Radar Signal Processing

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1 Signal Transmission

The transmitted signal as a function of time (seconds) is given by

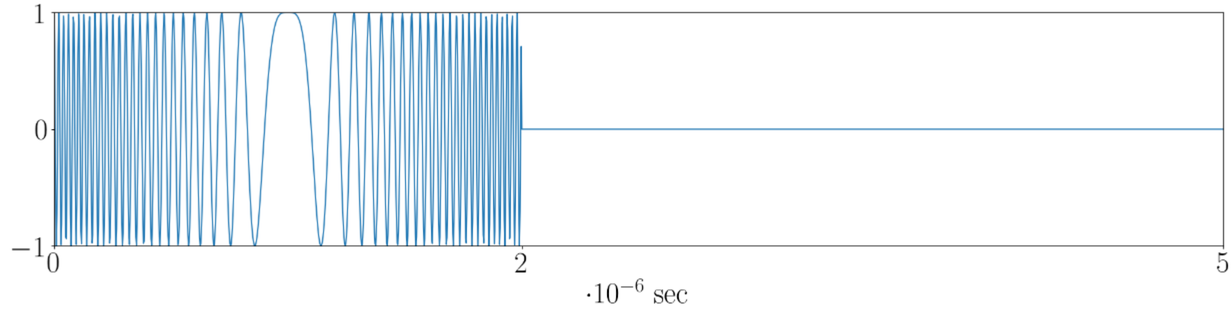
$$s_{\mathrm{Tx}}\left(t\right)=\sum_{n=0}^{N}h\left(t-n\cdot\mathrm{PRI}\right)\exp\left(i2\pi f_{c}t\right),$$

for f_c the carrier frequency (Hz), PRI the time length (seconds) of a single PRI, and N the number of PRIs in the CPI. The function h represents a single chirp and is given by

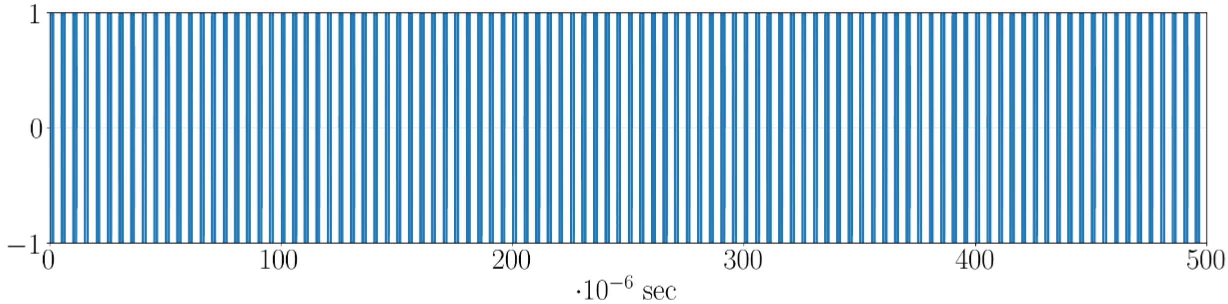
$$h\left(t\right)=\begin{cases}\exp\left(i\pi\cdot\frac{B}{\mathrm{pw}}\left(t-\frac{\mathrm{pw}}{2}\right)^{2}\right), & 0\leq t<\mathrm{pw},\\ 0, & \text{otherwise},\end{cases}$$

for B the bandwidth (Hz) and pw the pulse width (seconds).

