Nodal Analysis

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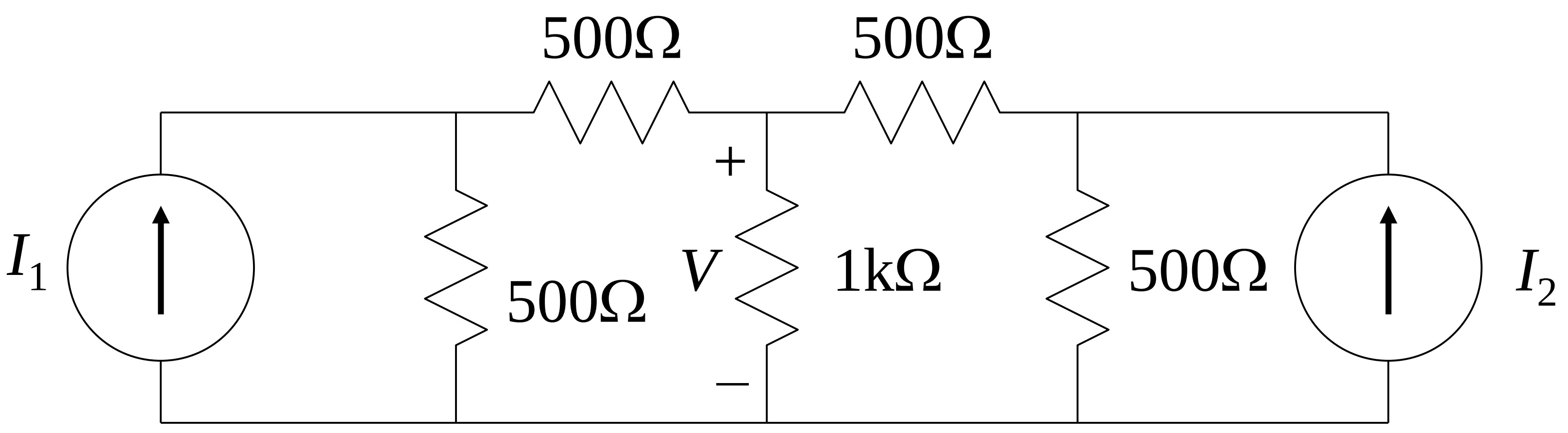
Summing Circuit



$$\textit{Solution:}\;\; V = 167I_1 + 167I_2$$

Can you analyze this circuit using the techniques of Chapter 2?

Not This One!

- There are no series or parallel resistors to combine.
- We do not have a single loop or a double node circuit.
- We need a more powerful analysis technique:

Nodal Analysis

Why Nodal or Loop Analysis?

- The analysis techniques in Chapter 2 (voltage divider, equivalent resistance, etc.) provide an intuitive approach to analyzing circuits.
- They cannot analyze all circuits.
- They cannot be easily automated by a computer.

