Exercise 4:

Foundations of Mathematical, WS24

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This is **exercise** 4 for Foundations of Mathematical, WS24. Generated on 2024-12-16 with 10 problems per section.

2025-01-27

1. Problems

1.1. Vector Arithmetic

1.1.1. Addition

Find the sum of the following vectors $\mathbf{u}$ and $\mathbf{v}$

1. $\mathbf{u} = \begin{bmatrix} -2 \\ -2 \\ -3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 5 \\ 2 \\ -7 \end{bmatrix}$
2. $\mathbf{u} = \begin{bmatrix} -2 \\ -2 \\ 10 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 0 \\ -1 \\ -8 \end{bmatrix}$
3. $\mathbf{u} = \begin{bmatrix} 8 \\ -5 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -1 \\ -7 \\ 9 \end{bmatrix}$
4. $\mathbf{u} = \begin{bmatrix} -8 \\ -2 \\ -4 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 2 \\ -10 \\ -8 \end{bmatrix}$
5. $\mathbf{u} = \begin{bmatrix} -8 \\ 4 \\ 8 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -3 \\ -4 \\ 9 \end{bmatrix}$
6. $\mathbf{u} = \begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -1 \\ -6 \\ 0 \end{bmatrix}$
7. $\mathbf{u} = \begin{bmatrix} 10 \\ 1 \\ 5 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 5 \\ 7 \\ -1 \end{bmatrix}$
8. $\mathbf{u} = \begin{bmatrix} 4 \\ -9 \\ 1 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix}$
9. $\mathbf{u} = \begin{bmatrix} 2 \\ -8 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -8 \\ -6 \\ 9 \end{bmatrix}$
10. $\mathbf{u} = \begin{bmatrix} -3 \\ -5 \\ 10 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 10 \\ 10 \\ -1 \end{bmatrix}$

1.1.2. Subtraction

Find the difference of the following vectors $\mathbf{u}$ and $\mathbf{v}$

1. $\mathbf{u} = \begin{bmatrix} 5 \\ -8 \\ -3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -10 \\ -1 \\ -9 \end{bmatrix}$
2. $\mathbf{u} = \begin{bmatrix} 2 \\ 2 \\ -9 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 5 \\ 2 \\ -3 \end{bmatrix}$
3. $\mathbf{u} = \begin{bmatrix} 0 \\ 4 \\ -6 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 3 \\ 5 \\ 4 \end{bmatrix}$
4. $\mathbf{u} = \begin{bmatrix} -7 \\ 9 \\ -9 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix}$
5. $\mathbf{u} = \begin{bmatrix} 7 \\ -6 \\ -9 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 9 \\ -4 \\ -5 \end{bmatrix}$

6. $\mathbf{u} = \begin{bmatrix} 5 \\ 3 \\ 3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -1 \\ 6 \\ -5 \end{bmatrix}$
7. $\mathbf{u} = \begin{bmatrix} -5 \\ -9 \\ 1 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 5 \\ 4 \\ -4 \end{bmatrix}$
8. $\mathbf{u} = \begin{bmatrix} 4 \\ 9 \\ 9 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -9 \\ -5 \\ -7 \end{bmatrix}$
9. $\mathbf{u} = \begin{bmatrix} 7 \\ 9 \\ 4 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 4 \\ -1 \\ -1 \end{bmatrix}$
10. $\mathbf{u} = \begin{bmatrix} 1 \\ 10 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 10 \\ 3 \\ 8 \end{bmatrix}$

