{bmatrix}$ 7: $\begin{bmatrix} -10 & 2 & -7 \\ 7 & -5 & -17 \\ 15 & -10 & 1 \end{bmatrix}$ 8: $\begin{bmatrix} 9 & 10 & -4 \\ -16 & -15 & 7 \\ -9 & -1 & 11 \end{bmatrix}$ 9: $\begin{bmatrix} -14 & -6 & 0 \\ -14 & 0 & -2 \\ 12 & 0 & 0 \end{bmatrix}$ 10: $\begin{bmatrix} -12 & -15 & 4 \\ -3 & -4 & -5 \\ -5 & 13 & 9 \end{bmatrix}$

2.2.2. Subtraction

1: $\begin{bmatrix} 5 & 3 & 10 \\ -12 & -15 & -6 \\ 3 & -1 & -12 \end{bmatrix}$ 2: $\begin{bmatrix} 3 & -9 & -7 \\ 11 & 4 & 3 \\ 4 & 17 & 6 \end{bmatrix}$ 3: $\begin{bmatrix} -3 & -11 & 10 \\ 10 & 7 & -9 \\ -10 & -3 & 15 \end{bmatrix}$ 4: $\begin{bmatrix} 8 & -1 & -5 \\ -1 & -10 & 2 \\ -5 & -3 & 10 \end{bmatrix}$ 5: $\begin{bmatrix} 1 & -9 & 4 \\ -6 & 12 & -8 \\ -1 & 0 & 6 \end{bmatrix}$

6: $\begin{bmatrix} 10 & 2 & 4 \\ -12 & -1 & -10 \\ 16 & 9 & 4 \end{bmatrix}$ 7: $\begin{bmatrix} 9 & -12 & 13 \\ -9 & 12 & 5 \\ -3 & 8 & 5 \end{bmatrix}$ 8: $\begin{bmatrix} 4 & 4 & 6 \\ 0 & 10 & 1 \\ -11 & 1 & -2 \end{bmatrix}$ 9: $\begin{bmatrix} 0 & -3 & 4 \\ 11 & 14 & 12 \\ -12 & 10 & 7 \end{bmatrix}$ 10: $\begin{bmatrix} 1 & -2 & 13 \\ 15 & 9 & 3 \\ 0 & -10 & 16 \end{bmatrix}$

2.2.3. Multiplication

1: $\begin{bmatrix} -113 & 62 & 44 \\ -69 & 1 & -2 \\ 3 & 118 & 109 \end{bmatrix}$ 2: $\begin{bmatrix} -137 & 66 & 63 \\ 43 & -103 & 16 \\ -141 & 58 & 69 \end{bmatrix}$ 3: $\begin{bmatrix} 35 & -94 & -7 \\ -105 & 12 & 57 \\ 115 & -36 & -66 \end{bmatrix}$ 4: $\begin{bmatrix} -85 & -73 & -91 \\ 47 & 44 & 2 \\ 16 & 136 & 40 \end{bmatrix}$ 5: $\begin{bmatrix} -48 & -36 & -90 \\ 30 & 40 & 65 \\ 9 & 49 & 2 \end{bmatrix}$

6: $\begin{bmatrix} -24 & 6 & -20 \\ 6 & -14 & 88 \\ -9 & 71 & -116 \end{bmatrix}$ 7: $\begin{bmatrix} -4 & 27 & -31 \\ 31 & -63 & -46 \\ -15 & 32 & 53 \end{bmatrix}$ 8: $\begin{bmatrix} -6 & 13 & 1 \\ 66 & 12 & 4 \\ -60 & -12 & 44 \end{bmatrix}$ 9: $\begin{bmatrix} -8 & 55 & -31 \\ 16 & 58 & -43 \\ 0 & 144 & -90 \end{bmatrix}$ 10: $\begin{bmatrix} 46 & -69 & 55 \\ -51 & 63 & -123 \\ -54 & 60 & -97 \end{bmatrix}$

2.3. Matrix Properties

2.3.1. Properties

Solution

Row Operations:

Step 1: $r_3 := r_3 - (2)r_1$ $\left[\begin{array}{ccc|ccc} 1 & 4 & 4 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & -6 & -6 & -2 & 0 & 1 \end{array}\right]$

Step 2: $r_1 := r_1 - (4)r_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -4 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & -6 & -6 & -2 & 0 & 1 \end{array}\right]$

Step 3: $r_3 := r_3 - (-6)r_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -4 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 & 6 & 1 \end{array}\right]$

