Eisenstein's Lemma: Let $p > 2$ be prime and $a \in \mathbb{Z}$, such that $p \nmid a$. For each $x = 1, 2, \ldots, \frac{p-1}{2}$, write

$$(*) \quad ax = p\,q_x + r_x, \qquad 1 \le r_x \le p-1.$$

Set $\boxed{T(a,p) := q_1 + q_2 + \cdots + q_{\frac{p-1}{2}}}$.

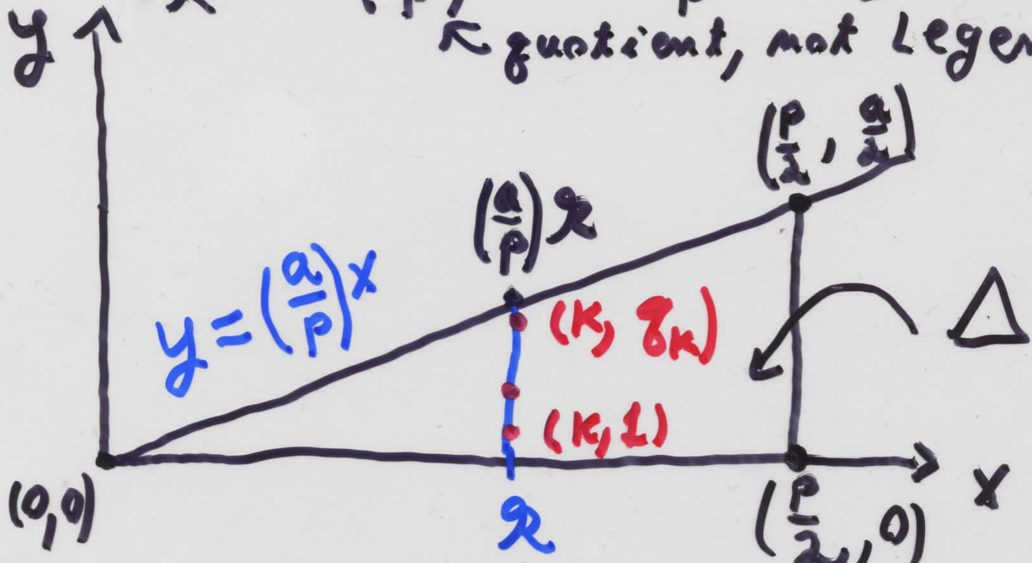
If a is ODD, then $\boxed{\left(\frac{a}{p}\right) = (-1)^{T(a,p)}.}$

Legendre

Geometric Meaning of $T(a,p)$:

$$(*) \iff q_x = \left(\frac{a}{p}\right)x - \frac{r_x}{p} = \left\lfloor \left(\frac{a}{p}\right)x \right\rfloor.$$

quotient, not Legendre symbol



Prop 11.5.2: Let $p > 2$ be prime and let $a \in \mathbb{Z}$ such that $p \nmid a$. Then the number of lattice points in the interior of the triangle $\Delta(a,p)$ with vertices $(0,0)$, $\left(\frac{p}{2}, 0\right)$, $\left(\frac{p}{2}, \frac{a}{2}\right)$ is equal to $T(a,p)$.