Set Theory

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Discrete mathematics is the part of mathematics devoted to the study of discrete objects. (Here discrete means consisting of distinct or unconnected elements.)

The kinds of problems solved using discrete mathematics include:

- What is the probability of winning a lottery?
- Is there a link between two computers in a network?
- How can I encrypt a message so that no unintended recipient can read it?
- What is the shortest path between two cities using a transportation system?
- How can a list of integers be sorted so that the integers are in increasing order?
- How many steps are required to do such a sorting?
- How can a circuit that adds two integers be designed?
- How many valid Internet addresses are there?

WHY STUDY DISCRETE MATHEMATICS?

There are several important reasons for studying discrete mathematics.

- ✓First, through this course you can develop your mathematical maturity: that is, your ability to understand and create mathematical arguments.

- ✓Second, discrete mathematics provides the mathematical foundations for many computer science courses including data structures, algorithms, database theory, automata theory, formal languages, compiler theory, computer security, and operating systems. Students find these courses much more difficult when they have not had the appropriate mathematical foundations from discrete math.

- ✓It is not possible to arrange each of the topic as a separate course organized by mathematics department

Sets

- ✓Sets are used to group objects together. Often, but not always, the objects in a set have similar properties.

- ✓For instance, all the students who are currently enrolled in your school make up a set. Likewise, all the students currently taking a course in discrete mathematics at any school make up a set.

- ✓In addition, those students enrolled in your school who are taking a course in discrete mathematics form a set that can be obtained by taking the elements common to the first two collections.

DEFINITION 1

A set is an unordered collection of objects, called elements or members of the set. A set is said to contain its elements. We write $a \in A$ to denote that a is an element of the set A. The Notation $\in$ denotes that a is not an element of the set A.

The set V of all vowels in the English alphabet can be written as

$V = \{a, e, i, o, u\}$.

Some notations

N = {0, 1, 2, 3, . . .}, the set of **natural numbers**
Z = {. . . , −2, −1, 0, 1, 2, . . .}, the set of **integers**
Z+ = {1, 2, 3, . . .}, the set of **positive integers**
Q = $\{p/q \mid p \in \mathbf{Z}, q \in \mathbf{Z}, \text{ and } q \neq 0\}$, the set of **rational numbers**
R, the set of **real numbers**
R+, the set of **positive real numbers**
C, the set of **complex numbers**.

$O = \{x \mid x$ is an odd positive integer less than 10$\}$,
or,
$O = \{x \in \mathbf{Z^+} \mid x$ is odd and $x < 10\}$.

List the elements of each set where **N = {1, 2, 3, …}.**

(*a*) $A = \{x \in \mathbf{N} / 3 < x < 9\}$

(*b*) $B = \{x \in \mathbf{N} / x \text{ is even}, x < 11\}$

(*c*) $C = \{x \in \mathbf{N} / 4 + x = 3\}$

(*a*) *A* consists of the positive integers between 3 and 9; hence $A = \{4, 5, 6, 7, 8\}$.

(*b*) *B* consists of the even positive integers less than 11; hence $B = \{2, 4, 6, 8, 10\}$.

(*c*) No positive integer satisfies $4 + x = 3$; hence $C = \varnothing$, the empty set.

Let $A = \{2, 3, 4, 5\}$.

(*a*) Show that *A* is not a subset of $B = \{x \in \mathbf{N} / x \text{ is even}\}$.

(*b*) Show that *A* is a proper subset of $C = \{1, 2, 3, \ldots, 8, 9\}$.

(*a*) It is necessary to show that at least one element in *A* does not belong to *B*. Now $3 \in A$ and, since *B* consists of even numbers, 3 *B*; hence *A* is not a subset of *B*.

(*b*) Each element of A belongs to C so $A \subseteq C$. On the other hand, $1 \in C$ but 1 A. Hence $A \neq C$. Therefore *A* is a proper subset of *C*.

DEFINITION 2

Two sets are **equal** if and only if they have the same elements. Therefore, if A and B are sets, then A and B are equal if and only if $\forall x(x \in A \leftrightarrow x \in B)$.We write $A = B$ if A and B are equal sets.

DEFINITION 3

The set A is a subset of B if and only if every element of A is also an element of B. We use the notation $A \subseteq B$ to indicate that A is a subset of the set B.

We see that $A \subseteq B$ if and only if the quantification $\forall x(x \in A \rightarrow x \in B)$ is true.

Given a set *S*, the power set of *S* is the set of all subsets of the set *S*. The power set of *S* is denoted by *P(S)*.

Q1. What is the power set of the set {0, 1, 2}?

Solution: The power set $P(\{0, 1, 2\})$ is the set of all subsets of $\{0, 1, 2\}$. Hence, $P(\{0, 1, 2\}) = \{\emptyset, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, \{0, 1, 2\}\}$.

Q.2 What is the power set of the empty set? What is the power set of the set {∅}?

