

Sec 3.1 page 96 prob 13:

Let $a, b, c \in \mathbb{Z}$. Prove that

$$a|b \text{ and } b|c \Rightarrow a|c.$$

Proof: Assume that $a|b$ and $b|c$. Then there exist integers g_1, g_2 such that $b = ag_1$ and $c = bg_2$. Thus $c = a(\underbrace{g_1 g_2}_{\in \mathbb{Z}})$. Hence, by definition, $a|c$. Since there exist an integer $g = g_1 g_2$ such that $c = ag$. □