Methods of Analysis for DC Networks

By Ariful Islam
Dept. of EEE

University of Dhaka

OBJECTIVES

- Become familiar with the terminal characteristics of a current source and how to solve for the voltages and currents of a network using current sources and/or current sources and voltage sources.
- Be able to apply branch-current analysis and mesh analysis to find the currents of network with one or more independent paths.
- Be able to apply nodal analysis to find all the terminal voltages of any series-parallel network with one or more independent sources.
- Become familiar with bridge network configurations and how to perform Δ - Y or Y - Δ conversions.

CURRENT SOURCES

- In previous chapters, the voltage source was the only source appearing in the circuit analysis.
- This was primarily because voltage sources such as the battery and supply are the most common in our daily lives and in the laboratory environment.
- We now turn our attention to a second type of source, called the **current source**.

CURRENT SOURCES

- Although current sources are available as laboratory supplies, they appear extensively in the modeling of electronic devices such as the transistor.
- Their characteristics and their impact on the currents and voltages of a network must therefore be clearly understood if electronic systems are to be properly investigated.

CURRENT SOURCES

- The current source is often described as the *dual* of the voltage source.
- Just as a battery provides a fixed voltage to a network, a current source establishes a fixed current in the branch where it is located.

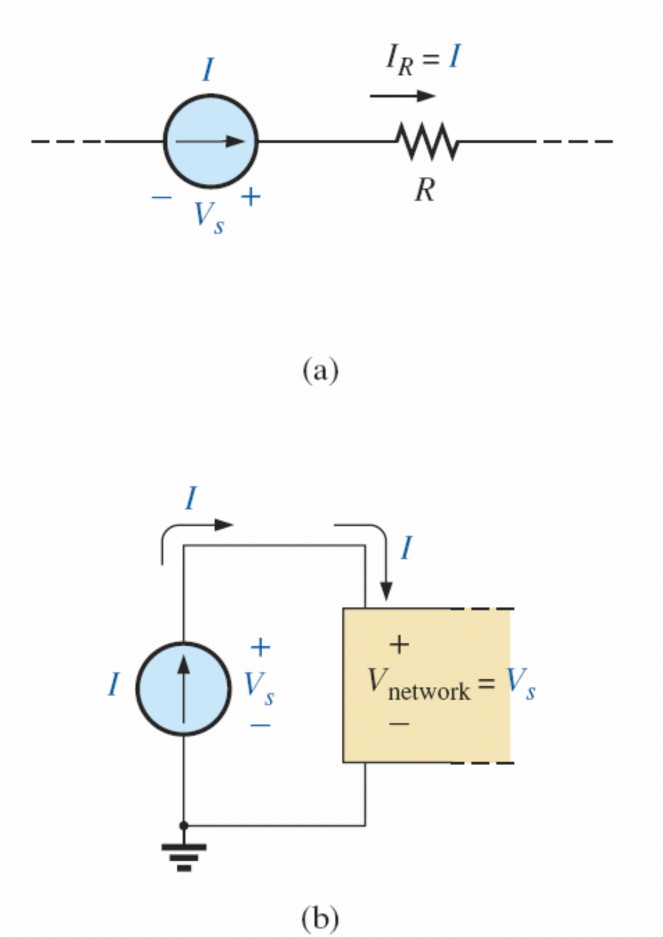


FIG. 8.1 *Introducing the current source symbol.*

CURRENT SOURCES

- In general, ***a current source determines the direction and magnitude of the current in the branch where it is located.***
- Furthermore, ***the magnitude and the polarity of the voltage across a current source are each a function of the network to which the voltage is applied.***

CURRENT SOURCES

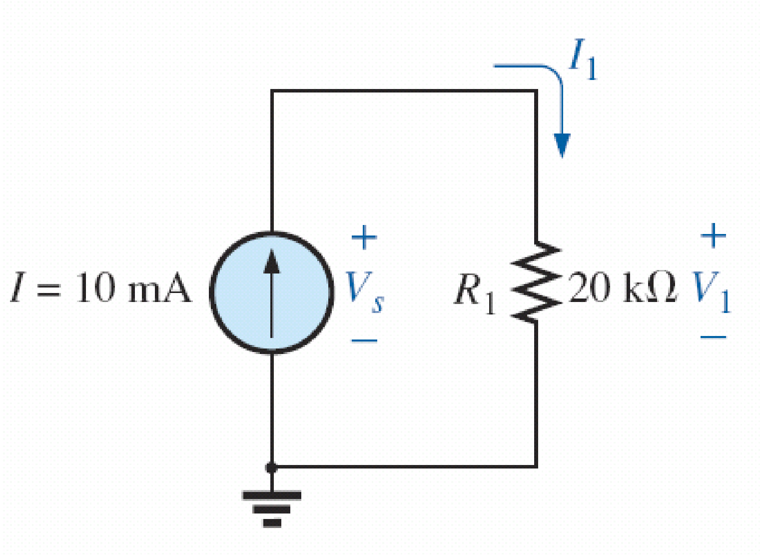


FIG. 8.2 *Circuit for Example 8.1.*

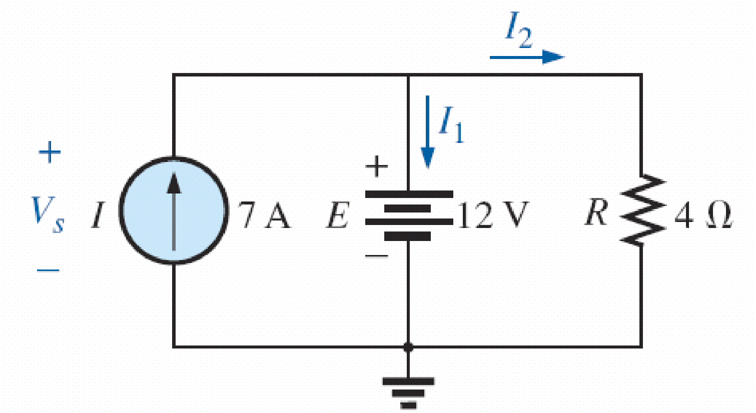


FIG. 8.3 *Network for Example 8.2.*

CURRENT SOURCES

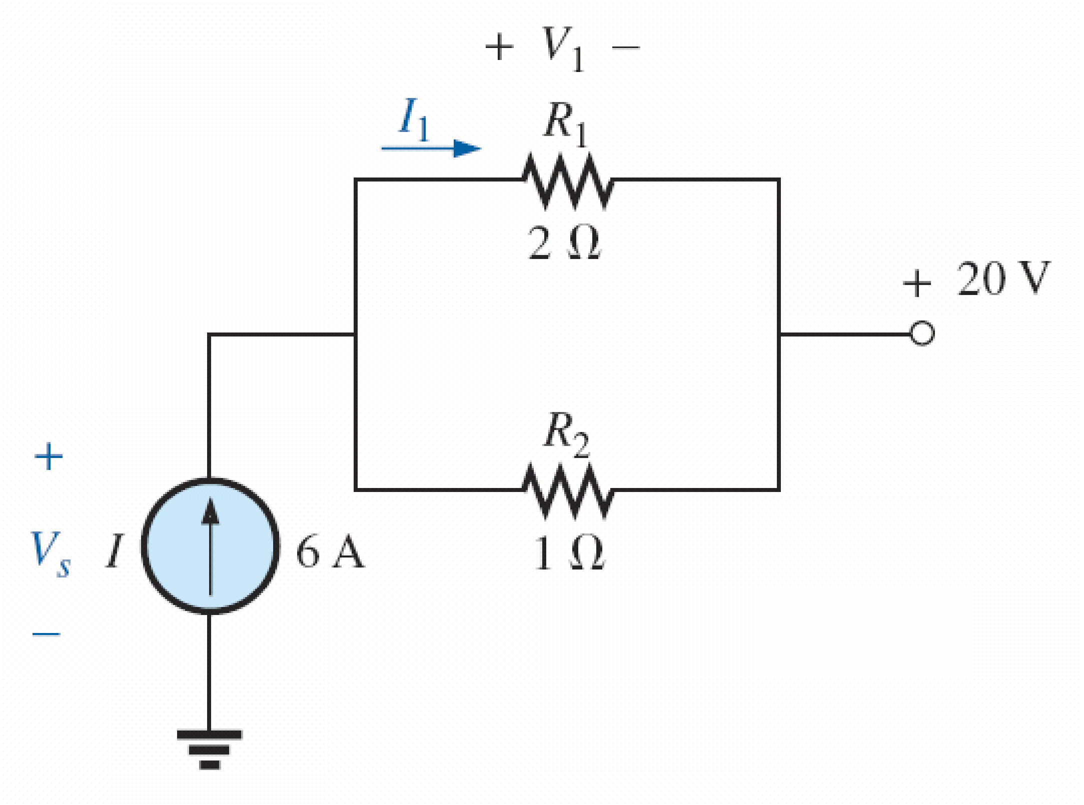


FIG. 8.4 *Example 8.3.*

SOURCE CONVERSIONS

- The current source appearing in the previous section is called an *ideal source* due to the absence of any internal resistance.
- In reality, all sources—whether they are voltage sources or current sources—have some internal resistance in the relative positions shown in Fig. 8.5.

