Sheikh Hasina University, Netrokona

Department of Computer Science and Engineering

CSE-2205: Introduction to Mechatronics

Lec-22: Thermal System Models

Mechatronics: Electronic Control Systems in Mechanical Engineering by W. Bolton

Md. Ariful Islam

Assistant Professor

Dept. of Robotics and Mechatronics Engineering

University of Dhaka

&

Adjunct Faculty

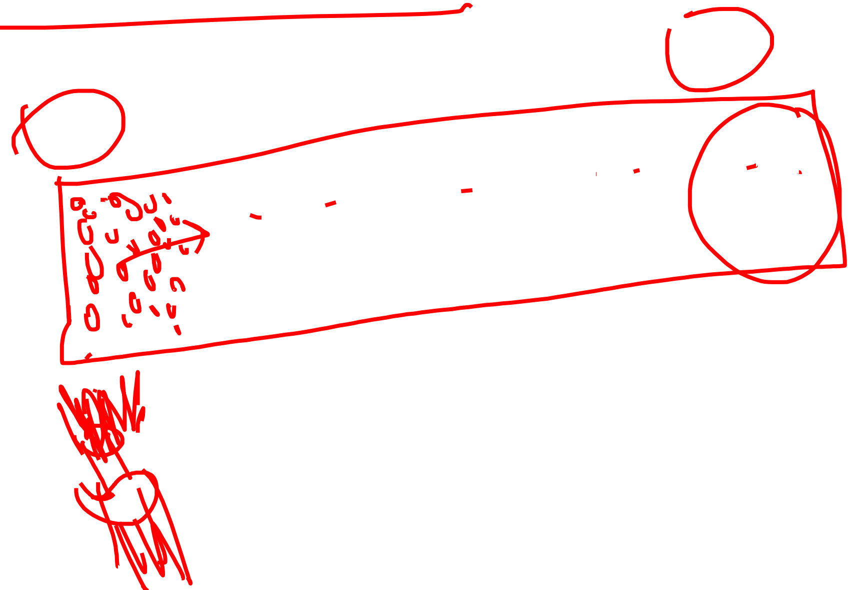
Sheikh Hasina University, Netrokona

Department of Computer Science and Engineering

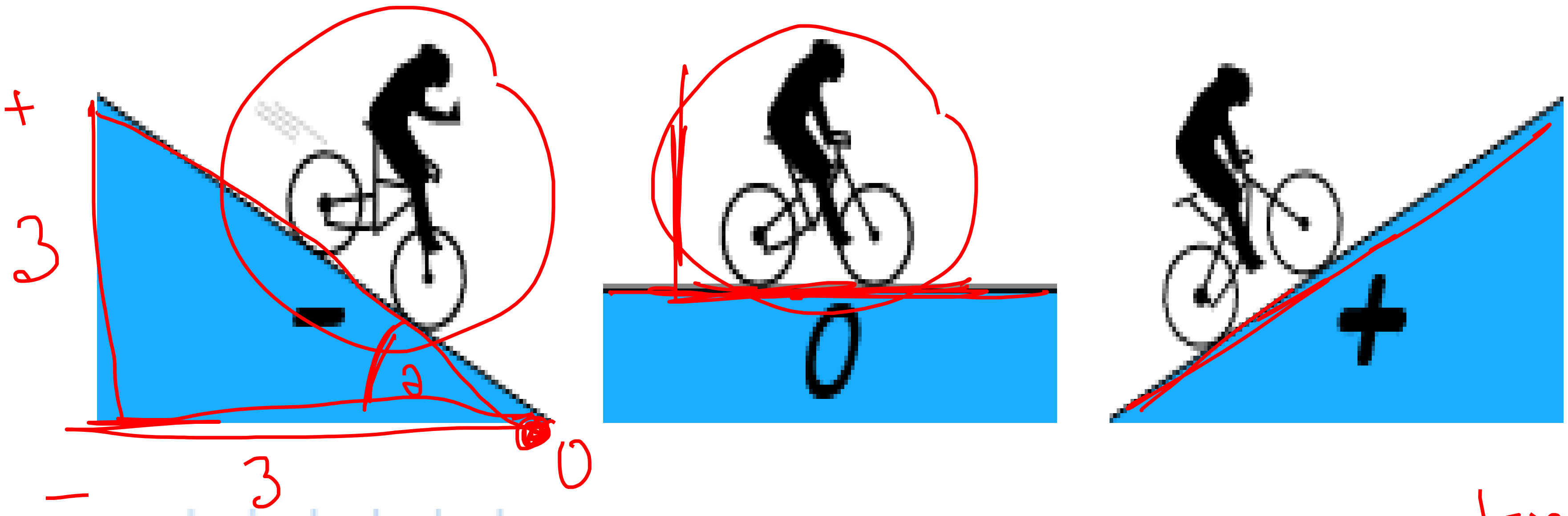
Q. Formal definition of conduction.

Transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones

- Takes place in solids, liquids, or gases
- Rate of heat conduction depends on geometry of the medium, its thickness, and the material of the medium, as well as the temperature difference across the medium.

