

Euclid's Lemma: Let $d, k, m \in \mathbb{Z}$.

If $d | km$ and $\gcd(d, m) = 1$, then $d | k$.

Theorem 5.4.1: Let $a, b \in \mathbb{Z}$, not both zero.

Set $d := \gcd(a, b)$. Assume that (x_0, y_0) is a particular solution of the Diophantine eq

$$ax + by = c.$$

Then the general solution is

$$\left\{ \left(x_0 - k \frac{b}{d}, y_0 + k \frac{a}{d} \right) : k \in \mathbb{Z} \right\}.$$

Ex: $(-4, 6)$ is a particular sol'n of

$$154x + 105y = 14.$$

$d := \gcd(154, 105) = 7$. So, the general sol'n is

$$\left\{ \left(-4 - k \frac{105}{7}, 6 + k \frac{154}{7} \right) : k \in \mathbb{Z} \right\}$$