Solution: The empty set has exactly one subset, namely, itself. Consequently, $P(\emptyset) = \{\emptyset\}$.

The set $\{\emptyset\}$ has exactly two subsets, namely, $\emptyset$ and the set $\{\emptyset\}$ itself. Therefore, $P(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$.

Determine the power set *P(A)* of *A = {a, b, c, d}.*

The elements of *P(A)* are the subsets of *A.* Hence
P(A) = [A, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a}, {b}, {c}, {d}, ∅]

Let A and B be sets. The Cartesian product of A and B, denoted by:
$A \times B$, is the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$. Hence,
$A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$.

Q3. What is the Cartesian product of $A = \{1, 2\}$ and $B = \{a, b, c\}$?
Solution: The Cartesian product $A \times B$ is
$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$.

Q4. Show that the Cartesian product $B \times A$ is not equal to the Cartesian product $A \times B$.
Solution: The Cartesian product $B \times A$ is
$B \times A = \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\} \neq A \times B$.

Q.5 What is the Cartesian product $A \times B \times C$, where $A = \{0, 1\}$, $B = \{1, 2\}$, and $C = \{0, 1, 2\}$?

Solution: The Cartesian product $A \times B \times C$ consists of all ordered triples (a, b, c), where $a \in A$,
$b \in B$, and $c \in C$. Hence,
$A \times B \times C = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 0), (0, 2, 1), (0, 2, 2), (1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 2, 0), (1, 2, 1), (1, 2, 2)\}$.

If $A = \emptyset$ and $B = \{2, 4\}$. Find $A \times B$ and $B \times A$

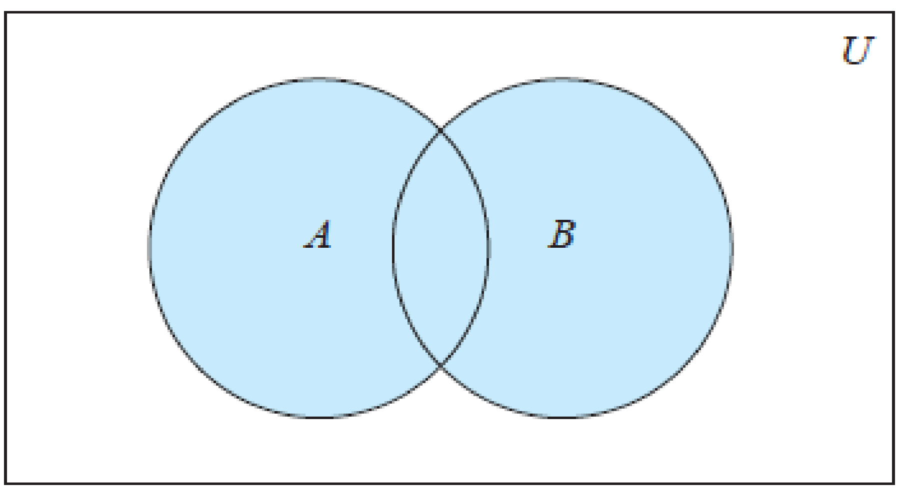
Ans. $A \times B = B \times A = \emptyset$

Let A and B be sets. The union of the sets A and B, denoted by $A \cup B$, is the set that contains those elements that are either in A or in B, or in both.

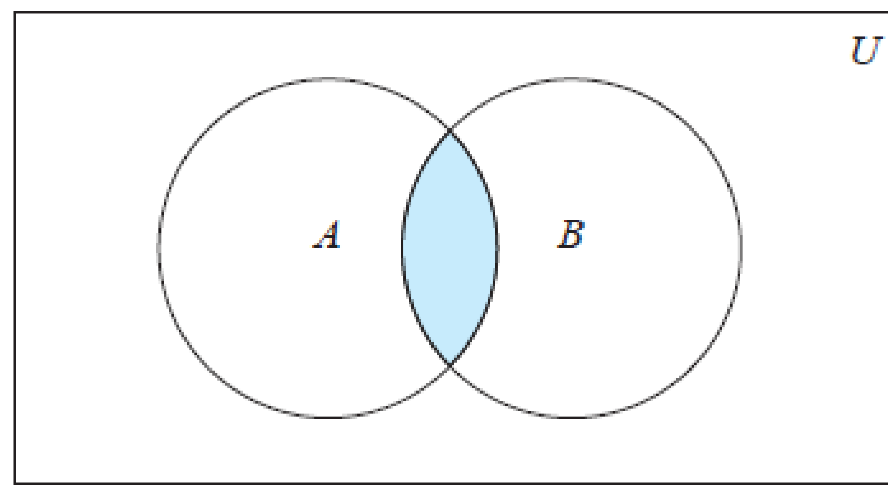
The union of the sets {1, 3, 5} and {1, 2, 3} is the set {1, 2, 3, 5}; that is, $\{1, 3, 5\} \cup \{1, 2, 3\} = \{1, 2, 3, 5\}$.