1.1.3. Scalar Multiplication

Find the scalar product of the following vector $\mathbf{u}$ and scalar k

1. $\mathbf{u} = \begin{bmatrix} 0 \\ 0 \\ -6 \end{bmatrix}$ $9\mathbf{v}$.
2. $\mathbf{u} = \begin{bmatrix} 8 \\ -6 \\ -8 \end{bmatrix}$ $-9\mathbf{v}$.
3. $\mathbf{u} = \begin{bmatrix} 6 \\ 10 \\ -8 \end{bmatrix}$ $-8\mathbf{v}$.
4. $\mathbf{u} = \begin{bmatrix} 6 \\ -6 \\ -8 \end{bmatrix}$ $-2\mathbf{v}$.
5. $\mathbf{u} = \begin{bmatrix} 5 \\ 5 \\ -3 \end{bmatrix}$ $-3\mathbf{v}$.
6. $\mathbf{u} = \begin{bmatrix} 9 \\ -5 \\ -6 \end{bmatrix}$ $5\mathbf{v}$.
7. $\mathbf{u} = \begin{bmatrix} 8 \\ -6 \\ 9 \end{bmatrix}$ $4\mathbf{v}$.
8. $\mathbf{u} = \begin{bmatrix} 10 \\ -6 \\ 5 \end{bmatrix}$ $-7\mathbf{v}$.
9. $\mathbf{u} = \begin{bmatrix} -3 \\ 6 \\ 6 \end{bmatrix}$ $-6\mathbf{v}$.
10. $\mathbf{u} = \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}$ $6\mathbf{v}$.

1.2. Matrix Arithmetic

1.2.1. Addition

Find the sum of the following matrices A and B

1. $A = \begin{bmatrix} 1 & 7 & 8 \\ -2 & -2 & 3 \\ -4 & 7 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & -4 & -6 \\ -7 & 2 & 7 \\ -4 & 6 & -8 \end{bmatrix}$

2. $A = \begin{bmatrix} -8 & -4 & 6 \\ -2 & -2 & -7 \\ 1 & -5 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} -7 & 2 & -8 \\ -9 & -3 & -1 \\ -1 & 2 & -8 \end{bmatrix}$
3. $A = \begin{bmatrix} 9 & 8 & -7 \\ 0 & -9 & -10 \\ -4 & -1 & -9 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & -5 & 7 \\ 8 & 0 & -6 \\ -10 & 3 & -8 \end{bmatrix}$
4. $A = \begin{bmatrix} 9 & -6 & -1 \\ 4 & -5 & -4 \\ 2 & -9 & -8 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 & -9 \\ 7 & -5 & 5 \\ 7 & 6 & 7 \end{bmatrix}$
5. $A = \begin{bmatrix} 2 & 6 & -10 \\ 0 & 8 & -6 \\ -4 & 8 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -2 & -9 \\ 1 & 6 & 4 \\ -8 & 9 & 1 \end{bmatrix}$
6. $A = \begin{bmatrix} 0 & -1 & 6 \\ -3 & -6 & 2 \\ -5 & 9 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & -4 \\ -2 & -4 & -2 \\ -2 & -1 & -6 \end{bmatrix}$
7. $A = \begin{bmatrix} 6 & 6 & 2 \\ -4 & 3 & 8 \\ 0 & 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -3 & 3 \\ 2 & 1 & -3 \\ -5 & -10 & -2 \end{bmatrix}$
8. $A = \begin{bmatrix} -7 & 9 & 9 \\ 4 & -2 & 4 \\ -10 & -9 & -8 \end{bmatrix}$ and $B = \begin{bmatrix} -6 & -6 & -7 \\ 7 & -7 & 4 \\ -4 & -10 & -7 \end{bmatrix}$
9. $A = \begin{bmatrix} -2 & 2 & 5 \\ -7 & -5 & -2 \\ 5 & 2 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} -7 & -3 & -9 \\ -10 & -8 & -4 \\ 3 & -2 & 5 \end{bmatrix}$
10. $A = \begin{bmatrix} 8 & -1 & 1 \\ 6 & -8 & 9 \\ -1 & 4 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 6 & -4 \\ -2 & 3 & 7 \\ 2 & -4 & -3 \end{bmatrix}$

