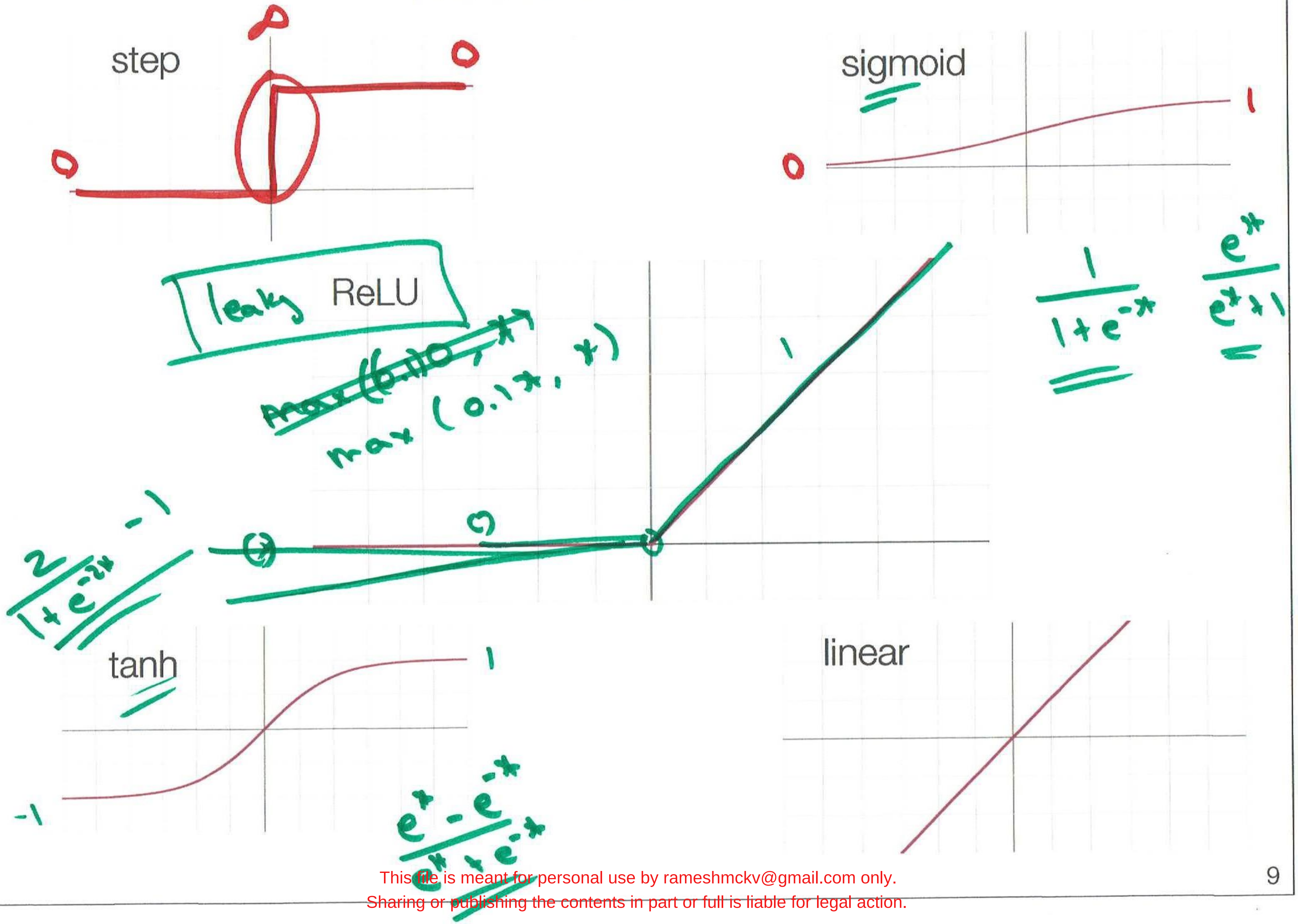
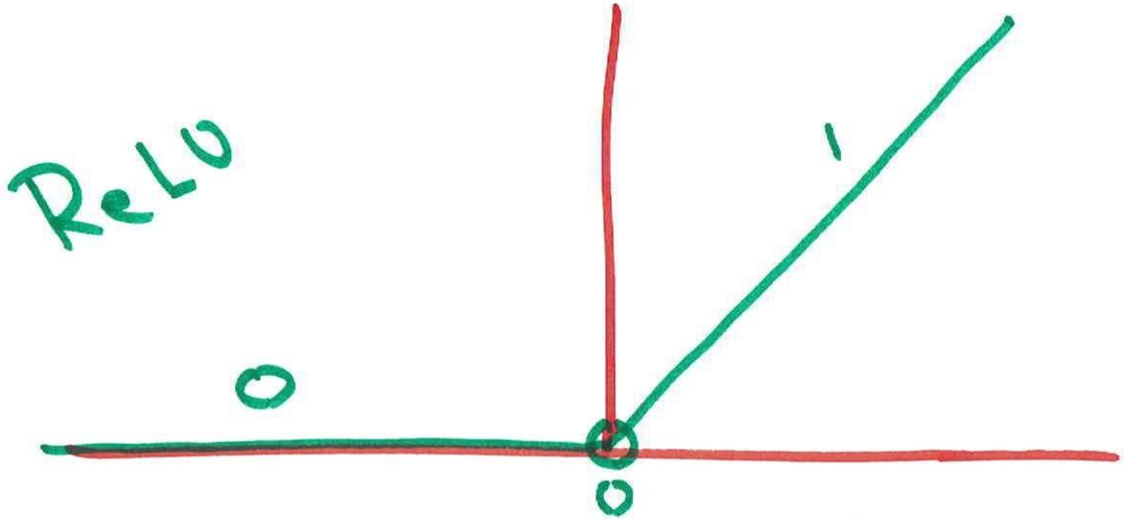
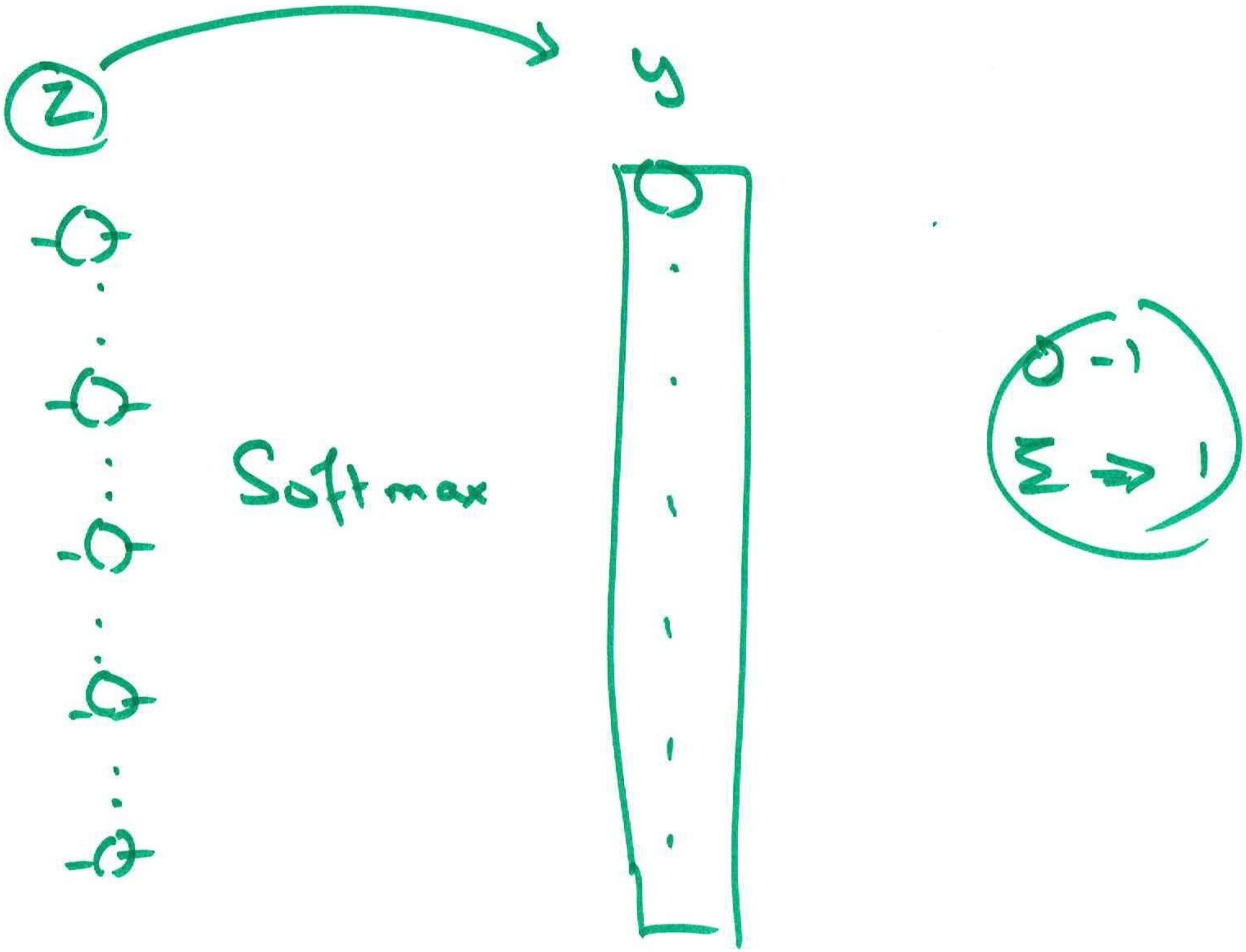
Activation Functions