SOURCE CONVERSIONS

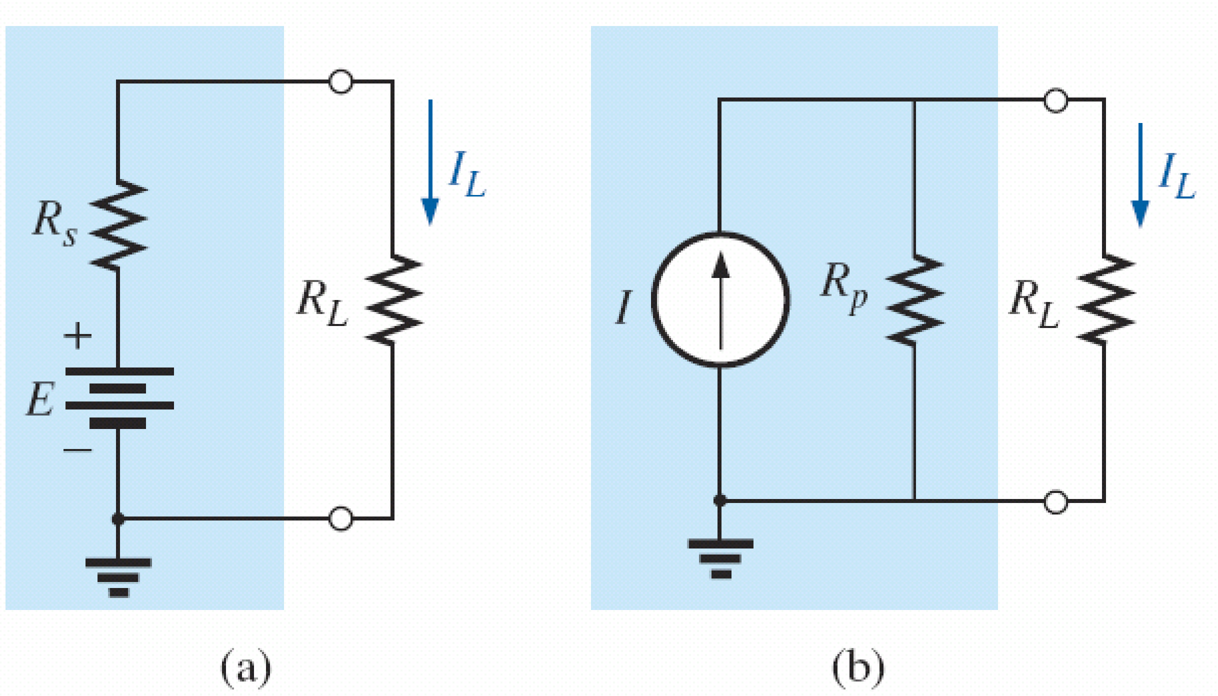


FIG. 8.5 *Practical sources: (a) voltage; (b) current.*

SOURCE CONVERSIONS

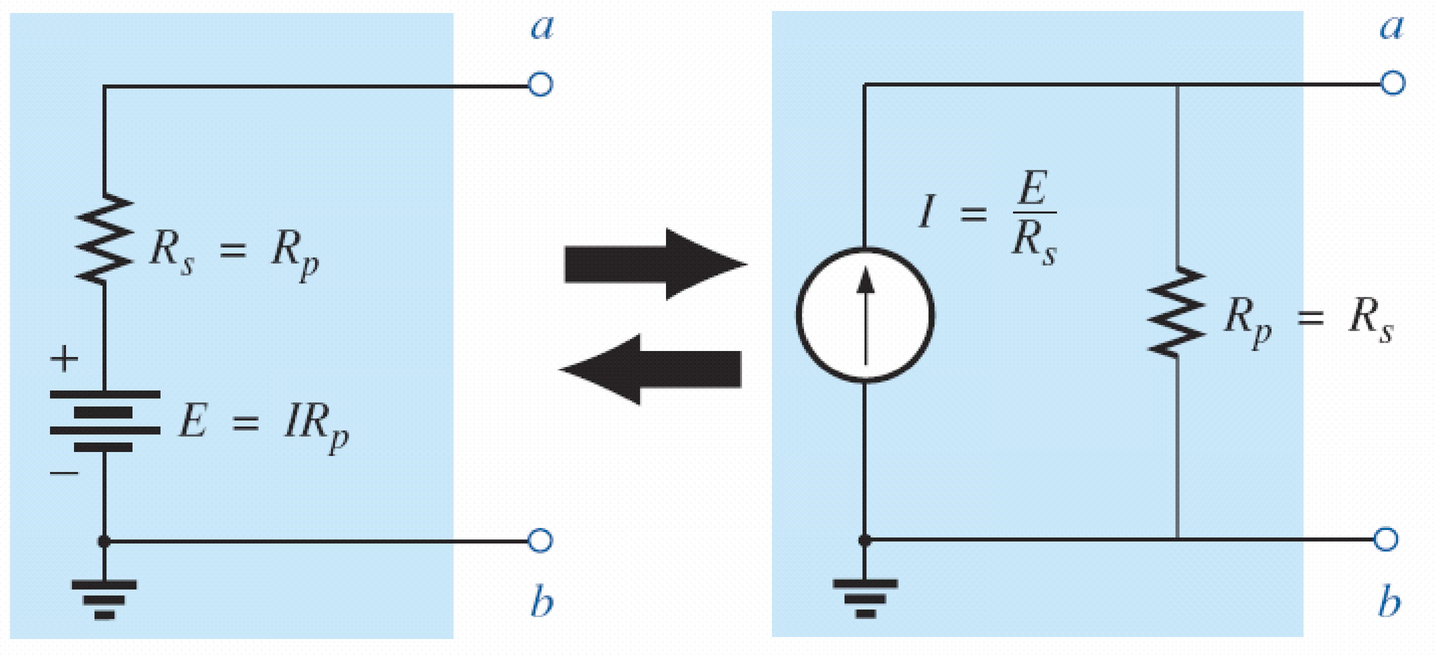


FIG. 8.6 *Source conversion.*

SOURCE CONVERSIONS

- It is important to realize, that ***the equivalence between a current source and a voltage source exists only at their external terminals.***