Let A and B be sets. The intersection of the sets A and B, denoted by $A \cap B$, is the set containing those elements in both A and B.

The intersection of the sets {1, 3, 5} and {1, 2, 3} is the set {1, 3}; that is, $\{1, 3, 5\} \cap \{1, 2, 3\} = \{1, 3\}$.



$A \cup B$ is shaded.

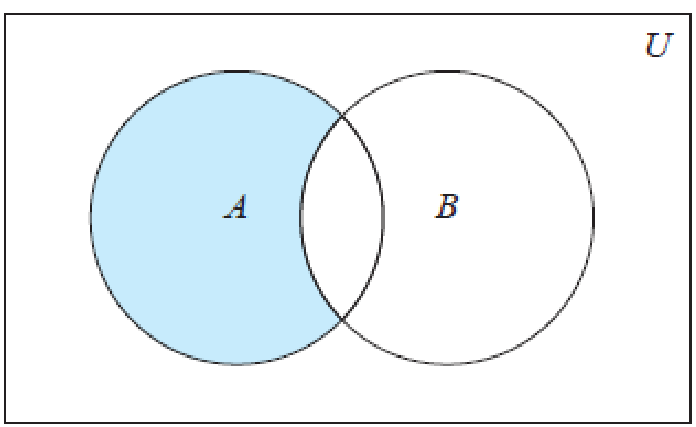


$A \cap B$ is shaded.

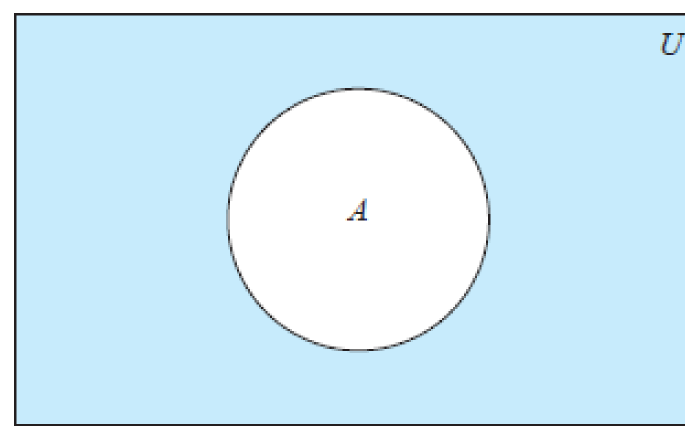
Let $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 4, 6, 8, 10\}$. Because $A \cap B = \emptyset$, A and B are disjoint.

Let A and B be sets. The difference of A and B, denoted by $A - B$, is the set containing those elements that are in A but not in B. The difference of A and B is also called the complement of B with respect to A.

Let U be the universal set. The complement of the set A, denoted by A^c, is the complement of A with respect to U. Therefore, the complement of the set A is $U - A$.



$A - B$ is shaded.



$\overline{A}$ is shaded.

Let $A = \{a, e, i, o, u\}$ (where the universal set is the set of letters of the English alphabet). Then

$A^c = \{b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z\}$.

Let A be the set of positive integers greater than 10 (with universal set the set of all positive integers).

Then

$A^c = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

Let **U =** $\{1,2, \ldots, 9\}$ be the universal set, and let
$A = \{1, 2, 3, 4, 5\}$, $C = \{5, 6, 7, 8, 9\}$, $E = \{2, 4, 6, 8\}$,
$B = \{4, 5, 6, 7\}$, $D = \{1, 3, 5, 7, 9\}$, $F = \{1, 5, 9\}$.
Find: (a) $A \cup B$ and $A \cap B$; (b) $A \cup C$ and $A \cap C$; (c) $D \cup F$ and $D \cap F$.

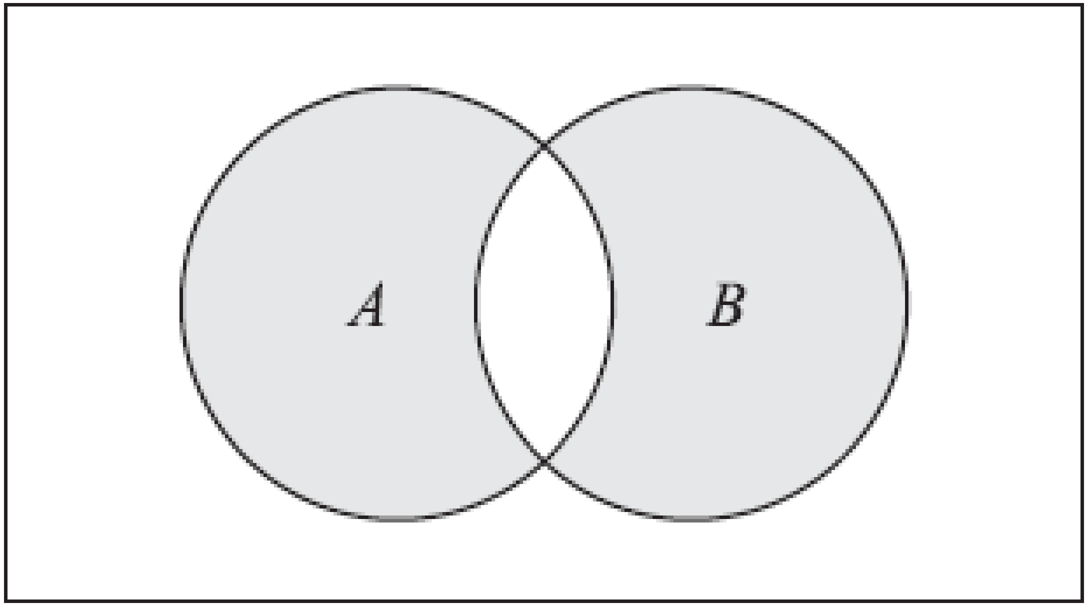
Recall that the union $X \cup Y$ consists of those elements in either X or Y (or both), and that the intersection $X \cap Y$ consists of those elements in both X and Y.

