Sheikh Hasina University, Netrokona
Department of Computer Science and Engineering

CSE-2205: Introduction to Mechatronics

Lec-20: Electrical System Models

Mechatronics: Electronic Control Systems in Mechanical Engineering by W. Bolton

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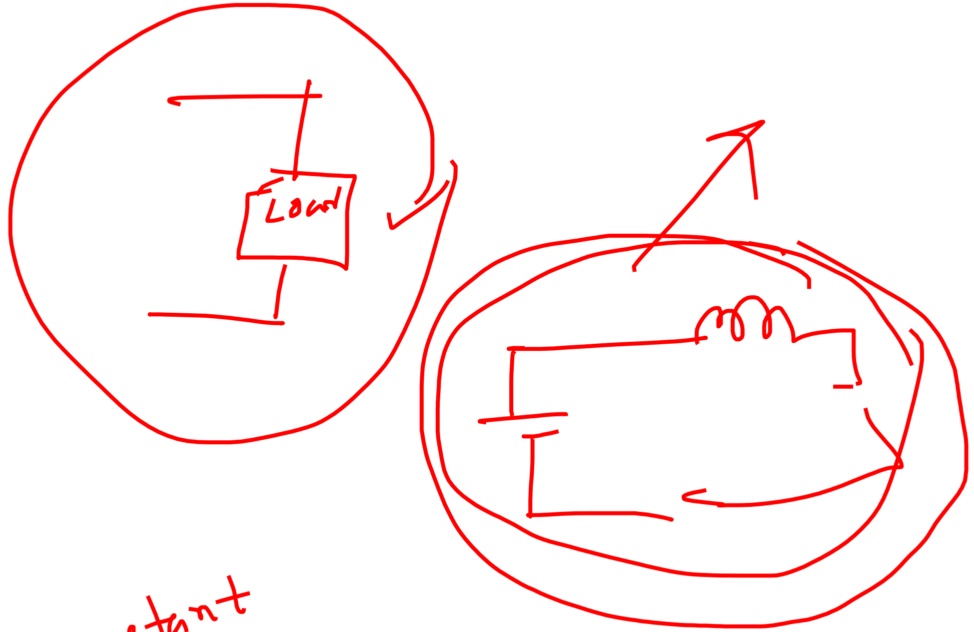
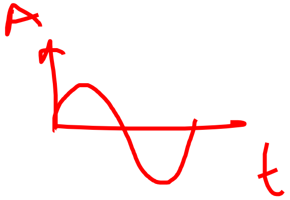
Electrical building blocks:

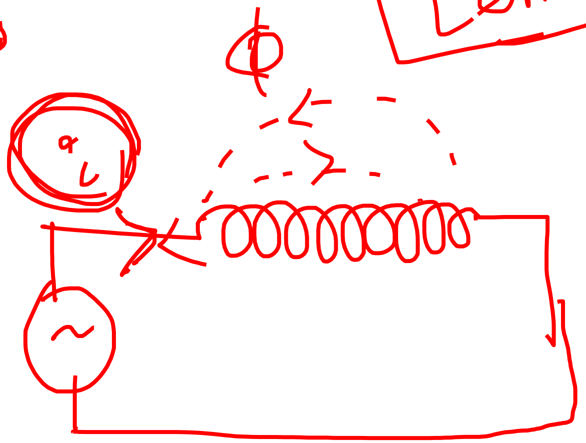
- → inductors
- → capacitors
- → Resistors



Lenz law

1) Inductors

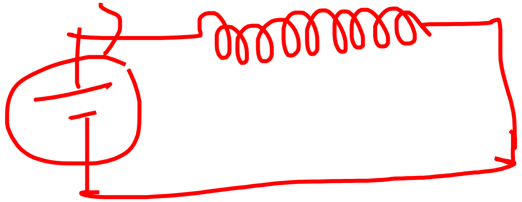




$$EMF \propto -\frac{di}{dt}$$

$$v = L\frac{di}{dt} \quad \text{---(i)}$$

i = constant



$$EMF \propto \frac{d}{dt}(i) \rightarrow \text{constant}$$

$$EMF \propto 0$$

$$v = L\frac{di}{dt}$$

$$\Rightarrow L\,di = v\,dt$$

$$\Rightarrow di = \frac{1}{L}v\,dt$$

$$\Rightarrow \boxed{i = \frac{1}{L}\int v\,dt} \quad -\text{(ii)}$$

model of inductor block.