1.2.2. Subtraction

Find the difference of the following matrices A and B

1. $A = \begin{bmatrix} 3 & -4 & 5 \\ 2 & -5 & -10 \\ -9 & -6 & -9 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 2 & -9 \\ -4 & 8 & -8 \\ 9 & -3 & -9 \end{bmatrix}$
2. $A = \begin{bmatrix} -1 & 7 & -3 \\ 9 & -7 & -1 \\ -9 & -10 & -9 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -8 & 0 \\ 8 & 7 & -6 \\ -1 & 9 & 6 \end{bmatrix}$
3. $A = \begin{bmatrix} 0 & -6 & 6 \\ 9 & -6 & -3 \\ -2 & -2 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 8 & -10 \\ 9 & 2 & 1 \\ -4 & 9 & -4 \end{bmatrix}$
4. $A = \begin{bmatrix} 3 & 8 & 5 \\ 3 & -9 & -8 \\ 0 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & -9 & -9 \\ 6 & 1 & 7 \\ 1 & -6 & -9 \end{bmatrix}$
5. $A = \begin{bmatrix} -6 & 6 & -4 \\ -6 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -7 & -3 \\ -2 & 8 & -10 \\ -4 & 1 & 7 \end{bmatrix}$
6. $A = \begin{bmatrix} -8 & -4 & -4 \\ -10 & 8 & 3 \\ 9 & -1 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 8 & 2 \\ -2 & -1 & -4 \\ -3 & -8 & -2 \end{bmatrix}$
7. $A = \begin{bmatrix} 4 & -6 & -9 \\ -8 & 8 & 7 \\ 6 & 7 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 8 & 6 \\ -1 & 2 & 6 \\ 5 & -10 & 0 \end{bmatrix}$
8. $A = \begin{bmatrix} -5 & 5 & -1 \\ 8 & -8 & -8 \\ 3 & -5 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -8 & 6 \\ 5 & -7 & 5 \\ -6 & 1 & -10 \end{bmatrix}$

9. $A = \begin{bmatrix} -7 & 4 & 5 \\ -10 & 1 & 9 \\ 9 & 1 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 7 & 6 \\ 5 & 2 & -8 \\ 7 & 1 & 5 \end{bmatrix}$

10. $A = \begin{bmatrix} -8 & -2 & -2 \\ 1 & 3 & -8 \\ 8 & 9 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 8 & -6 \\ 1 & -4 & -10 \\ 9 & 1 & -2 \end{bmatrix}$

1.2.3. Multiplication

Find the product of the following matrices A and B

1. $A = \begin{bmatrix} 7 & 7 & -6 \\ -10 & -6 & 4 \\ 7 & -6 & -8 \end{bmatrix}$ and $B = \begin{bmatrix} -10 & 5 & -6 \\ 9 & -9 & -8 \\ 5 & 5 & -1 \end{bmatrix}$

2. $A = \begin{bmatrix} -2 & -3 & 7 \\ 0 & 1 & 9 \\ -9 & 6 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 1 & -3 \\ 6 & -5 & -7 \\ -10 & 8 & -8 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & 9 & 0 \\ 2 & 5 & -9 \\ -8 & 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -7 & -9 \\ -8 & -8 & -2 \\ 3 & 1 & -10 \end{bmatrix}$

4. $A = \begin{bmatrix} 7 & 8 & 2 \\ 3 & 0 & -6 \\ -3 & -4 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & -7 & 4 \\ 3 & -8 & 7 \\ -3 & 9 & 0 \end{bmatrix}$

5. $A = \begin{bmatrix} -6 & -6 & -5 \\ -9 & -7 & -8 \\ -2 & 7 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 4 & -2 \\ 5 & -3 & 9 \\ -9 & 3 & -2 \end{bmatrix}$

6. $A = \begin{bmatrix} -1 & -6 & 8 \\ -1 & -10 & -10 \\ -2 & 1 & -9 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -4 & 9 \\ -8 & -1 & -5 \\ 2 & -6 & -6 \end{bmatrix}$

7. $A = \begin{bmatrix} -2 & 1 & -7 \\ -3 & -2 & -10 \\ -8 & -1 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 & 4 \\ -1 & -8 & -6 \\ 5 & -8 & -5 \end{bmatrix}$