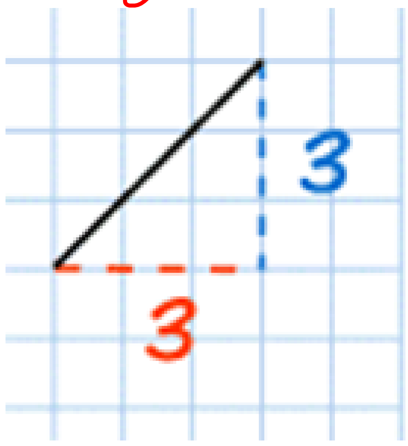


Gradient or slope



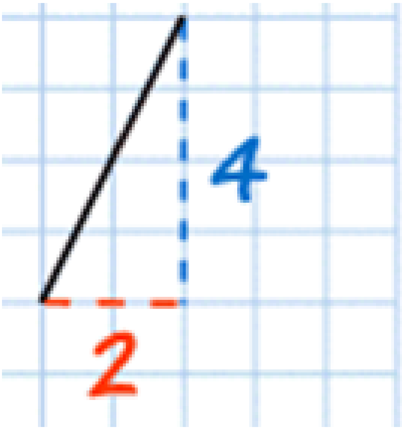
$$\tan\theta = \frac{0}{\;} = 0$$

$$\tan\theta = \frac{3}{-3} = -1$$

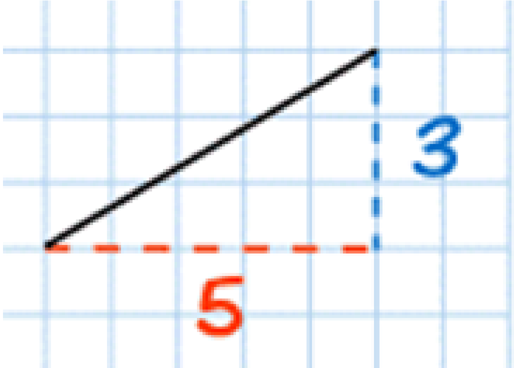


The Gradient $= \frac{3}{3} = 1$

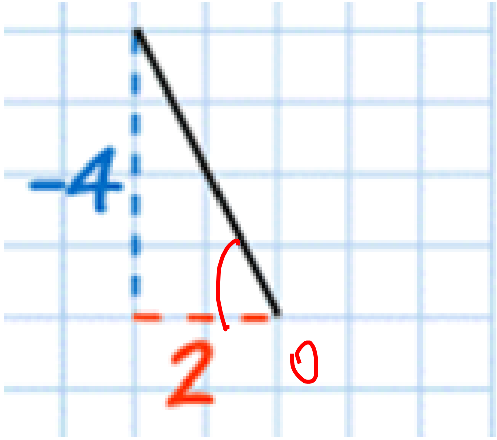
So the Gradient is equal to 1



$$\text{The Gradient} = \frac{4}{2} = 2$$

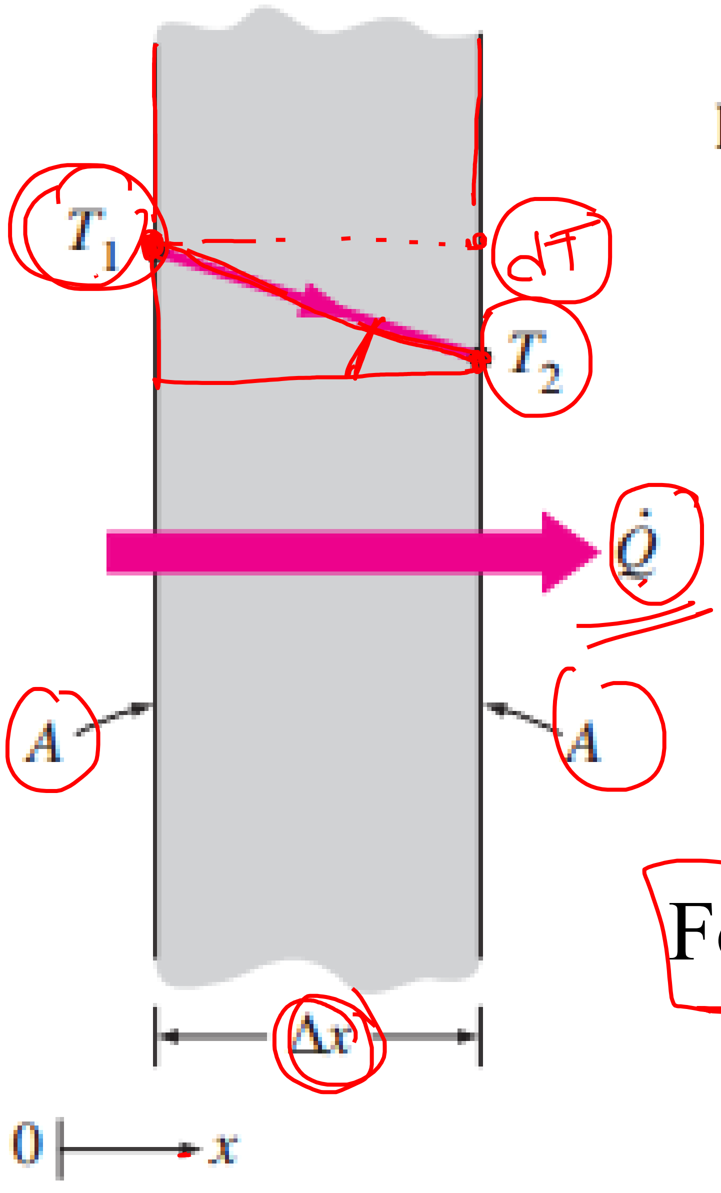


$$\text{The Gradient} = \frac{3}{5} = 0.6$$



$$\text{Gradient} = \frac{-4}{2} = -2$$

Q. Rate of heat conduction



$$\text{Rate of heat conduction} \propto \frac{\text{(Area)(Temperature difference)}}{\text{Thickness}}$$

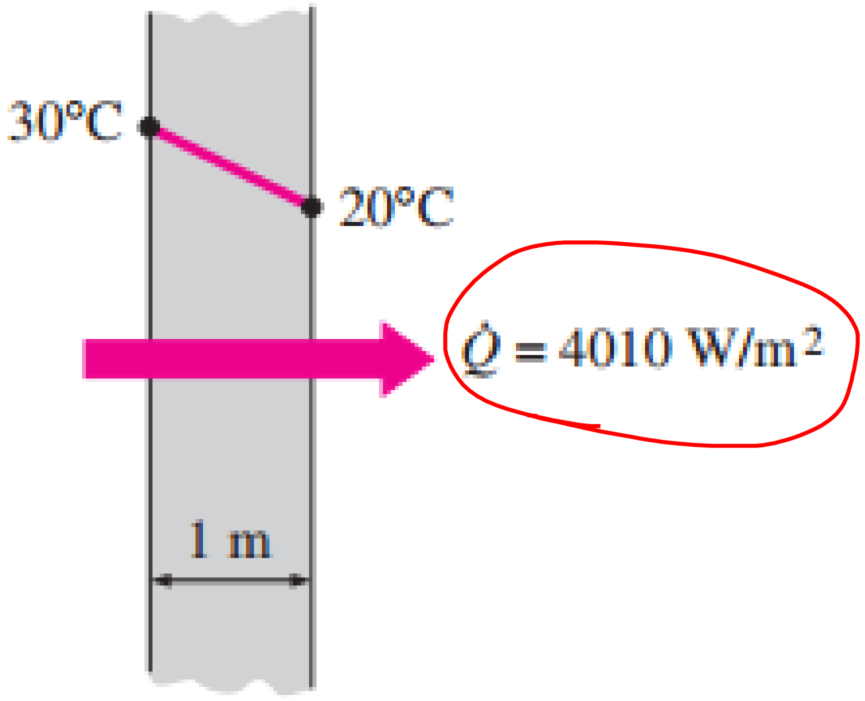
$$\dot{Q}_{\text{cond}} = kA\,\frac{T_1 - T_2}{\Delta x}$$

thermal conductivity

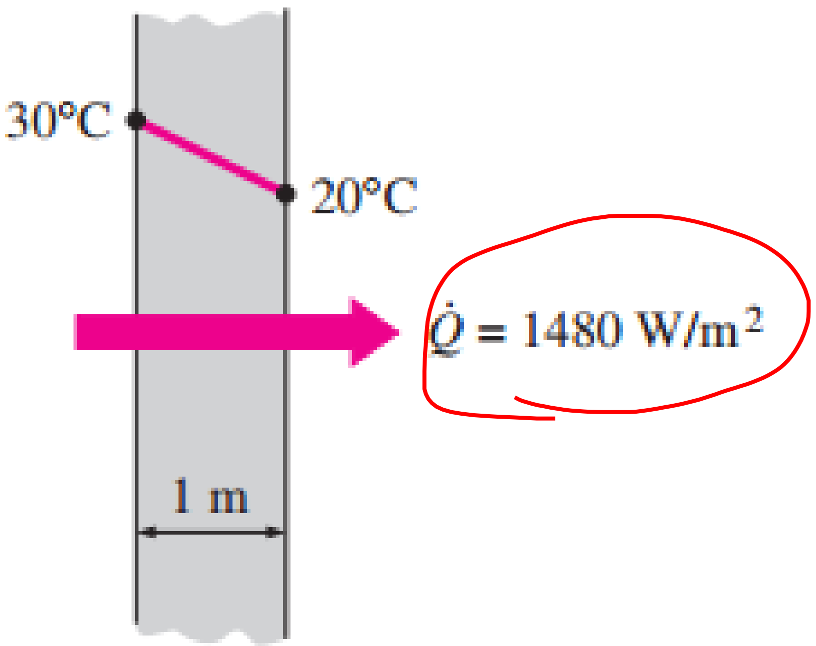
$$\dot{Q}_{\text{cond}} = -kA\,\frac{dT}{dx} \qquad \text{(W)}$$