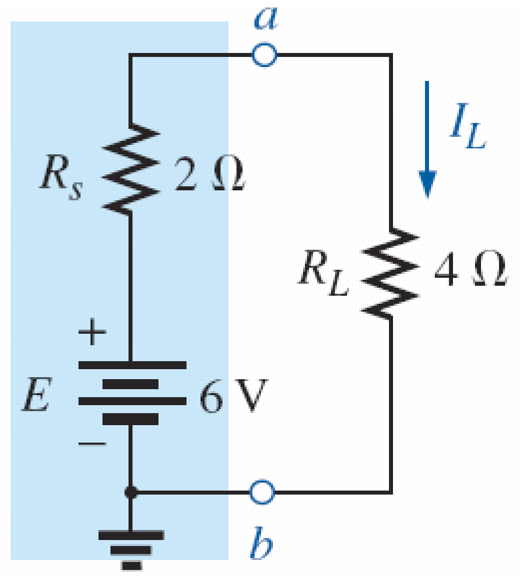


FIG. 8.7 *Practical voltage source and load for Example 8.4.*

SOURCE CONVERSIONS

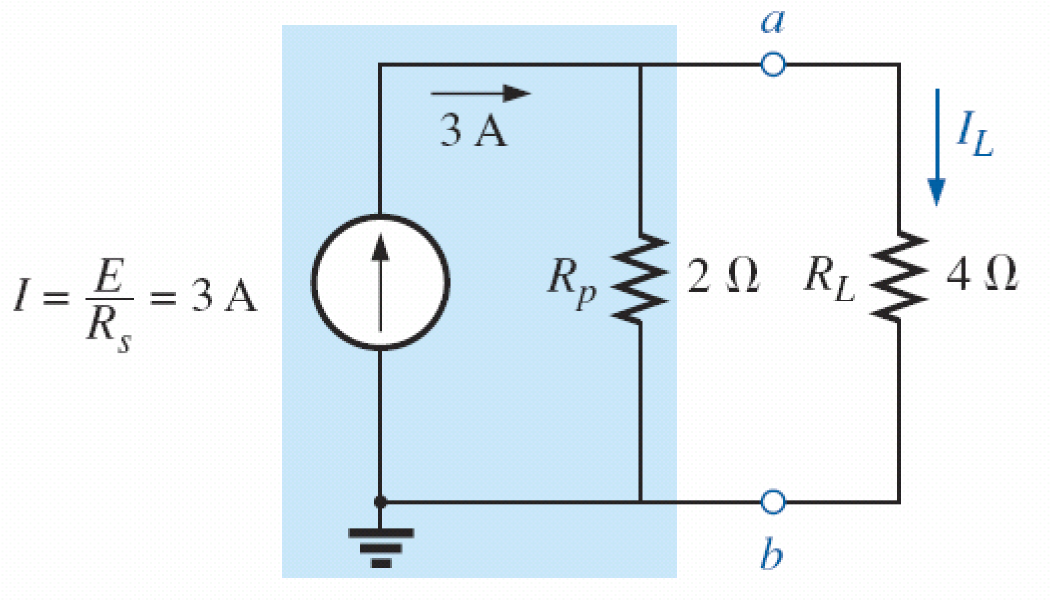


FIG. 8.8 *Equivalent current source and load for the voltage source in Fig. 8.7.*

SOURCE CONVERSIONS

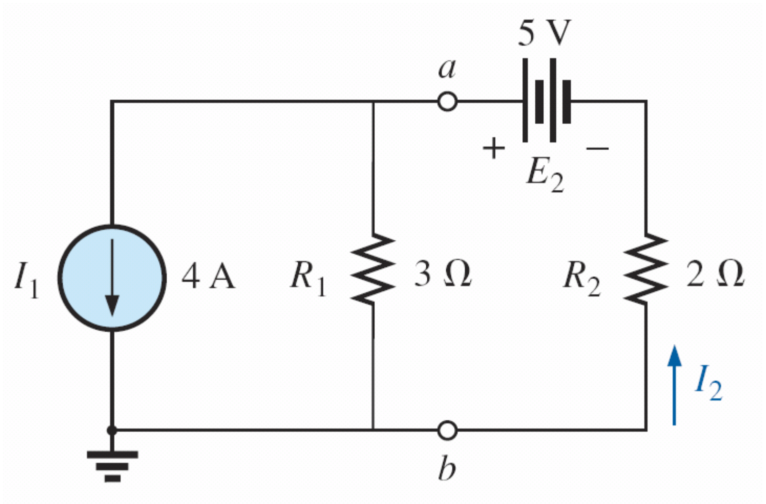


FIG. 8.9 *Two-source network for Example 8.5.*

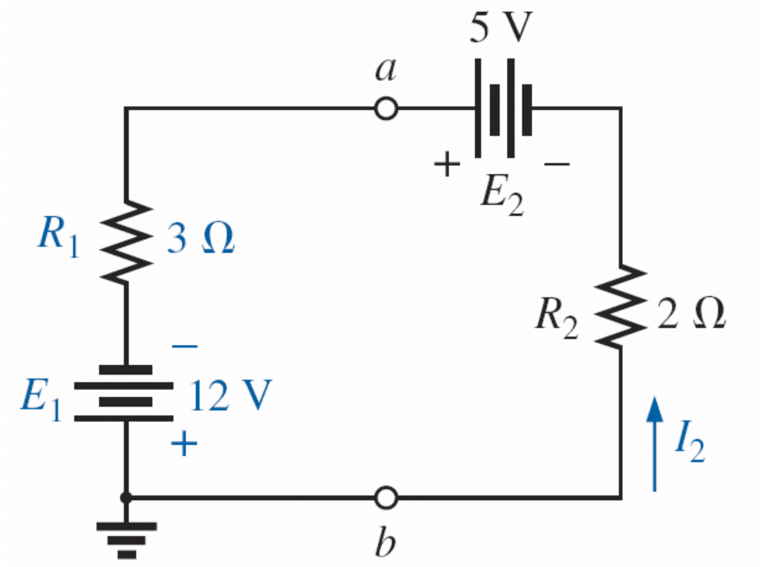


FIG. 8.10 *Network in Fig. 8.9 following the conversion of the current source to a voltage source.*

CURRENT SOURCES IN PARALLEL

- We found that voltage sources of different terminal voltages cannot be placed in parallel because of a violation of Kirchhoff’s voltage law.
  - Similarly, ***current sources of different values cannot be placed in series due to a violation of Kirchhoff’s current law.***

CURRENT SOURCES IN PARALLEL

- However, current sources can be placed in parallel just as voltage sources can be placed in series.
  - In general, ***two or more current sources in parallel can be replaced by a single current source having a magnitude determined by the difference of the sum of the currents in one direction and the sum in the opposite direction. The new parallel internal resistance is the total resistance of the resulting parallel resistive elements.***