8. $A = \begin{bmatrix} 7 & 4 & 6 \\ 6 & 8 & -3 \\ 9 & -7 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -10 & 6 \\ -6 & -2 & -2 \\ 5 & -9 & 9 \end{bmatrix}$

9. $A = \begin{bmatrix} -1 & 9 & 1 \\ -9 & 6 & -10 \\ -1 & -8 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -7 & 3 & 1 \\ 3 & 2 & 9 \\ -6 & 0 & 7 \end{bmatrix}$

10. $A = \begin{bmatrix} 9 & 7 & -1 \\ -6 & -2 & -2 \\ 7 & 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & 2 & 7 \\ -8 & -9 & -4 \\ -4 & -1 & -2 \end{bmatrix}$

1.3. Matrix Properties

1.3.1. Properties

For each matrix A, find:

a) rank(A)

b) nullity(A)

c) det(A)

d) A^{-1} (if exists)

e) basis of ker(A)

1. $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix}$

2. $A = \begin{bmatrix} -1 & -1 & 8 \\ 0 & 1 & -2 \\ -1 & 0 & 5 \end{bmatrix}$
3. $A = \begin{bmatrix} 1 & 3 & -1 \\ -1 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
4. $A = \begin{bmatrix} -5 & 11 & -10 \\ -6 & 13 & -11 \\ 2 & -4 & 3 \end{bmatrix}$
5. $A = \begin{bmatrix} 1 & 0 & -1 \\ -2 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$
6. $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
7. $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 5 & -4 \\ 3 & 6 & -3 \end{bmatrix}$
8. $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 3 & -4 \\ -1 & -2 & 3 \end{bmatrix}$
9. $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -2 \\ -1 & -2 & 3 \end{bmatrix}$
10. $A = \begin{bmatrix} 3 & 6 & 6 \\ 0 & 1 & -1 \\ 1 & 4 & 0 \end{bmatrix}$

1.3.2. RREF

Find the Reduced Row Echelon Form of the following matrix A

1. $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$
2. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
3. $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & -2 \\ 0 & 1 & -1 \end{bmatrix}$
4. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & -2 & 5 \end{bmatrix}$
5. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
6. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 0 \end{bmatrix}$
7. $A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$
8. $A = \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 0 \end{bmatrix}$

9. $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

10. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

2. Solutions

2.1. Vector Arithmetic

2.1.1. Addition

$$\begin{bmatrix} 3 \\ 0 \\ -10 \end{bmatrix} \quad \begin{bmatrix} -2 \\ -3 \\ 2 \end{bmatrix} \quad \begin{bmatrix} 7 \\ -12 \\ 11 \end{bmatrix} \quad \begin{bmatrix} -6 \\ -12 \\ -12 \end{bmatrix} \quad \begin{bmatrix} -11 \\ 0 \\ 17 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ -4 \\ -2 \end{bmatrix} \quad \begin{bmatrix} 15 \\ 8 \\ 4 \end{bmatrix} \quad \begin{bmatrix} 7 \\ -13 \\ 2 \end{bmatrix} \quad \begin{bmatrix} -6 \\ -14 \\ 11 \end{bmatrix} \quad \begin{bmatrix} 7 \\ 5 \\ 9 \end{bmatrix}$$

2.1.2. Subtraction

$$\begin{bmatrix} 15 \\ -7 \\ 6 \end{bmatrix} \quad \begin{bmatrix} -3 \\ 0 \\ -6 \end{bmatrix} \quad \begin{bmatrix} -3 \\ -1 \\ -10 \end{bmatrix} \quad \begin{bmatrix} -9 \\ 6 \\ -4 \end{bmatrix} \quad \begin{bmatrix} -2 \\ -2 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} 6 \\ -3 \\ 8 \end{bmatrix} \quad \begin{bmatrix} -10 \\ -13 \\ 5 \end{bmatrix} \quad \begin{bmatrix} 13 \\ 14 \\ 16 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 10 \\ 5 \end{bmatrix} \quad \begin{bmatrix} -9 \\ 7 \\ -6 \end{bmatrix}$$