$1 \cdot di$

$= \text{i}$

Energy stored by inductor:

$$E = -\frac{d\phi}{dt}$$

$$\phi \propto I$$

$$\phi = LI$$

inductance

$$V = L\frac{dI}{dt}$$

$$P = VI = L\frac{dI}{dt}\cdot I$$

~~2) Capacitors~~

$$P = -L\frac{dI}{dt}\cdot I$$

P = rate of change of energy (work done)

$$\frac{dW}{dt} = -LI\frac{dI}{dt}$$

integrating

$$\Rightarrow W = -\int L\, I \frac{dI}{\cancel{dt}}\,\cancel{dt}$$

$$\Rightarrow W = -\int L\cdot I\cdot dI$$

$$W = -L\int_{0}^{I_0} I\, dI$$

$$W = -L\int_{I_0}^{I} I\, dI = -\frac{1}{2}LI^2 + \frac{1}{2}LI_0^2$$

$$W = -L\,\frac{1}{2}I^2$$

$$\boxed{W = -\frac{1}{2}LI^2}$$

I_0

Summarize:

Inductor:

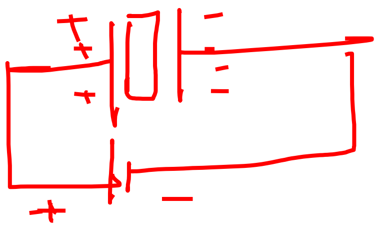
$$v = L \frac{di}{dt}$$

$$\boxed{i = \frac{1}{L} \int v\, dt}$$

$$E = W = \frac{1}{2} L i^2$$

2) Capacitor:

q



v

$q \propto v$

$q = Cv$ capacitance

$\Rightarrow v = \frac{q}{C}$

$$\boxed{v = \frac{1}{C}\int i\,dt} \quad i$$

e

$i = \frac{dq}{dt}$

$\Rightarrow dq = i\,dt$

$\Rightarrow \int dq = \int i\,dt$

$q = \int i\,dt$

$\Rightarrow$

$q \propto v$

$q = cv$

$\Rightarrow v = \frac{q}{c}$

Differentiate w.r.t time

$\Rightarrow \frac{dv}{dt} = \frac{1}{c} \frac{dq}{dt}$

$\Rightarrow \frac{dv}{dt} = \frac{1}{c} \cdot i$

$\Rightarrow \boxed{i = c \frac{dv}{dt}}$

Energy stored by capacitors:

$q \propto v$

$q = cv \qquad \therefore v = \frac{q}{c}$

$\Rightarrow c = \frac{q}{v}$

$\boxed{w = F S}$

Workdone

$W = \int_0^q V \, dq$

$= \int_0^q \frac{q}{c} \, dq$

$$W = \frac{1}{C}\int_0^q q\,dq$$

$$= \frac{1}{2}\frac{q^2}{C}$$

$$q = CV$$

$$W = \frac{1}{2}\frac{C^2V^2}{C}$$

$$\boxed{W = \frac{1}{2}CV^2}$$

Summarize:

$$v = \frac{1}{C}\int i\,dt$$

$$i = C\frac{dv}{dt}$$

$$E = W = \frac{1}{2}Cv^2$$

3) Resistor:

$v = Ri$

$i = \frac{v}{R}$

does not store energy, but dissipates it

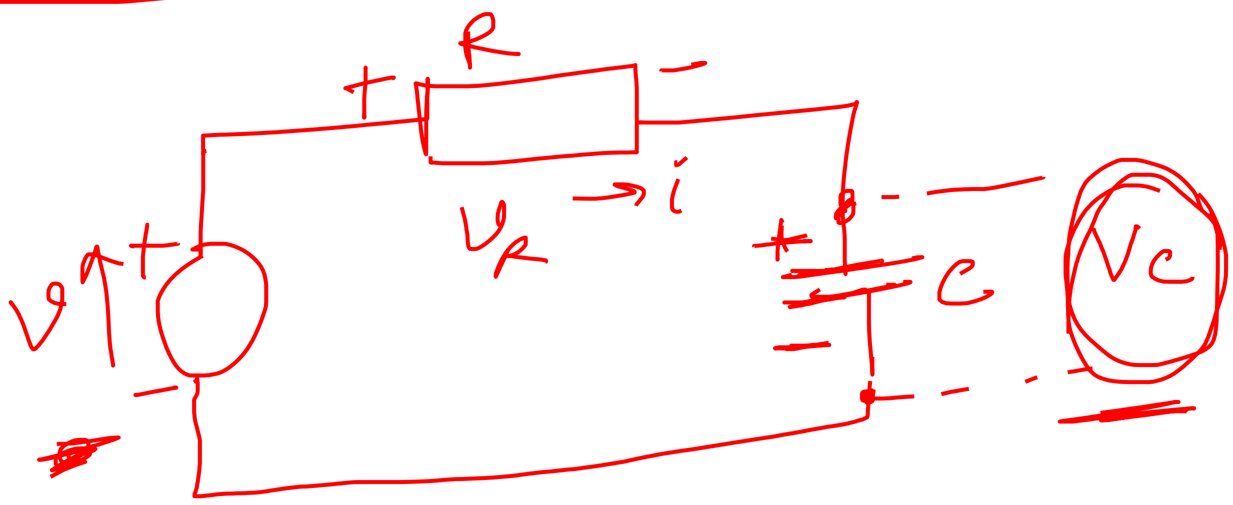
power dissipated by a resistor:

$p = vi$

$= v \cdot \frac{v}{R}$

$\boxed{p = \frac{v^2}{R}}$

Building up a model for an electrical system:



KVL:

$$v - V_R - v_c = 0$$

$$\Rightarrow v = v_R + v_c$$

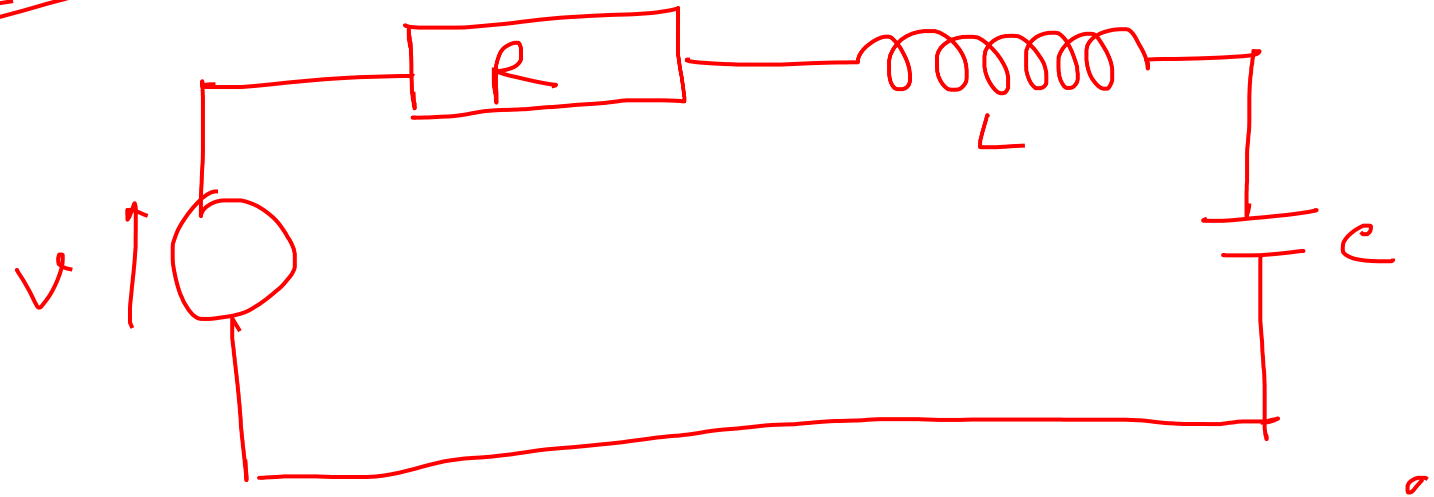
$$\Rightarrow v = Ri + v_c$$

$$v = Ri + v_c$$

$$= R \cdot C \frac{dv_c}{dt} + v_c$$

$$\boxed{\Rightarrow v = RC \frac{dv_c}{dt} + v_c}$$

Example: v & v_c



$$i = C\frac{dv_c}{dt}$$

KVL:

$$v - v_R - v_L - v_c = 0$$

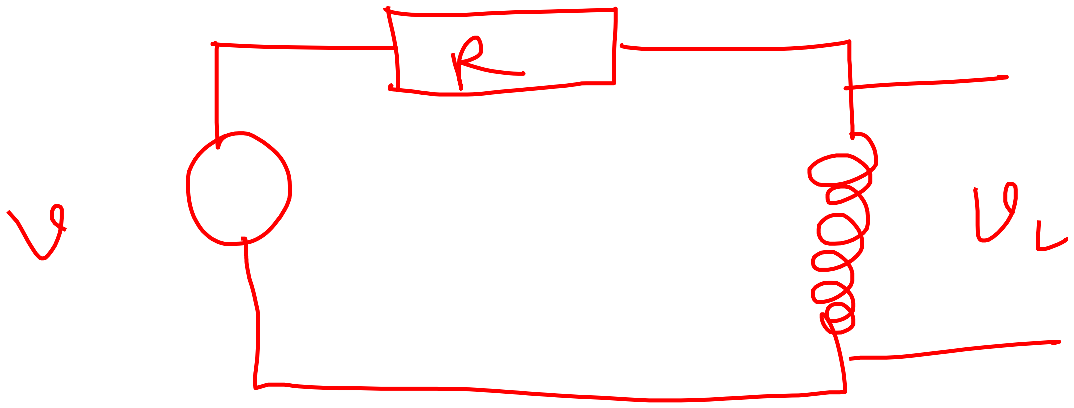
$$\Rightarrow v = v_R + v_L + v_c$$

$$v = iR + L\frac{di}{dt} + v_c$$

$$= C\frac{dv_c}{dt}R + L\frac{d}{dt}\left(C\frac{dv_c}{dt}\right) + v_c$$

$$\boxed{v = RC\frac{dv_c}{dt} + LC\frac{d^2v_c}{dt^2} + v_c}$$

Example: input & inductor.