CURRENT SOURCES IN PARALLEL

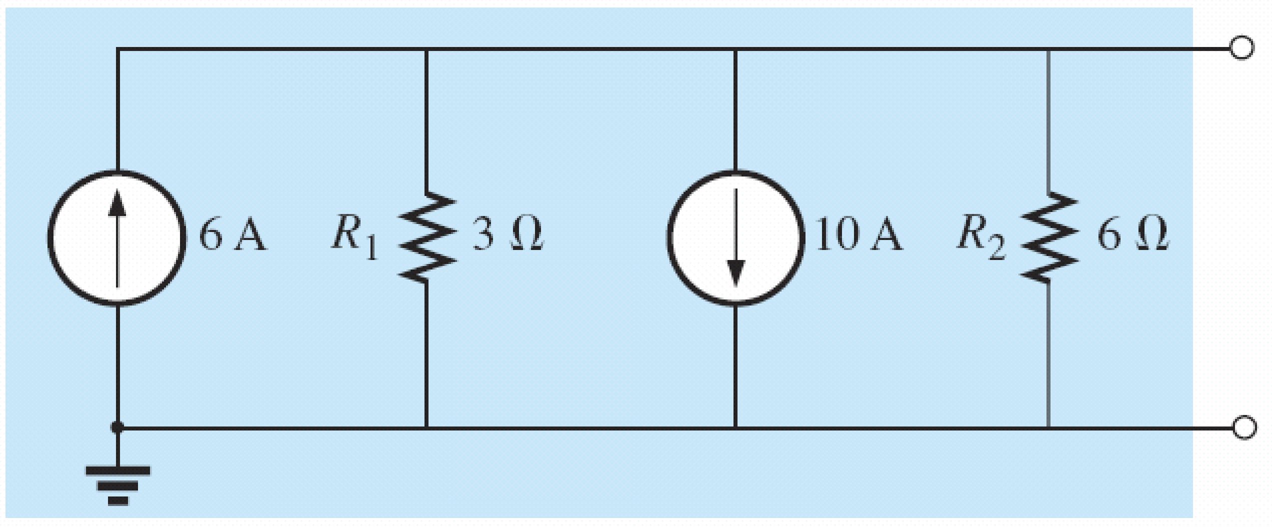


FIG. 8.11 *Parallel current sources for Example 8.6.*

CURRENT SOURCES IN PARALLEL

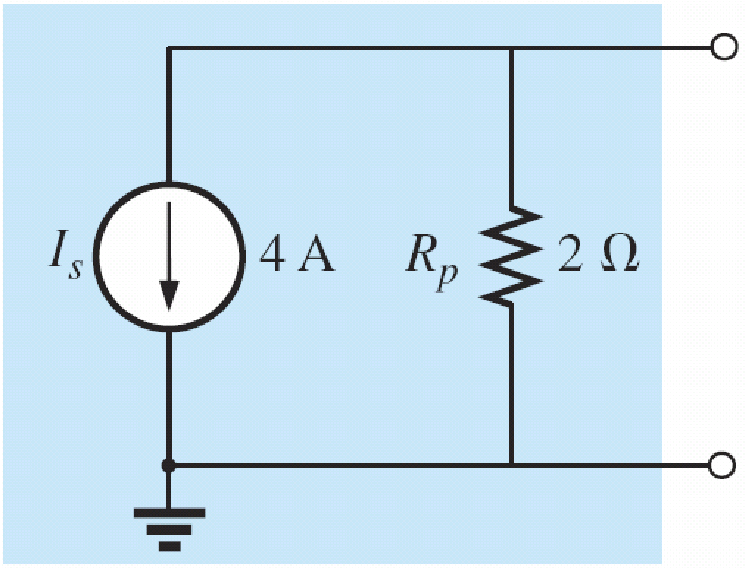


FIG. 8.12 *Reduced equivalent for the configuration of Fig. 8.11.*

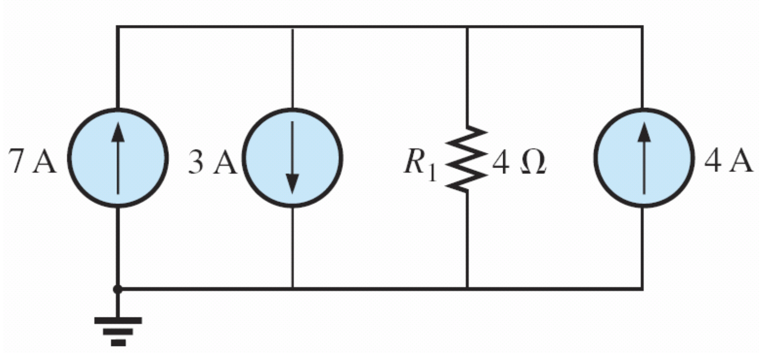


FIG. 8.13 *Parallel current sources for Example 8.7.*

CURRENT SOURCES IN PARALLEL

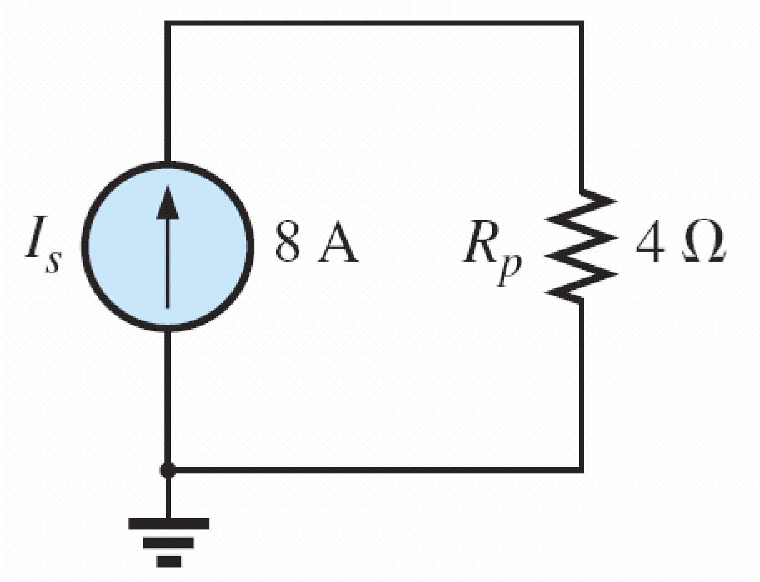


FIG. 8.14 *Reduced equivalent for Fig. 8.13.*

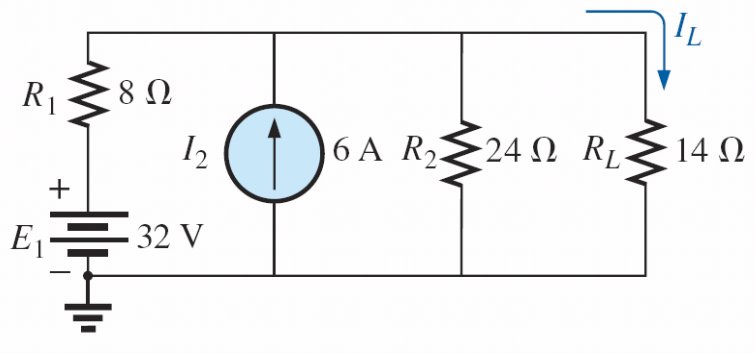


FIG. 8.15 *Example 8.8.*

CURRENT SOURCES IN PARALLEL

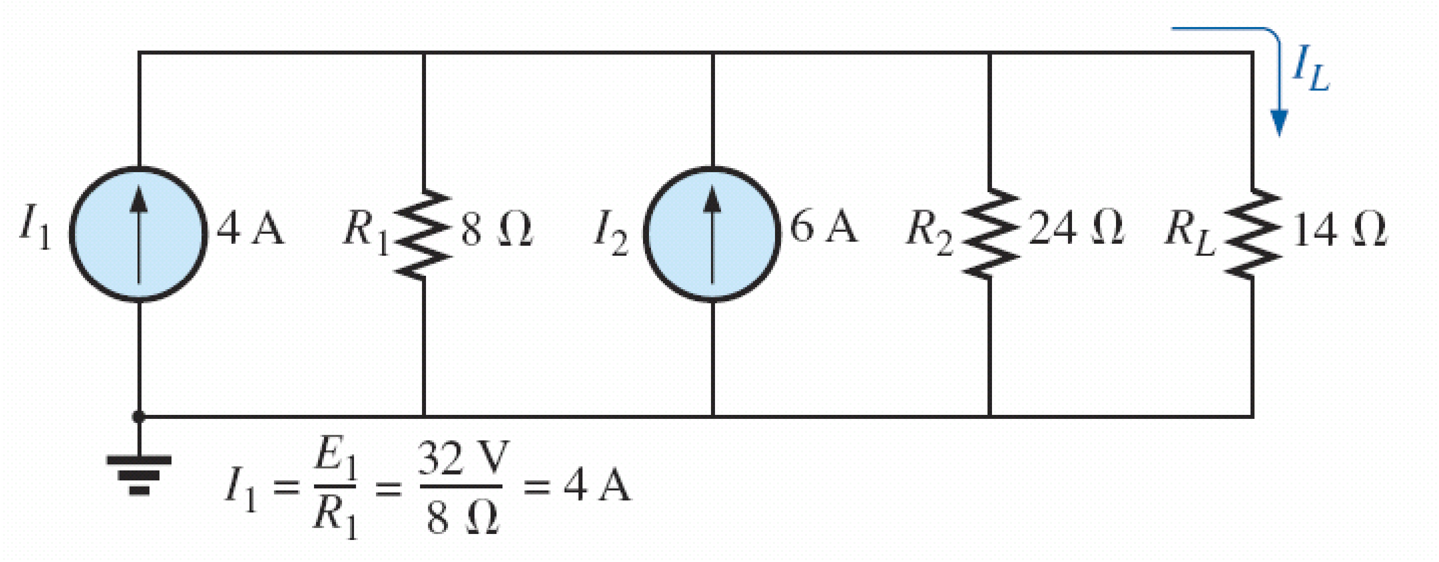


FIG. 8.16 *Network in Fig. 8.15 following the conversion of the voltage source to a current source.*

CURRENT SOURCES IN PARALLEL

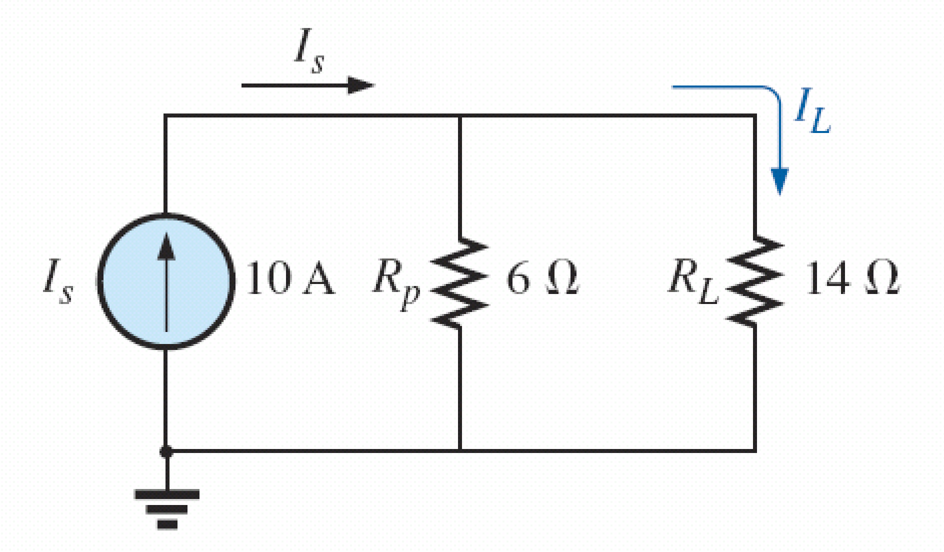


FIG. 8.17 *Network in Fig. 8.16 reduced to its simplest form.*

CURRENT SOURCES IN SERIES

- The current through any branch of a network can be only single-valued.
- For the situation indicated at point *a* in Fig. 8.18, we find by application of Kirchhoff's current law that the current leaving that point is greater than that entering—an impossible situation.
  - Therefore, ***current sources of different current ratings are not connected in series,*** just as voltage sources of different voltage ratings are not connected in parallel.

