Thus we can write

$$T - T_1 = T - nT_2 = T - n\left(I_2 \frac{d^2\theta_2}{dt^2}\right)$$

and so

$$T - n\left(I_2 \frac{d^2\theta_2}{dt^2}\right) = I_1 \frac{d^2\theta_1}{dt^2}$$

Since $\theta_2 = n\theta_1$, $d\theta_2/dt = nd\theta_1/dt$ and $d^2\theta_2/dt^2 = nd^2\theta_1/dt$ then

$$T - n^2\left(I_2 \frac{d^2\theta_1}{dt^2}\right) = I_1 \frac{d^2\theta_1}{dt^2}$$

$$(I_1 + n^2 I_2)\frac{d^2\theta_1}{dt^2} = T$$

Without the gear train we would have had simply

$$I_1 \frac{d^2\theta_1}{dt^2} = T$$

Thus the moment of inertia of the load is reflected back to the other side of the gear train as an additional moment of inertia term $n^2 I_2$.

17.3 Electrical system building blocks

The basic building blocks of electrical systems are inductors, capacitors and resistors (Figure 17.8).

For an **inductor** the potential difference v across it at any instant depends on the rate of change of current (di/dt) through it:

$$v = L\frac{di}{dt}$$

where L is the inductance. The direction of the potential difference is in the opposite direction to the potential difference used to drive the current through the inductor, hence the term back e.m.f. The equation can be rearranged to give

$$i = \frac{1}{L}\int v\,dt$$

For a **capacitor**, the potential difference across it depends on the charge q on the capacitor plates at the instant concerned:

$$v = \frac{q}{C}$$

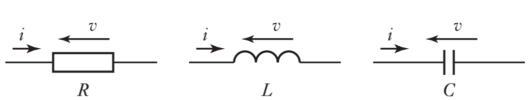


Figure 17.8 Electrical building blocks.

where C is the capacitance. Since the current i to or from the capacitor is the rate at which charge moves to or from the capacitor plates, i.e. $i = \mathrm{d}q/\mathrm{d}t$, then the total charge q on the plates is given by

$$q = \int i\,\mathrm{d}t$$

and so

$$v = \frac{1}{C}\int i\,\mathrm{d}t$$

Alternatively, since $v = q/C$ then

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{1}{C}\frac{\mathrm{d}q}{\mathrm{d}t} = \frac{1}{C}i$$

and so

$$i = C\frac{\mathrm{d}v}{\mathrm{d}t}$$

For a **resistor,** the potential difference v across it at any instant depends on the current i through it

$$v = Ri$$

where R is the resistance.

Both the inductor and capacitor store energy which can then be released at a later time. A resistor does not store energy but just dissipates it. The energy stored by an inductor when there is a current i is

$$E = \frac{1}{2}Li^2$$

The energy stored by a capacitor when there is a potential difference v across it is

$$E = \frac{1}{2}Cv^2$$

The power P dissipated by a resistor when there is a potential difference v across it is

$$P = iv = \frac{v^2}{R}$$

Table 17.2 summarises the equations defining the characteristics of the electrical building blocks when the input is current and the output is potential difference. Compare them with the equations given in Table 17.1 for the mechanical system building blocks.

Table 17.2 Electrical building blocks.

Building block	Describing equation	Energy stored or power dissipated
Inductor	$i = \frac{1}{L}\int v\,\mathrm{d}t$	$E = \frac{1}{2}Li^2$
	$v = L\frac{\mathrm{d}i}{\mathrm{d}t}$	
Capacitor	$i = C\frac{\mathrm{d}v}{\mathrm{d}t}$	$E = \frac{1}{2}Cv^2$
Resistor	$i = \frac{v}{R}$	$P = \frac{v^2}{R}$

17.3.1 Building up a model for an electrical system

The equations describing how the electrical building blocks can be combined are **Kirchhoff 's laws**, and can be expressed as outlined below:

Law 1: the total current flowing towards a junction is equal to the total current flowing from that junction, i.e. the algebraic sum of the currents at the junction is zero.
Law 2: in a closed circuit or loop, the algebraic sum of the potential differences across each part of the circuit is equal to the applied e.m.f.

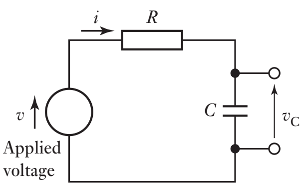


Figure 17.9
Resistor–capacitor system.

Now consider a simple electrical system consisting of a resistor and capacitor in series, as shown in Figure 17.9. Applying Kirchhoff's second law to the circuit loop gives

$$v = v_R + v_C$$

where v_R is the potential difference across the resistor and v_C that across the capacitor. Since this is just a single loop, the current i through all the circuit elements will be the same. If the output from the circuit is the potential difference across the capacitor, v_C, then since $v_R = iR$ and $i = C(dv_C/dt)$,

$$v = RC\frac{dv_C}{dt} + v_C$$

This gives the relationship between the output v_C and the input v and is a first-order differential equation.

Figure 17.10 shows a resistor–inductor–capacitor system. If Kirchhoff 's second law is applied to this circuit loop,

$$v = v_R + v_L + v_C$$

where v_R is the potential difference across the resistor, v_L that across the inductor and v_C that across the capacitor. Since there is just a single loop, the current i will be the same through all circuit elements. If the output from the circuit is the potential difference across the capacitor, v_C, then since $v_R = iR$ and $v_L = L(di/dt)$

$$v = iR + L\frac{di}{dt} + v_C$$

But $i = C(dv_C/dt)$ and so

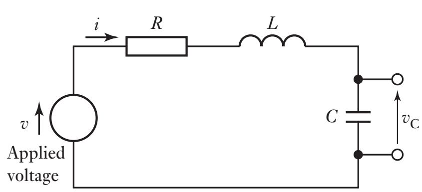
$$\frac{di}{dt} = C\frac{d(dv_C/dt)}{dt} = C\frac{d^2v_C}{dt^2}$$

Hence

$$v = RC\frac{dv_C}{dt} + LC\frac{d^2v_C}{dt^2} + v_C$$

This is a second-order differential equation.