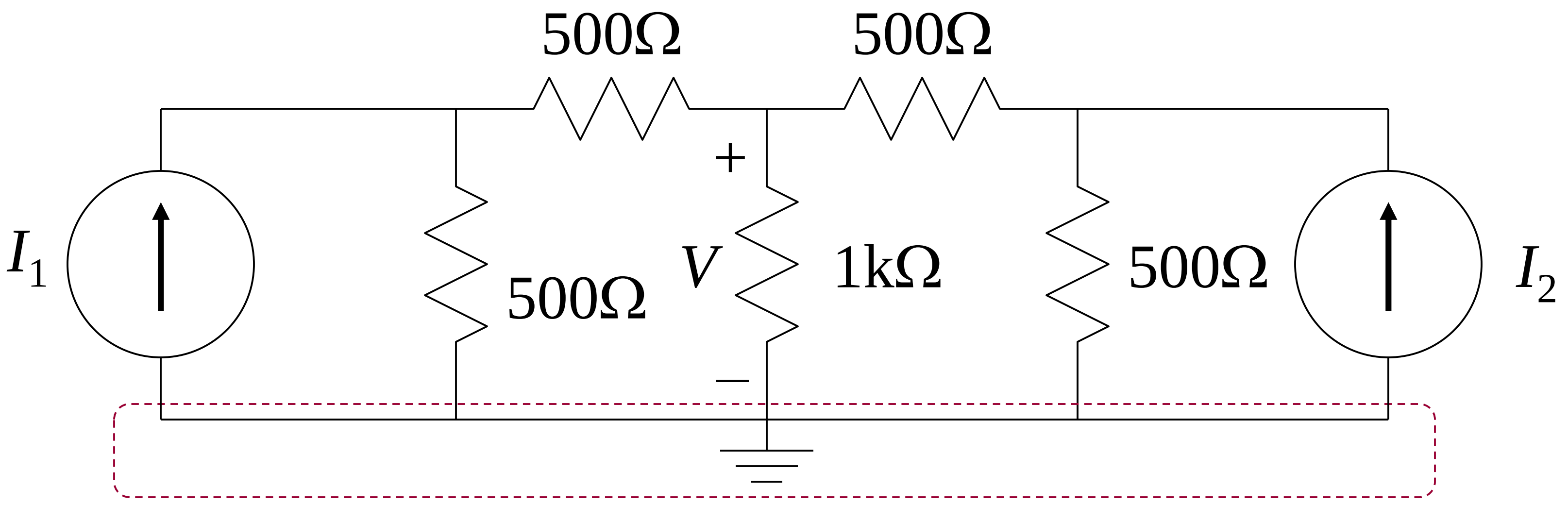
Node and Loop Analysis

- Node analysis and loop analysis are both circuit analysis methods which are systematic and apply to most circuits.
- Analysis of circuits using node or loop analysis requires solutions of systems of linear equations.
- These equations can usually be written by inspection of the circuit.

Steps of Nodal Analysis

1. **Choose a reference node.**
2. Assign node voltages to the other nodes.
3. Apply KCL to each node other than the reference node; express currents in terms of node voltages.
4. Solve the resulting system of linear equations.

Reference Node



The reference node is called the *ground* node.

Steps of Nodal Analysis

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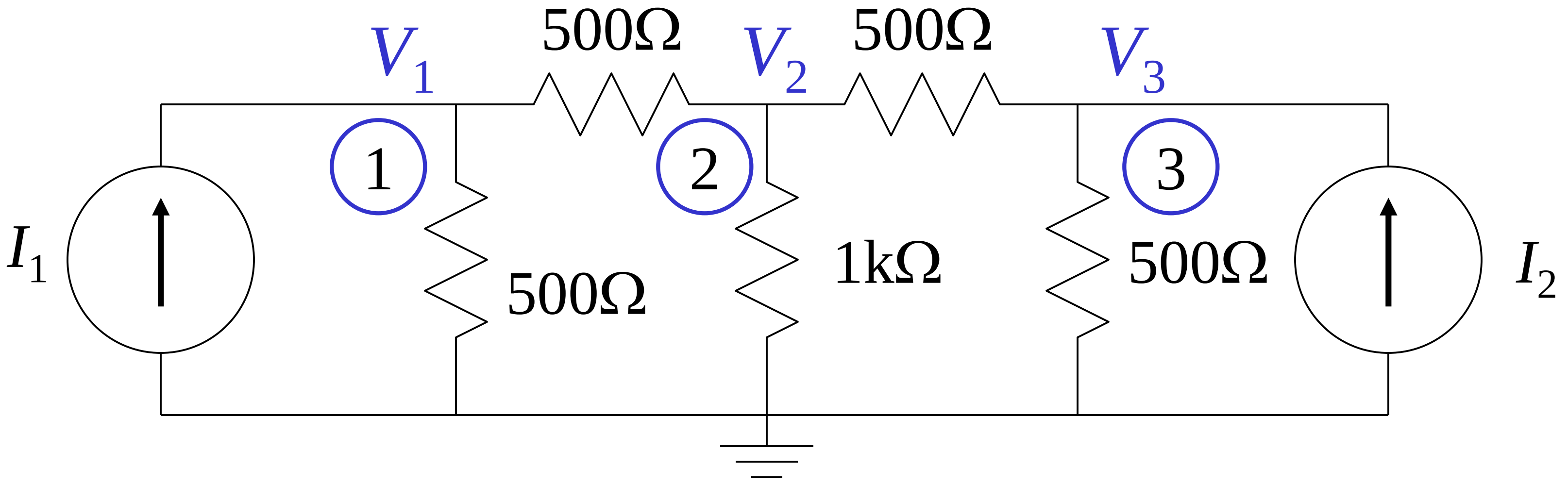
Node Voltages



V_1, V_2, and V_3 are unknowns for which we solve using KCL.

Steps of Nodal Analysis

1. Choose a reference node.
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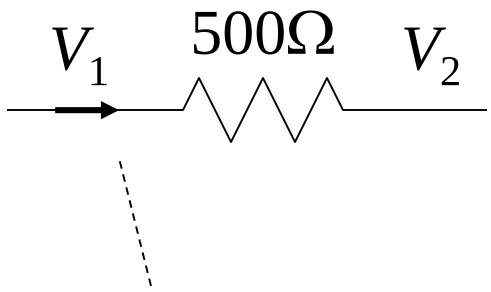
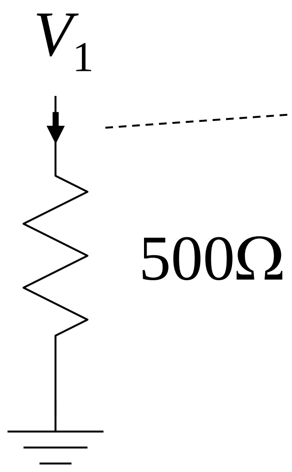
4. Solve the resulting system of linear equations.