2.1.3. Scalar Multiplication

1: $\begin{bmatrix} 0 \\ 0 \\ -54 \end{bmatrix}$ 2: $\begin{bmatrix} -72 \\ 54 \\ 72 \end{bmatrix}$ 3: $\begin{bmatrix} -48 \\ -80 \\ 64 \end{bmatrix}$ 4: $\begin{bmatrix} -12 \\ 12 \\ 16 \end{bmatrix}$ 5: $\begin{bmatrix} -15 \\ -15 \\ 9 \end{bmatrix}$

6: $\begin{bmatrix} 45 \\ -25 \\ -30 \end{bmatrix}$ 7: $\begin{bmatrix} 32 \\ -24 \\ 36 \end{bmatrix}$ 8: $\begin{bmatrix} -70 \\ 42 \\ -35 \end{bmatrix}$ 9: $\begin{bmatrix} 18 \\ -36 \\ -36 \end{bmatrix}$ 10: $\begin{bmatrix} -6 \\ 18 \\ -12 \end{bmatrix}$

2.2. Matrix Arithmetic

2.2.1. Addition

1: $\begin{bmatrix} -2 & 3 & 2 \\ -9 & 0 & 10 \\ -8 & 13 & -13 \end{bmatrix}$ 2: $\begin{bmatrix} -15 & -2 & -2 \\ -11 & -5 & -8 \\ 0 & -3 & -13 \end{bmatrix}$ 3: $\begin{bmatrix} 16 & 3 & 0 \\ 8 & -9 & -16 \\ -14 & 2 & -17 \end{bmatrix}$ 4: $\begin{bmatrix} 7 & -3 & -10 \\ 11 & -10 & 1 \\ 9 & -3 & -1 \end{bmatrix}$ 5: $\begin{bmatrix} 2 & 4 & -19 \\ 1 & 14 & -2 \\ -12 & 17 & 2 \end{bmatrix}$

6: $\begin{bmatrix} 2 & -2 & 2 \\ -5 & -10 & 0 \\ -7 & 8 & -9 \end{bmatrix}$ 7: $\begin{bmatrix} 9 & 3 & 5 \\ -2 & 4 & 5 \\ -5 & -8 & 0 \end{bmatrix}$ 8: $\begin{bmatrix} -13 & 3 & 2 \\ 11 & -9 & 8 \\ -14 & -19 & -15 \end{bmatrix}$ 9: $\begin{bmatrix} -9 & -1 & -4 \\ -17 & -13 & -6 \\ 8 & 0 & -1 \end{bmatrix}$ 10: $\begin{bmatrix} 12 & 5 & -3 \\ 4 & -5 & 16 \\ 1 & 0 & 6 \end{bmatrix}$

2.2.2. Subtraction

1: $\begin{bmatrix} -3 & -6 & 14 \\ 6 & -13 & -2 \\ -18 & -3 & 0 \end{bmatrix}$ 2: $\begin{bmatrix} -5 & 15 & -3 \\ 1 & -14 & 5 \\ -8 & -19 & -15 \end{bmatrix}$ 3: $\begin{bmatrix} 3 & -14 & 16 \\ 0 & -8 & -4 \\ 2 & -11 & 13 \end{bmatrix}$ 4: $\begin{bmatrix} -5 & 17 & 14 \\ -3 & -10 & -15 \\ -1 & 7 & 14 \end{bmatrix}$ 5: $\begin{bmatrix} -5 & 13 & -1 \\ -4 & -6 & 11 \\ 6 & -1 & -4 \end{bmatrix}$