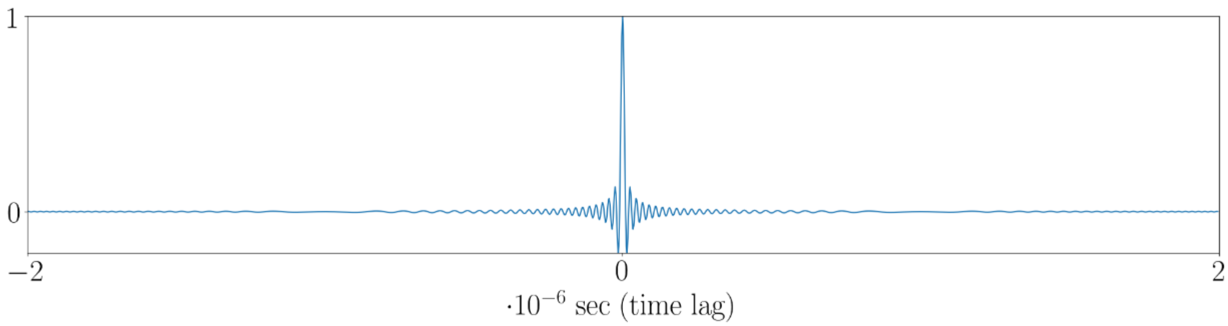
In the attached code, which is pure NumPy, we constructed the transmitted signal using solely the equations defined above. In order to generate the continuous transmitted signal, we divided each PRI into 2000 time segments (i.e., time "samples"), that include the values of pwμsec and PRIμsec. Now, given the parameters $\mathrm{PRI}=5\mu$sec, $\mathrm{pw}=2\mu$sec, $\mathrm{CPI}=0.5$msec, $B=100$MHz and $f_c=40$GHz, we scale them into Hz and seconds, and plot the transmitted signal over a single PRI (including the reception time of length $5-2=3\mu$sec).



As for the entire CPI, it consists of $N = \text{CPI}/\text{PRI} = 100$ pulses, where all pulses are assumed to be the same.



When plotting the auto-correlation function of a transmitted pulse, we see a relatively week correlation of the signal with time shifted versions of itself (where 1 indicates perfect positive correlation).



2 Signal Reception

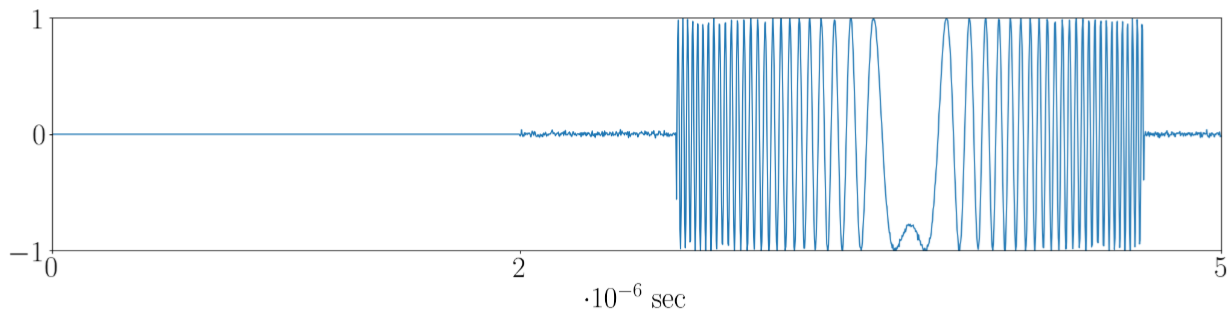
The received signal, as a function of time (seconds), is modeled as

$$s_{\text{Rx}}(t) = w(t) + \sum_{l=1}^{L} \sigma_l \cdot s_{\text{Tx}}(t - \tau_l(t)),$$

for L the number of targets, $\tau_l(t) = \frac{2R_l(t)}{C}$ the time delay due to the target's speed where $R_l(t)$ is the target's range (meters) as a function of time, σ_l the amplitude for each target, $w(t)$ thermal complex AWGN, and C the speed of light (meters per second).

Assuming each target moves at a constant speed v_l (could be negative for approaching targets) on the line between the target and the radar, then we can model $R_l(t) = R_l^0 + v_l \cdot t$, where R_l^0 is the initial range of target l.

