Graphing functions:

Graphs may be straight line or curves

Curves:

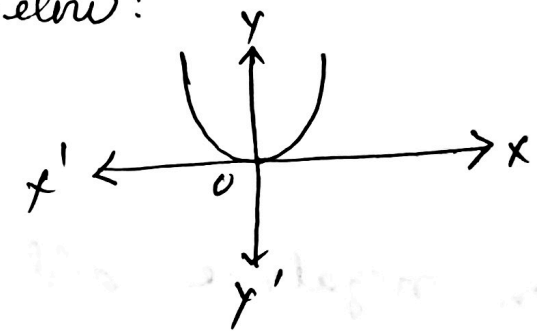
Graphs of $y = x^n$, n being any positive integer.

Case-1:

when n is positive even integer

Let $y = x^{2m}$ i.e $y = x^2$, $y = x^4$, $y = x^6$, ... $y = x^{2m}$

then the graph of y below:

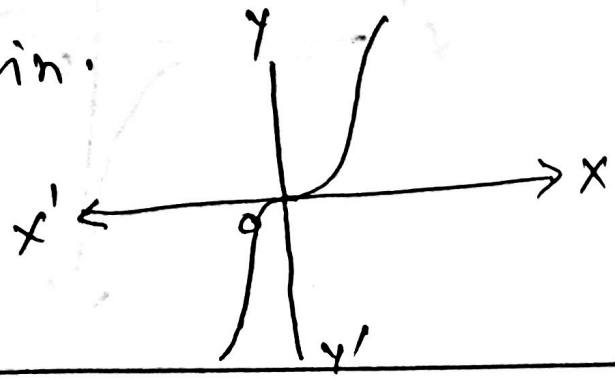


Case-2:

when n is a positive odd integer:

Let $n = 2m+1$ then $y = x^n = x^{2m+1}$ i.e $y = x^3/x^5/x^7/\ldots/x^{2m+1}$

y is positive or negative according as x is positive or negative. So there will be no branch of the curve in the 2nd and 4th quadrants. It is symmetrical about origin.



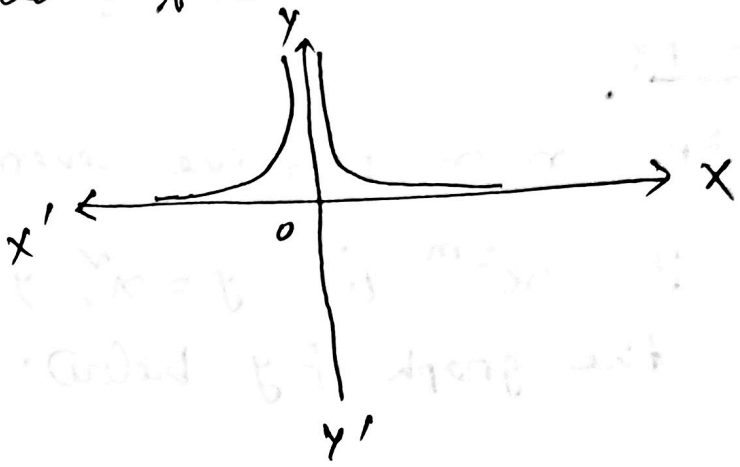
Case-3:

When n is negative even integer.

Let $n = -2m$, where m is a positive integer.

Then $y = x^n = x^{-2m} = \frac{1}{x^{2m}}$

Then function is defined for all values of x except at $x = 0$.

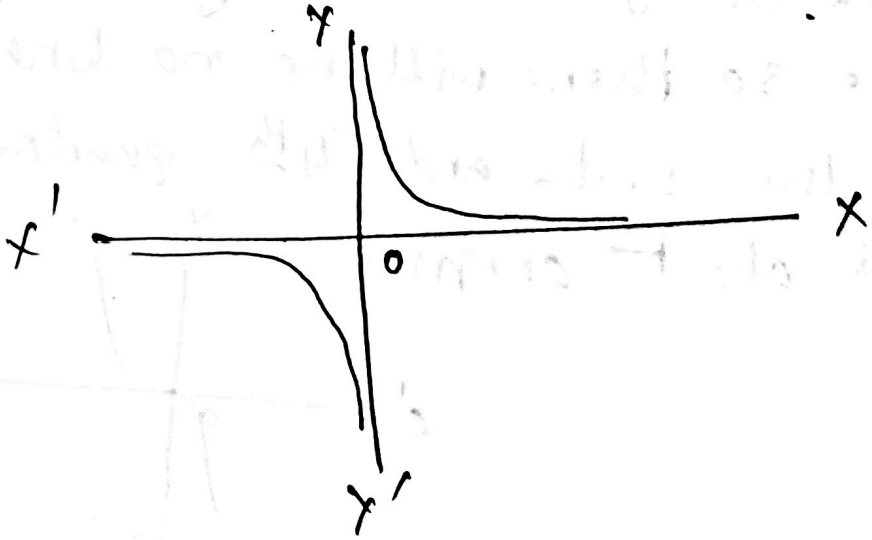


Case-4:

When n is negative odd integer:

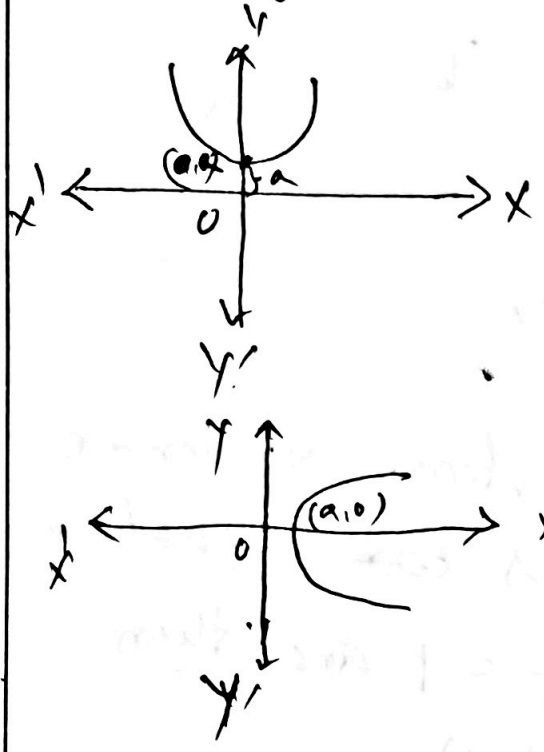
Let $n = -(2m+1)$ Then $y = x^n = x^{-(2m+1)} = \frac{1}{x^{2m+1}}$.

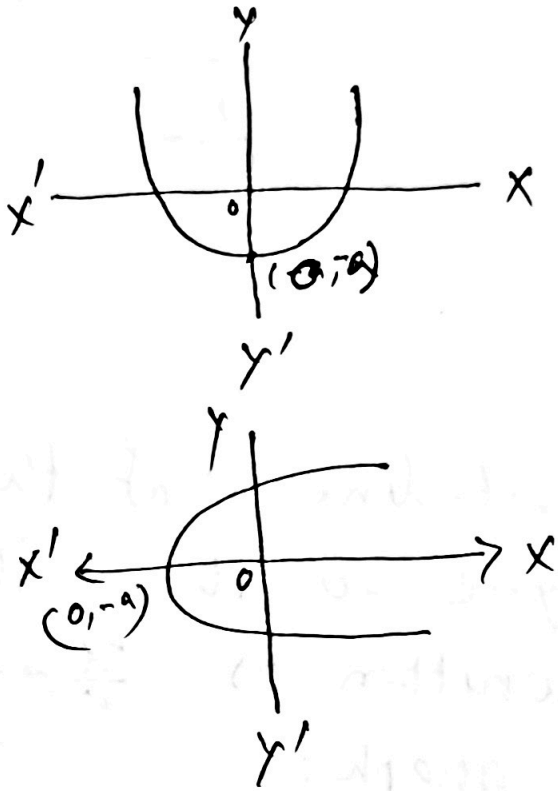
y is positive or negative according as x is positive or negative. So there is no branch of the graph in the 2nd and 4th quadrants.



Special cases:

i) When $y = x^2 + a$ or $y = x^2 - a$ or $x = y^2 + a$ or $x = y^2 - a$ then the graphs are below respectively.



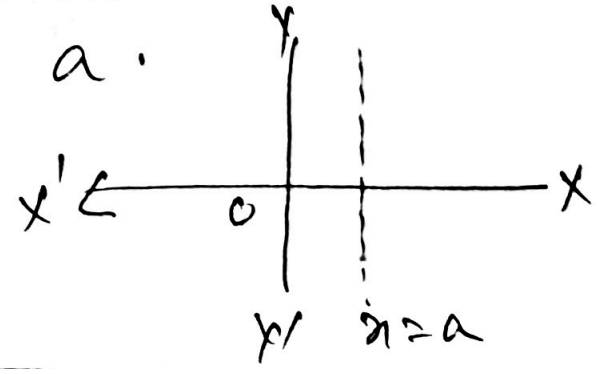


Straight line:

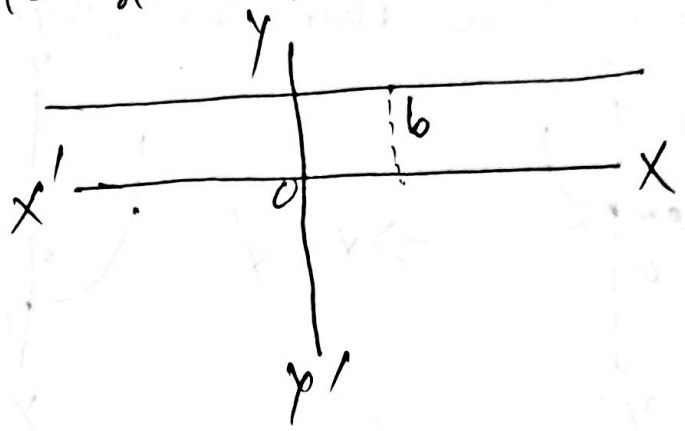
If the function $y = f(x)$ is a straight line, then we know that, there are six types of straight line such as

$x = a$, $y = b$, $ax + by + c = 0$, $y = mx$, $x = 0$, $y = 0$

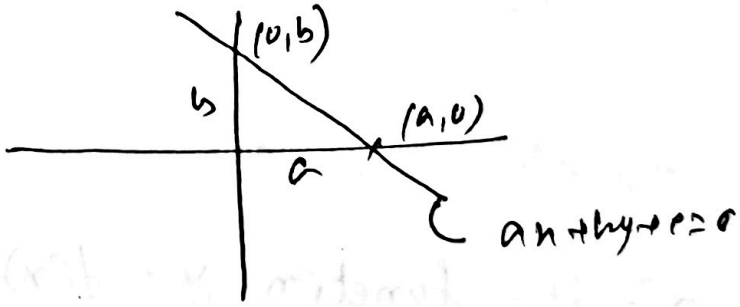
i) $x = a$ which is an equation of parallel to y-axis then we draw the st. line of parallel to y-axis from the distance a.



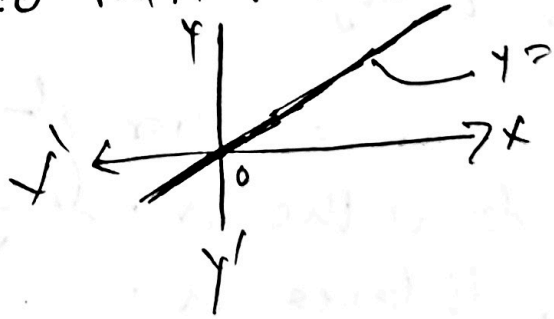
(ii) $y = b$ which is know as the equation of st. line parallel to x-axis. So we drawn the st. line of parallel to x-axis from the distance b



(iii) If the st. line is of the form $y = mx + c$ or $ax + by + c = 0$ then this equation can be written as $\frac{x}{a} + \frac{y}{b} = 1$ and then drawn the graph:



iv) If the st. line is of the form $y = mx$ or $ax + by = 0$ then this st. line can be written as



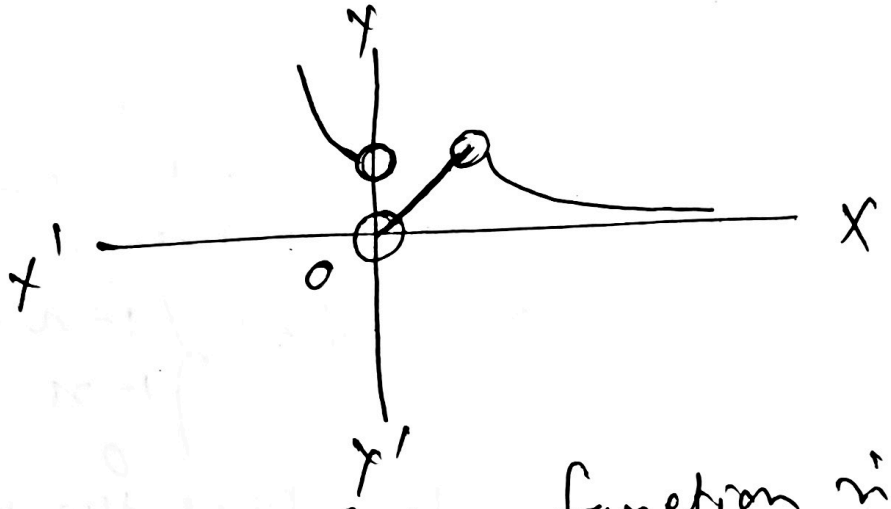
putting $x = *$ then find $y = *$

Ex:- Draw the graph of the function and also find Domain and Range.