Currents and Node Voltages

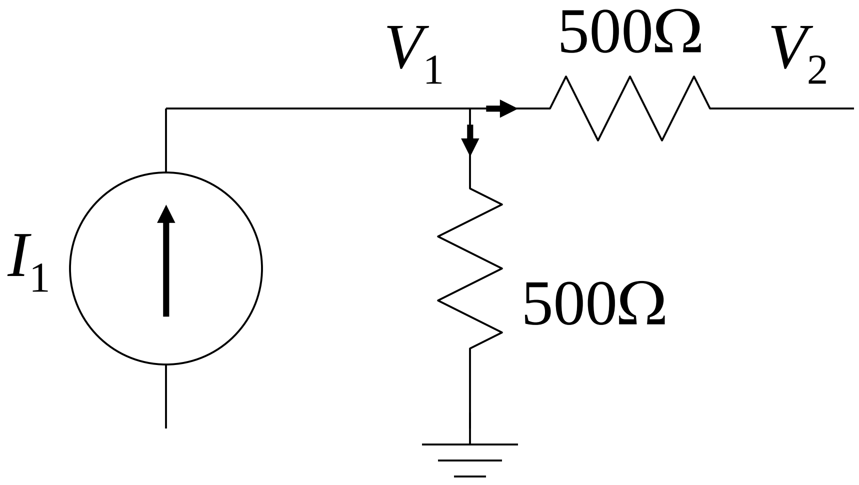


$$\frac{V_1 - V_2}{500\Omega}$$



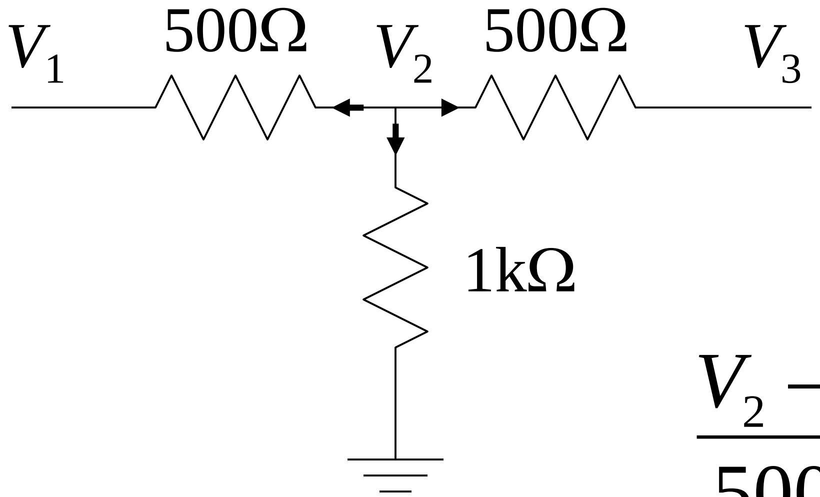
$$\frac{V_1}{500\Omega}$$

KCL at Node 1



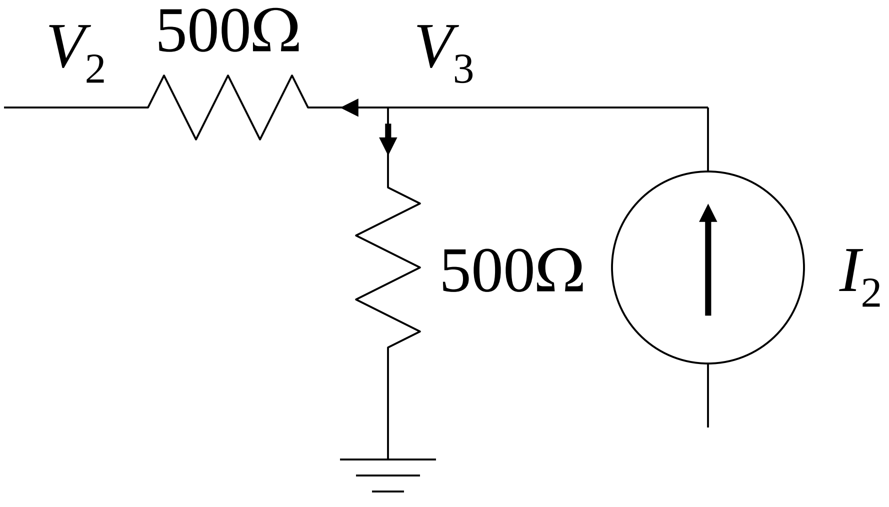
$$I_1 = \frac{V_1 - V_2}{500\Omega} + \frac{V_1}{500\Omega}$$

KCL at Node 2



$$\frac{V_2 - V_1}{500\Omega} + \frac{V_2}{1k\Omega} + \frac{V_2 - V_3}{500\Omega} = 0$$

KCL at Node 3



$$\frac{V_3 - V_2}{500\Omega} + \frac{V_3}{500\Omega} = I_2$$

Steps of Nodal Analysis

1. Choose a reference node.
2. Assign node voltages to the other nodes.
3. Apply KCL to each node other than the reference node; express currents in terms of node voltages.
4. **Solve the resulting system of linear equations.**

System of Equations

- Node 1:

$$V_1\left(\frac{1}{500\Omega}+\frac{1}{500\Omega}\right)-\frac{V_2}{500\Omega}=I_1$$