For example, given the parameters (for a single target) of $R^0 = 400\text{m}$, $v = 100\frac{\text{m}}{\text{sec}}$, $\sigma = 1$ and noise variance of 0.01^2, we can plot the first PRI of the received signal:



2.1 Target Detection Using Range-Doppler Map

We construct a range-Doppler map for target detection. In order to generate the map, we process the received signal over the entire CPI as follows (see attached code):

1. Stacking the reception segments (one for each PRI) as rows of a complex matrix, denoted as $\mathbf{R}$. Notice that each such segment is a vector with entries corresponding to the time samples. Hence, if such segment is constructed by $r \geq 0$ samples, then the matrix $\mathbf{R}$ has N rows and r columns (i.e., $\mathbf{R} \in \mathbb{C}^{N \times r}$).

2. Correlating each row of the matrix $\mathbf{R}$ (after down-converting the carrier frequency to base-band) with the transmitted signal complex vector (the transmission time segment of a single PRI), denoted by $\mathbf{e}$. Assuming that the transmitted signal is constructed by $e \geq 0$ time samples, then the vector $\mathbf{e}$ is of length e (i.e., $\mathbf{e} \in \mathbb{C}^{e}$).

   We denote by $\mathbf{R}_i \in \mathbb{C}^r$ the i-th row of $\mathbf{R}$. The correlation of $\mathbf{R}_i$ (after down-conversion) with $\mathbf{e}$, is given by the operations

   $$\texttt{convolve}\left(\texttt{conjugate}\left(\texttt{flip}\left(\mathbf{e}\right)\right), \mathbf{R}_i \odot \mathbf{o}\right) \in \mathbb{C}^{e+r-1},$$

   where $\mathbf{o} \in \mathbb{C}^r$ is a vector for down-conversion and $\odot$ is element-wise multiplication. The operation `flip` outputs a vector in its reversed order, `conjugate` is element-wise complex conjugation, and `convolve` is the convolution operation of two vectors.

   As the output of the correlation process is a vector of length $e + r - 1$ (due to the convolution operation), then the complex matrix obtained after correlation has N rows and $e + r - 1$ columns.

3. Applying `fft` and then `fftshift` to each of the $e + r - 1$ columns of the correlation matrix. The obtained complex matrix is of size $N \times (e + r - 1)$.

4. Applying $10 \log_{10}$ on the absolute values of each element of the last matrix (conversion to dB). The resulting real matrix $\mathbf{M} \in \mathbb{R}^{N \times (e+r-1)}$ is the range-Doopler map, up to scaling of its axes.

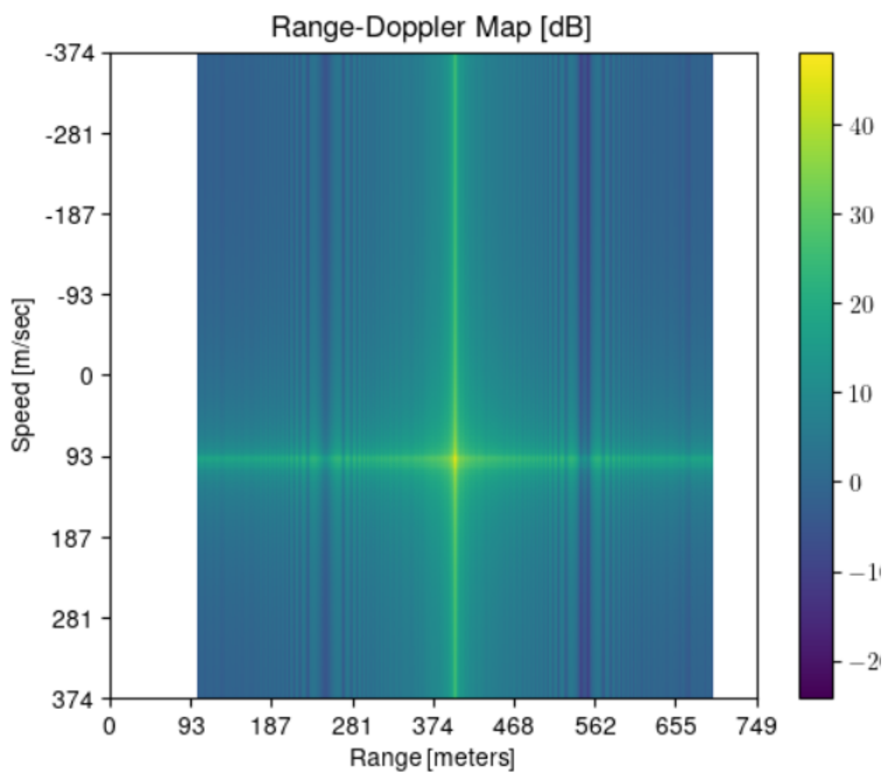
5. Scaling the range: the current x-axis is indexed 0 to $e+r-1$. Multiply its indices by
$$\frac{1}{e+r-1}\cdot\frac{1}{2\text{PRI}}\cdot C\cdot\text{PRI}^2,$$
to get the range in meters (from 0 to the maximal detection range).