CURRENT SOURCES IN SERIES

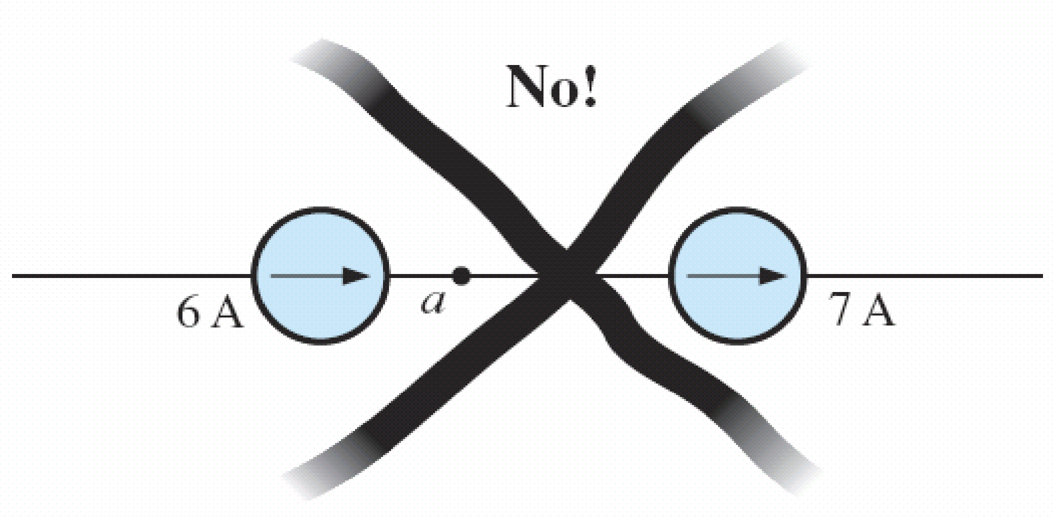


FIG. 8.18 *Invalid situation.*

BRANCH-CURRENT ANALYSIS
Branch-Current Analysis Procedure

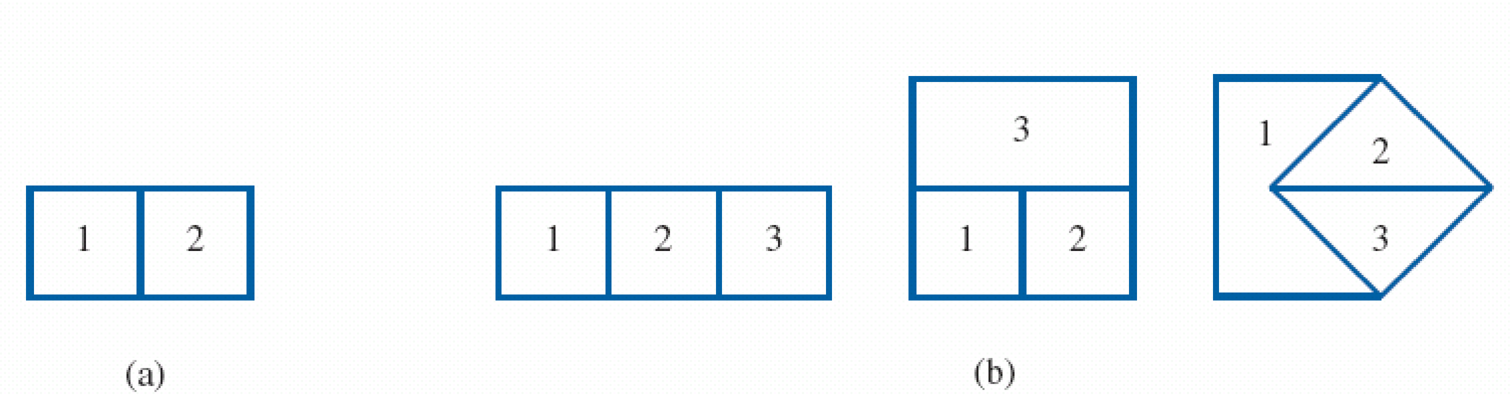


FIG. 8.20 *Determining the number of independent closed loops.*

BRANCH-CURRENT ANALYSIS
Branch-Current Analysis Procedure

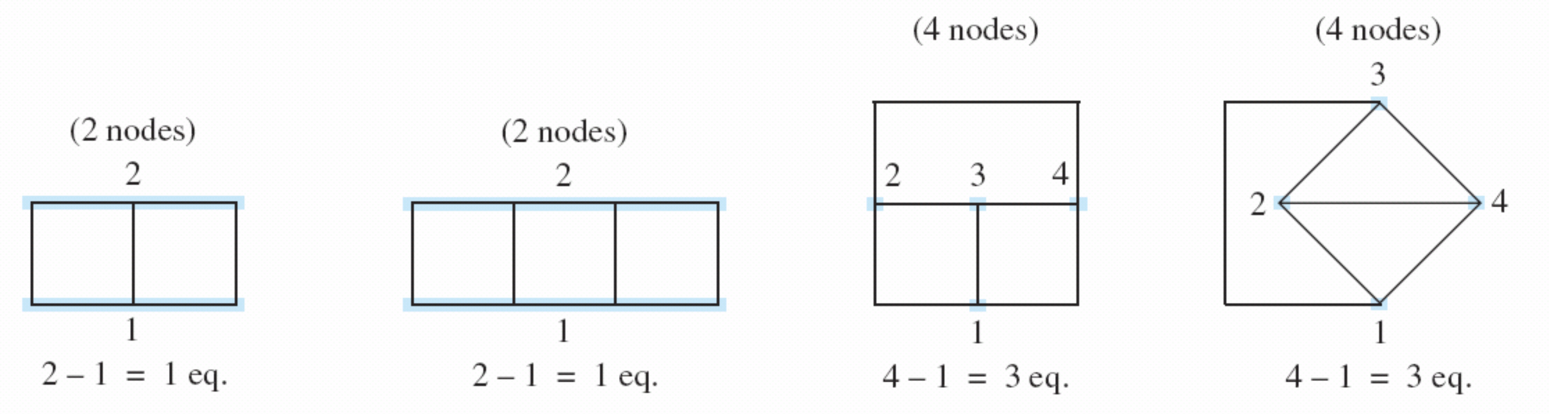


FIG. 8.21 *Determining the number of applications of Kirchhoff 's current law required.*

MESH ANALYSIS (GENERAL APPROACH)

- The next method to be described—**mesh analysis**—is actually an extension of the branch-current analysis approach just introduced.
- By defining a unique array of currents to the network, the information provided by the application of Kirchhoff's current law is already included when we apply Kirchhoff's voltage law. In other words, there is no need to apply step 4 of the branch-current method.
- The currents to be defined are called **mesh** or **loop currents.**

MESH ANALYSIS (GENERAL APPROACH)

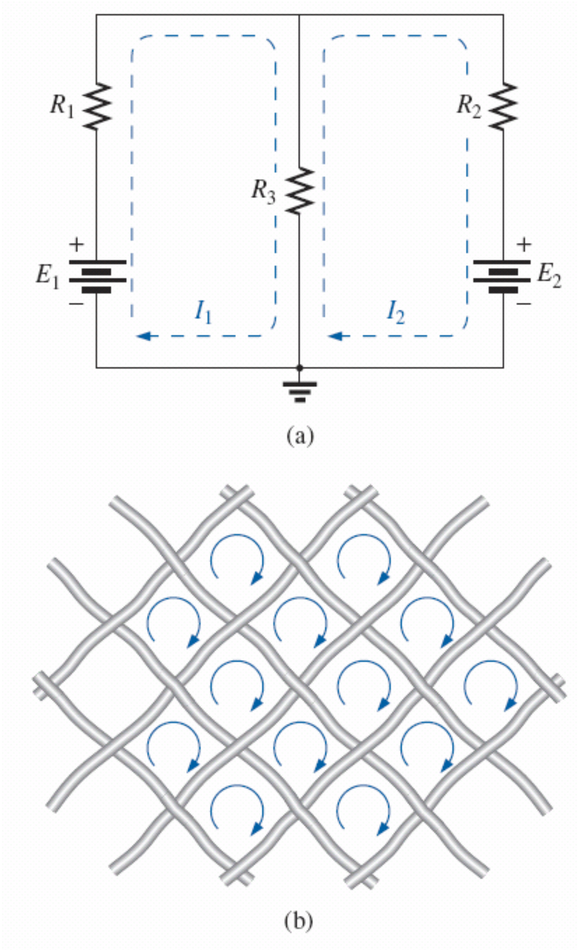


FIG. 8.27 *Defining the mesh (loop) current: (a) "two-window" network; (b) wire mesh fence analogy.*