- Node 2:

$$-\frac{V_1}{500\Omega}+V_2\left(\frac{1}{500\Omega}+\frac{1}{1\text{k}\Omega}+\frac{1}{500\Omega}\right)-\frac{V_3}{500\Omega}=0$$

System of Equations

- Node 3:

$$-\frac{V_2}{500\Omega}+V_3\left(\frac{1}{500\Omega}+\frac{1}{500\Omega}\right)=I_2$$

Equations

- These equations can be written by inspection.
- The left side of the equation:
  - The node voltage is multiplied by the sum of *conductances* of all resistors connected to the node.
  - Other node voltages are multiplied by the conductance of the resistor(s) connecting to the node and subtracted.

Equations

- The right side of the equation:
  - The right side of the equation is the sum of currents from sources entering the node.

Matrix Notation

- The three equations can be combined into a single matrix/vector equation.

$$\begin{bmatrix} \frac{1}{500\Omega}+\frac{1}{500\Omega} & -\frac{1}{500\Omega} & 0 \\ -\frac{1}{500\Omega} & \frac{1}{500\Omega}+\frac{1}{1\text{k}\Omega}+\frac{1}{500\Omega} & -\frac{1}{500\Omega} \\ 0 & -\frac{1}{500\Omega} & \frac{1}{500\Omega}+\frac{1}{500\Omega} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \\ I_2 \end{bmatrix}$$

Matrix Notation

- The equation can be written in matrix-vector form as

$$\mathbf{Av} = \mathbf{i}$$

- The solution to the equation can be written as

$$\mathbf{v} = \mathbf{A}^{-1}\,\mathbf{i}$$

Solving the Equation with MATLAB

$$I_1 = 3\text{mA}, I_2 = 4\text{mA}$$

```
>> A = [1/500+1/500 -1/500 0;
 -1/500 1/500+1/1000+1/500 -1/500;
 0 -1/500 1/500+1/500];
>> i = [3e-3; 0; 4e-3];
```

Solving the Equation

```
>> v = inv(A)*i
 v =
    1.3333
    1.1667
    1.5833
```

$$V_1 = 1.33\text{V},\ V_2 = 1.17\text{V},\ V_3 = 1.58\text{V}$$

Nodal Analysis: Example 6.2, using circuit values.

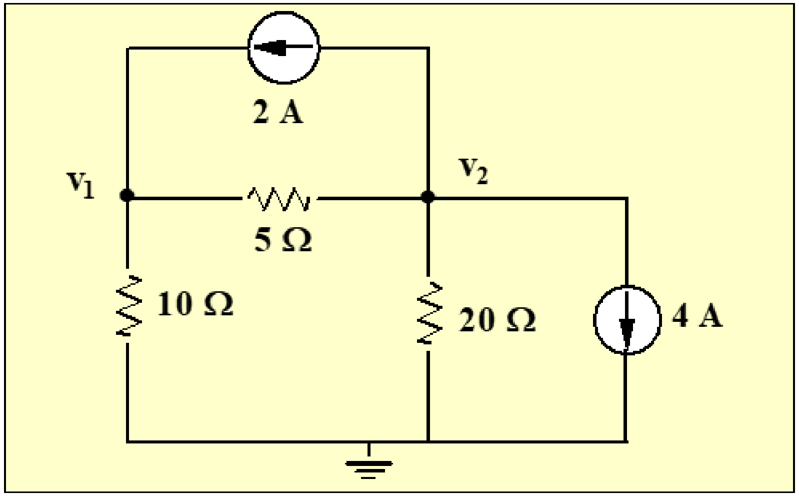


Figure 6.3: Circuit for Example 6.2.

Find V_1 and V_2.

Basic Circuits

Nodal Analysis: Example 6.2, using circuit values.

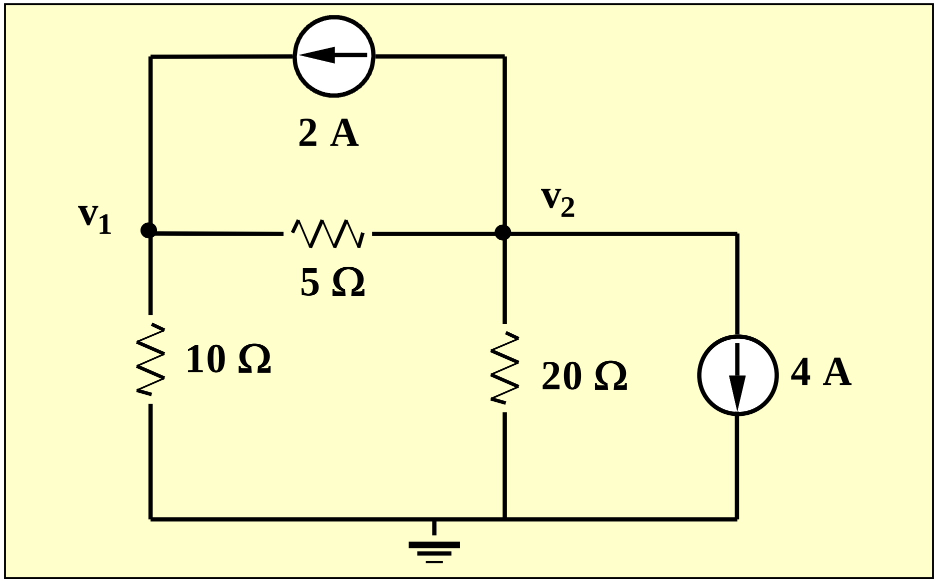


Figure 6.3: Circuit for Example 6.2.