$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

$$f(x) = \max(0, x)$$


z
y
Softmax
0-1
Σ ⇒ 1

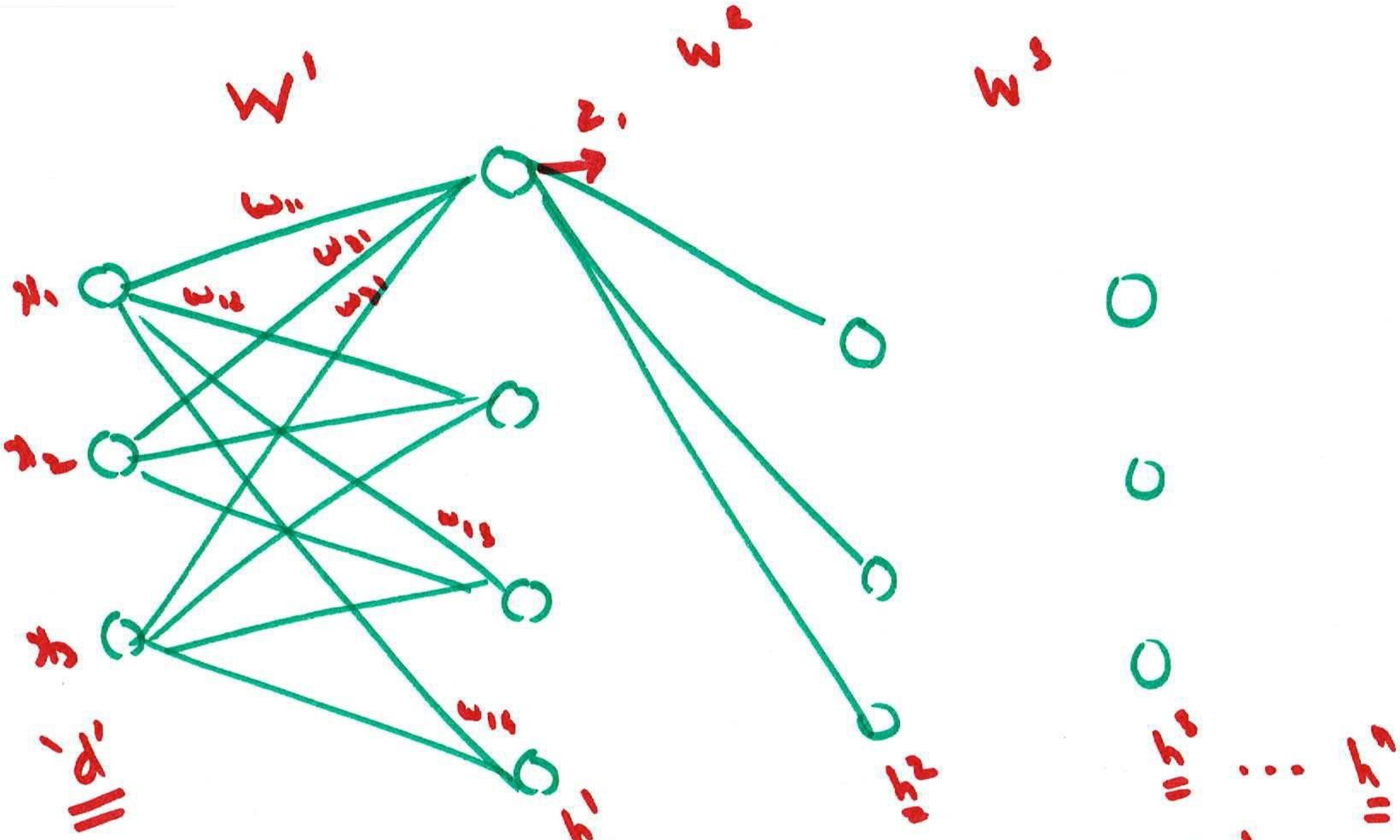
z

y

$$\boxed{y_i = \frac{e^{z_i}}{\sum e^{z_i}}}$$

$$w^{new} = w^{old} - \eta \nabla_w l(w)$$

$$= w^{old} - \frac{1}{N} \eta \sum \nabla_w l_i(w)$$



$$z_1 = f(w_{11}x_1 + w_{21}x_2 + w_{31}x_3 + b)$$

$$\boxed{z_j = f\left(\sum_i w_{ij}x_i + b_j\right)}$$

$$W^1 = \begin{pmatrix} w_{11} & w_{21} & w_{31} & \cdots & w_{d1} \\ w_{12} & w_{22} & w_{32} & & \\ & & \ddots & & \\ w_{1j} & w_{2j} & w_{3j} & \cdots & w_{dj} \\ w_{1h^1} & \cdots & \cdots & \cdots & w_{dh^1} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{pmatrix}$$

$h^1 \times d$ matrix

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{pmatrix}$$

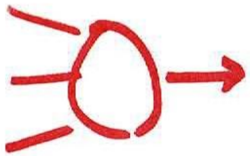
$\sum w_{ij} x_i + b$

$$\boxed{\hat{y} = f^n(\ldots f^3(W^3 f^2(W^2 f^1(W^1 X + b^1) + b^2) + b^3)) \ldots)}$$

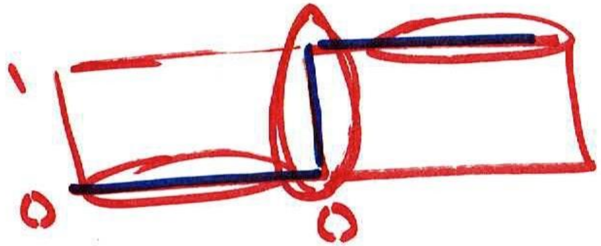
$\hat{y}$: $h^n \times 1$

$\underbrace{\underbrace{W^1}_{h^1 \times d} \underbrace{X}_{d \times 1}}_{h^1 \times 1}$, $\underbrace{b^1}_{h^1 \times 1}$ → $h^1 \times 1$

$\underbrace{\underbrace{W^2}_{h^2 \times h^1} \quad h^1 \times 1}_{h^2 \times 1}$, b^2: $h^2 \times 1$



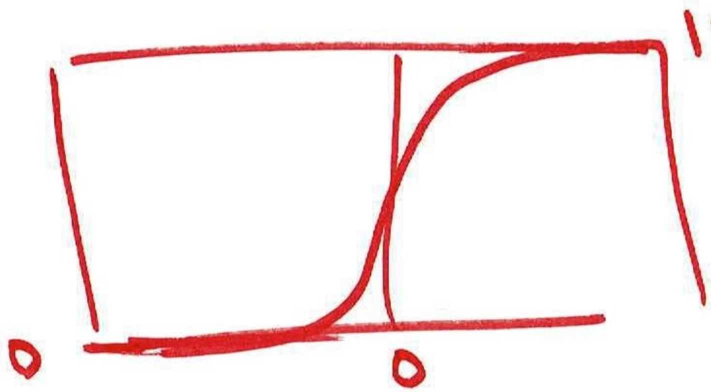
$$\underline{\mathrm{Step}\,(w_1x_1+w_2x_2+w_3x_3+b)}$$