Figure 17.10
Resistor–inductor–capacitor system.



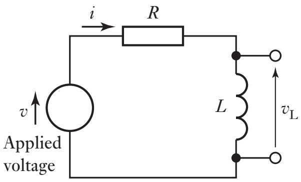


Figure 17.11
Resistor–inductor system.

As a further illustration, consider the relationship between the output, the potential difference across the inductor of v_L, and the input v for the circuit shown in Figure 17.11. Applying Kirchhoff 's second law to the circuit loop gives

$$v = v_R + v_L$$

where v_R is the potential difference across the resistor R and v_L that across the inductor. Since $v_R = iR$,

$$v = iR + v_L$$

and

$$i = \frac{1}{L}\int v_L \mathrm{d}t$$

then the relationship between the input and output is

$$v = \frac{R}{L}\int v_L \mathrm{d}t + v_L$$

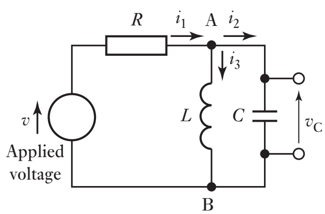


Figure 17.12
Resistor–capacitor–inductor system.

As another example, consider the relationship between the output, the potential difference v_C across the capacitor, and the input v for the circuit shown in Figure 17.12. Applying Kirchhoff 's first law to node A gives

$$i_1 = i_2 + i_3$$

but

$$i_1 = \frac{v - v_A}{R}$$

$$i_2 = \frac{1}{L}\int v_A \, \mathrm{d}t$$

$$i_3 = C\frac{\mathrm{d}v_A}{\mathrm{d}t}$$

Hence

$$\frac{v - v_A}{R} = \frac{1}{L}\int v_A \, \mathrm{d}t + C\frac{\mathrm{d}v_A}{\mathrm{d}t}$$

But $v_C = v_A$. Hence, with some rearrangement,

$$v = RC\frac{\mathrm{d}v_C}{\mathrm{d}t} + v_C + \frac{R}{L}\int v_C \, \mathrm{d}t$$

17.3.2 Electrical and mechanical analogies

The building blocks for electrical and mechanical systems have many similarities (Figure 17.13). For example, the electrical resistor does not store energy but dissipates it, with the current i through the resistor being given by $i = v/R$, where R is a constant, and the power P dissipated by $P = v^2/R$. The mechanical analogue of the resistor is the dashpot. It also does not store energy but dissipates it, with the force F being related to the velocity v by $F = cv$, where c is a constant, and the power P dissipated by $P = cv^2$. Both these sets of equations have similar forms. Comparing them, and taking the

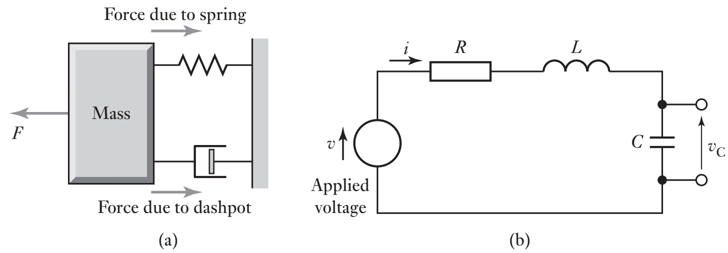


Figure 17.13 Analogous systems.

current as being analogous to the force, then the potential difference is analogous to the velocity and the dashpot constant c to the reciprocal of the resistance, i.e. $(1/R)$. These analogies between current and force, potential difference and velocity, hold for the other building blocks with the spring being analogous to inductance and mass to capacitance.

The mechanical system in Figure 17.1(a) and the electrical system in Figure 17.1(b) have input/output relationships described by similar differential equations:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F \quad \text{and} \quad RC\frac{dv_C}{dt} + LC\frac{d^2v_C}{dt^2} + v_C = v$$

The analogy between current and force is the one most often used. However, another set of analogies can be drawn between potential difference and force.

17.4 Fluid system building blocks

In fluid flow systems there are three basic building blocks which can be considered to be the equivalent of electrical resistance, capacitance and inductance. Fluid systems can be considered to fall into two categories: hydraulic, where the fluid is a liquid and is deemed to be incompressible; and pneumatic, where it is a gas which can be compressed and consequently shows a density change.

Hydraulic resistance is the resistance to flow which occurs as a result of a liquid flowing through valves or changes in a pipe diameter (Figure 17.14(a)). The relationship between the volume rate of flow of liquid q through the resistance element and the resulting pressure difference $(p_1 - p_2)$ is

$$p_1 - p_2 = Rq$$

where R is a constant called the hydraulic resistance. The bigger the resistance, the bigger the pressure difference for a given rate of flow. This equation, like that for the electrical resistance and Ohm's law, assumes a linear relationship. Such hydraulic linear resistances occur with orderly flow through capillary tubes and porous plugs, but non-linear resistances occur with flow through sharp-edged orifices or if flow is turbulent.

Hydraulic capacitance is the term used to describe energy storage with a liquid where it is stored in the form of potential energy. A height of liquid in a container (Figure 17.14(b)), i.e. a so-called pressure head, is one form of such a storage. For such a capacitance, the rate of change of volume V in the