Fourier's law of heat conduction

Q. Effect of thermal conductivity



(*a*) Copper (*k* = 401 W/m·°C)



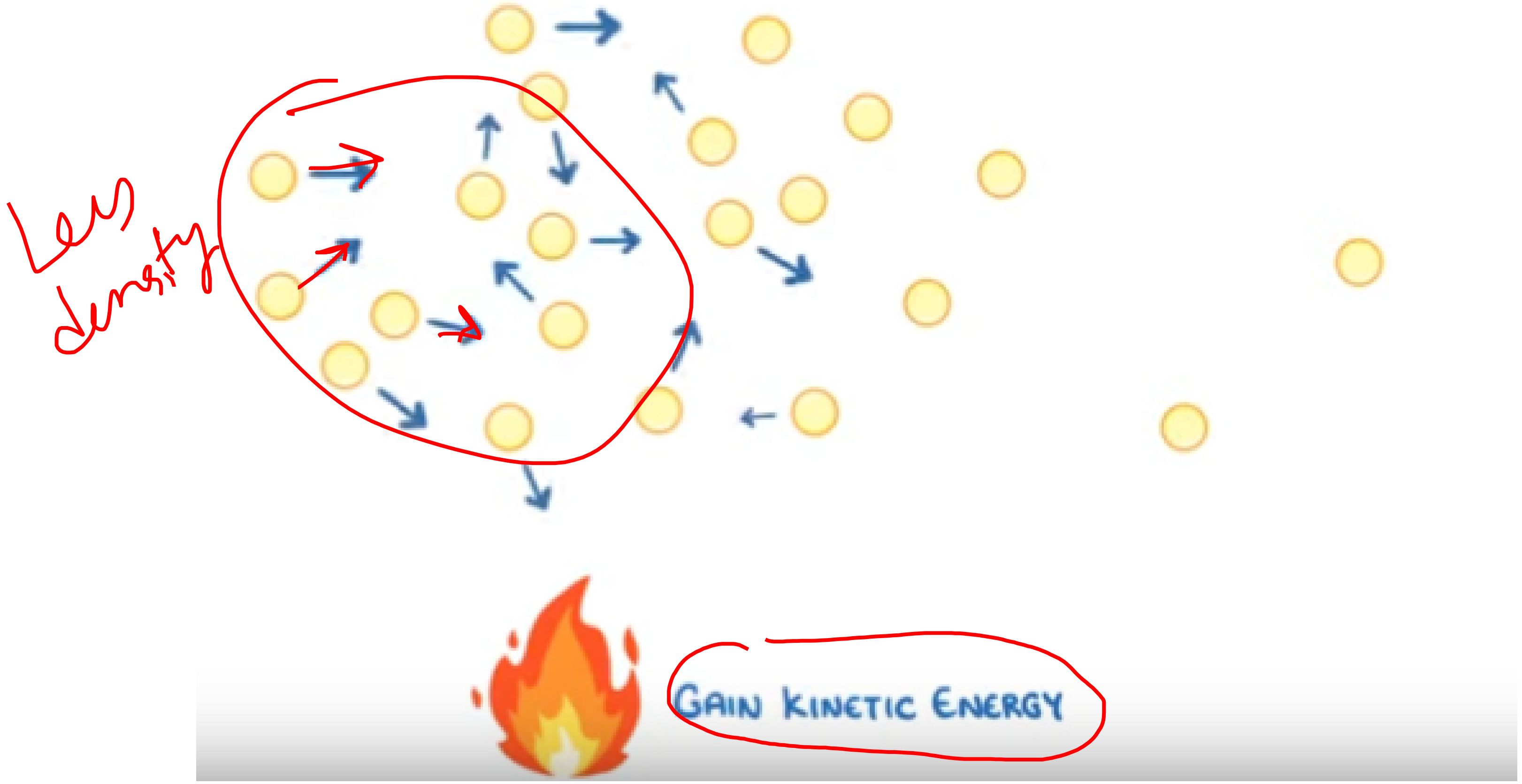
(*b*) Silicon (*k* = 148 W/m·°C)

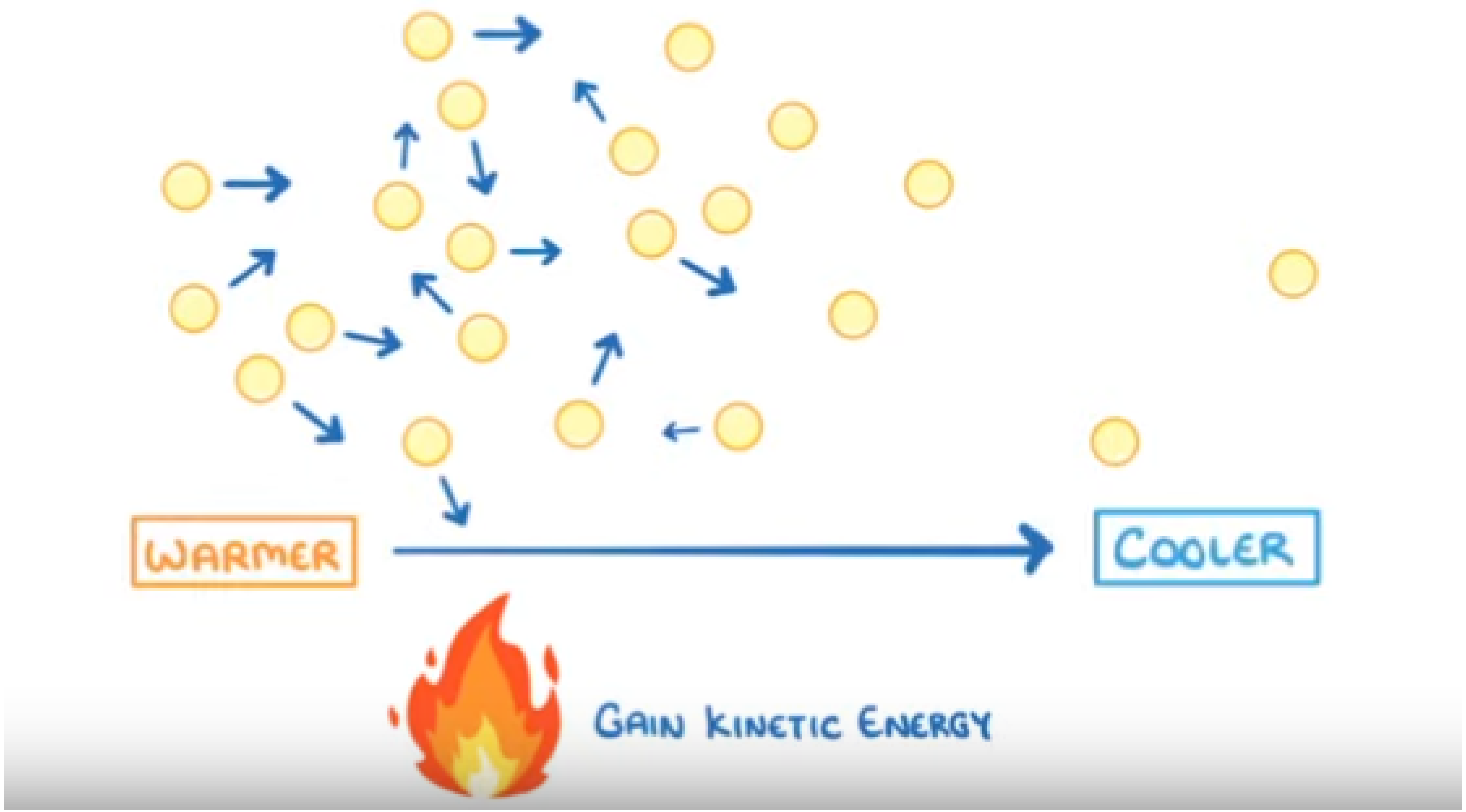
Q. Significance of thermal conductivity.