6: $\begin{bmatrix} -13 & -12 & -6 \\ -8 & 9 & 7 \\ 12 & 7 & -1 \end{bmatrix}$ 7: $\begin{bmatrix} 4 & -14 & -15 \\ -7 & 6 & 1 \\ 1 & 17 & 6 \end{bmatrix}$ 8: $\begin{bmatrix} -1 & 13 & -7 \\ 3 & -1 & -13 \\ 9 & -6 & 4 \end{bmatrix}$ 9: $\begin{bmatrix} -6 & -3 & -1 \\ -15 & -1 & 17 \\ 2 & 0 & 2 \end{bmatrix}$ 10: $\begin{bmatrix} -9 & -10 & 4 \\ 0 & 7 & 2 \\ -1 & 8 & 2 \end{bmatrix}$

2.2.3. Multiplication

1: $\begin{bmatrix} -37 & -58 & -92 \\ 66 & 24 & 104 \\ -164 & 49 & 14 \end{bmatrix}$ 2: $\begin{bmatrix} -82 & 69 & -29 \\ -84 & 67 & -79 \\ 23 & -7 & -47 \end{bmatrix}$ 3: $\begin{bmatrix} -72 & -79 & -27 \\ -67 & -63 & 62 \\ -10 & 42 & 48 \end{bmatrix}$ 4: $\begin{bmatrix} 67 & -95 & 84 \\ 39 & -75 & 12 \\ -30 & 44 & -40 \end{bmatrix}$ 5: $\begin{bmatrix} -9 & -21 & -32 \\ 1 & -39 & -29 \\ 72 & -44 & 77 \end{bmatrix}$

6: $\begin{bmatrix} 59 & -38 & -27 \\ 55 & 74 & 101 \\ -36 & 61 & 31 \end{bmatrix}$ 7: $\begin{bmatrix} -42 & 52 & 21 \\ -57 & 102 & 50 \\ -48 & 64 & -1 \end{bmatrix}$ 8: $\begin{bmatrix} 6 & -132 & 88 \\ -63 & -49 & -7 \\ 32 & -58 & 50 \end{bmatrix}$ 9: $\begin{bmatrix} 28 & 15 & 87 \\ 141 & -15 & -25 \\ -47 & -19 & -38 \end{bmatrix}$ 10: $\begin{bmatrix} -97 & -44 & 37 \\ 54 & 8 & -30 \\ -39 & 6 & 47 \end{bmatrix}$

2.3. Matrix Properties

2.3.1. Properties

Solution

Row Operations:

Step 1: $r_3 := r_3 - r_1$ $\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$

Step 2: $r_1 := r_1 - (2)r_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$

Step 3: $r_1 := r_1 - (2)r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -2 & -2 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$

Step 4: $r_2 := r_2 - (-1)r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -2 & -2 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$

Results:

a) $\text{rank}(A) = 3$

b) $\text{nullity}(A) = 0$

c) $\det(A) = 0$

d) $A^{-1} = \left[\begin{array}{ccc} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{array}\right]$

e) $\ker(A) = \{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_1 := -1r_1$ $\left[\begin{array}{ccc|ccc} 1 & 1 & -8 & -1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ -1 & 0 & 5 & 0 & 0 & 1 \end{array}\right]$

Step 2: $r_3 := r_3 - (-1)r_1$ $\left[\begin{array}{ccc|ccc} 1 & 1 & -8 & -1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 1 & -3 & -1 & 0 & 1 \end{array}\right]$

Step 3: $r_1 := r_1 - r_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & -6 & -1 & -1 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 1 & -3 & -1 & 0 & 1 \end{array}\right]$

Step 4: $r_3 := r_3 - r_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & -6 & -1 & -1 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 & 1 \end{array}\right]$

Step 5: $r_3 := -1r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & -6 & -1 & -1 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{array}\right]$

Step 6: $r_1 := r_1 - (-6)r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & 5 & -6 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{array}\right]$

Step 7: $r_2 := r_2 - (-2)r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & 5 & -6 \\ 0 & 1 & 0 & 2 & 3 & -2 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{array}\right]$