MESH ANALYSIS (GENERAL APPROACH)

Mesh Analysis Procedure

1. ***Assign a distinct current in the clockwise direction to each independent, closed loop of the network. It is not absolutely necessary to choose the clockwise direction for each loop current. In fact, any direction can be chosen for each loop current with no loss in accuracy, as long as the remaining steps are followed properly. However, by choosing the clockwise direction as a standard, we can develop a shorthand method (Section 8.8) for writing the required equations that will save time and possibly prevent some common errors.***

MESH ANALYSIS (GENERAL APPROACH)

Mesh Analysis Procedure

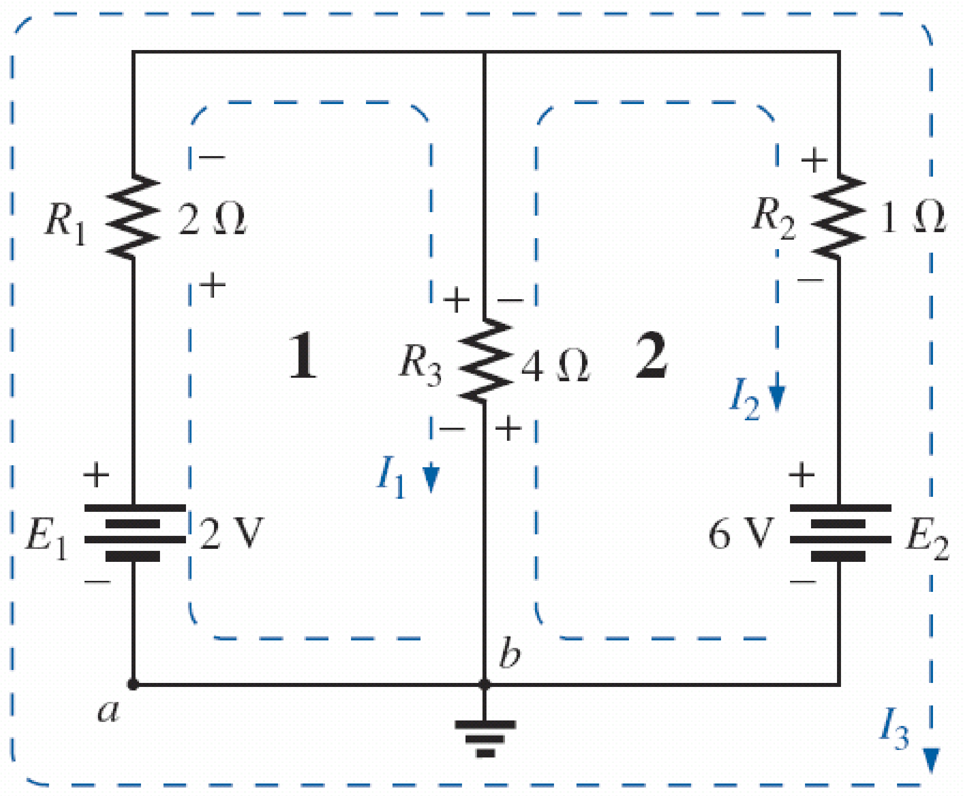


FIG. 8.28 *Defining the mesh currents for a "two-window" network.*

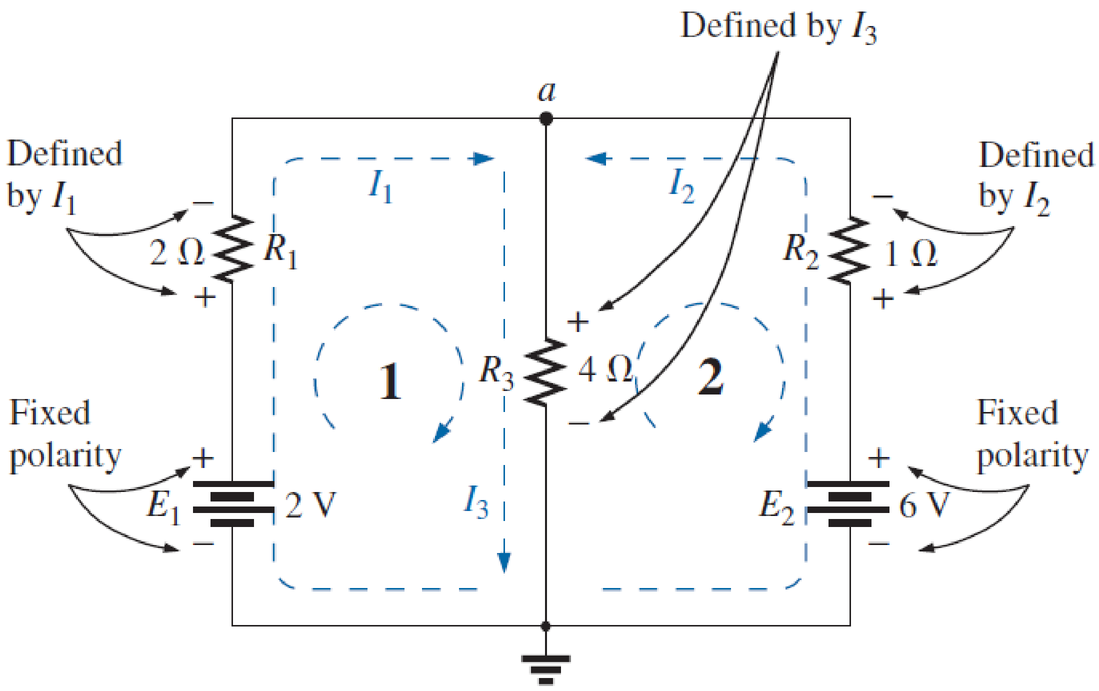
MESH ANALYSIS (GENERAL APPROACH)

Mesh Analysis Procedure

2. ***Indicate the polarities within each loop for each resistor as determined by the assumed direction of loop current for that loop. Note the requirement that the polarities be placed within each loop. This requires, as shown in Fig. 8.28, that the 4Ω resistor have two sets of polarities across it.***

MESH ANALYSIS (GENERAL APPROACH)

Mesh Analysis Procedure



MESH ANALYSIS (GENERAL APPROACH)

Mesh Analysis Procedure

3. ***Apply Kirchhoff's voltage law around each closed loop in the clockwise direction. Again, the clockwise direction was chosen to establish uniformity and prepare us for the method to be introduced in the next section.***
   - ***a. If a resistor has two or more assumed currents through it, the total current through the resistor is the assumed current of the loop in which Kirchhoff's voltage law is being applied, plus the assumed currents of the other loops passing through in the same direction, minus the assumed currents through in the opposite direction.***
   - ***b. The polarity of a voltage source is unaffected by the direction of the assigned loop currents.***

MESH ANALYSIS (GENERAL APPROACH)

Mesh Analysis Procedure

Rise in potential

$$\text{loop 1:} \quad \Sigma_{\circlearrowright} V = +E_1 - V_{R_1} - V_{R_3} = 0$$

Drop in potential

and

Rise in potential

$$\text{loop 2:} \quad \Sigma_{\circlearrowright} V = +V_{R_3} + V_{R_2} - E_2 = 0$$

Drop in potential

$$\text{loop 1:} \quad \Sigma_{\circlearrowright} V = \underbrace{+2\text{ V}}_{\substack{\text{Battery}\\ \text{potential}}} - \underbrace{(2\ \Omega)I_1}_{\substack{\text{Voltage drop}\\ \text{across } 2\ \Omega\\ \text{resistor}}} - \underbrace{(4\ \Omega)I_3}_{\substack{\text{Voltage drop}\\ \text{across } 4\ \Omega\\ \text{resistor}}} = 0$$

$$\text{loop 2:} \quad \Sigma_{\circlearrowright} V = (4\ \Omega)I_3 + (1\ \Omega)I_2 - 6\text{ V} = 0$$

MESH ANALYSIS (GENERAL APPROACH)
Mesh Analysis Procedure

4. ***Solve the resulting simultaneous linear equations for the assumed loop currents.***

