Project: Rest-to-Rest control of a 2-DoF Robot

An often occurring control task in industry is to compute optimal input trajectories for robots to reach a given end position while consuming minimal energy or to reach the end position in minimal time. In this project we want to tackle this challenges by controlling a Two degree of Freedom (2-DoF) robot shown in Figure 1. The dynamic model of the robot is given by a vector-valued second order differential equation

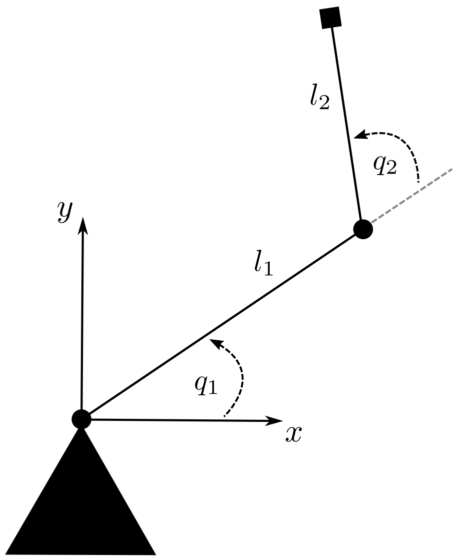

Figure 1: 2-DoF robot.

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u \tag{1}$$

in terms of the joint angles $q = (q_1, q_2)^\top \in \mathbb{R}^2$ and the input torques $u = (u_1, u_2)^\top \in \mathbb{R}^2$. The coefficient matrices are given by

$$\begin{aligned}
B(q) &= \begin{pmatrix} b_1 + b_2\cos(q_2) & b_3 + b_4\cos(q_2) \\ b_3 + b_4\cos(q_2) & b_5 \end{pmatrix} \in \mathbb{R}^{2\times 2}, \\
C(q,\dot{q}) &= -c_1 \sin(q_2) \begin{pmatrix} \dot{q}_1 & \dot{q}_1 + \dot{q}_2 \\ -\dot{q}_1 & 0 \end{pmatrix} \in \mathbb{R}^{2\times 2}, \\
g(q) &= \Big(g_1\cos(q_1) + g_2\cos(q_1+q_2), \quad g_2\cos(q_1+q_2)\Big)^\top \in \mathbb{R}^{1\times 2}.
\end{aligned}$$

If you are not familiar with robot modeling you can have a look at [1] to recall the basic modeling steps.

Modeling

a) What would be your approach to obtain model (1) from basic physical laws? Why is the model formulated in q_1-q_2-coordinates and not in x-y-coordinates? Explain briefly.

b) Model (1) is given as an implicit system of second-order differential equations. Reformulate it as an explicit first-order system in state space form $\dot{x} = f(x,u)$. Choose an appropriate state vector x.

Table 1: Parameters for system (1).

b_1	200.0	[kg m^2/rad]	b_2	50.0	[kg m^2/rad]
b_3	23.5	[kg m^2/rad]	b_4	25.0	[kg m^2/rad]
b_5	122.5	[kg m^2/rad]	c_1	-25.0	[Nms^{-2}]
g_1	784.8	[Nm]	g_2	245.3	[Nm]
l_1	0.5	[m]	l_2	0.5	[m]

Open-loop Optimal Control

a) Formulate at least two OCPs that drive the robot from the initial position $q = (-5, -4)^\top \,\mathrm{rad},\ \dot{q} = (0, 0)^\top \,\mathrm{rad/s}$ to the upper position $q = (\frac{\pi}{2}, 0)^\top \,\mathrm{rad},\ \dot{q} = (0, 0)^\top \,\mathrm{rad/s}$ in $3\mathrm{s}$. Thereby, the input constraints $u \in [-1000 \ \ 1000]\,\mathrm{Nm}$ and the state constraints $\dot{q} \in [-\frac{3}{2}\pi \ \ \frac{3}{2}\pi]\,\mathrm{rad/s}$ should be satisfied for all t. Motivate your problem formulation.

b) Solve one of the OCPs of a) numerically. Use different numbers of sampling points and at least two different integrators. Compare the simulation results. Plot your results in the q_1-q_2-plane and the x-y-plane.

c) Now formulate a problem that reaches the end position from a) in minimum time.

d) Reformulate the OCP of c) as a fixed end-time problem by using the time transformation $t := \tau\, t_e$.

e) Solve your problem formulation from d) numerically. What is the minimum time needed? Try as well a problem formulation to find a compromise between minimal time and minimal control energy.

Model Predictive Control

a) What type of uncertainties/disturbances can you imagine for the robot? Formulate them mathematically.

b) Design an MPC controller for the fixed end-time problem a) from the previous exercise.

c) Assume incorrect parameters in the model of $b_1 = 180\,\mathrm{kg\,m^2/rad}$ and $b_2 = 45\,\mathrm{kg\,m^2/rad}$. (When would this happen in practice?) Simulate the open loop and the closed loop system. Plot your results and describe the differences.

d) Now assume an additive Gaussian measurement error of appropriate variance. Implement this situation and simulate your closed-loop system for the fixed end-time problem. What do you observe? Plot your results.

e) Analyze the stability properties of your designed MPC controller.

References

[1] Siciliano, B., Sciavicco, L., Villani, L., & Oriolo, G. (2010). "Robotics: Modelling, Planning and Control". Springer Science & Business Media.