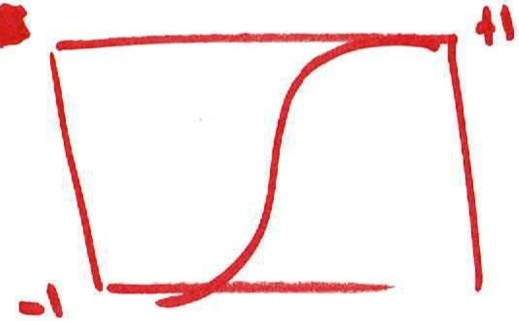


Sigmoid

$$\sigma(x)=\frac{1}{1+e^{-x}}$$

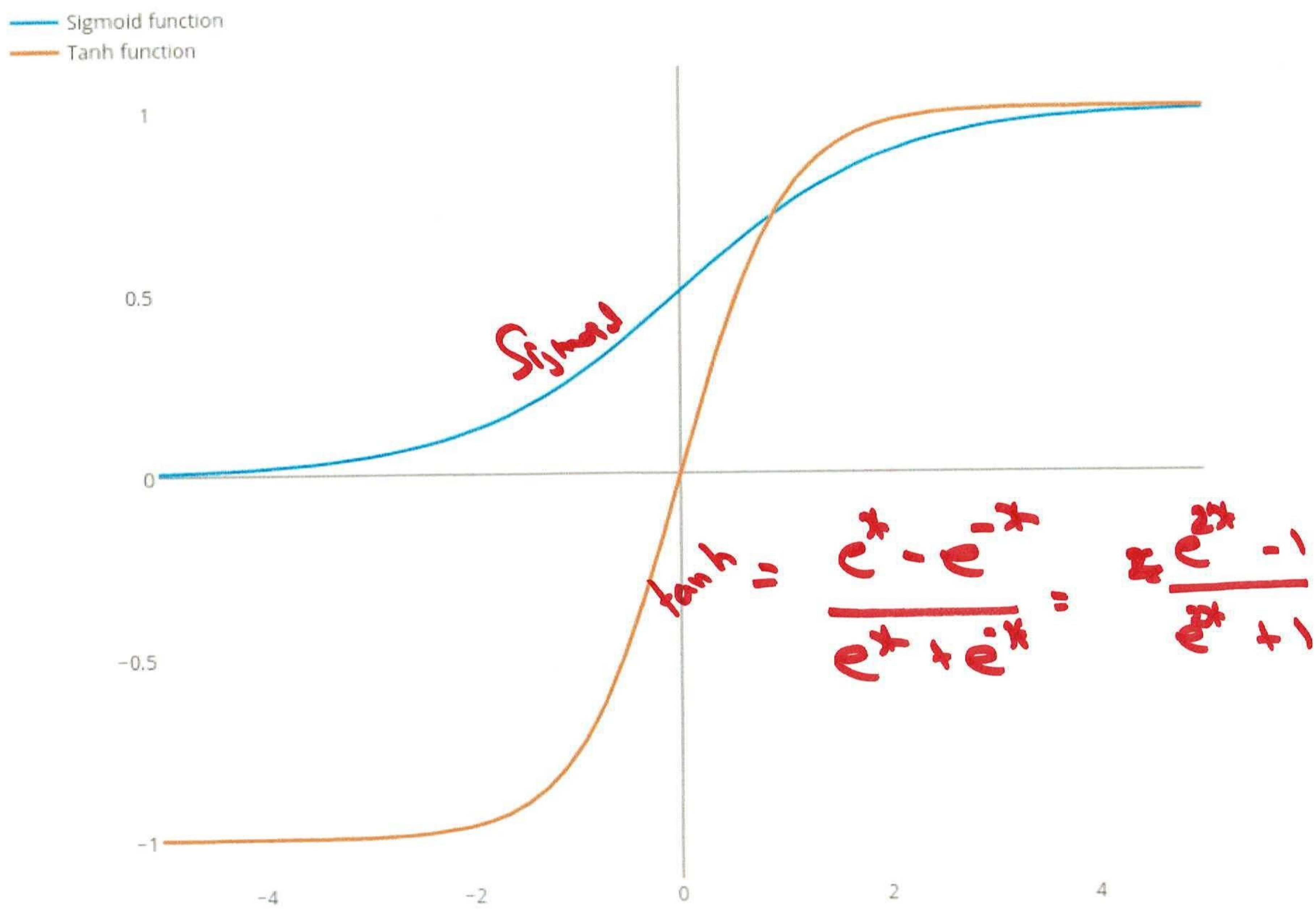
$$=\frac{e^{x}}{e^{x}+1}$$



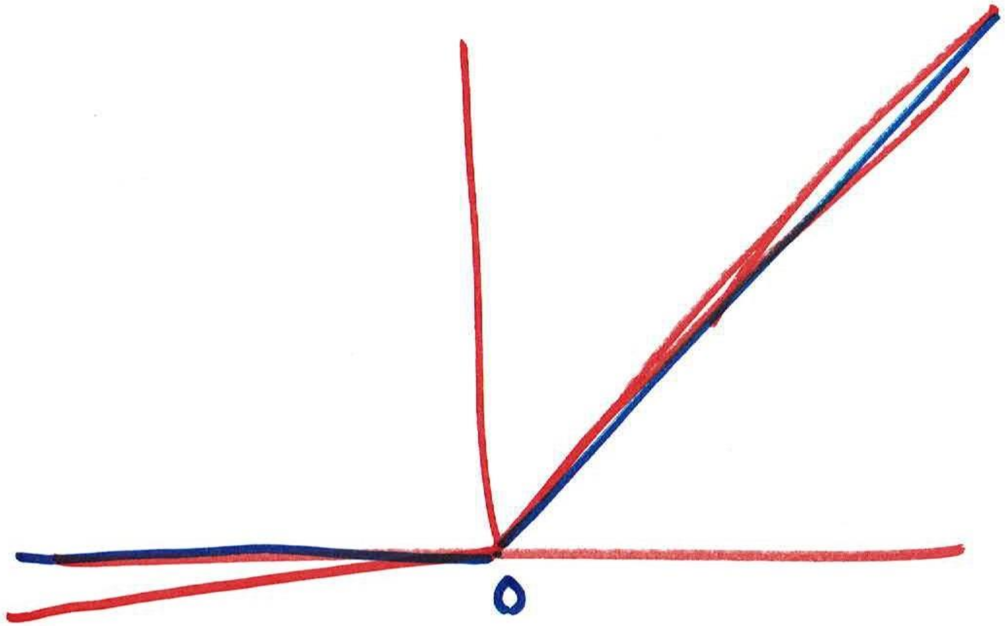


$$\boxed{\cancel{2\sigma(x)}-1}$$

$$\boxed{\tanh = \underbrace{2\sigma(2x)}-1}$$


Sigmoid function
Tanh function
1
0.5
0
−0.5
−1
−4
−2
0
2
4
Sigmoid
tanh = $\frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$