6. Scaling the speed: the current y-axis is indexed 0 to N. Shift its indices by $N/2$ and then multiply them by
$$\frac{2}{N}\cdot\frac{1}{2\text{PRI}}\cdot\frac{-C}{2f_c},$$
to get the speed in $\frac{\text{m}}{\text{sec}}$ (from negative maximal speed to positive maximal speed).

After obtaining the range-Doppler map, we can detect a single target by extracting the cell with the greatest value (to detect several targets, a different extraction method should be considered, as the cell with the second greatest value may still represent the first target).

2.1.1 A Single Target

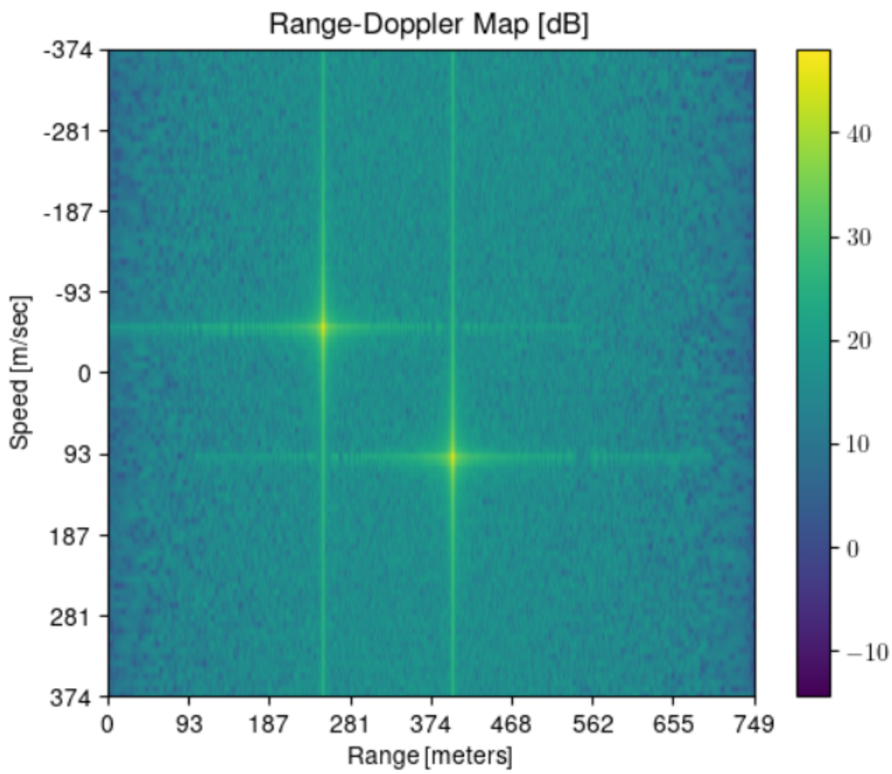
For example, given the parameters $R^0 = 400\text{m}$, $v = 100\frac{\text{m}}{\text{sec}}$, $\sigma = 1$ and no noise, we construct the range-Doppler map and extract the target's estimated speed and range:




```
Detection of target #1:
→ Range is 400.0482149649824 meters away
→ Speed is 97.43254885 meters per second
```