$$f(x) = \begin{cases} x^2+1 & \text{when } x<0 \\ x & \text{" } 0\le x\le 1 \\ \frac{1}{x} & \text{" } x>1 \end{cases}$$

Sol^n Given that the function

$y = x^2+1$ $\quad y = x$ $\quad y = \frac{1}{x} = x^{-1}$



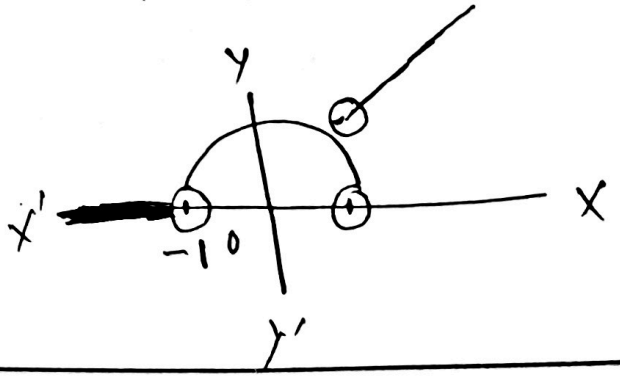
The domain of the function is

$(-\infty, 0) \cup [0,1] \cup (1,\infty) = (-\infty, \infty) = \mathbb{R}$

And the range is $[0,\infty)$

Ex:

$$f(x) = \begin{cases} 0 & \text{when } x \le -1 \\ \sqrt{1-x^2} & \text{" } -1<x<1 \\ x & \text{" } x\ge 1 \end{cases}$$



Ex: Sketch / Draw the graph of the function $f(x) = \frac{x^2-9}{x-3}$

2. " " " " " " . $f(x) = |x| + |x+1|$. Also find the domain and range.

3. " " " " " $f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ x+2, & x > 0 \end{cases}$

4. " " " " " $f(x) = \begin{cases} 2x-3; & x < 1 \\ -x^2; & x \geq 1 \end{cases}$

Find domain and range

5. " " " " $f(x) = |x| + |x-1|$ and also find domain and range.

6. " " " " " $f(x) = \begin{cases} 1+x \text{ when } -1 \leq x \leq 0 \\ 1-x \text{ " } 0 < x \leq 1 \\ 0 \text{ " } x > 1 \end{cases}$

Determine the range and domain of the function.

7. " " " " " $f(x) = |x+1| + |x-1|$

Also find domain and range.

8. " " " " " $f(x) = \begin{cases} 0, & |x| > 1 \text{ i.e. } x < -1 \text{ or } x > 1 \\ 1+x, & -1 \leq x \leq 0 \\ 1-x, & 0 < x \leq 1 \end{cases}$

9. " " " " " $f(x) = \begin{cases} x^2, & 0 < x < 1 \\ x, & 1 \leq x \leq 2 \\ \frac{1}{4}x^2, & 2 < x < 5. \end{cases}$

Also find domain and range.

Ex:10. Sketch the graph of the function $f(x) = |x+1| + |x-2|$. Also find domain and range.

11. " " " " " " $f(x) = |x+1| + |x| + |x-1|$
Also find domain and range.

12. " " " " " " $f(x) = |x| + |x-1| + |x-2|$
Also find domain and range

13. " " " " " " $f(x) = |x+1| + |x| + |x-1|$
Also find domain and range.

14. " " " " " $y = a^x$ when $a > 1$
" $0 < a < 1$

15. " " " " " $f(x) = \begin{cases} x^2, & x < 0 \\ x, & 0 \le x \le 1 \\ \frac{1}{x}, & x > 1 \end{cases}$

16. " " " " " $f(x) = \begin{cases} 2-x, & x > 1 \\ x, & 0 < x \le 1 \\ -x, & x \le 0 \end{cases}$

17. " " " " " $f(x) = \begin{cases} x^2+1, & x > 1 \\ x^2, & -1 \le x \le 1 \\ x^2-1, & x < -1 \end{cases}$

18. " " " " " $f(x) = \begin{cases} x, & 1 \le x \le 2 \\ 2, & -1 < x < 1 \\ x^2+1, & -3 \le x \le -1 \end{cases}$