Results:

a) rank(A) = 3

b) nullity(A) = 0

c) det(A) = 0

d) $A^{-1} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix}$

e) ker(A) = $\{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_2 := r_2 - (-1)r_1$ $\left[\begin{array}{ccc|ccc} 1 & 3 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$

Step 2: $r_1 := r_1 - (3)r_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & -2 & -3 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$

Results:

a) rank(A) = 2

b) nullity(A) = 1

c) det(A) = 0

d) A^{-1} = does not exist

e) ker(A) = span $\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

Solution

Row Operations:

Step 1: $r_1 := -1/5r_1$ $\left[\begin{array}{ccc|ccc} 1 & -11/5 & 2 & -1/5 & 0 & 0 \\ -6 & 13 & -11 & 0 & 1 & 0 \\ 2 & -4 & 3 & 0 & 0 & 1 \end{array}\right]$

Step 2: $r_2 := r_2 - (-6)r_1$ $\left[\begin{array}{ccc|ccc} 1 & -11/5 & 2 & -1/5 & 0 & 0 \\ 0 & -1/5 & 1 & -6/5 & 1 & 0 \\ 2 & -4 & 3 & 0 & 0 & 1 \end{array}\right]$

Step 3: $r_3 := r_3 - (2)r_1$ $\left[\begin{array}{ccc|ccc} 1 & -11/5 & 2 & -1/5 & 0 & 0 \\ 0 & -1/5 & 1 & -6/5 & 1 & 0 \\ 0 & 2/5 & -1 & 2/5 & 0 & 1 \end{array}\right]$

Step 4: $r_2 := -5r_2$ $\left[\begin{array}{ccc|ccc} 1 & -11/5 & 2 & -1/5 & 0 & 0 \\ 0 & 1 & -5 & 6 & -5 & 0 \\ 0 & 2/5 & -1 & 2/5 & 0 & 1 \end{array}\right]$

Step 5: $r_1 := r_1 - (-11/5)r_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & -9 & 13 & -11 & 0 \\ 0 & 1 & -5 & 6 & -5 & 0 \\ 0 & 2/5 & -1 & 2/5 & 0 & 1 \end{array}\right]$

Step 6: $r_3 := r_3 - (2/5)r_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & -9 & 13 & -11 & 0 \\ 0 & 1 & -5 & 6 & -5 & 0 \\ 0 & 0 & 1 & -2 & 2 & 1 \end{array}\right]$

Step 7: $r_1 := r_1 - (-9)r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & 7 & 9 \\ 0 & 1 & -5 & 6 & -5 & 0 \\ 0 & 0 & 1 & -2 & 2 & 1 \end{array}\right]$

Step 8: $r_2 := r_2 - (-5)r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & 7 & 9 \\ 0 & 1 & 0 & -4 & 5 & 5 \\ 0 & 0 & 1 & -2 & 2 & 1 \end{array}\right]$

Results:

a) rank(A) = 3

b) nullity(A) = 0

c) det(A) = 0

d) $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 3 \\ -2 & 2 & 1 \end{bmatrix}$

e) ker(A) = $\{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_2 := r_2 - (-2)r_1$ $\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

Step 2: $r_1 := r_1 - (-1)r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

Step 3: $r_2 := r_2 - r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 2 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

Results:

a) rank(A) = 3

b) nullity(A) = 0

c) det(A) = 0

d) $A^{-1} = \begin{bmatrix} 5 & 2 & -2 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

e) ker(A) = $\{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_1 := r_1 - (-1)r_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

Step 2: $r_1 := r_1 - r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

Results:

a) rank(A) = 3

b) nullity(A) = 0

c) det(A) = 0

d) $A^{-1} = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

e) ker(A) = $\{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_2 := r_2 - (2)r_1$ $\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 3 & 6 & -3 & 0 & 0 & 1 \end{array}\right]$

Step 2: $r_3 := r_3 - (3)r_1$ $\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 1 \end{array}\right]$

Step 3: $r_1 := r_1 - (2)r_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 5 & -2 & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 1 \end{array}\right]$