ReLU

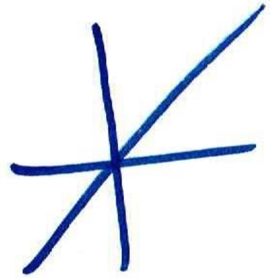


$$= \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

$$= \max(0, x)$$

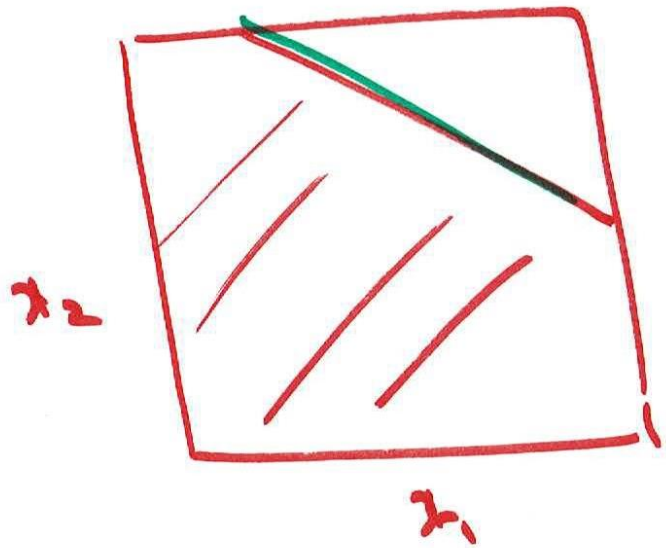
$\max(0.01x, x)$

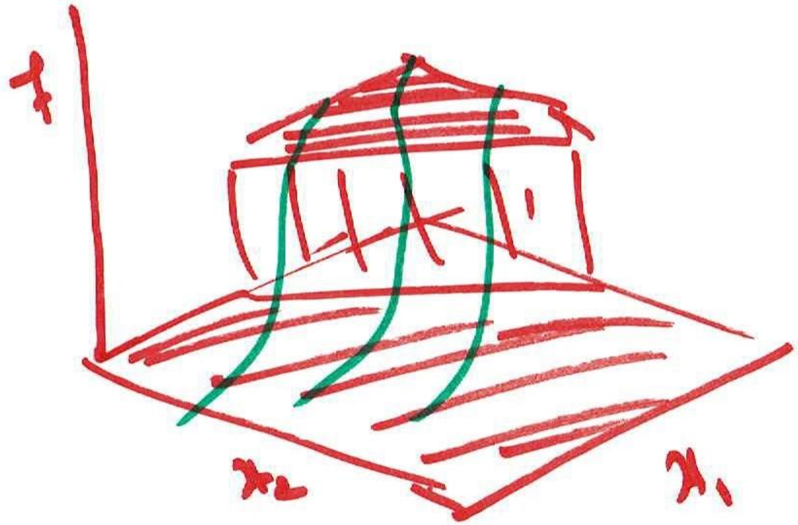
linear = x

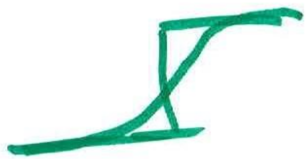


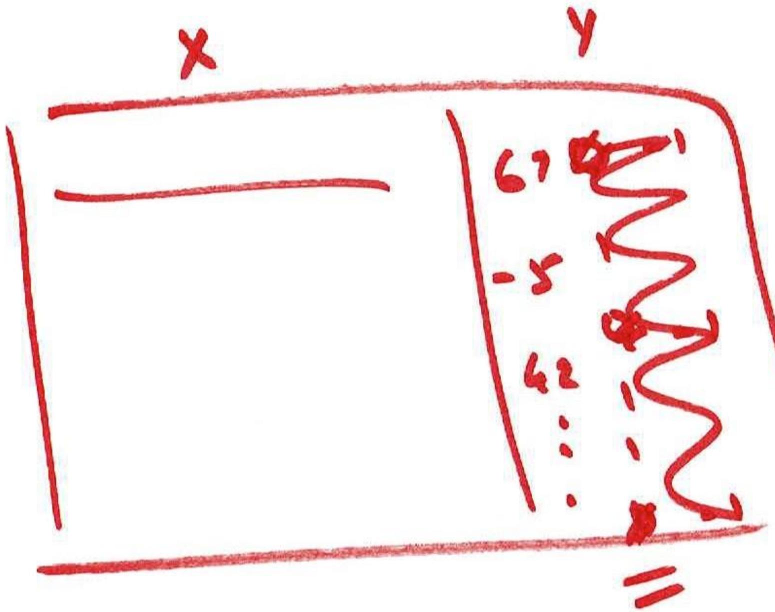
$$\text{Step} = f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