Results:

a) rank(A) = 2

b) nullity(A) = 1

c) det(A) = 0

d) A^{-1} = does not exist

e) $\ker(A) = \text{span}\left\{\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}\right\}$

Solution

Row Operations:

Step 1: $r_1 := r_1 - (-1)r_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 6 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

Step 2: $r_1 := r_1 - (6)r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -6 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

Step 3: $r_2 := r_2 - r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -6 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

Results:

a) rank(A) = 3

b) nullity(A) = 0

c) det(A) = 0

d) $A^{-1} = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

e) $\ker(A) = \{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_3 := r_3 - r_1$ $\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$

Step 2: $r_1 := r_1 - (-1)r_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$

Step 3: $r_1 := r_1 - (4)r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & 1 & -4 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$

Step 4: $r_2 := r_2 - (2)r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & 1 & -4 \\ 0 & 1 & 0 & 2 & 1 & -2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$

Results:

a) rank(A) = 3

b) nullity(A) = 0

c) det(A) = 0

d) $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

e) ker(A) = $\{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_1 \leftrightarrow r_3$ $\left[\begin{array}{ccc|ccc} -1 & 2 & -1 & 0 & 0 & 1 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 2 & -3 & 1 & 0 & 0 \end{array}\right]$

Step 2: $r_1 := -1r_1$ $\left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 0 & 0 & -1 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 2 & -3 & 1 & 0 & 0 \end{array}\right]$

Step 3: $r_1 := r_1 - (-2)r_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 0 & 2 & -1 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 2 & -3 & 1 & 0 & 0 \end{array}\right]$

Step 4: $r_3 := r_3 - (2)r_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 0 & 2 & -1 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 0 \end{array}\right]$

Step 5: $r_1 := r_1 - (-3)r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -4 & -1 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 0 \end{array}\right]$

Step 6: $r_2 := r_2 - (-2)r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -4 & -1 \\ 0 & 1 & 0 & 2 & -3 & 0 \\ 0 & 0 & 1 & 1 & -2 & 0 \end{array}\right]$

Results:

a) rank(A) = 3

b) nullity(A) = 0

c) det(A) = 0

d) $A^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & -2 & 0 \end{bmatrix}$

e) ker(A) = $\{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_1 \leftrightarrow r_2$ $\left[\begin{array}{ccc|ccc} 1 & 2 & -5 & 0 & 1 & 0 \\ 0 & -1 & 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

Step 2: $r_2 := -1r_2$ $\left[\begin{array}{ccc|ccc} 1 & 2 & -5 & 0 & 1 & 0 \\ 0 & 1 & -3 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

Step 3: $r_1 := r_1 - (2)r_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 2 & 1 & 0 \\ 0 & 1 & -3 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

Step 4: $r_1 := r_1 - r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & -1 \\ 0 & 1 & -3 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

Step 5: $r_2 := r_2 - (-3)r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & -1 \\ 0 & 1 & 0 & -1 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

Results:

a) rank(A) = 3

b) nullity(A) = 0

c) det(A) = 0

d) $A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

e) ker(A) = $\{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_3 := r_3 - (-2)r_1$ $\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 1 \end{array}\right]$

Step 2: $r_1 := r_1 - (2)r_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 5 & 1 & -2 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 1 \end{array}\right]$

Results:

a) rank(A) = 2

b) nullity(A) = 1

c) det(A) = 0

d) A^{-1} = does not exist

e) $\ker(A) = \text{span}\left\{\begin{bmatrix}-1\\2\\1\end{bmatrix}\right\}$

Solution

Row Operations:

Step 1: $r_3 := r_3 - (-1)r_1$ $\left[\begin{array}{ccc|ccc}1&-2&-3&1&0&0\\0&1&1&0&1&0\\0&-2&-2&1&0&1\end{array}\right]$

Step 2: $r_1 := r_1 - (-2)r_2$ $\left[\begin{array}{ccc|ccc}1&0&-1&1&2&0\\0&1&1&0&1&0\\0&-2&-2&1&0&1\end{array}\right]$

Step 3: $r_3 := r_3 - (-2)r_2$ $\left[\begin{array}{ccc|ccc}1&0&-1&1&2&0\\0&1&1&0&1&0\\0&0&0&1&2&1\end{array}\right]$

Results:

a) rank(A) = 2

b) nullity(A) = 1

c) det(A) = 0

d) A^{-1} = does not exist

e) $\ker(A) = \text{span}\left\{\begin{bmatrix}1\\-1\\1\end{bmatrix}\right\}$

Solution

Row Operations:

Step 1: $r_1 := 1/5r_1$ $\left[\begin{array}{ccc|ccc}1&-12/5&-12/5&1/5&0&0\\0&1&1&0&1&0\\2&-4&-4&0&0&1\end{array}\right]$

Step 2: $r_3 := r_3 - (2)r_1$ $\left[\begin{array}{ccc|ccc}1&-12/5&-12/5&1/5&0&0\\0&1&1&0&1&0\\0&4/5&4/5&-2/5&0&1\end{array}\right]$

Step 3: $r_1 := r_1 - (-12/5)r_2$ $\left[\begin{array}{ccc|ccc}1&0&0&1/5&12/5&0\\0&1&1&0&1&0\\0&4/5&4/5&-2/5&0&1\end{array}\right]$

Step 4: $r_3 := r_3 - (4/5)r_2$ $\left[\begin{array}{ccc|ccc}1&0&0&1/5&12/5&0\\0&1&1&0&1&0\\0&0&0&-2/5&-4/5&1\end{array}\right]$

Results:

a) rank(A) = 2

b) nullity(A) = 1

c) det(A) = 0

d) A^{-1} = does not exist

e) $\ker(A) = \text{span}\left\{\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}\right\}$

Solution

Row Operations:

Step 1: $r_1 := 1/5 r_1$ $\left[\begin{array}{ccc|ccc} 1 & 7/5 & 1 & 1/5 & 0 & 0 \\ 2 & 3 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

Step 2: $r_2 := r_2 - (2) r_1$ $\left[\begin{array}{ccc|ccc} 1 & 7/5 & 1 & 1/5 & 0 & 0 \\ 0 & 1/5 & 0 & -2/5 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

Step 3: $r_2 := 5 r_2$ $\left[\begin{array}{ccc|ccc} 1 & 7/5 & 1 & 1/5 & 0 & 0 \\ 0 & 1 & 0 & -2 & 5 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

Step 4: $r_1 := r_1 - (7/5) r_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 3 & -7 & 0 \\ 0 & 1 & 0 & -2 & 5 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

Step 5: $r_1 := r_1 - r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -7 & -1 \\ 0 & 1 & 0 & -2 & 5 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

Results:

a) rank(A) = 3

b) nullity(A) = 0

c) det(A) = 0

d) $A^{-1} = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

e) $\ker(A) = \{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_3 := r_3 - (-1) r_1$ $\left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$

Step 2: $r_1 := r_1 - (2) r_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & -2 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$

Step 3: $r_1 := r_1 - r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -2 & -1 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$

Step 4: $r_2 := r_2 - (-2)r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -2 & -1 \\ 0 & 1 & 0 & 2 & 1 & 2 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$

Results:

a) rank(A) = 3

b) nullity(A) = 0

c) det(A) = 0

d) $A^{-1} = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

e) ker(A) = $\{\mathbf{0}\}$

2.3.2. RREF

Solution

Elementary Row Operations:

(1) $r_3 := r_3 - (2)r_1$

$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

(2) $r_1 := r_1 - r_3$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

Result:

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

Solution

Elementary Row Operations:

(1) $r_1 := r_1 - r_2$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

(2) $r_1 := r_1 + (-1)r_2$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

(3) $r_3 := r_3 - r_2$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_2 \coloneqq r_2 + (-1)r_3$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

(2) $r_1 \coloneqq r_1 + (-1)r_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

(3) $r_2 \coloneqq r_2 + (-2)r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_2 \coloneqq r_2 + (-1)r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

(2) $r_3 \coloneqq r_3 + (-1)r_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(3) $r_1 \coloneqq r_1 - r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_2 \coloneqq r_2 - (2)r_1$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 2 & -3 & 0 \end{bmatrix}$$

(2) $r_3 := r_3 + (-2)r_1$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

(3) $r_3 := r_3 - r_2$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_2 := r_2 - r_1$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(2) $r_1 := r_1 - (2)r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_1 := r_1 + (-1)r_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(2) $r_2 := r_2 - r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

(3) $r_2 := r_2 + (-1)r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_3 := r_3 + (-2)r_1$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

(2) $r_3 := r_3 + (-1)r_2$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_3 := r_3 - (2)r_1$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

(2) $r_2 := r_2 + (-1)r_3$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_1 := r_1 + (-2)r_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(2) $r_2 := r_2 + (-2)r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$