(a) $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$ and $A \cap B = \{4, 5\}$
(b) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} = U$ and $A \cap C = \{5\}$
(c) $D \cup F = \{1, 3, 5, 7, 9\} = D$ and $D \cap F = (1, 5, 9) = F$

The symmetric difference of sets A and B, denoted by $A \oplus B$, consists of those elements which belong to A or B but not to both. That is,

$A \oplus B = (A \cup B) \setminus (A \cap B)$ or $A \oplus B = (A \setminus B) \cup (B \setminus A)$

Figure below is a Venn diagram in which $A \oplus B$ is shaded.



$A \oplus B$ is shaded

Suppose **U = N** = {1, 2, 3, . . .} is the universal set.

Let

$A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6, 7\}$, $C = \{2, 3, 8, 9\}$, $E = \{2, 4, 6, \ldots\}$

(Here E is the set of even integers.) Then:

$A^C = \{5, 6, 7, \ldots\}$, $B^C = \{1, 2, 8, 9, 10, \ldots\}$, $E^C = \{1, 3, 5, 7, \ldots\}$

That is, E^C is the set of odd positive integers. Also:

$A \backslash B = \{1, 2\}$, $A \backslash C = \{1, 4\}$, $B \backslash C = \{4, 5, 6, 7\}$, $A \backslash E = \{1, 3\}$,

$B \backslash A = \{5, 6, 7\}$, $C \backslash A = \{8, 9\}$, $C \backslash B = \{2, 8, 9\}$, $E \backslash A = \{6, 8, 10, 12, \ldots\}$.

Furthermore:

$A \oplus B = (A \backslash B) \cup (B \backslash A) = \{1, 2, 5, 6, 7\}$, $B \oplus C = \{2, 4, 5, 6, 7, 8, 9\}$,

$A \oplus C = (A \backslash C) \cup (B \backslash C) = \{1, 4, 8, 9\}$, $A \oplus E = \{1, 3, 6, 8, 10, \ldots\}$.

TABLE 1 Set Identities.

Identity	*Name*
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws

Use set builder notation and logical equivalences to establish the first De Morgan law $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

Solution: We can prove this identity with the following steps.

$$
\begin{aligned}
\overline{A \cap B} &= \{x \mid x \notin A \cap B\} && \text{by definition of complement} \\
&= \{x \mid \neg(x \in (A \cap B))\} && \text{by definition of does not belong symbol} \\
&= \{x \mid \neg(x \in A \wedge x \in B)\} && \text{by definition of intersection} \\
&= \{x \mid \neg(x \in A) \vee \neg(x \in B)\} && \text{by the first De Morgan law for logical equivalences} \\
&= \{x \mid x \notin A \vee x \notin B\} && \text{by definition of does not belong symbol} \\
&= \{x \mid x \in \overline{A} \vee x \in \overline{B}\} && \text{by definition of complement} \\
&= \{x \mid x \in \overline{A} \cup \overline{B}\} && \text{by definition of union} \\
&= \overline{A} \cup \overline{B} && \text{by meaning of set builder notation}
\end{aligned}
$$

Let A, B, and C be sets. Show that

$$\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}.$$

Solution: We have

$$\begin{aligned}
\overline{A \cup (B \cap C)} &= \overline{A} \cap (\overline{B \cap C}) && \text{by the first De Morgan law} \\
&= \overline{A} \cap (\overline{B} \cup \overline{C}) && \text{by the second De Morgan law} \\
&= (\overline{B} \cup \overline{C}) \cap \overline{A} && \text{by the commutative law for intersections} \\
&= (\overline{C} \cup \overline{B}) \cap \overline{A} && \text{by the commutative law for unions.}
\end{aligned}$$

The *union* of a collection of sets is the set that contains those elements that are members of at least one set in the collection.

We use the notation

$$A_1 \cup A_2 \cup \cdots \cup A_n = \bigcup_{i=1}^{n} A_i$$

to denote the union of the sets $A_1, A_2, \ldots, A_n$.