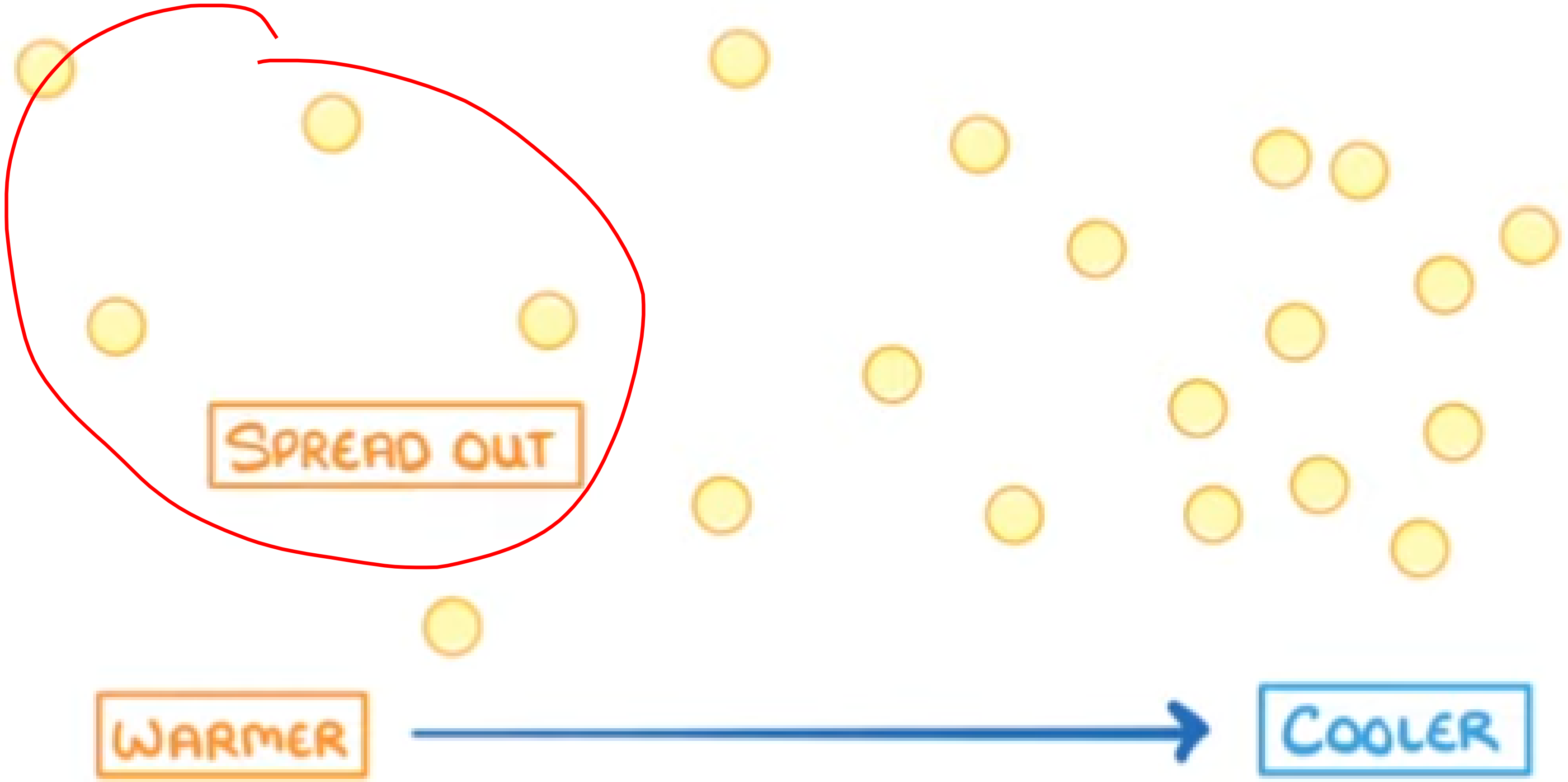
A high value for thermal conductivity indicates that the material is a good heat conductor, and a low value indicates that the material is a poor heat conductor or insulator.

Convection

Convection is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of conduction and fluid motion.