Find V_1 and V_2.

At v_1:

$$\frac{V_1}{10} + \frac{V_1 - V_2}{5} = 2 \qquad \text{Eq 6.7}$$

At v_2:

$$\frac{V_2 - V_1}{5} + \frac{V_2}{20} = -6 \qquad \text{Eq 6.8}$$

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Basic Circuits

Nodal Analysis: Example 6.2: Clearing Equations;

From Eq 6.7:

$$V_1 + 2V_1 - 2V_2 = 20$$

or

$$3V_1 - 2V_2 = 20 \qquad \text{Eq 6.9}$$

From Eq 6.8:

$$4V_2 - 4V_1 + V_2 = -120$$

or

$$-4V_1 + 5V_2 = -120 \qquad \text{Eq 6.10}$$

Solution: $V_1 = -20\text{ V}, \quad V_2 = -40\text{ V}$

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Basic Circuits

Nodal Analysis: Example 6.3: With voltage source.

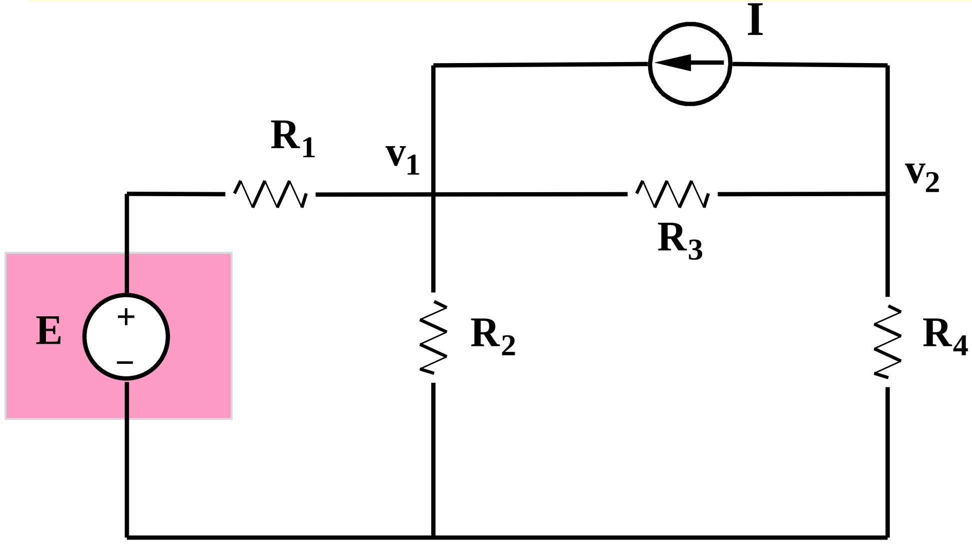

Figure 6.4: Circuit for Example 6.3.

At V_1:

$$\frac{V_1 - E}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - V_2}{R_3} = I$$ **Eq 6.11**

At V_2:

$$\frac{V_2}{R_4} + \frac{V_2 - V_1}{R_3} = -I$$ **Eq 6.12**

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Basic Circuits

Nodal Analysis: Example 6.3: Continued.

Collecting terms in Equations (6.11) and (6.12) gives

$$\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)V_1-\left(\frac{1}{R_3}\right)V_2=I+\frac{E}{R_1} \qquad \text{Eq 6.13}$$

$$-\left(\frac{1}{R_2}\right)V_1+\left(\frac{1}{R_3}+\frac{1}{R_4}\right)V_2=-I \qquad \text{Eq 6.14}$$