The *intersection* of a collection of sets is the set that contains those elements that are members of all the sets in the collection.

We use the notation

$$A_1 \cap A_2 \cap \cdots \cap A_n = \bigcap_{i=1}^{n} A_i$$

to denote the intersection of the sets $A_1, A_2, \ldots, A_n$.

Example

For $i = 1, 2, \ldots,$ let $A_i = \{i, i+1, i+2, \ldots\}$. Then,

$$\bigcup_{i=1}^{n} A_i = \bigcup_{i=1}^{n} \{i, i+1, i+2, \ldots\} = \{1, 2, 3, \ldots\},$$

and

$$\bigcap_{i=1}^{n} A_i = \bigcap_{i=1}^{n} \{i, i+1, i+2, \ldots\} = \{n, n+1, n+2, \ldots\} = A_n.$$

Example

Suppose that $A_i = \{1, 2, 3, \ldots, i\}$ for $i = 1, 2, 3, \ldots$. Then,

$$\bigcup_{i=1}^{\infty} A_i = \bigcup_{i=1}^{\infty} \{1, 2, 3, \ldots, i\} = \{1, 2, 3, \ldots\} = \mathbf{Z}^+$$

and

$$\bigcap_{i=1}^{\infty} A_i = \bigcap_{i=1}^{\infty} \{1, 2, 3, \ldots, i\} = \{1\}.$$

Example

Given, A_1={1, 5}, A_2 = {1,2,4,6}, A_3 = {3, 4,7}, B = {2,4} and I = {1, 2,3}. Determine the following sets.

$$\bigcup_{i \in I} A_i = \{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7\}$$

$$\bigcap_{i \in I} A_i = \phi$$

$$B \cup \left(\bigcap_{i \in I} A_i \right) = \{2,\ 4\}$$

$$\bigcap_{i \in I} (B \cup A_i) = \{1,\ 2,\ 4,\ 5\} \cap \{1,\ 2,\ 4,\ 5,\ 6\} \cap \{2,\ 3,\ 4,\ 7\} = \{2,\ 4\}$$

Partitions

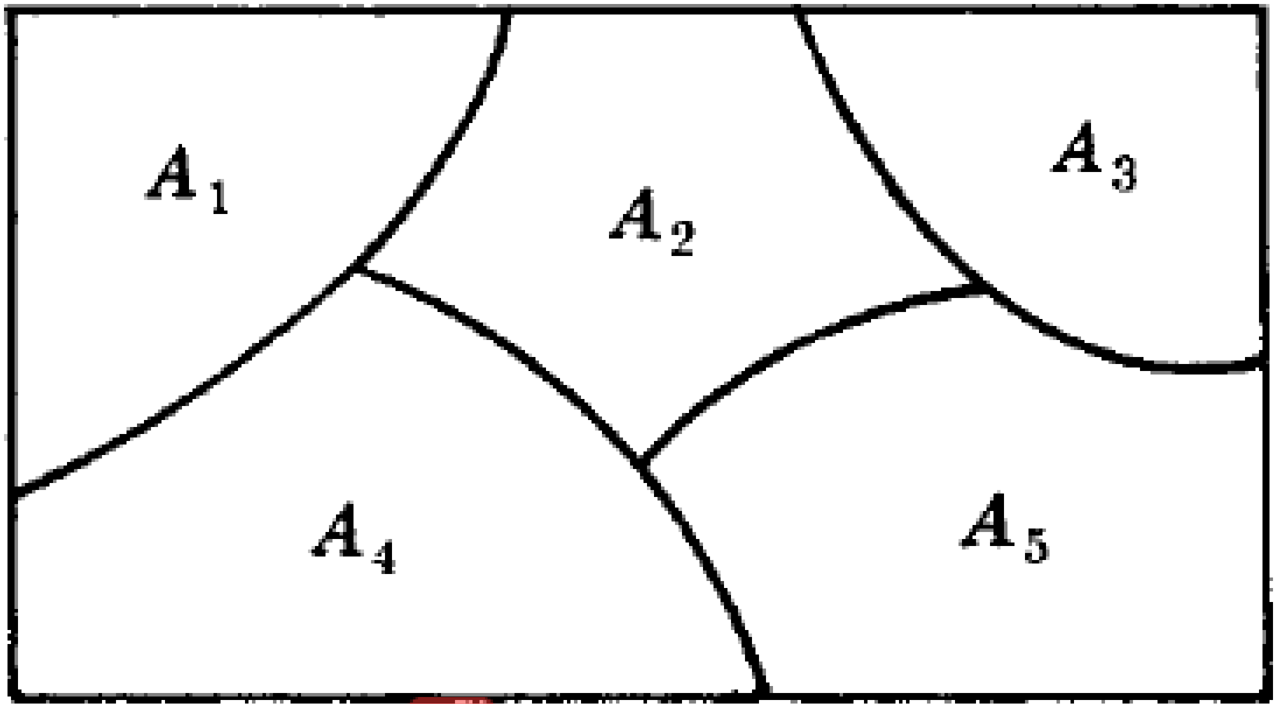
Let S be a nonempty set. A partition of S is a subdivision of S into nonoverlapping, nonempty subsets.