$$f(w_1 x_1 + w_2 x_2 + b)$$





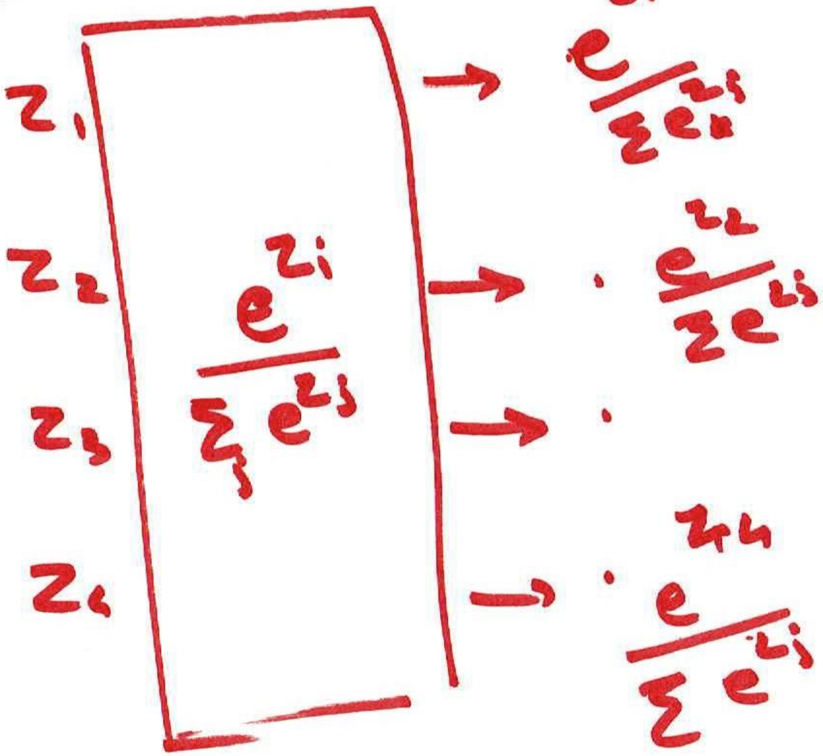




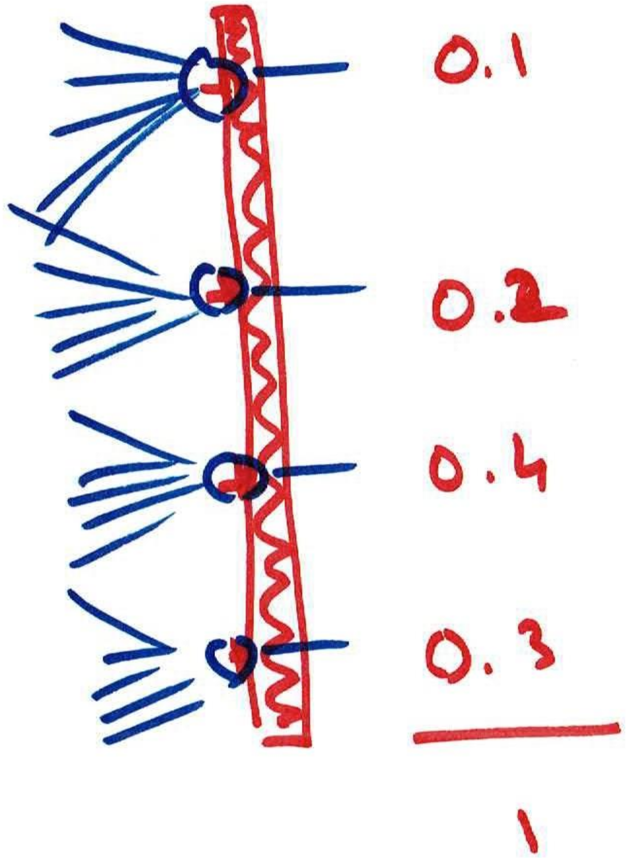
	Output nodes	hidden layer
Classification →	Sigmoid, tanh, Softmax	Sigmoid ✓
		tanh ✓
		$\boxed{\text{ReLU ✓}}$
		~~linear~~
Reg →	linear	

$$\boxed{\hat{a} + \hat{b}\,(a + b x)}$$

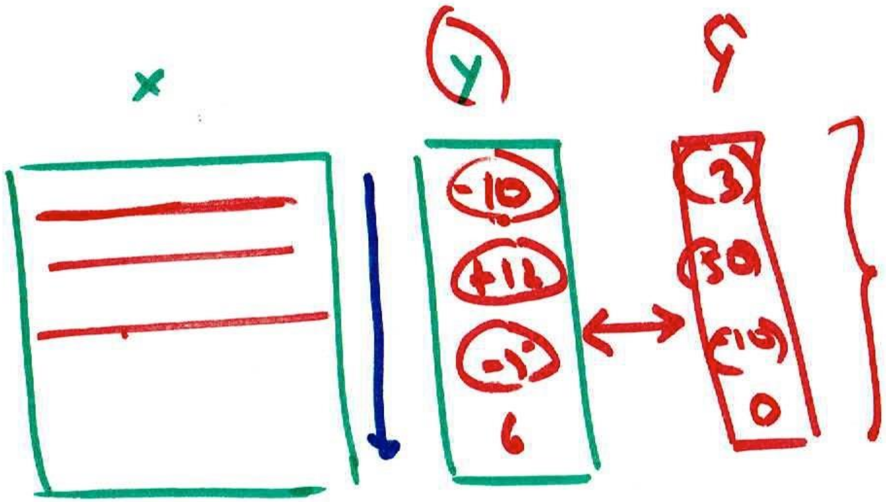
Softmax

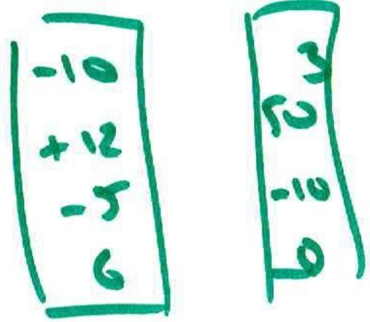


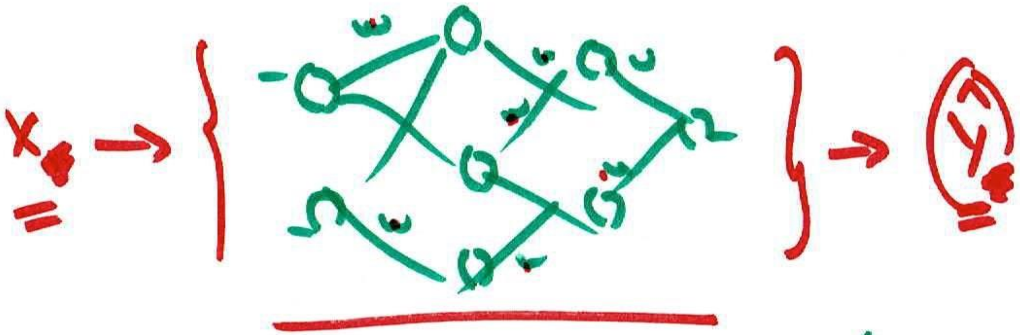
z_1
z_2
z_3
z_4
$\frac{e^{z_i}}{\sum_j e^{z_j}}$
$\frac{e^{z_1}}{\sum e^{z_j}}$
$\frac{e^{z_2}}{\sum e^{z_j}}$
$\frac{e^{z_4}}{\sum e^{z_j}}$