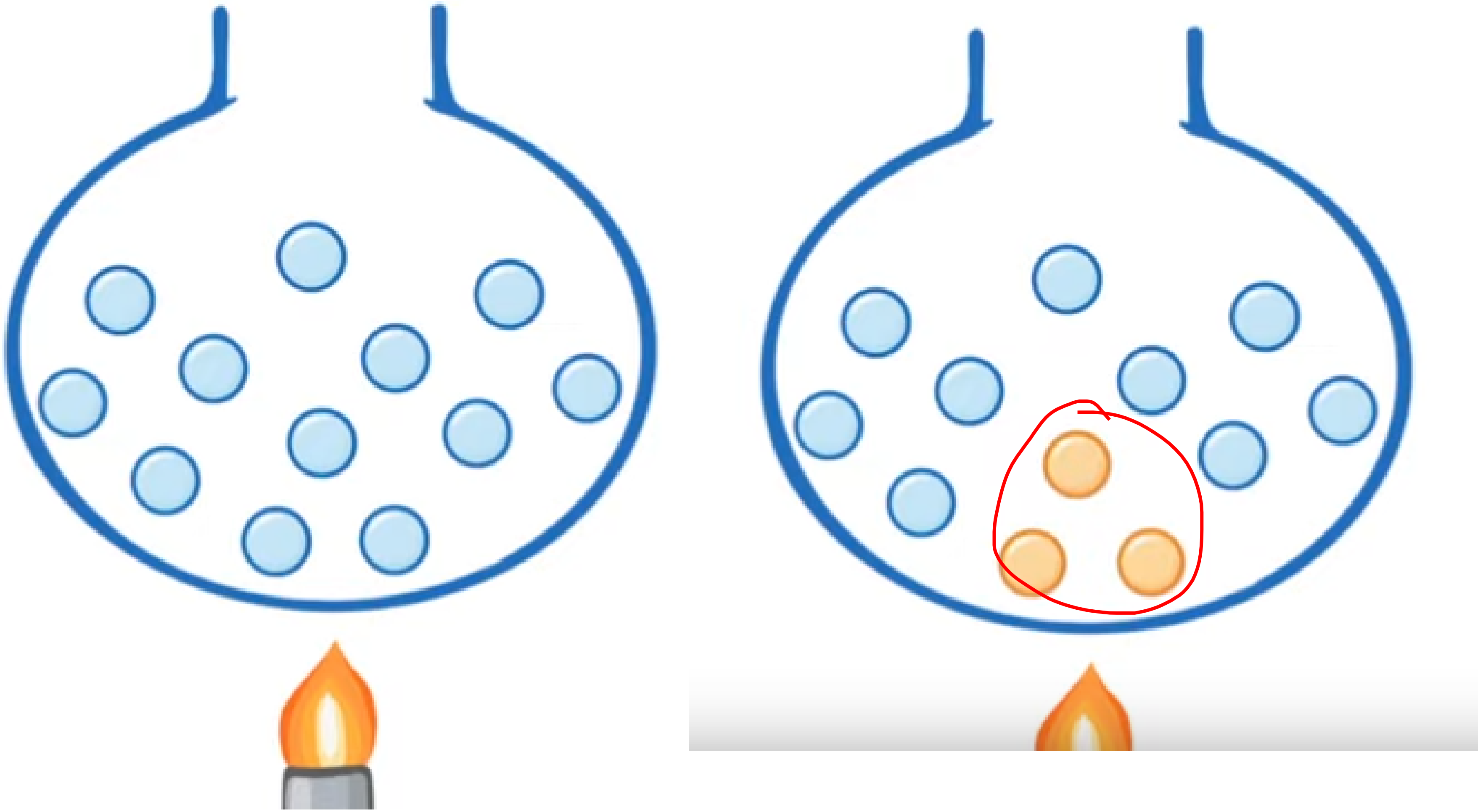


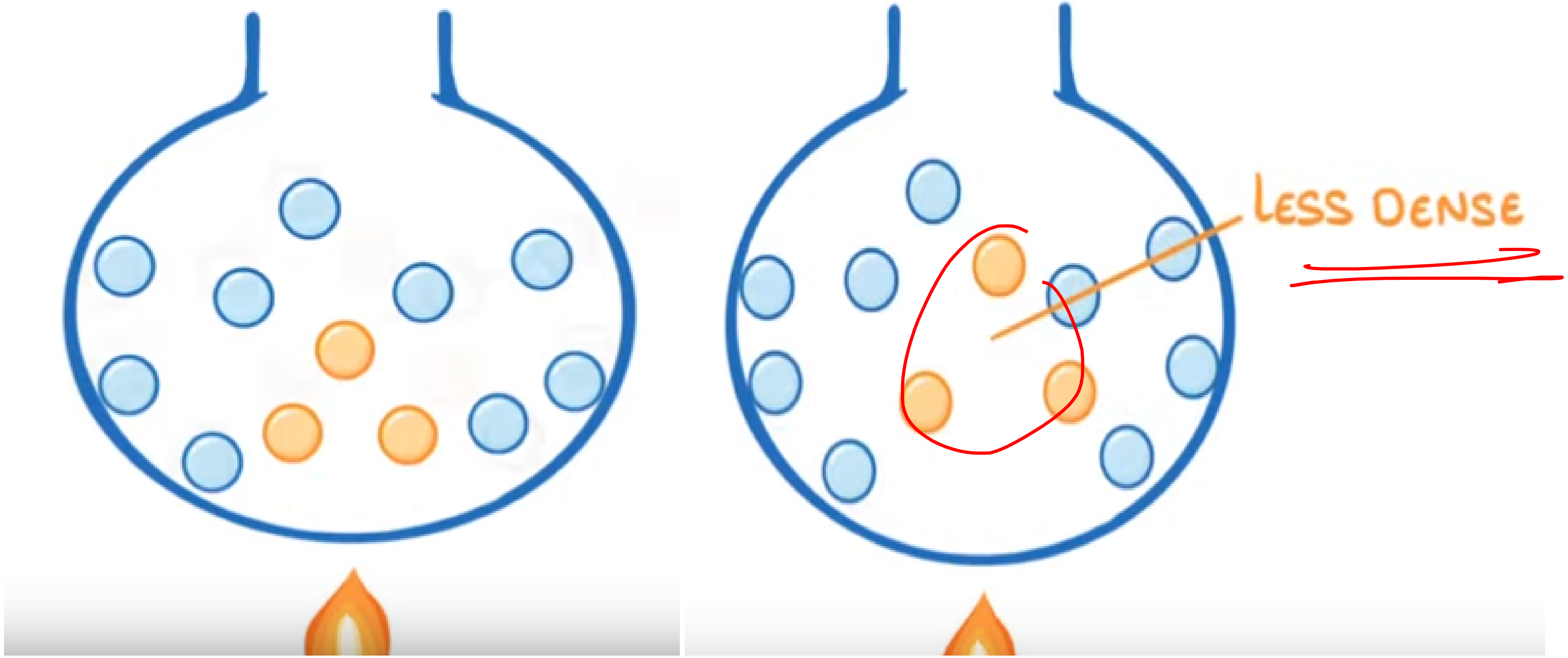

WARMER
COOLER
GAIN KINETIC ENERGY

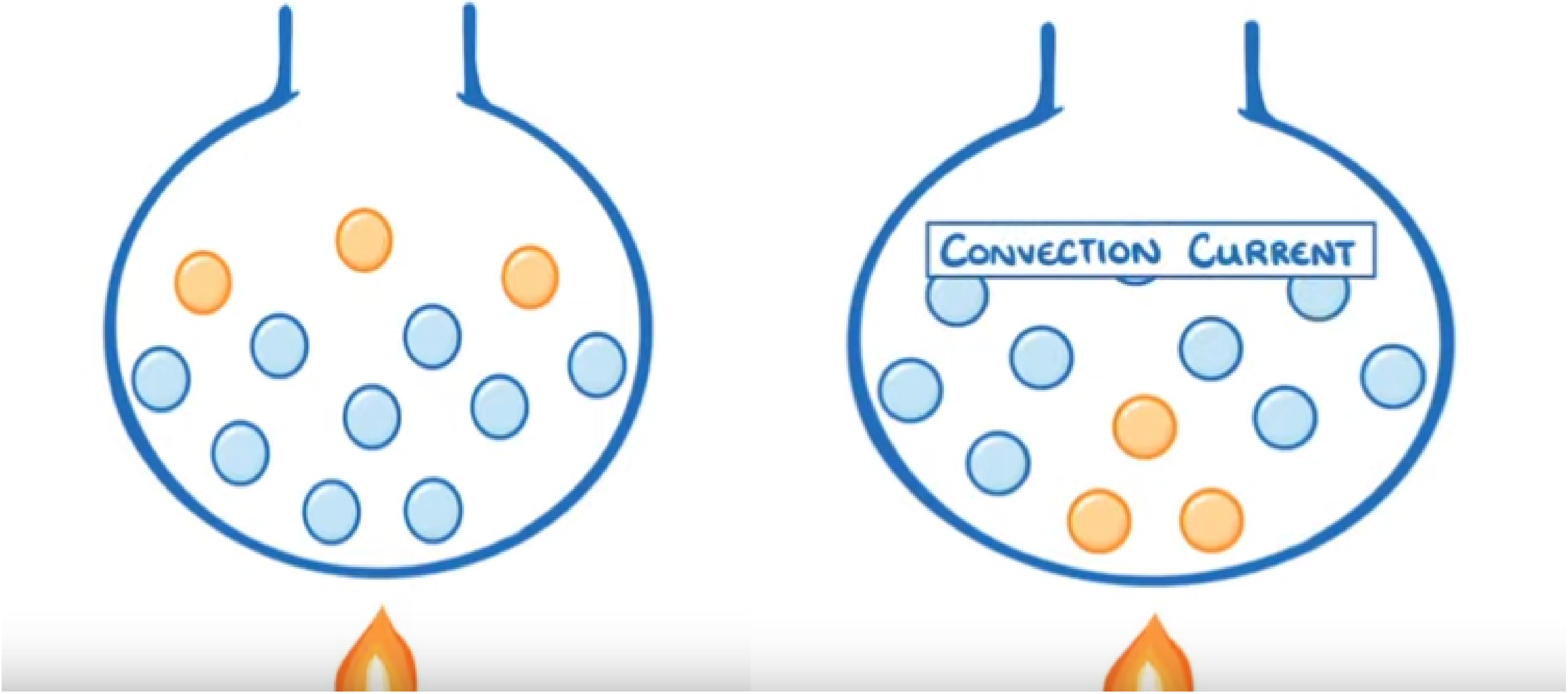


Expands

Less dense




LESS DENSE


CONVECTION CURRENT

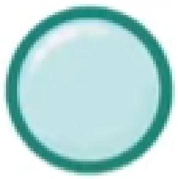
CONDUCTION

- ONLY THE ENERGY IS TRANSFERRED

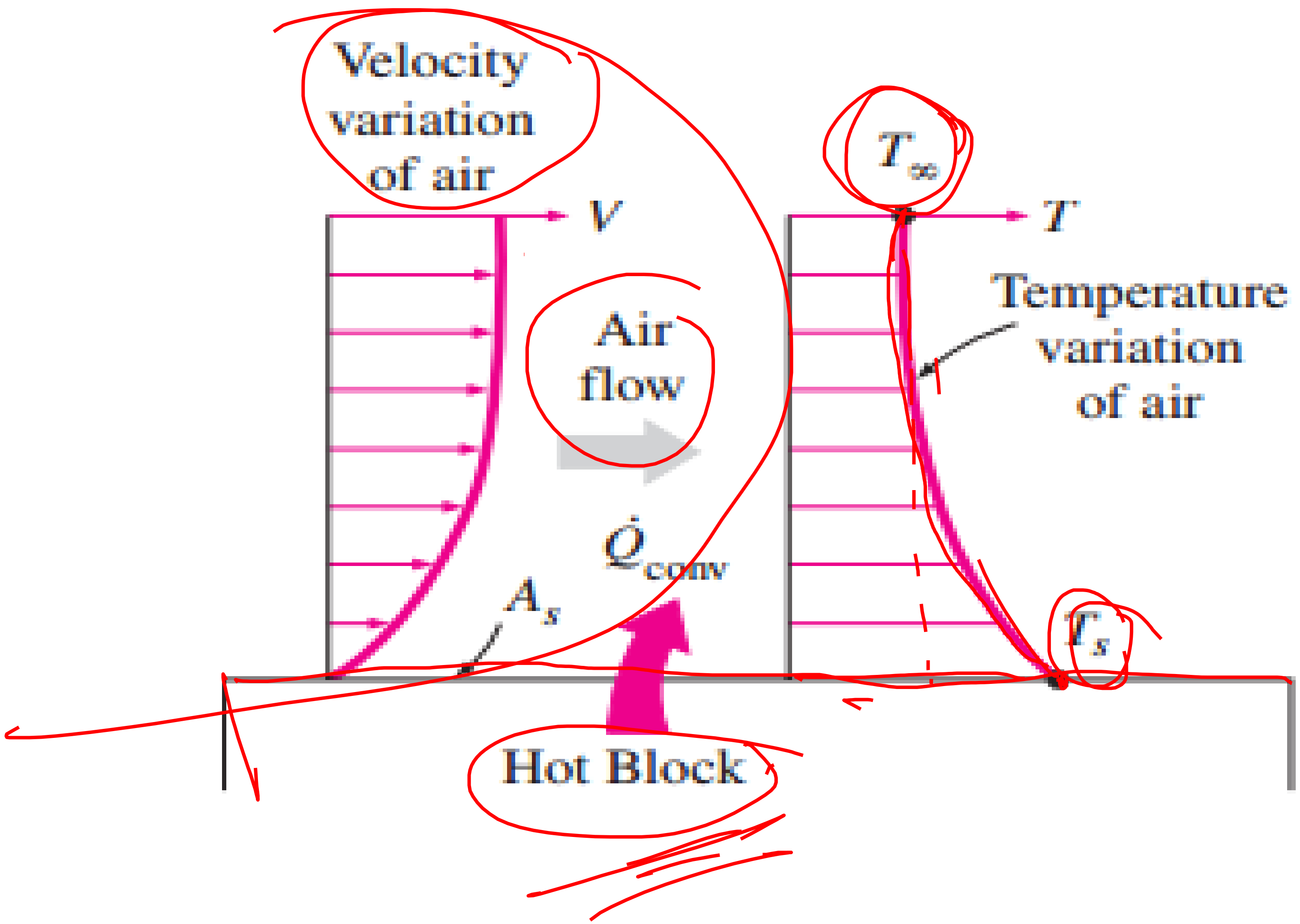
CONVECTION

- THE PARTICLES MOVE

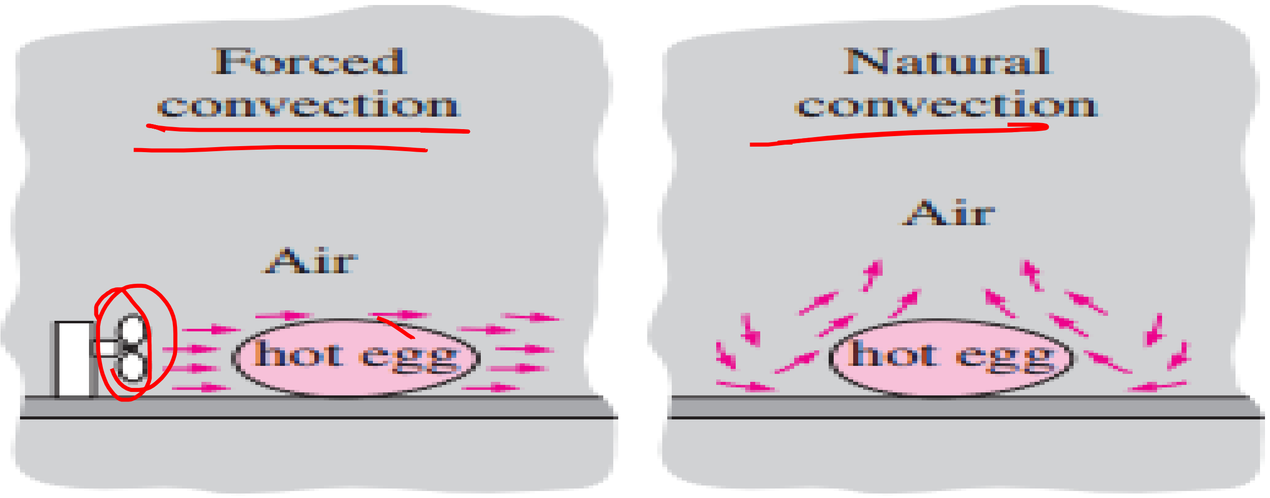
PARTICLES



GAINING KINETIC ENERGY



Velocity variation of air
V
Air flow
$\dot{Q}_{conv}$
A_s
T_∞
T
Temperature variation of air
T_s
Hot Block



The rate of convection heat transfer

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) \qquad \text{(W)}$$

