Edward Yaroslavsky 9/22/19 CS385 HW #2

I pledge my honor that I have abided by the Stevens Honor System.

E.Y.

4a) The algorithm computes the summation of the squares of all the numbers from 1 to n. If n is 0, the output will also be 0.

b) Multiplication is the basic operation.

c) The basic operation is executed n-1 times.

d) The efficiency class of this algorithm is $\Theta(n)$.

e) $\Theta(n)$ is the best efficiency class for this algorithm. In order for this algorithm to be completed, it has to go through $\sum_{i=1}^{n} 1$ multiplications, or n multiplications. Therefore, since this algorithm can't be made into a constant efficiency level and is able to be bounded, it falls under the $\Theta(n)$ efficiency level.

1a) $x(n) = x(n-1) + 5, \ n > 1 \quad x(1) = 0$

1) $x(n-1) = x(n-2) + 5$

$x(n) = x(n-2) + 10$

2) $x(n-2) = x(n-3) + 5$

$x(n) = x(n-3) + 15$

3) $x(n) = x(n-k) + 5k$

4) $n - k = 1$

$n = k+1$

$k = n-1$

5) $x(n) = x(n-(n-1)) + 5(n-1)$

$= \underbrace{x(1)}_{0} + 5n - 5 \Rightarrow \boxed{x(n) = 5n - 5, \ n > 1}$

b) $x(n) = 3x(n-1), \ n > 1 \quad x(1) = 4$

1) $x(n-1) = 3x(n-2)$

$x(n) = 3 \cdot 3x(n-2) = 9x(n-2)$

2) $x(n-2) = 3x(n-3)$

$x(n) = 9 \cdot 3x(n-3) = 27x(n-3)$

3) $x(n) = 3^k \cdot x(n-k)$

4) $n - k = 1$

$n = k+1$

$k = n-1$

5) $x(n) = 3^{n-1} \cdot x(n-(n-1))$

$= 3^{n-1} \cdot \underbrace{x(1)}_{4}$

$\Rightarrow \boxed{x(n) = 4 \cdot 3^{n-1}, \ n > 1}$

c) $x(n-1)+n,\ n>0 \quad x(0)=0$

1) $x(n-1)=x(n-2)+n-1$

$x(n)=x(n-2)+n-1+n = x(n-2)+2n-1$

2) $x(n-2)=x(n-3)+n-2$

$x(n)=x(n-3)+n-2+2n-1 = x(n-3)+3n-3$

3) $x(n)=x(n-k)+kn-\frac{k(k-1)}{2}$

4) $n-k=0$

$n=k$

$k=n$

5) $x(n)=x(n-n)+n(n)-\frac{n(n-1)}{2}$

$= \underbrace{x(0)}_{0}+n^2-\frac{n^2-n}{2}$

$\Rightarrow \boxed{x(n)=\frac{n^2}{2}+\frac{n}{2},\ n>0}$

d) $x(n)=x(n/2)+n,\ n>1 \quad x(1)=1$

1) $x(\frac{n}{2})=x(\frac{n}{4})+\frac{n}{2}$

$x(n)=x(\frac{n}{4})+\frac{n}{2}+n = x(\frac{n}{4})+\frac{3n}{2}$

2) $x(\frac{n}{4})=x(\frac{n}{8})+\frac{n}{4}$

$x(n)=x(\frac{n}{8})+\frac{n}{4}+\frac{3n}{2} = x(\frac{n}{8})+\frac{7n}{8}$

3) $x(n)=x(\frac{n}{2^k})+\left(\frac{2^k-1}{2^{k-1}}\right)n$

4) $\frac{n}{2^k}=1$

$2^k=n$

$k=\log_2 n$

5) $x(n)=x(\frac{n}{n})+\frac{2(n-1)n}{n}$

$= \underbrace{x(1)}_{1}+2n-2$

$\Rightarrow \boxed{x(n)=2n-1,\ n>1}$

$\Rightarrow \boxed{x(2^k)=2^{k+1}-1,\ 2^k>1}$

e) $x(n) = x(n/3) + 1,\ n > 1 \quad x(1) = 1$

1) $x(\frac{n}{3}) = x(\frac{n}{9}) + 1$
$x(n) = x(\frac{n}{9}) + 2$

2) $x(\frac{n}{9}) = x(\frac{n}{27}) + 1$
$x(n) = x(\frac{n}{27}) + 3$

3) $x(n) = x(\frac{n}{3^k}) + k$

4) $\frac{n}{3^k} = 1$
$3^k = n$
$k = \log_3 n$

5) $x(n) = x(\frac{n}{n}) + \log_3 n$
$= \underbrace{x(1)}_{1} + \log_3 n \quad \Rightarrow \boxed{\begin{array}{l} x(n) = 1 + \log_3 n,\ n > 1 \\ x(3^k) = 1 + k,\ 3^k > 1 \end{array}}$

3a) $S(n) = S(n-1) + 2,\ n > 1 \quad S(1) = 0$

1) $S(n-1) = S(n-2) + 2$
$S(n) = S(n-2) + 4$

2) $S(n-2) = S(n-3) + 2$
$S(n) = S(n-3) + 6$

3) $S(n) = S(n-k) + 2k$

4) $n - k = 1$
$n = k + 1$
$k = n - 1$

5) $S(n) = S(n-(n-1)) + 2(n-1)$
$\underbrace{S(1)}_{0} + 2n - 2 \quad \Rightarrow \quad S(n) = 2n-2,\ n > 1$

This algorithm's basic operation is executed $2n-2$ times.

b) Based off a non-recursive algorithm, we would have
$\sum_{i=2}^{n} 2 = 2\sum_{i=2}^{n} 1 = 2n - 2$ executions, the same as the recursive algorithm.

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