2.1.2 Several Targets

We can also plot the map and detect several targets. In this case, in order to detect the l-th target, we can (for example) take the cell with l-th maximal value while "deleting" the neighboring cells of the targets already detected. For instance, given two targets with parameters $(R_1^0, v_1, \sigma_1) = (400, 100, 1)$ and $(R_2^0, v_2, \sigma_2) = (250, -50, 1)$, and noise variance of 0.1^2, we get




```
Detection of target #1:
→ Range is 400.0482149649824 meters away
→ Speed is 97.43254885 meters per second

Detection of target #2:
→ Range is 249.70207232116053 meters away
→ Speed is -52.46368015000001 meters per second
```

2.1.3 Reducing the Detection Error

The error for target detection can be up to half the size of a single cell. Following the above derivations, the size of a single cell for detection is

$$\frac{C \cdot \mathrm{PRI}}{2\,(e + r - 1)}\text{meters} \quad \times \quad \frac{C}{2 f_c \cdot \mathrm{CPI}}\frac{\mathrm{m}}{\mathrm{sec}},$$

where we recall that $e + r$ is the number of time samples of a single PRI.

Decreasing the error can be achieved by decreasing the size of a cell. With the calculated ratios above, we see that this can be achieved by increasing f_c, increasing CPI, increasing the number of time samples, and/or decreasing PRI. Of course, all these actions bear costs.

Another strategy to reduce the error, which does not require changing the transmission parameters, can also be derived. By inspecting the size of a cell for range detection as given above, we see that it does not depend on the number of PRIs in the CPI (it only depends on the time length of a single PRI and the number of its time samples). Therefore, each PRI in the entire CPI produces a (possibly different) range estimation. Since we have N such PRIs, we have N range estimations (which roughly corresponds to a highlighted column in the range-Doppler map). Therefore, we can estimate an optimal range x out of these N rage estimations d_i by taking their average $x = \frac{1}{N}\sum_{i=0}^{N-1} d_i$, or by solving the following (possibly weighted) optimization problem

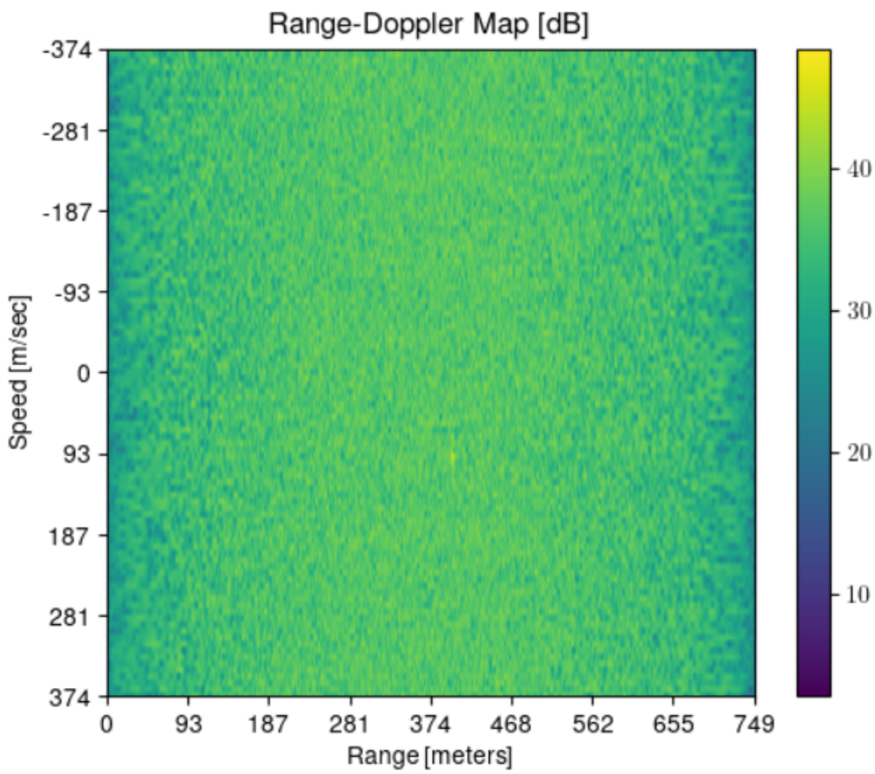
$$\underset{x\in\mathbb{R}}{\text{minimize}} \quad \sum_{i=0}^{N-1} \omega_i \left(x - d_i\right)^2 .$$