Newton's law of cooling

$Q_{convec} \propto A_s (T_s - T_\infty)$

$\boxed{Q_{con} = h A_s (T_s - T_\infty)}$

Thermal system building blocks:

$\boxed{\text{Resistance}}$

$\boxed{\text{Capacitance}}$

$$\boxed{q = \frac{T_1 - T_2}{R}}$$

Thermal resistance

electrical equivalent

$$V = IR$$

$$\Rightarrow I = \frac{V}{R}$$

Resistance:

Value of R

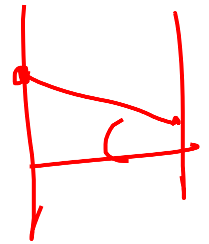
depends on

mode of heat transfer

✓ i) Conduction ( )

✓ 2) convection ( )

Conduction:



$$q = -KA\frac{dT}{dx}$$

$$q = \frac{T_1 - T_2}{R}$$

Equating

$$KA\frac{dT}{dx} = \frac{T_1 - T_2}{R}$$

$$\Rightarrow KA\frac{\cancel{T_1 - T_2}}{L} = \frac{\cancel{T_1 - T_2}}{R}$$

$$\Rightarrow \cancel{R} = \Rightarrow \frac{KA}{L} = \frac{1}{R}$$

$$\therefore R = \frac{L}{KA}$$

Convection:

$$q = Ah(T_1 - T_2)$$

$$q = \frac{T_1 - T_2}{R}$$

Equating

$$Ah(T_1 - T_2) = \frac{T_1 - T_2}{R}$$

$$\Rightarrow \boxed{R = \frac{1}{Ah}}$$

Capacitance;

$q_1 \rightarrow$

$q_2 \rightarrow$

Rate of change of internal energy = $q_1 - q_2$

=

specific heat content

internal energy change = $mc \times$ rate of change of temp

$$q_1 - q_2 = mc \frac{dT}{dt}$$

$$q_1 - q_2 = C \frac{dT}{dt}$$

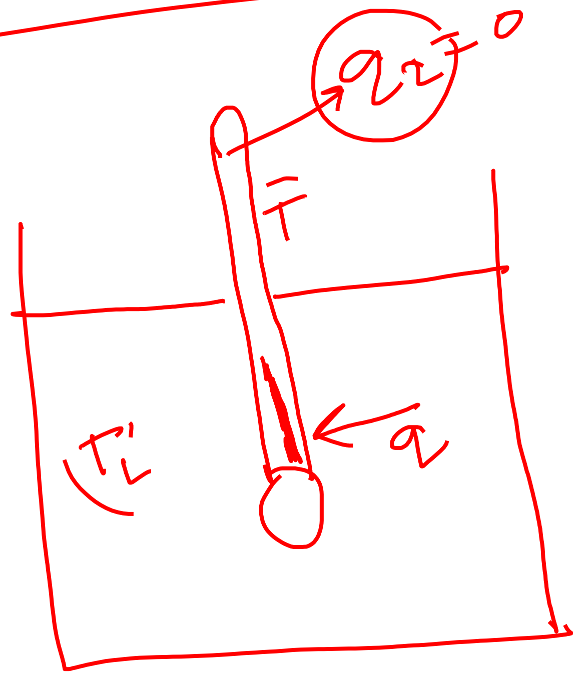
$$\boxed{C = mc}$$

thermal capacitance

Building up a model for thermal system:

Example

$T_L > T$



q_1

$q_2 = 0$

i) R

ii) C

$$q = \frac{T_L - T}{R}$$

$$q_1 - q_2 = C\frac{dT}{dt}$$

$$q_1 = C\frac{dT}{dt}$$

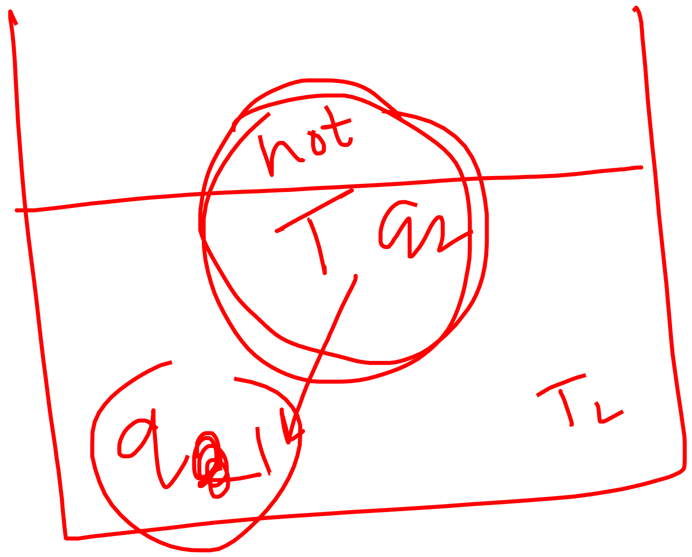
$$q = C\frac{dT}{dt}$$

$$q = c\frac{dT}{dt}$$

$$\frac{T_L - T}{R} = c\frac{dT}{dt}$$

$$\Rightarrow T_L - T = Rc\frac{dT}{dt}$$

$$\Rightarrow \boxed{Rc\frac{dT}{dt} + T = T_L}$$



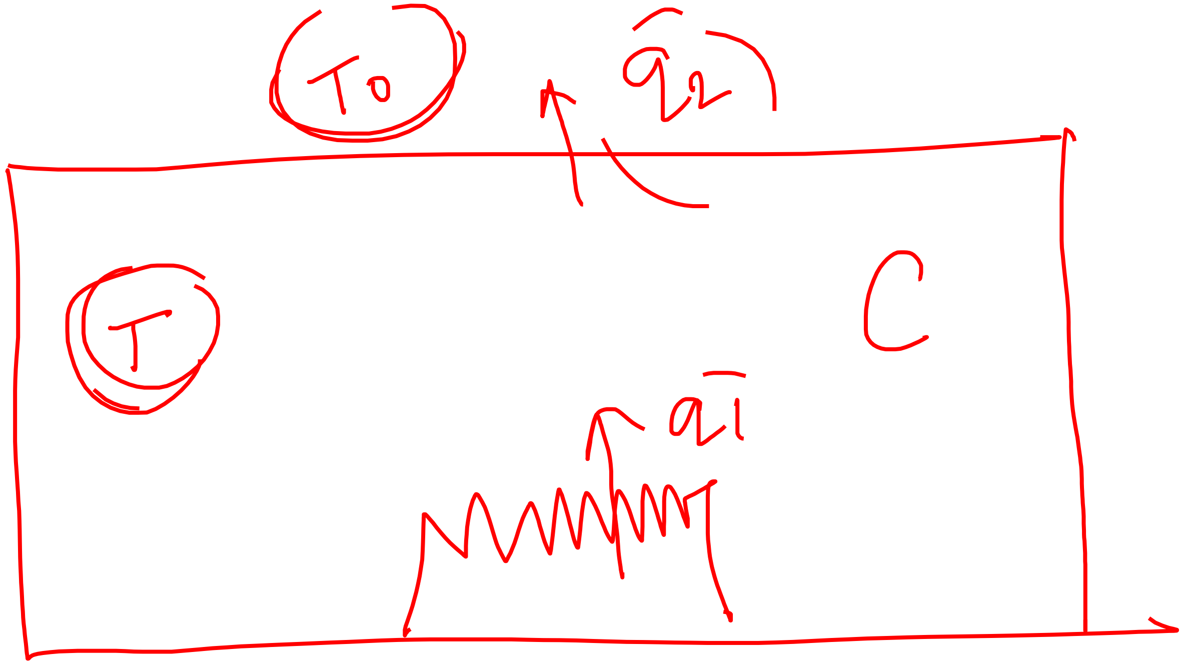
$$T > T_L$$

$$q = \frac{T - T_L}{R}$$

$$q_1 - q_2 = C\frac{dT}{dt}$$

$$q_1 = 0$$

Example:



Derive an equation describing how the room temp will change with time!

$$q_1 - q_2 = C \frac{dT}{dt} \quad \text{—(i)}$$

$$q_2 = \frac{T - T_0}{R} \quad \text{—(ii)}$$

From eqn (i)

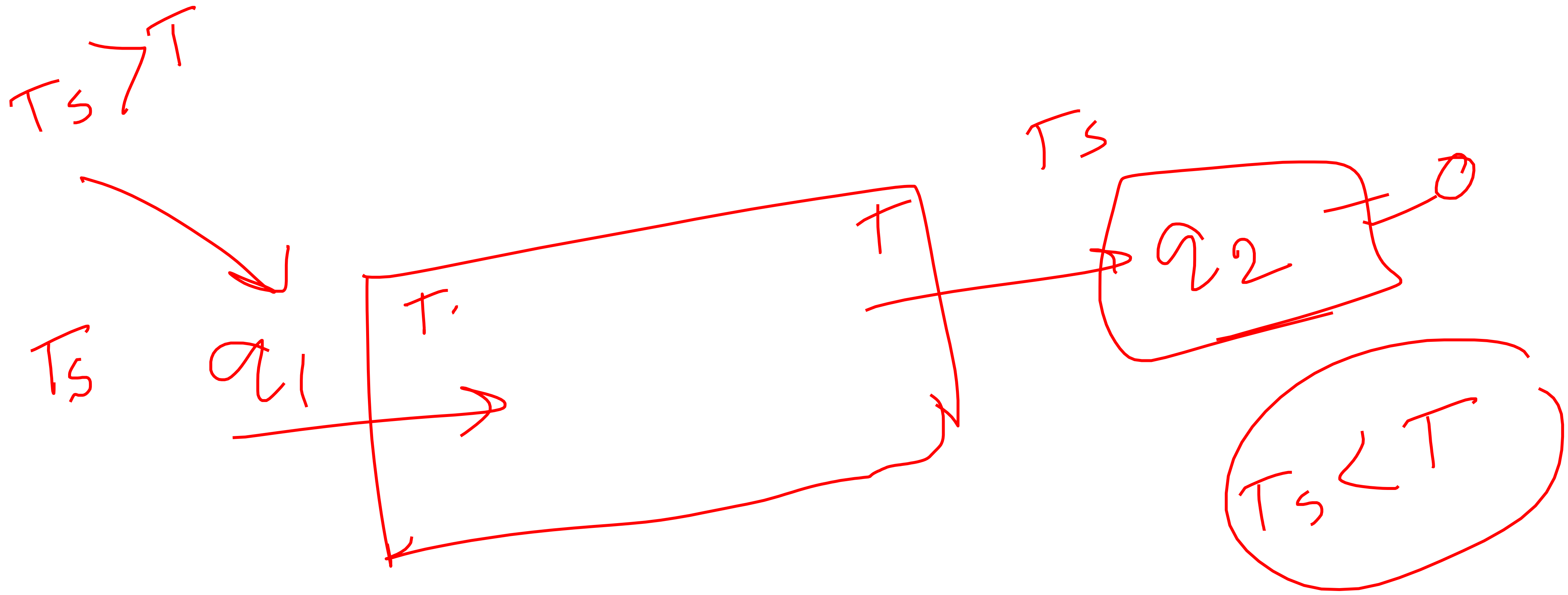
$$q_1 - q_2 = C\frac{dT}{dt}$$

$$\Rightarrow q_1 - \frac{T - T_0}{R} = C\frac{dT}{dt}$$

$$\Rightarrow \frac{Rq_1 - T + T_0}{R} = C\frac{dT}{dt}$$

$$\Rightarrow Rq_1 - T + T_0 = RC\frac{dT}{dt}$$

$$\Rightarrow \boxed{RC\frac{dT}{dt} + T = Rq_i + T_0}$$



Ts > T
Ts
a1
T1
T
Ts
a2 ≠ 0
Ts < T