Sheikh Hasina University, Netrokona
Department of Computer Science and Engineering
CSE-2205: Introduction to Mechatronics

ρ = const

Hydraulic = liquid
incompressible

Pneumatic = gas
air
compressible

$\rho \neq$ constant

Lec-21: Fluid System Models

Mechatronics: Electronic Control Systems in Mechanical Engineering by W. Bolton

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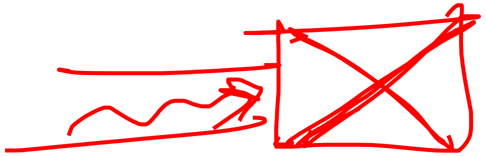
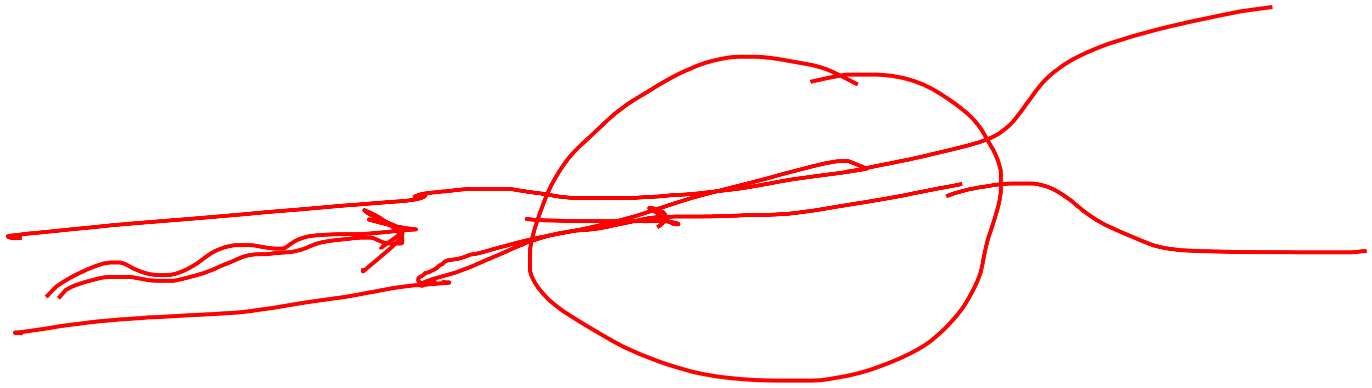
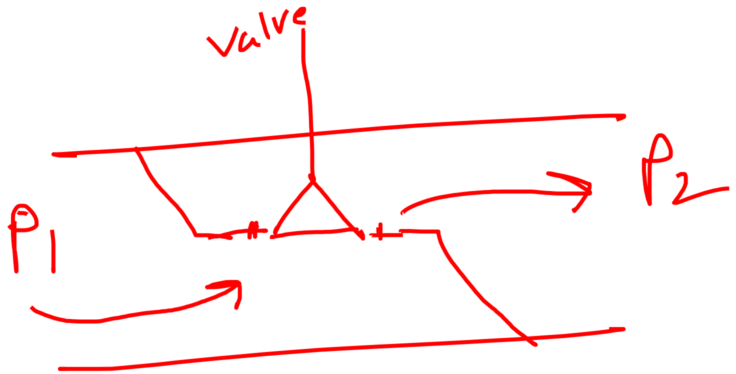
Fluid system building block:

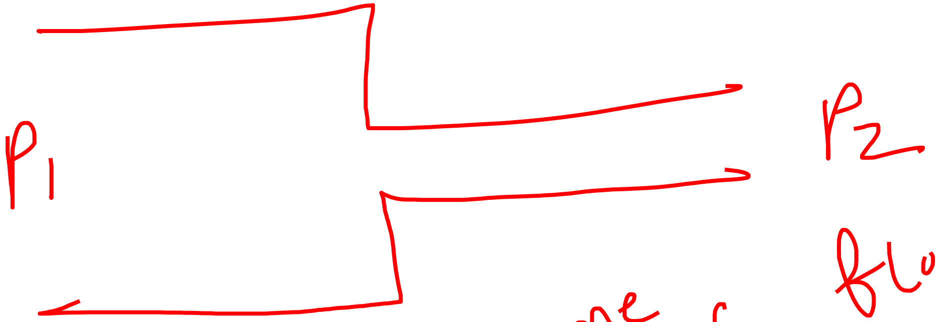
Hydraulic system (liquid)

1) Hydraulic Resistance:









$P_1 - P_2 \propto q$ → Volume rate of flow of liquid

$\Rightarrow \boxed{P_1 - P_2 = R q}$ → hydraulic Resistance

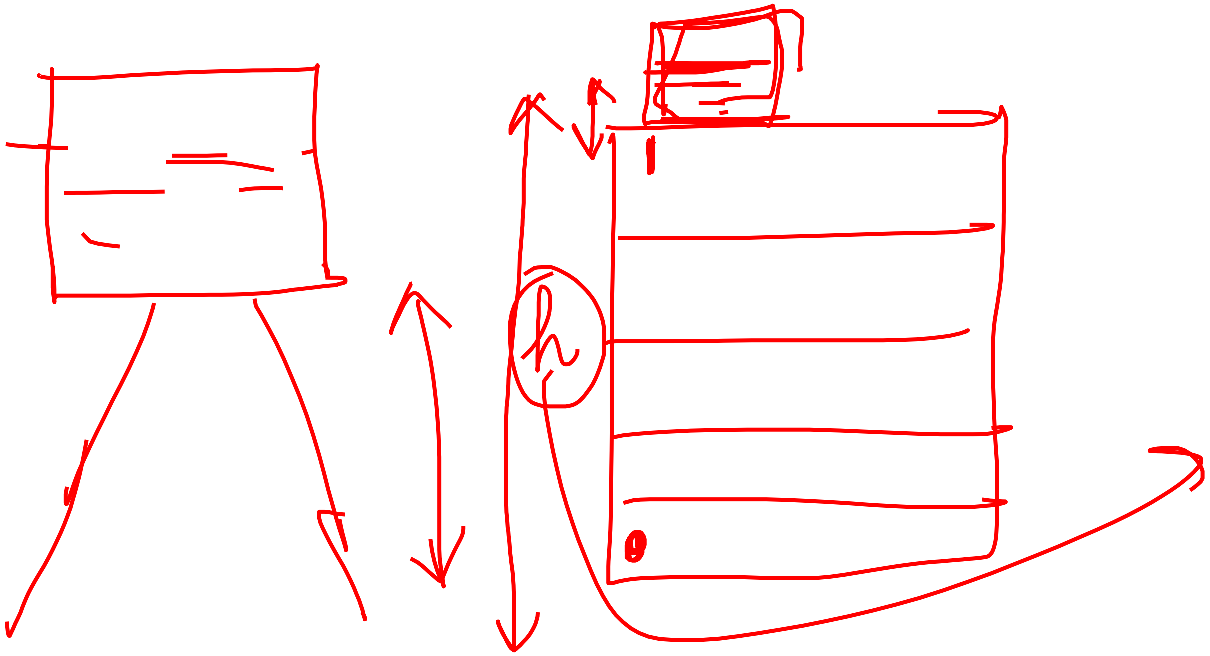
ΔP $\boxed{P_1 - P_2} = \acute{R}Q$

$V = I\acute{R}$

$\boxed{R \uparrow}$

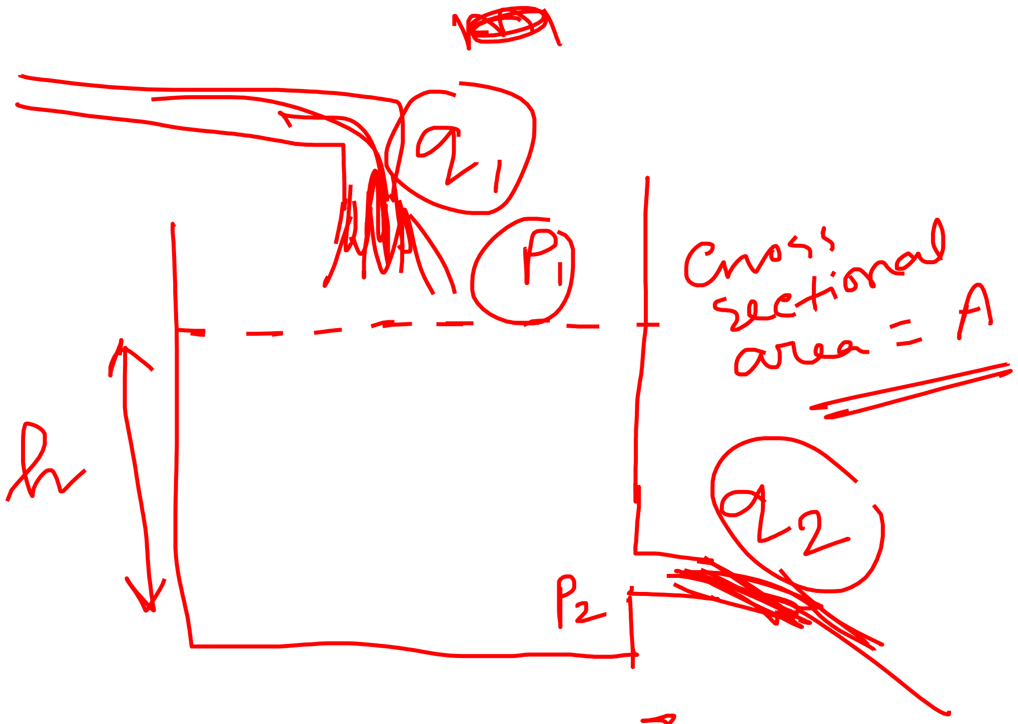
$\underline{V} = IR$

ii) Hydraulic Capacitance:



potential energy

Pressure head



$$P = h\rho g$$

$$h = \frac{P}{\rho g}$$

$$\frac{dV}{dt}$$

Volume

$$q_1 - q_2 = \frac{dV}{dt}$$

$$q_1 - q_2 = \frac{d(Ah)}{dt}$$

$$q_1 - q_2 = A\frac{dh}{dt}$$

$$q_1 - q_2 = A \frac{dh}{dt}$$

$$h = \frac{p}{\rho g}$$

$$\Rightarrow q_1 - q_2 = A \cdot \frac{d\left(\frac{p}{\rho g}\right)}{dt}$$

$$\Rightarrow q_1 - q_2 = \frac{A}{\rho g} \frac{dp}{dt}$$

C = hydraulic capacitance

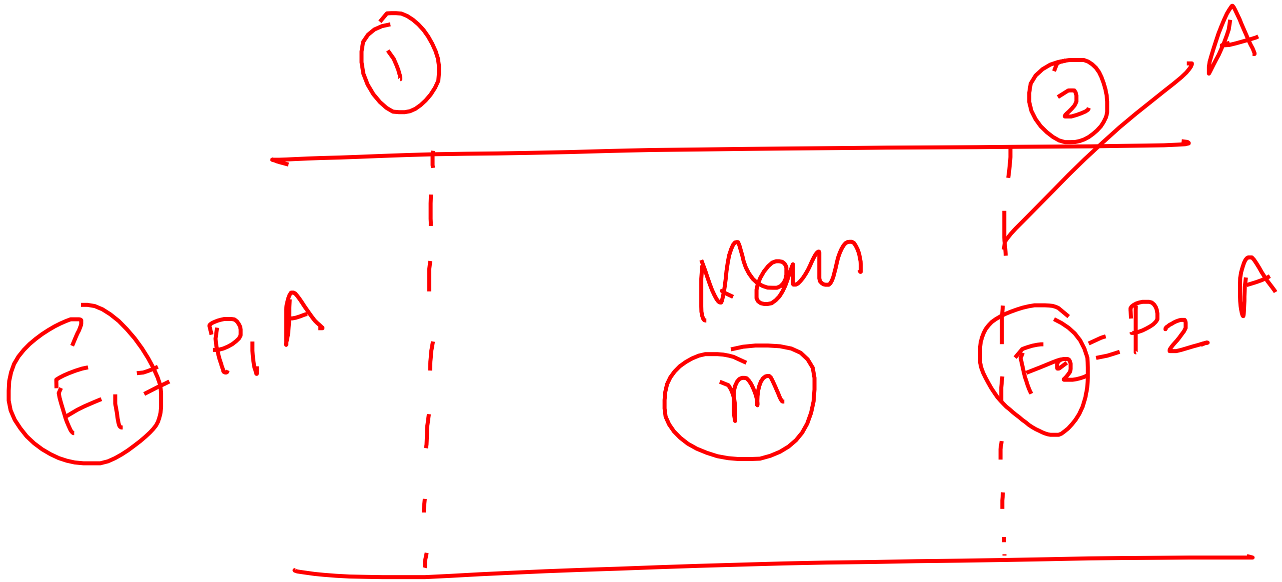
$$\boxed{q_1 - q_2 = C \frac{dp}{dt}}$$

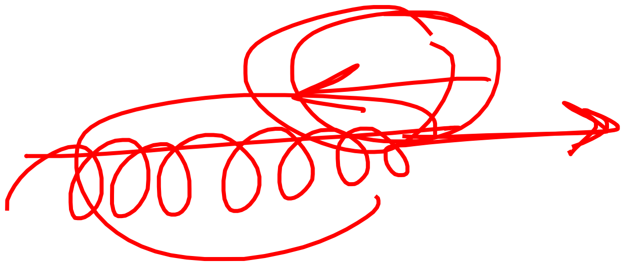
integral

$$\int q_1 - q_2 = C \int \frac{dp}{dt}$$

$$\Rightarrow p = \frac{1}{C} \int (q_1 - q_2)\, dt$$

(iii) Hydraulic inertance:





$$\text{Net force on the liquid} = F_1 - F_2$$

$$= P_1 A - P_2 A$$

$$ma = (P_1 - P_2)\, A$$

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$p = \frac{F}{A}$$

$$F = pA$$

$$ma = (P_1 - P_2)A$$

$$\Rightarrow (P_1 - P_2)A = m \cdot \frac{dv}{dt}$$ velocity

$$\rho = \frac{m}{V}$$

$$\Rightarrow (P_1 - P_2)A = \rho V \frac{dv}{dt}$$

$$\boxed{A_1 v_1 = A_2 v_2}$$

$$\Rightarrow m = \rho V$$

Av = volume rate of flow

q

$$\Rightarrow (P_1 - P_2)A = \rho \cdot A L \frac{dv}{dt}$$

$$V = A \times L$$

$$\Rightarrow (P_1 - P_2)A = \rho L \frac{d(Av)}{dt}$$

$$\Rightarrow (P_1 - P_2)A = \rho L \frac{dq}{dt}$$

$$\Rightarrow (P_1 - P_2) A = L\rho \frac{dq}{dt}$$

$$\Rightarrow P_1 - P_2 = \frac{L\rho}{A} \cdot \frac{dq}{dt}$$

$$\Rightarrow \underline{P_1} - \underline{P_2} = I \frac{dq}{dt}$$

hydraulic inertance

Pneumatic system:

resistance, Capacitance, inertance

Compressible

air

$\rho = \frac{m}{V}$

$\dot{m}$

1) Pneumatic Resistance:

$P_1 - P_2 = Rq$

$P_1 - P_2 = R \cdot \frac{dm}{dt}$

$$\boxed{P_1 - P_2 = R \cdot \dot{m}}$$

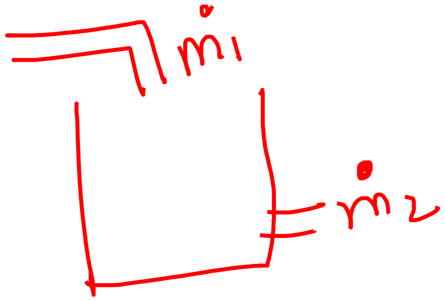
2) Pneumatic Capacitance:

$\rho = \frac{m}{V}$

$\Rightarrow m = \rho V$

rate of change of mass in container $= \frac{dm}{dt}$

$$= \frac{d(\rho V)}{dt}$$



$$\dot{m}_1 - \dot{m}_2 = \rho \frac{dV}{dt} + V \frac{d\rho}{dt}$$

$$\Rightarrow \frac{dm_1}{dt} - \frac{dm_2}{dt} = \rho \frac{dV}{dt} + V \frac{d\rho}{dt} \quad \text{(i)}$$

$$\frac{dV}{dt} = \frac{dV}{d\rho} \cdot \frac{d\rho}{dt}$$

$$pV = mRT$$

$$p = \frac{m}{V} RT$$

$$p = \rho R T$$

$$\rho = \frac{p}{RT}$$

$$\frac{d\rho}{dt} = \frac{1}{RT} \frac{dp}{dt}$$

from eqn(1)

$$\frac{dm_1}{dt} - \frac{dm_2}{dt} = \rho\frac{dV}{dt} + V\cdot\frac{d\rho}{dt}$$

$$= \rho\cdot\frac{dV}{dp}\cdot\frac{dp}{dt} + V\cdot\frac{1}{RT}\cdot\frac{dp}{dt}$$

$$\frac{dm_1}{dt} - \frac{dm_2}{dt} = \left(\rho\frac{dV}{dp} + \frac{V}{RT}\right)\frac{dp}{dt}$$

$$C_1 = \rho\frac{dV}{dp} \qquad C_2 = \frac{V}{RT}$$

$$\boxed{\frac{dm_1}{dt} - \frac{dm_2}{dt} = (c_1+c_2)\frac{dP}{dt}}$$

$P = P_1 - P_2$

$$\int \frac{dP}{dt} = \frac{1}{c_1+c_2}\int\left(\frac{dm_1}{dt} - \frac{dm_2}{dt}\right)$$

$$P_1 - P_2 = \frac{1}{c_1+c_2}\int\left(\dot{m}_1 - \dot{m}_2\right)$$

iii) Pnematic inertance:

Net force on the gas = $F_1 - F_2$

$$ma = P_1 A - P_2 A$$

$$m \cdot \frac{dv}{dt} = (P_1 - P_2) A$$

$$\Rightarrow (P_1 - P_2) A = \frac{d(mv)}{dt}$$

$$mv = \rho V \cdot v$$

$$= \rho . V \cdot \frac{q}{A}$$

$$= \rho . \frac{V}{A} . q$$

$$= \rho . L . q$$

$$m = \rho V$$

$$\boxed{A v = q}$$

$$v = \frac{q}{A}$$

$$\rho . q = \dot{m}$$

$$(P_1 - P_2) A = \frac{d(\rho L q)}{dt}$$

$$P_1 - P_2 = \frac{L}{A} \left(\frac{d q \rho}{dt} \right)$$

$$= \frac{L}{A} \left(\frac{d \dot{m}}{dt} \right)$$

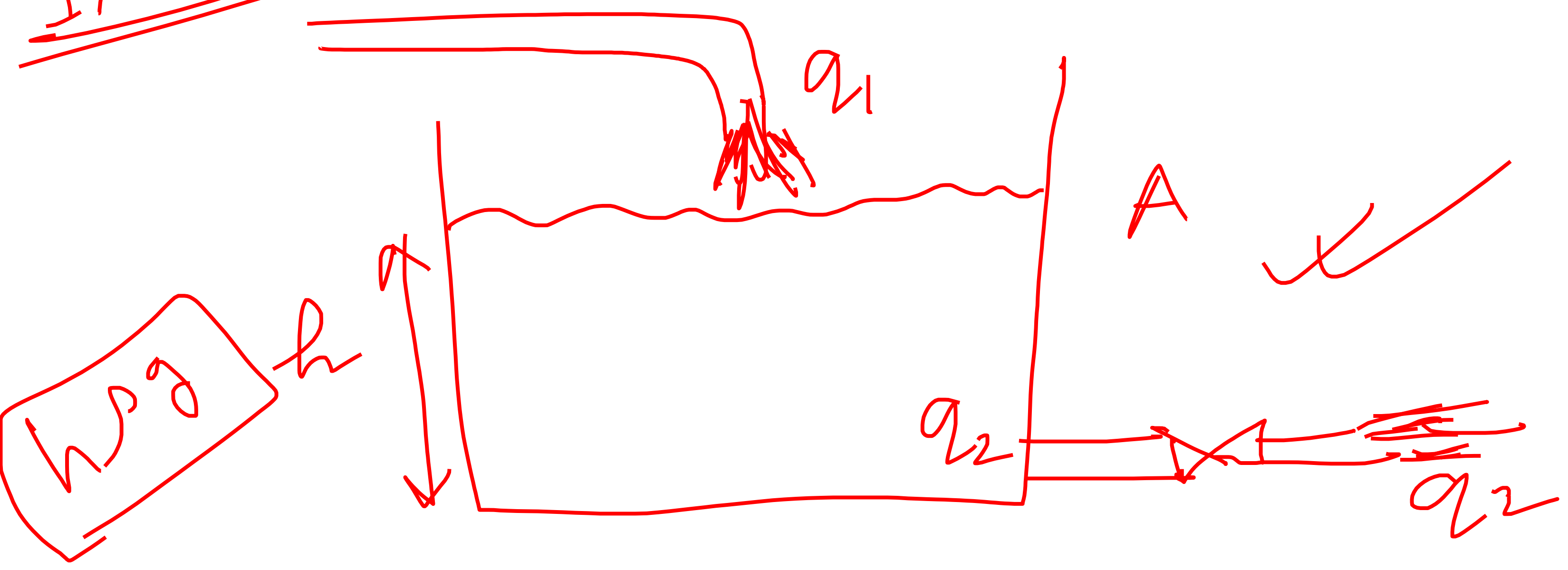
$$P_1 - P_2 = \frac{L}{A} \frac{d\dot{m}}{dt}$$

$$\boxed{P_1 - P_2 = I \frac{d\dot{m}}{dt}}$$

Pneumatic inertance

$$I = \frac{L}{A}$$

Example:



$\boxed{\text{Inertance} = \text{Neglect}}$

Capacitor

$$q_1 - q_2 = C\frac{dP}{dt} \quad \text{--- (i)}$$

Resistor:

$$P_1 - P_2 = R q_2 \quad \text{--- (ii)}$$

$$q_2 = \frac{P_1 - P_2}{R} = \frac{P}{R}$$

$$q_2 = \frac{P}{R}$$

$$q_2 = \frac{h\rho g}{R}$$

Eqn(i)