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Basic Circuits

Nodal Analysis: **Example 6.4: Numerical example with voltage source.**

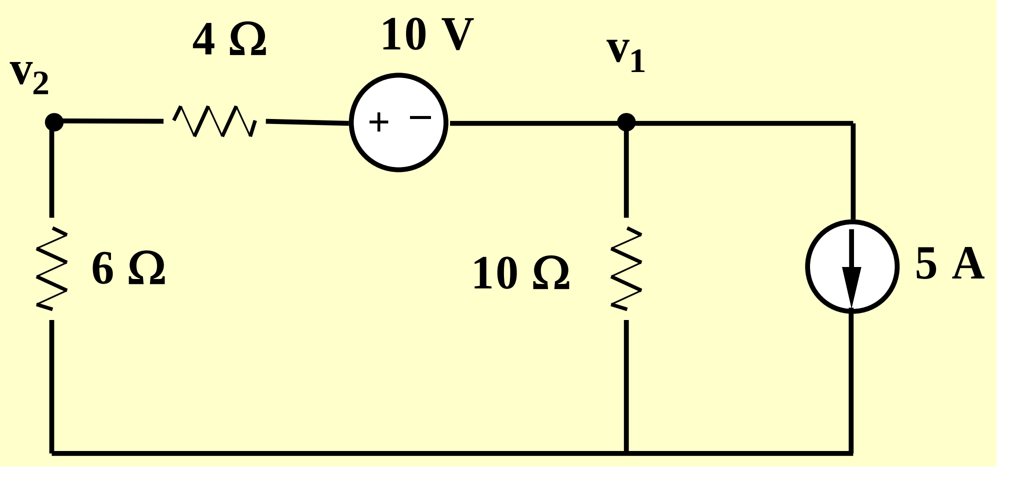

Figure 6.5: Circuit for Example 6.4.

What do we do first?

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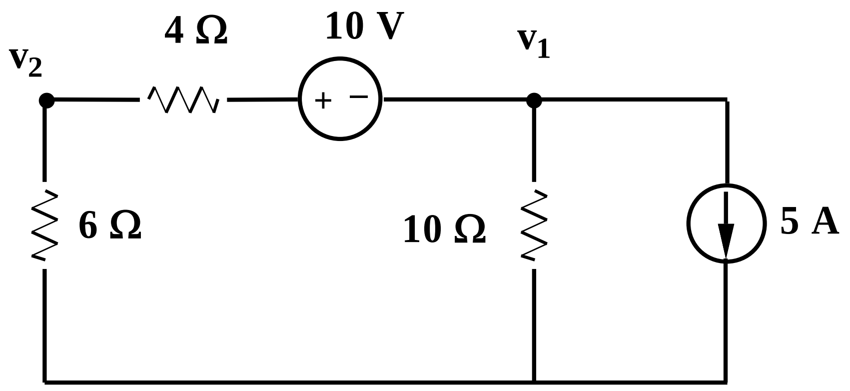
Basic Circuits

Nodal Analysis: Example 6.4: Continued



At v_1:

$$\frac{V_1}{10} + \frac{V_1 + 10 - V_2}{4} = -5 \qquad \text{Eq 6.15}$$

At v_2:

$$\frac{V_2}{6} + \frac{V_2 - 10 - V_1}{4} = 0 \qquad \text{Eq 6.16}$$

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Basic Circuits

Nodal Analysis: Example 6.4: Continued

Clearing Eq 6.15

$$4V_1 + 10V_1 + 100 - 10V_2 = -200$$

or

$$14V_1 - 10V_2 = -300 \qquad \text{Eq 6.17}$$

Clearing Eq 6.16

$$4V_2 + 6V_2 - 60 - 6V_1 = 0$$

or

$$-6V_1 + 10V_2 = 60 \qquad \text{Eq 6.18}$$

$$V_1 = -30\text{ V},\ V_2 = -12\text{ V},\ I_1 = -2\text{ A}$$

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Basic Circuits

Nodal Analysis: Example 6.5: Voltage super node.

Given the following circuit. Solve for the indicated nodal voltages.

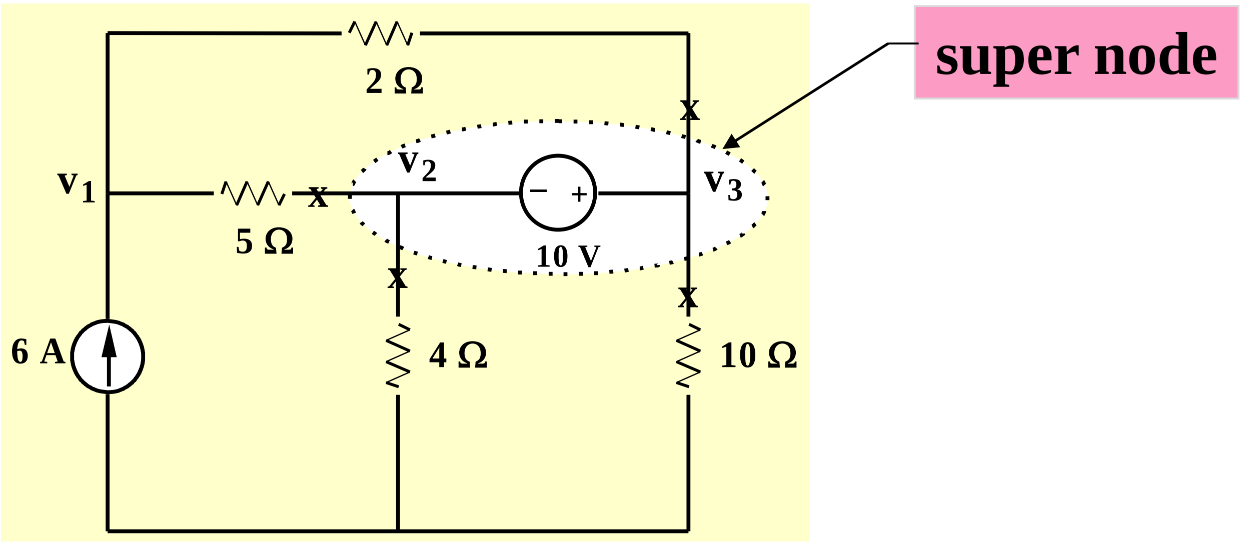

Figure 6.6: Circuit for Example 6.5.