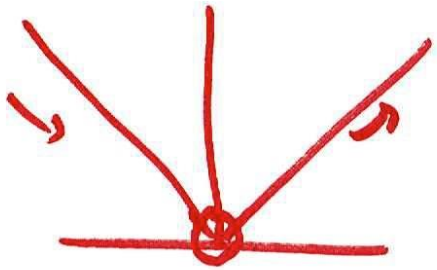


0.1
0.2
0.4
0.3
1









Loss Function $L(y, \hat{y})$

$$\boxed{\frac{1}{N}\sum_i \left(y_i - \hat{y}_i\right)^2}$$

L_2 loss
MSE
SSE

Reg

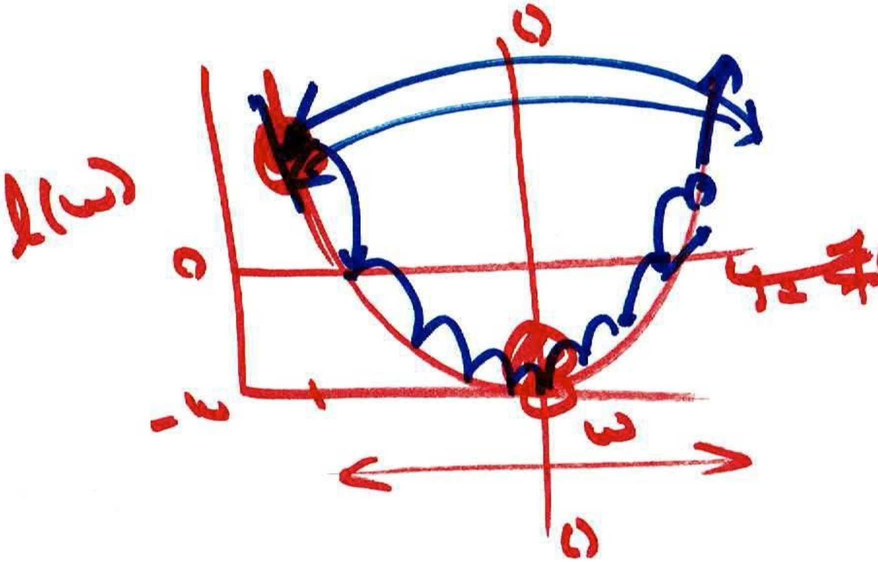
$$L(y, \hat{y}) = l(\omega)$$

Classification $L(y, \hat{y}) = -\left(y_i \log(\hat{y}_i) + (1-y_i)(\log(1-\hat{y}_i))\right)$