Precisely, a partition of S is a collection $\{A_i\}$ of nonempty subsets of S such that:

(i) Each subset in S belongs to one of the A_i.

(ii) The sets of $\{A_i\}$ are mutually disjoint; that is, if $A_j \neq A_k$ then $A_j \cap A_k = \varnothing$

The subsets in a partition are called cells. Figure shown below is a Venn diagram of a partition of the rectangular set S of points into five cells, A_1, A_2, A_3, A_4, A_5.



EXAMPLE

Consider the following collections of subsets of $S = \{1, 2, \ldots, 8, 9\}$:

(i) [{1, 3, 5}, {2, 6}, {4, 8, 9}]

(ii) [{1, 3, 5}, {2, 4, 6, 8}, {5, 7, 9}]

(iii) [{1, 3, 5}, {2, 4, 6, 8}, {7, 9}]

Then (i) is not a partition of *S* since 7 in *S* does not belong to any of the subsets. Furthermore, (ii) is not a partition of *S* since {1, 3, 5} and {5, 7, 9} are not disjoint. On the other hand, (iii) is a partition of *S*.

Example

Let $S = \{a, b, c, d, e, f, g\}$. Determine which of the following are partitions of S:

(a) $P_1 = [\{a, c, e\}, \{b\}, \{d, g\}]$, (c) $P_3 = [\{a, b, e, g\}, \{c\}, \{d, f\}]$,

(b) $P_2 = [\{a, e, g\}, \{c, d\}, \{b, e, f\}]$, (d) $P_4 = [\{a, b, c, d, e, f, g\}]$.

Ans.

(a) P_1 is not a partition of S since $f \in S$ does not belong to any of the cells.

(b) P_2 is not a partition of S since $e \in S$ belongs to two of the cells.

(c) P_3 is a partition of S since each element in S belongs to exactly one cell.

(d) P_4 is a partition of S into one cell, S itself.

Example

Find all partitions of $S = \{a, b, c, d\}$.

Note first that each partition of S contains either 1, 2, 3, or 4 distinct cells. The partitions are as follows:

(1) $[\{a, b, c, d\}]$

(2) $[\{a\}, \{b, c, d\}]$, $[\{b\}, \{a, c, d\}]$, $[\{c\}, \{a, b, d\}]$, $[\{d\}, \{a, b, c\}]$, $[\{a, b\}, \{c, d\}]$, $[\{a, c\}, \{b, d\}]$, $[\{a, d\}, \{b, c\}]$

(3) $[\{a\}, \{b\}, \{c, d\}]$, $[\{a\}, \{c\}, \{b, d\}]$, $[\{a\}, \{d\}, \{b, c\}]$, $[\{b\}, \{c\}, \{a, d\}]$, $[\{b\}, \{d\}, \{a, c\}]$, $[\{c\}, \{d\}, \{a, b\}]$

(4) $[\{a\}, \{b\}, \{c\}, \{d\}]$

There are 15 different partitions of S.

Let $\mathbf{N} = \{1, 2, 3, \ldots\}$ and, for each $n \in \mathbf{N}$, Let $A_n = \{n, 2n, 3n, \ldots\}$. Find:

(a) $A_3 \cap A_5$; (b) $A_4 \cap A_5$; (c) $\bigcup_{i \in Q} A_i$ where $Q = \{2, 3, 5, 7, 11, \ldots\}$ is the set of prime numbers.

(a) Those numbers which are multiples of both 3 and 5 are the multiples of 15; hence $A_3 \cap A_5 = A_{15}$.

(b) The multiples of 12 and no other numbers belong to both A_4 and A_6, hence $A_4 \cap A_6 = A_{12}$.

(c) Every positive integer except 1 is a multiple of at least one prime number; hence

$$\bigcup_{i \in Q} A_i = \{2, 3, 4, \ldots\}$$

Counting Elements in Finite Sets

The notation $n(S)$ or $|S|$ will denote the number of elements in a set S. (Some texts use $\#(S)$ or card(S) instead of $n(S)$.) Thus $n(A) = 26$, where A is the letters in the English alphabet, and $n(D) = 7$, where D is the days of the week. Also $n(\varnothing) = 0$ since the empty set has no elements.

The following lemma applies.

Lemma 1: Suppose A and B are finite disjoint sets. Then $A \cup B$ is finite and $n(A \cup B) = n(A) + n(B)$

Corollary 1: Let *A* and *B* be finite sets. Then

$n(A \backslash B) = n(A) - n(A \cap B)$

For example, suppose an art class A has 25 students and 10 of them are taking a biology class B. Then the number of students in class A which are not in class B is:

$n(A \backslash B) = n(A) - n(A \cap B) = 25 - 10 = 15$

Corollary 2: Let A be a subset of a finite universal set U. Then

$n(A^C) = n(U) - n(A)$

For example, suppose a class *U* with 30 students has 18 full-time students. Then there are 30−18 = 12 part-time students in the class *U*.