When a voltage source appears between two nodes, an easy way to handle this is to form a super node. The super node encircles the voltage source and the tips of the branches connected to the nodes.

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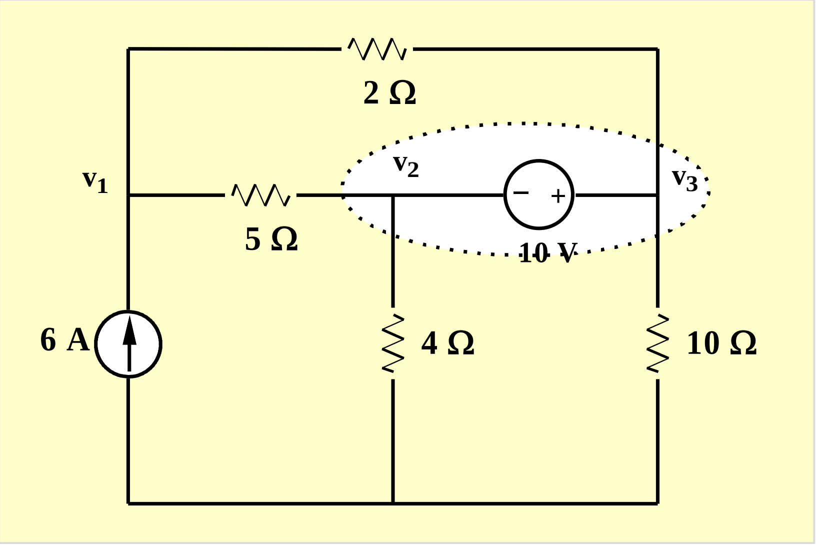
Basic Circuits

Nodal Analysis: Example 6.5: Continued.



Constraint Equation

$V_2 - V_3 = -10$ **Eq 6.19**

At V_1

$$\frac{V_1 - V_2}{5} + \frac{V_1 - V_3}{2} = 6$$ **Eq 6.20**

At super node

$$\frac{V_2 - V_1}{5} + \frac{V_2}{4} + \frac{V_3}{10} + \frac{V_3 - V_1}{2} = 0$$ **Eq 6.21**

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Basic Circuits

Nodal Analysis: Example 6.5: Continued.

Clearing Eq 6.19, 6.20, and 6.21:

$$7V_1 - 2V_2 - 5V_3 = 60 \qquad \text{Eq 6.22}$$

$$-14V_1 + 9V_2 + 12V_3 = 0 \qquad \text{Eq 6.23}$$

$$V_2 - V_3 = -10 \qquad \text{Eq 6.24}$$

Solving gives:

$$V_1 = 30\text{ V}, \quad V_2 = 14.29\text{ V}, \quad V_3 = 24.29\text{ V}$$

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Basic Circuits

Nodal Analysis: Example 6.6: With Dependent Sources.

Consider the circuit below. We desire to solve for the node voltages V_1 and V_2.