MESH ANALYSIS (GENERAL APPROACH)

Mesh Analysis Procedure

$$2 - 2I_1 - 4I_3 = 0 \qquad \text{Rewritten: } 2I_1 + 0 + 4I_3 = 2$$

$$4I_3 + 1I_2 - 6 = 0 \qquad 0 + I_2 + 4I_3 = 6$$

$$I_1 + I_2 = I_3 \qquad I_1 + I_2 - I_3 = 0$$

Using third-order determinants, we have

$$I_1 = \frac{\begin{vmatrix} 2 & 0 & 4 \\ 6 & 1 & 4 \\ 0 & 1 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & 0 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & -1 \end{vmatrix}} = \mathbf{-1\ A}$$

A negative sign in front of a branch current indicates only that the actual current is in the direction opposite to that assumed.

$$D = \begin{vmatrix} 2 & 0 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & -1 \end{vmatrix}$$

$$I_2 = \frac{\begin{vmatrix} 2 & 2 & 4 \\ 0 & 6 & 4 \\ 1 & 0 & -1 \end{vmatrix}}{D} = \mathbf{2\ A}$$

MESH ANALYSIS (GENERAL APPROACH)
Mesh Analysis Procedure

$$I_3 = \frac{\begin{vmatrix} 2 & 0 & 2 \\ 0 & 1 & 6 \\ 1 & 1 & 0 \end{vmatrix}}{D} = \mathbf{1\ A}$$

MESH ANALYSIS (GENERAL APPROACH)
Mesh Analysis Procedure

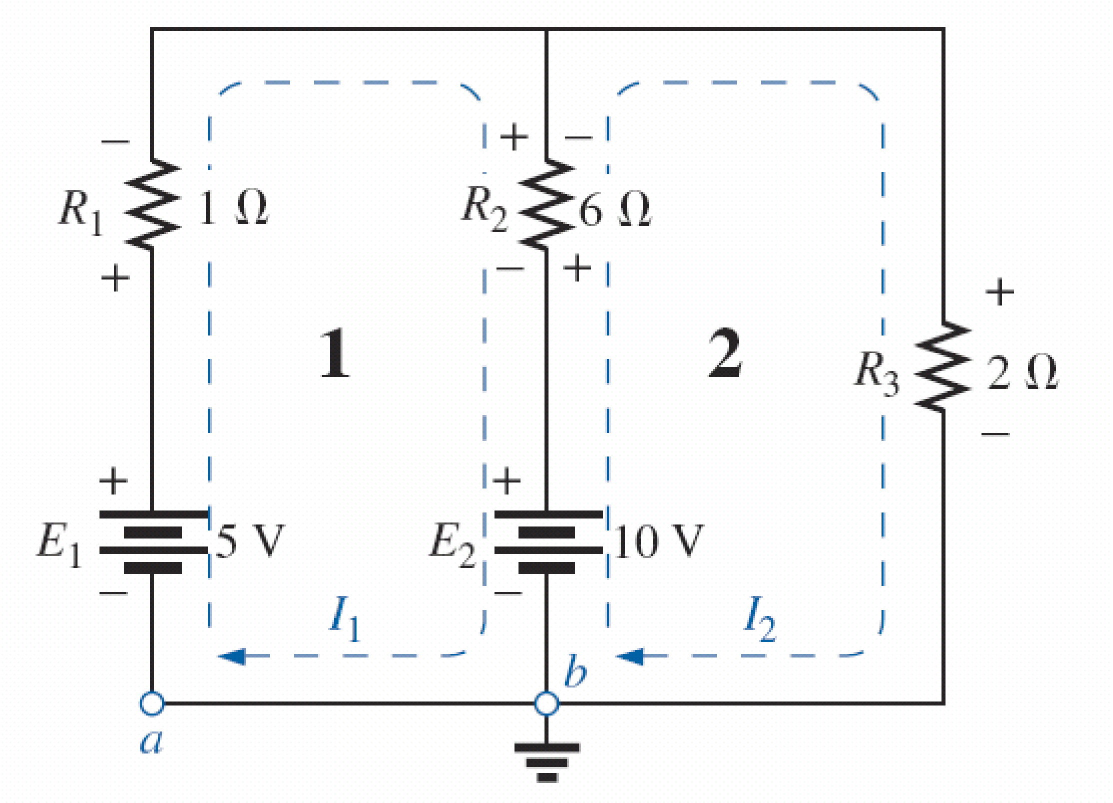


FIG. 8.29 *Example 8.12.*

MESH ANALYSIS (GENERAL APPROACH)

Mesh Analysis Procedure

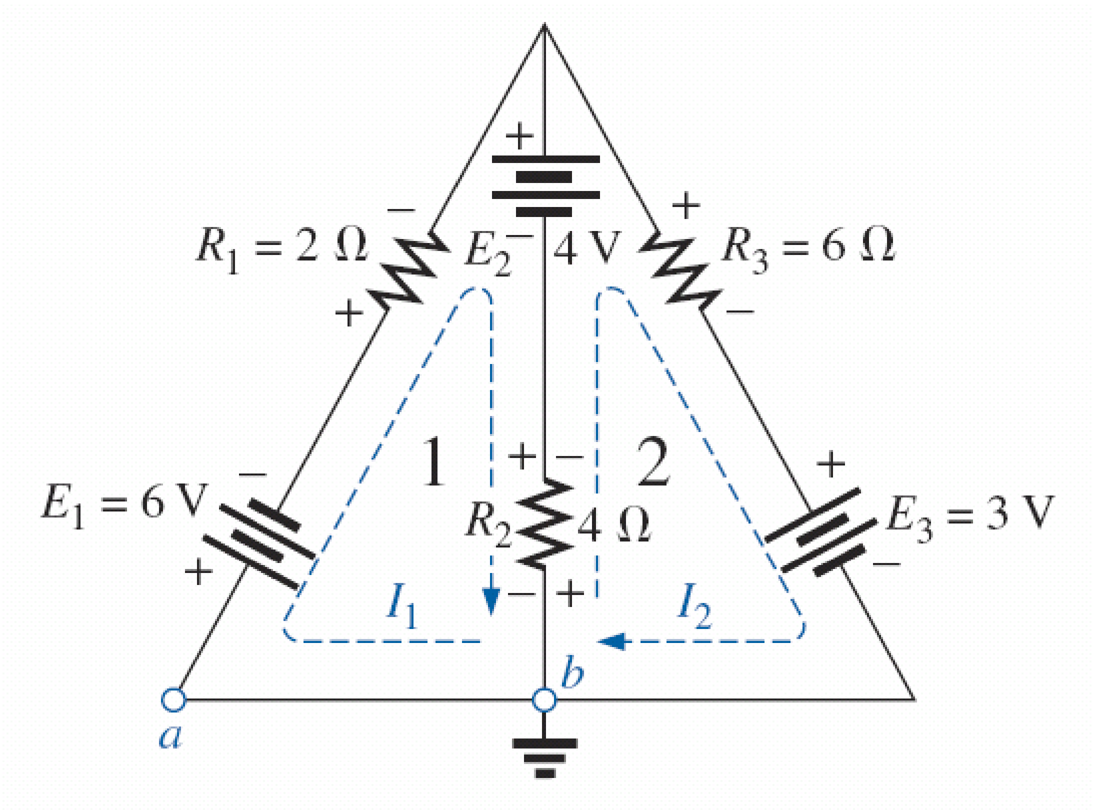


FIG. 8.30 *Example 8.13.*

MESH ANALYSIS (GENERAL APPROACH)
Supermesh Currents

- Occasionally, you will find current sources in a network without a parallel resistance.
- This removes the possibility of converting the source to a voltage source as required by the given procedure.