Cross entropy loss

How min $L(y, \hat{y})$ by ~~chag~~ Changing ~~to~~ my $W^1, W^2, \ldots W^n$

$$\begin{matrix} w_{11} & w_{12} & \ldots \end{matrix}$$



$$y_2 = x^2 - 10 = -10$$

$$\frac{dy}{dx} = 2x = 0$$

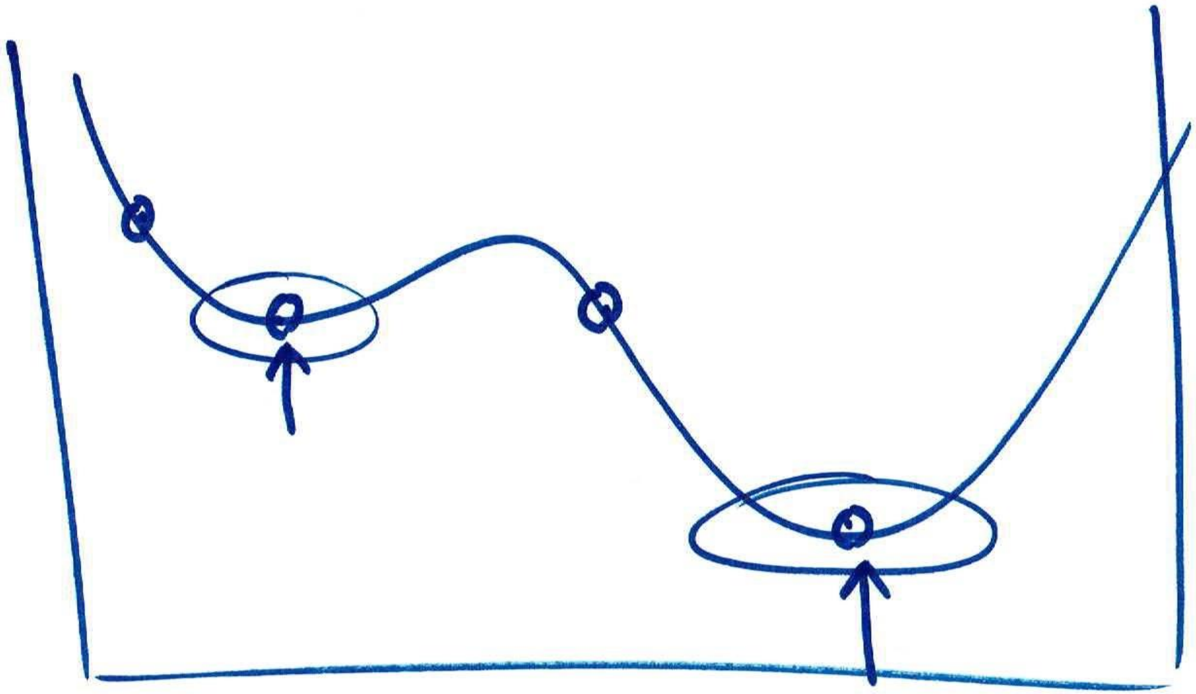
$$x = 0$$

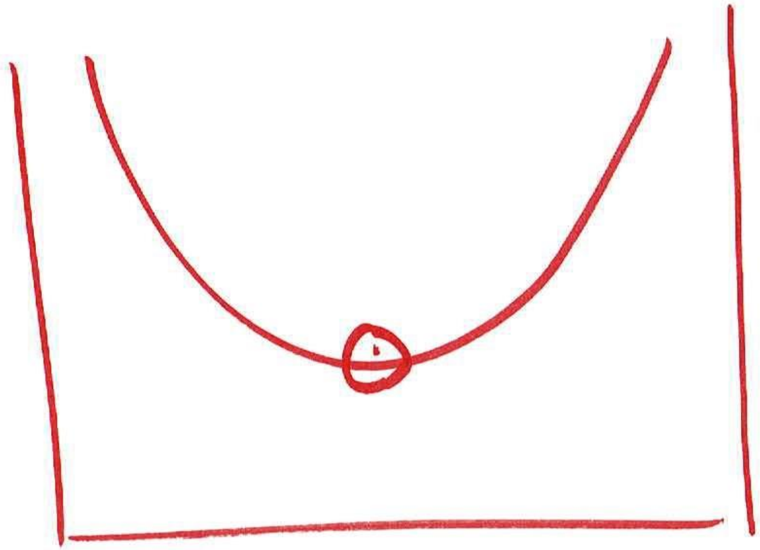
$$\frac{dL}{dw} = \boxed{\quad} = 0$$

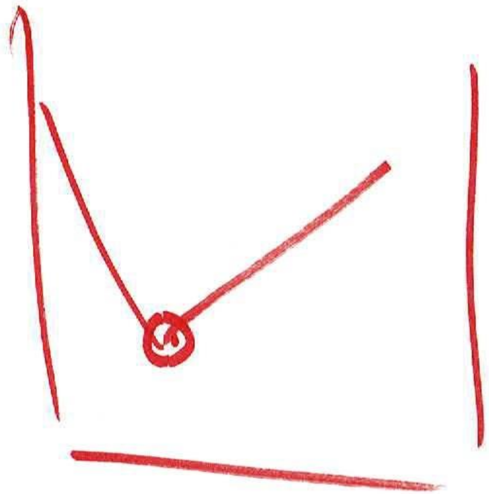
$$\frac{dl}{dw}$$

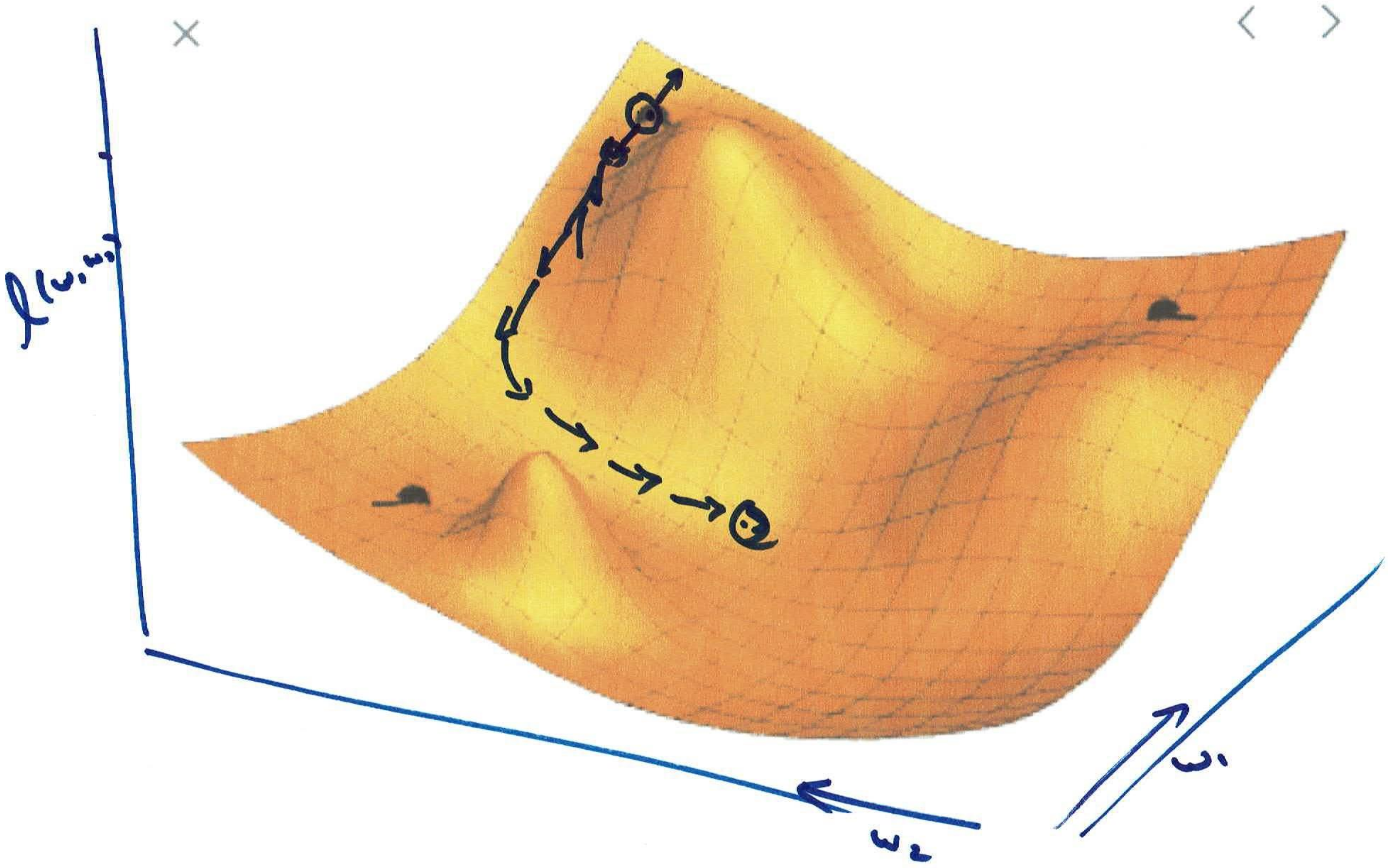
$$w^{new} = w - \eta \nabla_w l$$

learning rate









$l(w_1, w_2)$
w_1
w_2

$$w^{new} = w^{old} - \underline{\eta \nabla_w l(w)}$$

$$= w^{old} - \frac{1}{N} \eta \sum \nabla_w l_i(w)$$

SGD

$$w^{new} = w^{old} - \eta \nabla_w l_i(w)$$

$$\boxed{w^{new} = w^{old} - \frac{1}{N} \eta \underbrace{\sum}_{\text{over a min batch}} \nabla_w l_i(w)}$$

Loss

Function of w ($l(w)$)

$$L = \frac{1}{N} \sum \left(\underline{y_i} - \underbrace{\underline{f^n(\ldots f(w^2 f^1(w^1 x + b^1) + b^2))_i}}_{} \right)^2$$

Chain Rule

$$f(g(h(\ldots x \ldots)))$$

$$\frac{df}{dx} = \boxed{\frac{df}{dg} \cdot \frac{dg}{dh}} \cdot \frac{dh}{dx}$$

Back propagation