Similarly, the size of a cell for speed detection does not depend on the number of time samples, so we get $e + r - 1$ speed estimations, and we can therefore take their average or solve a similar optimization problem.

2.2 The Effect of SNR

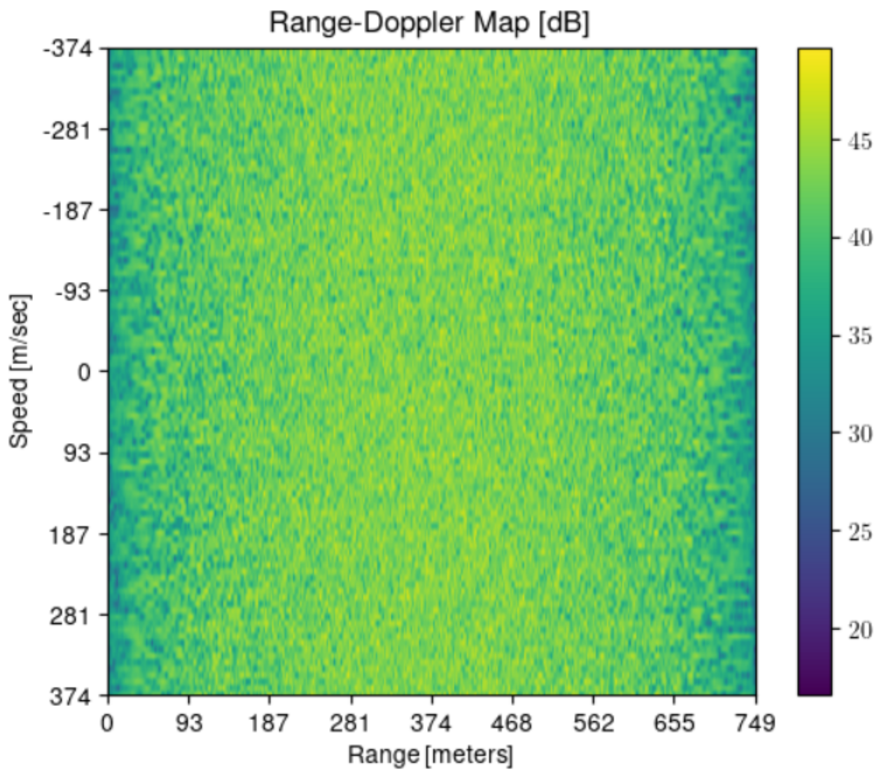
For a single target, the SNR is defined as $\frac{\mathrm{Var}(s_{\mathrm{Tx}})}{\mathrm{Var}(w)}$ (considering the real part of both s_{Tx} and w). As the SNR is defined as a ration, below we only change the values of $\mathrm{Var}\left(w\right)$ in order to test detection in different SNR scenarios.