MESH ANALYSIS (GENERAL APPROACH)
Supermesh Currents

- In such cases, you have a choice of two approaches.
  - The simplest and most direct approach is to place a resistor in parallel with the current source that has a much higher value than the other resistors of the network.
  - The other choice is to use the **supermesh approach.**

MESH ANALYSIS (GENERAL APPROACH)
Supermesh Currents

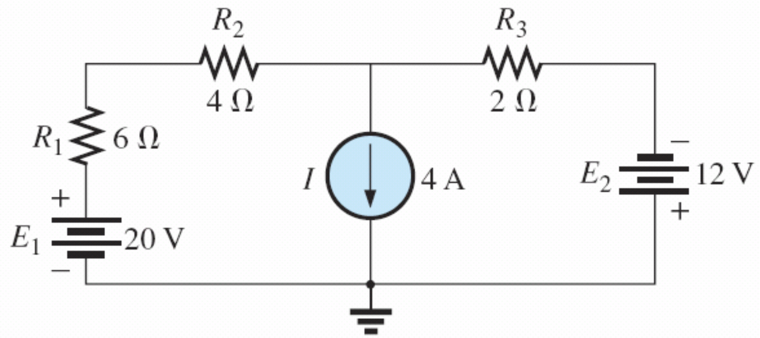


FIG. 8.31 *Example 8.14.*

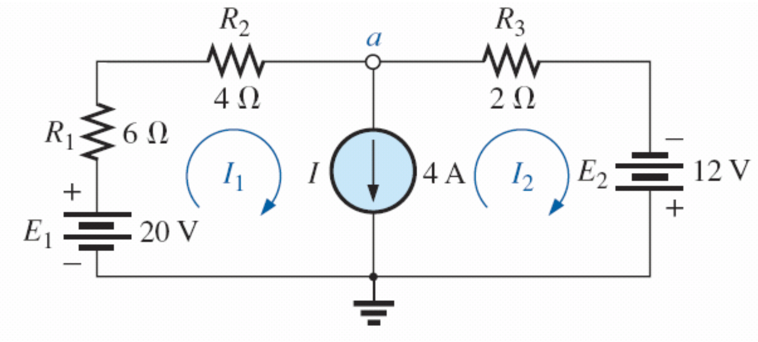


FIG. 8.32 *Defining the mesh currents for the network in Fig. 8.31.*

MESH ANALYSIS (GENERAL APPROACH)
Supermesh Currents

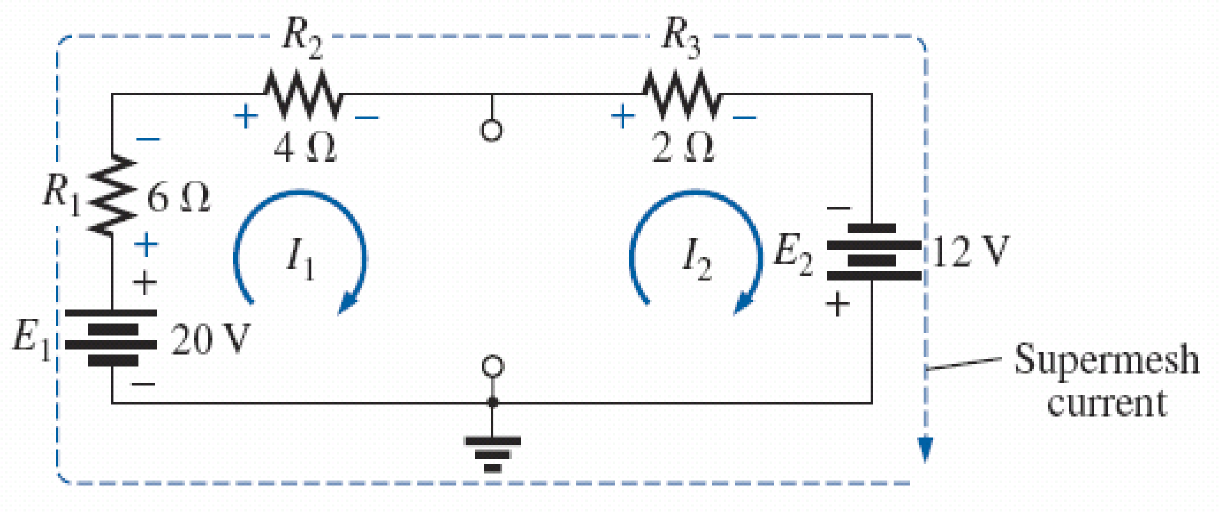


FIG. 8.33 *Defining the supermesh current.*

MESH ANALYSIS (GENERAL APPROACH)
Supermesh Currents

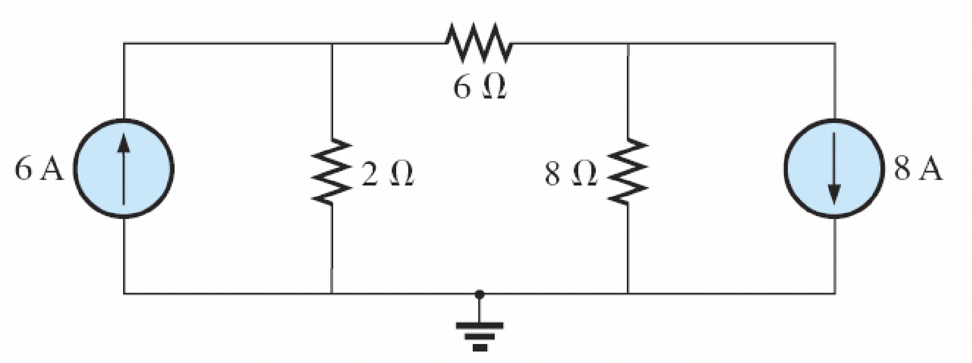


FIG. 8.34 *Example 8.15.*

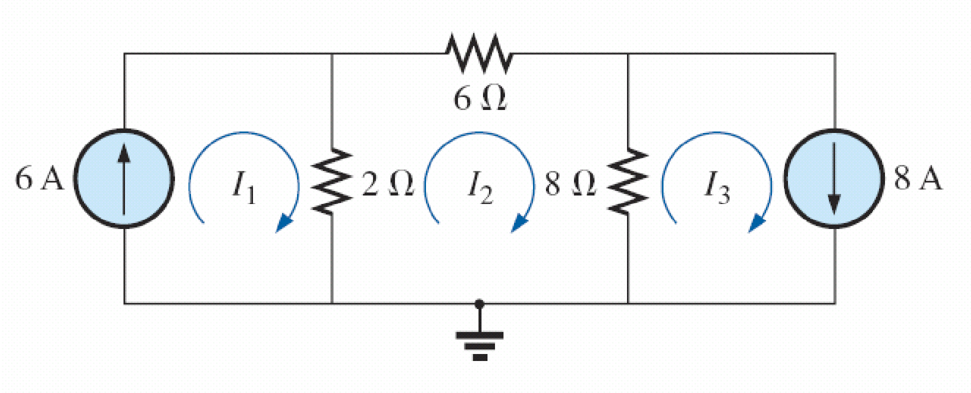


FIG. 8.35 *Defining the mesh currents for the network in Fig. 8.34.*

MESH ANALYSIS (GENERAL APPROACH)

Supermesh Currents

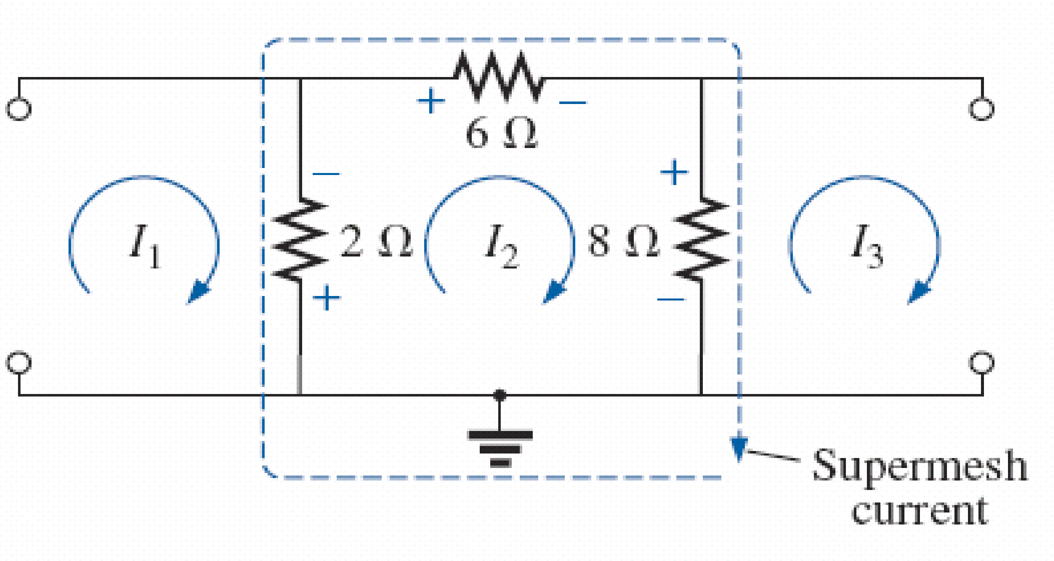


FIG. 8.36 *Defining the supermesh current for the network in Fig. 8.34.*