Inclusion–Exclusion Principle

There is a formula for $n(A \cup B)$ even when they are not disjoint, called the Inclusion–Exclusion Principle.

Suppose A and B are finite sets. Then $A \cup B$ and $A \cap B$ are finite and

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

That is, we find the number of elements in A or B (or both) by first adding $n(A)$ and $n(B)$ (inclusion) and then subtracting $n(A \cap B)$ (exclusion) since its elements were counted twice.

We can apply this result to obtain a similar formula for three sets:

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

EXAMPLE Suppose a list A contains the 30 students in a mathematics class, and a list B contains the 35 students in an English class, and suppose there are 20 names on both lists. Find the number of students:

(a) only on list A, (b) only on list B, (c) on list A or B (or both), (d) on exactly one list.

(a) List A has 30 names and 20 are on list B; hence 30 − 20 = 10 names are only on list A.

(b) Similarly, 35 − 20 = 15 are only on list B.

(c) We seek $n(A \cup B)$. By inclusion–exclusion,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ = 30 + 35 − 20 = 45.

In other words, we combine the two lists and then cross out the 20 names which appear twice.

(d) By (a) and (b), 10 + 15 = 25 names are only on one list; that is, $n(A \oplus B)$ = 25.

Example

Determine the validity of the following argument:

S_1: All my friends are musicians.

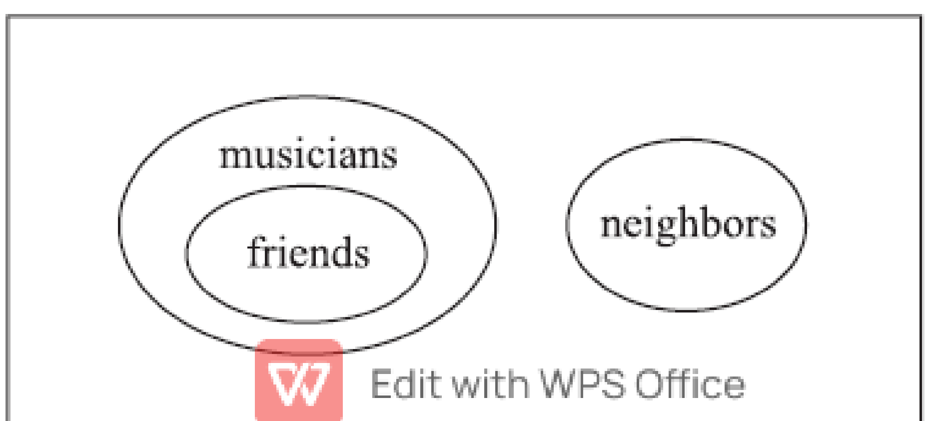
S_2: John is my friend.

S_3: None of my neighbours are musicians.

____

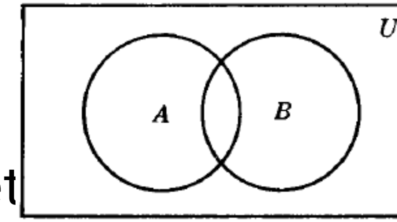
S : John is not my neighbour.

The premises S_1 and S_3 lead to the Venn diagram in Fig. below. By S_2, John belongs to the set of friends which is disjoint from the set of neigh[illegible] ion and so the argument is va



Example

Each student in Philosophy at **Jahangirngar University** requires to complete Science indicated as: *A* and Mathematics as *B*.



A poll of 140 students shows that:

60 completed *A*, 45 completed B, 20 completed both *A* and *B*.

Use a Venn diagram to find the number of students who have complet

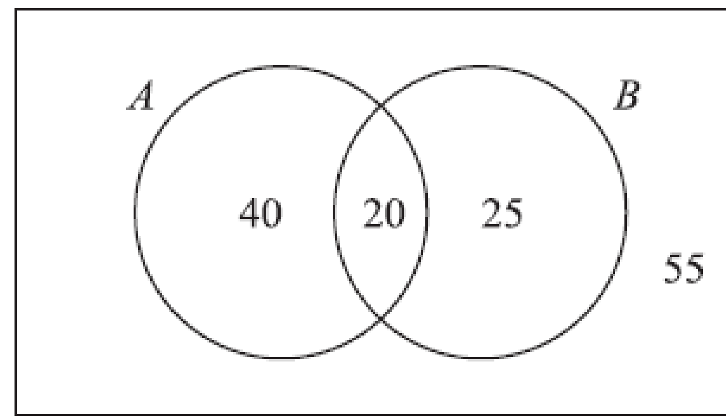
(a) At least one of *A* and *B*; (b) exactly one of *A* or *B*; (c) neither *A* nor *B*.

Ans.