KVL

$$v - v_R - v_L = 0$$

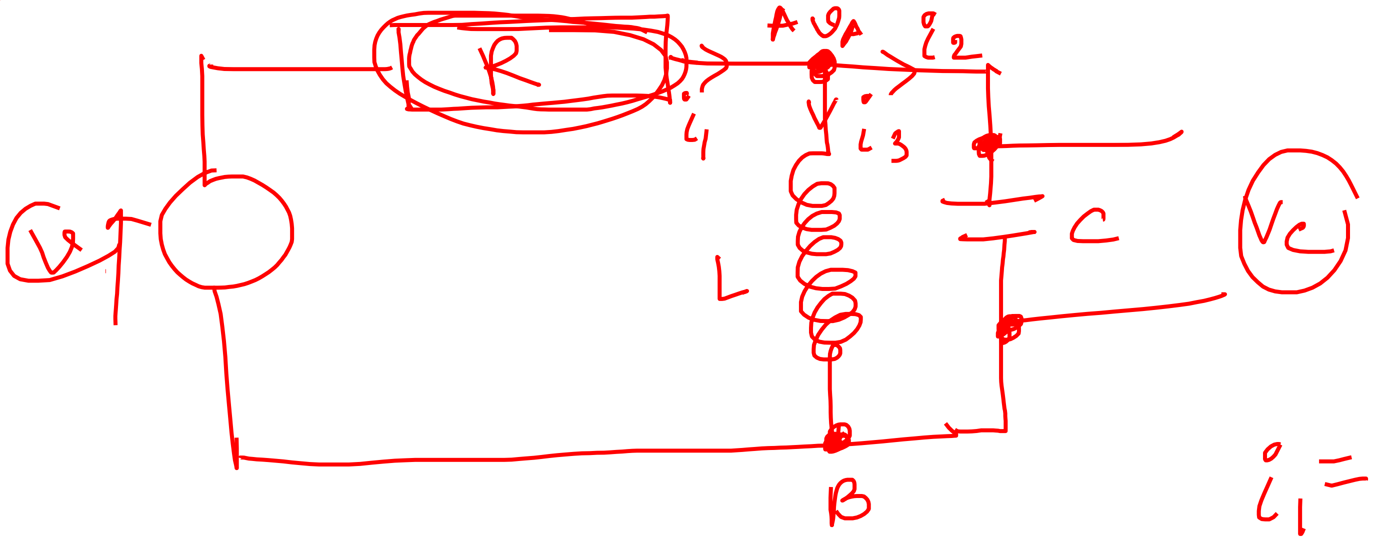
$$\Rightarrow v = v_R + v_L$$

$$\Rightarrow v = iR + v_L$$

$$v = \frac{1}{L}\int v_L\,dt \cdot R + v_L$$

$$\Rightarrow \boxed{v = \frac{R}{L}\int v_L\,dt + v_L}$$

Example:



$$i = \frac{V}{R}$$

KCL:

$$i_1 = i_2 + i_3$$

$$i_1 = \frac{v - v_A}{R}$$

$$i_2 = C \frac{dv_A}{dt}$$

$$i_3 = \frac{1}{L} \int v_A \, dt$$

$$i_1 = i_2 + i_3$$

$$\Rightarrow \frac{v - v_A}{R} = \frac{1}{L}\int v_A\,dt + C\frac{dv_A}{dt}$$

$$\Rightarrow v_A = v_C$$

$$\Rightarrow \frac{v - v_C}{R} = \frac{1}{L}\int v_C\,dt + C\frac{dv_C}{dt}$$

$$\Rightarrow \frac{v}{R} - \frac{v_C}{R} = \frac{1}{L}\int v_C\,dt + C\frac{dv_C}{dt}$$

$$\Rightarrow \frac{v}{R} = \frac{v_C}{R} + \frac{1}{L}\int v_C\,dt + C\frac{dv_C}{dt}$$

$$\Rightarrow \frac{v}{R} = \frac{1}{L}\int v_c\,dt$$

$$\Rightarrow \frac{v}{R} = \frac{V_c}{R} + \frac{1}{L}\int v_c\,dt + \frac{C\,dV_c}{dt}$$

$$\Rightarrow v = \frac{V_c}{\cancel{R}} \times \cancel{R} + \frac{R}{L}\int v_c\,dt + RC\,\frac{dV_c}{dt}$$

$$\Rightarrow \boxed{v = v_c + \frac{R}{L}\int v_c\,dt + RC\,\frac{dV_c}{dt}}$$

Electrical & Mechanical analogies!

dashpot

Resistor = Does not store energy

$F = cv$

~~Voltage~~:

$$P = \frac{V^2}{R}$$

$$P = cv^2$$

$$C = \frac{1}{R}$$

= velocity

voltage