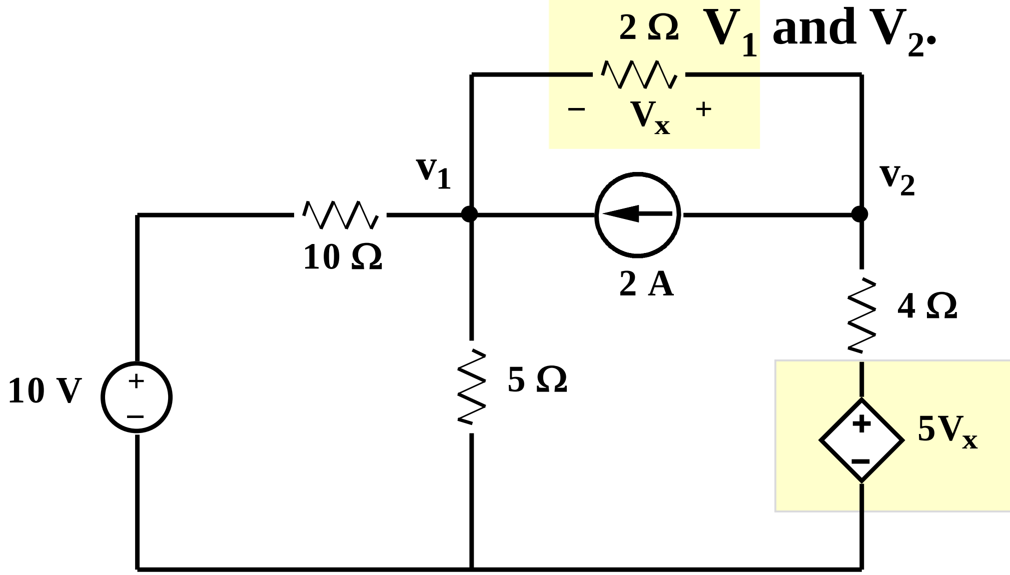


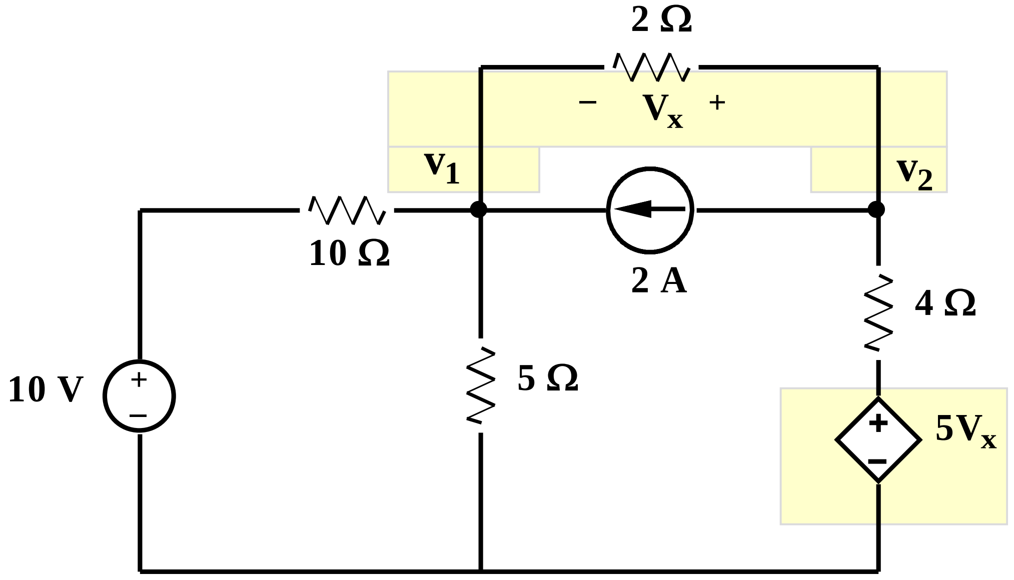
Figure 6.7: Circuit for Example 6.6.

In this case we have a dependent source, $5V_x$, that must be reckoned with. Actually, there is a constraint equation of

$$V_2 - V_x - V_1 = 0 \qquad \text{Eq 6.25}$$

Basic Circuits

Nodal Analysis: Example 6.6: With Dependent Sources.



At node V_1

$$\frac{V_1 - 10}{10} + \frac{V_1}{5} + \frac{V_1 - V_2}{2} = 2$$

At node V_2

$$\frac{V_2 - V_1}{2} + \frac{V_2 - 5V_x}{4} = -2$$

The constraint equation:

$$V_x = V_1 - V_2$$

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Basic Circuits

Nodal Analysis: Example 6.6: With Dependent Sources.

Clearing the previous equations and substituting the constraint $V_X = V_1 - V_2$ gives,

$$8V_1 - 5V_2 = 30$$ **Eq 6.26**

$$-7V_1 + 8V_2 = -8$$ **Eq 6.27**

which yields,

$$V_1 = 6.9V, \qquad V_2 = 5.03V$$

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NODAL ANALYSIS (GENERAL APPROACH)

Supernode

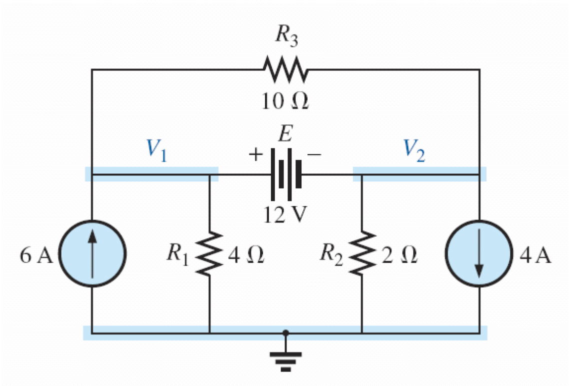

FIG. 8.53 *Example 8.22.*

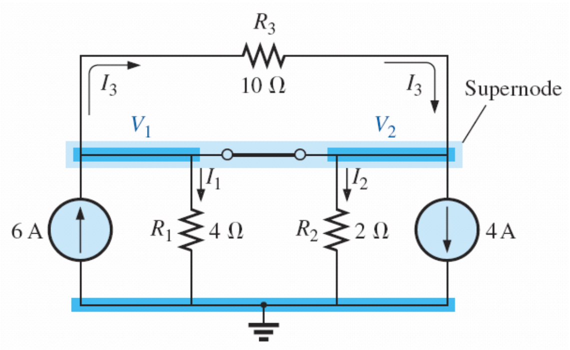

FIG. 8.54 *Defining the supernode for the network in Fig. 8.53.*