Let us translating the above data into set notation yields:

$n(A) = 60$, $n(B) = 45$, $n(A \cap B) = 20$, $n(U) = 140$

Corresponding Venn diagram of sets A and B is shown below



Now we get,

20 completed both A and B, so $n(A \cap B) = 20$.

60 − 20 = 40 completed A but not B, s o$n(A \setminus B) = 40$.

45 − 20 = 25 completed *B* but not *A*, so $n(B \setminus A) = 25$.

140 − 20 − 40 − 25 = 55 completed neither *A* nor *B*.

By the Venn diagram:

(*a) 20+ 40 + 25 = 85* completed *A* or *B*. Alternately, by the

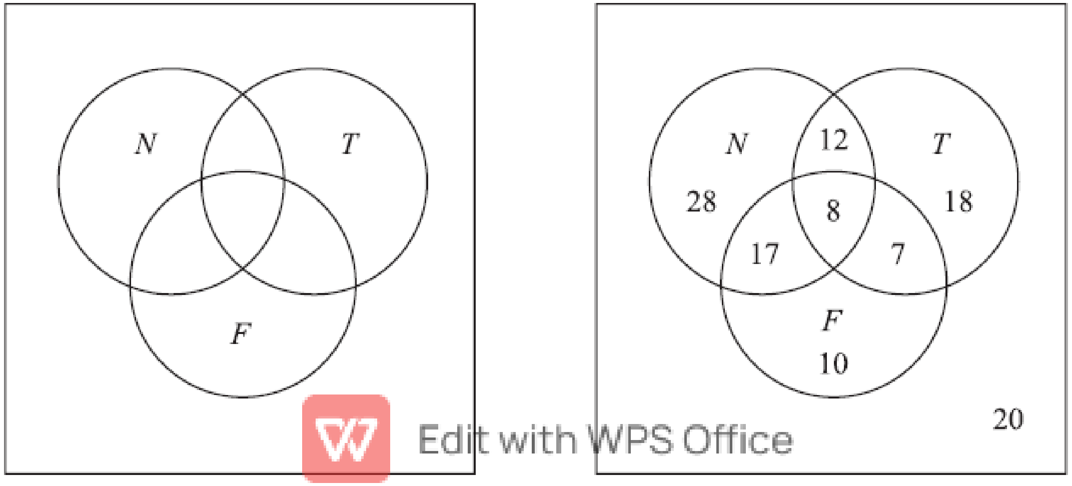
$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 60 + 45 - 20 = 85$

(b) 40+ 25 = 65 completed exactly one requirement. That is, $n(A \oplus B) = 65$.

(c) 55 completed neither requirement, i.e. $n(AC \cap BC) = n[(A \cup B)C] = 140 - 85 = 55$.

Example

In a survey of 120 people, it was found that:
65 read Newsweek magazine, 20 read both Newsweek and Time,
45 read Time, 25 read both Newsweek and Fortune,
42 read Fortune, 15 read both Time and Fortune,
8 read all three magazines.
(a) Find the number of people who read at least one of the three magazines.
(b) Fill in the correct number of people in each of the eight regions of the Venn diagram in Fig. a where *N, T*, and *F* denote the set of people who read Newsweek, Time, and Fortune, respectively.
(c) Find the number of people who read exactly one magazine.



(a) (b)

(a) We want to find $n(N \cup T \cup F)$.

$n(N \cup T \cup F) = n(N) + n(T) + n(F) - n(N \cap T) - n(N \cap F) - n(T \cap F) + n(N \cap T \cap F)$

$= 65 + 45 + 42 - 20 - 25 - 15 + 8 = 100$

(b) The required Venn diagram in Fig. b is obtained as follows:

8 read all three magazines,

20 − 8 = 12 read Newsweek and Time but not all three magazines,

25 − 8 = 17 read Newsweek and Fortune but not all three magazines,

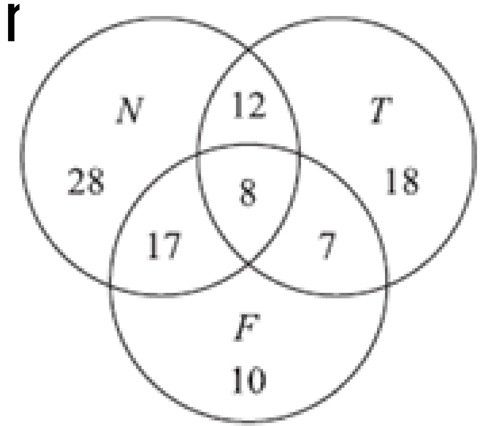
15 − 8 = 7 read Time and Fortune but not all thr

65 − 12 − 8 − 17 = 28 read only Newsweek,

45 − 12 − 8 − 7 = 18 read only Time,

42 − 17 − 8 − 7 = 10 read only Fortune,

120 − 100 = 20 read no magazine at all.



(c) 28+ 18 + 10 = 56 read exactly one of the magazines.