Under the single target setting of $R^0 = 400\mathrm{m}$, $v = 100\frac{\mathrm{m}}{\mathrm{sec}}$ and $\sigma = 1$, we get that $\log_{10}\left(\mathrm{SNR}\right) \approx 0$ for $\mathrm{Var}\left(w\right) \approx 10^2$. For such noise level, we indeed still able to (barely) detect the target by extracting the maximal cell:




```
Detection of target #1:
→ Range is 400.0482149649824 meters away
→ Speed is 97.43254885 meters per second
```

However, for larger values of $\mathrm{Var}(w)$ (i.e., $\log_{10}(\mathrm{SNR}) < 0$), we are less likely to correctly detect the target. For example, for $\mathrm{Var}(w) = 50^2$ we get $\log_{10}(\mathrm{SNR}) = -1.4$ and the target is not detected:




```
Detection of target #1:
→ Range is 305.1914217258629 meters away
→ Speed is -134.9066061 meters per second
```

As for the estimation error, we define it as $|R^0 - R^{\text{est}}| + |v^0 - v^{\text{est}}|$, where the superscript est denotes the estimation obtained from the map. For the single target setting described above, the estimation error remains somewhat constant on 2.61 for values of SNR that satisfy $\log_{10}(\text{SNR}) > 0$ (assuming the target is detected by extracting the maximal value from the map). For values of SNR that satisfy $\log_{10}(\text{SNR}) < 0$, the error increases monotonically.

Last, if for some value of SNR the detection may not be good enough, we can solve the optimization problems introduced in the previous sub-section, which do not require any changes in the parameters of the target (nor the radar).

3 Additional Aspects

1. Following the derivations above, the estimation depends on the number of time samples, the length of a single PRI, the length of the entire CPI, and the frequency f_c.

   In addition, the maximal detection range depends on the length of PRI, whereas the maximal speed also depends on f_c.

2. Given two targets with the same speed, we expect to distinguish between them for a range difference greater than $\frac{C \cdot \mathrm{PRI}}{2(e+r-1)}$ meters (follows from the ratios given above).

3. Given two targets with the same range, we expect to distinguish between them for a speed difference greater than $\frac{C}{2f_c \cdot \mathrm{PRI}}$ meters per second (follows directly from the ratios given above).

4. Given a value of PRI, it limits the maximal detection range, which is $\frac{C \cdot \mathrm{PRI}}{2}$ meters. In addition, it limits the maximal detection speed, which is $\frac{C}{2f_c \cdot \mathrm{PRI}}$ meters per second. It also limits the size of the detection cells (as discussed above).

5. Assume a target with 0 speed at an unknown range between 600 and 650 meters. In this case, there is no need to detect in ranges over 650 meters, so from the equation $\frac{C \cdot \mathrm{PRI}}{2} = 650$ we get $\mathrm{PRI} = 4.33$. Now, since the target is over 600 meters away, we can adjust the transmission time of a single PRI and by that to reduce the transmission time of the radar to a minimum. To this end, notice that the correlation operation yields $r - e$ zeros (where r is the number reception time samples and e is the number of transmission time samples). Since we can zero the first 600 elements of each row of the range-Doppler map without affecting the detection, then we have $\frac{C(r-e)}{2} = 600$. Since the signal is continuous, then we substitute $r - e$ with $\mathrm{PRI} - \mathrm{pw} - \mathrm{pw}$ and we get that $\mathrm{pw} = \frac{\mathrm{PRI}}{26} = 0.17\mu\mathrm{sec}$ (we recall that pw affects the SNR as the variance of the transmitted signal depends on pw through its norm).

6. If the target's speed is high, then in order to detect it we must decrease the length of PRI and/or decrease the frequency f_c, which also decreases the maximal detection range. Hence, we cannot detect very fast targets that are far away or very close to each other. Eventually, starting from some speed, we will not be able to detect the target.