$$q_1 - q_2 = C \frac{dP}{dt}$$

$$q_2 = \frac{h\rho g}{R}$$

$$P_1 - P_2 = q_2 R$$

$$\Rightarrow q_1 - q_2 = C \cdot \frac{\cancel{P}\, d(h\rho g)}{dt}$$

$$\Rightarrow q_1 - \frac{h\rho g}{R} = C \cdot \frac{d(h\rho g)}{dt}$$

$$C = \frac{A}{\rho g}$$

$$q_1 - \frac{h\rho g}{R} = \frac{A}{\rho g} \cdot \frac{d(h\rho g)}{dt}$$

$$\cancel{q_1 - 0}$$

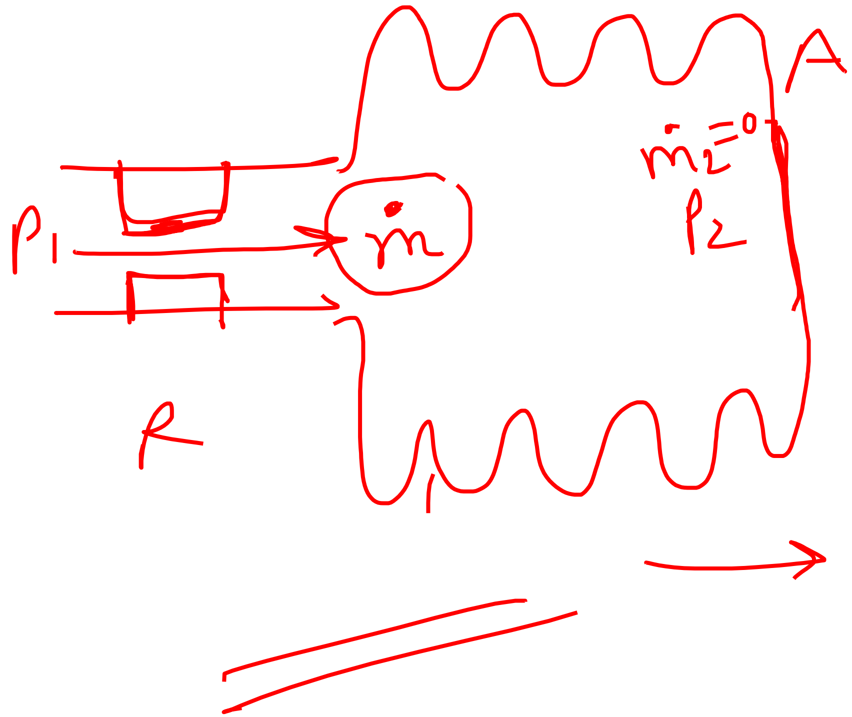
$$q_1 - \frac{h\rho g}{R} = \frac{A}{\cancel{\rho g}} \times \cancel{\rho g} \frac{dh}{dt}$$

$$q_1 - \frac{h\rho g}{R} = A\frac{dh}{dt}$$

$$\Rightarrow \boxed{q_1 = \frac{h\rho g}{R} + A\frac{dh}{dt}}$$

Models for hydro

Ex-- ple.



Resistance:

$$P_1 - P_2 = R\dot m \quad \text{--- (i)}$$

$$\Rightarrow \dot m = \frac{P_1 - P_2}{R}$$

Capacitance.

$$\dot m_1 - \underbrace{\dot m_2}_{=0} = (C_1 + C_2)\frac{dP_2}{dt} \quad \text{--- (ii)}$$

$$\dot m = C_1 + C_2 \frac{dP_2}{dt}$$

$$\frac{P_1 - P_2}{R} = C_1 + C_2 \frac{dP_2}{dt}$$

$$\frac{P_1 - P_2}{R} = C_1 + C_2 \frac{dP_2}{dt}$$

$$\Rightarrow \frac{P_1}{R} - \frac{P_2}{R} = (C_1 + C_2) \frac{dP_2}{dt}$$

$$\Rightarrow P_1 - P_2 = R(C_1 + C_2) \frac{dP_2}{dt}$$

$$\Rightarrow \boxed{P_1 = R(C_1 + C_2) \frac{dP_2}{dt} + P_2}$$

Pneumatic system model