Results:

a) rank(A) = 2

b) nullity(A) = 1

c) det(A) = 0

d) A^{-1} = does not exist

e) ker(A) = span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\}$

Solution

Row Operations:

Step 1: $r_2 := r_2 - r_1$ $\left[\begin{array}{ccc|ccc} 1 & 2 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ -1 & -2 & 3 & 0 & 0 & 1 \end{array}\right]$

Step 2: $r_3 := r_3 - (-1)r_1$ $\left[\begin{array}{ccc|ccc} 1 & 2 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$

Step 3: $r_1 := r_1 - (2)r_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 3 & -2 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$

Step 4: $r_1 := r_1 - (2)r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & -2 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$

Step 5: $r_2 := r_2 - (-2)r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & -2 \\ 0 & 1 & 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$

Results:

a) rank(A) = 3

b) nullity(A) = 0

c) det(A) = 0

d) $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

e) ker(A) = $\{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_3 := r_3 - (-1)r_1$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & -2 & 5 & 1 & 0 & 1 \end{array}\right]$

Step 2: $r_3 := r_3 - (-2)r_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 2 & 1 \end{array}\right]$

Step 3: $r_1 := r_1 - (2)r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -4 & -2 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 2 & 1 \end{array}\right]$

Step 4: $r_2 := r_2 - (-2)r_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -4 & -2 \\ 0 & 1 & 0 & 2 & 5 & 2 \\ 0 & 0 & 1 & 1 & 2 & 1 \end{array}\right]$

Results:

a) rank(A) = 3

b) nullity(A) = 0

c) det(A) = 0

d) $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

e) ker(A) = $\{\mathbf{0}\}$

Solution

Row Operations:

Step 1: $r_1 := 1/3r_1$ $\left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1/3 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 1 & 4 & 0 & 0 & 0 & 1 \end{array}\right]$

Step 2: $r_3 := r_3 - r_1$ $\left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1/3 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 2 & -2 & -1/3 & 0 & 1 \end{array}\right]$

Step 3: $r_1 := r_1 - (2)r_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 1/3 & -2 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 2 & -2 & -1/3 & 0 & 1 \end{array}\right]$

Step 4: $r_3 := r_3 - (2)r_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 1/3 & -2 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1/3 & -2 & 1 \end{array}\right]$

Results:

a) rank(A) = 2

b) nullity(A) = 1

c) det(A) = 0

d) A^{-1} = does not exist

e) ker(A) = span $\left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \right\}$

2.3.2. RREF

Solution

Elementary Row Operations:

(1) $r_1 := r_1 + (-1)r_2$

$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

(2) $r_2 := r_2 + (-1)r_1$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

(3) $r_3 := r_3 + (-1)r_1$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Result:

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Solution

Elementary Row Operations:

(1) $r_3 := r_3 + (-1)r_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(2) $r_1 \coloneqq r_1 + (-2)r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_2 \coloneqq r_2 - r_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 1 & -1 \end{bmatrix}$$

(2) $r_3 \coloneqq r_3 + (-1)r_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

(3) $r_2 \coloneqq r_2 - r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_3 \coloneqq r_3 - (2)r_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

(2) $r_2 \coloneqq r_2 - (2)r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_3 := r_3 - r_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

(2) $r_3 := r_3 + (-2)r_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_3 := r_3 - (2)r_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(2) $r_2 := r_2 - r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_2 := r_2 - (2)r_3$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

(2) $r_2 := r_2 - r_3$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_1 := r_1 + (-1)r_2$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 0 \end{bmatrix}$$

(2) $r_3 := r_3 - (2)r_2$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(3) $r_2 := r_2 - r_3$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_2 := r_2 + (-1)r_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(2) $r_1 := r_1 + (-2)r_3$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

Elementary Row Operations:

(1) $r_3 := r_3 + (-2)r_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 0 \end{bmatrix}$$

(2) $r_3 := r_3 - (2)r_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$