Sheikh Hasina University, Netrokona
Department of Computer Science and Engineering
CSE-2205: Introduction to Mechatronics

Lec-19: System Models

Mechatronics: Electronic Control Systems in Mechanical Engineering by W. Bolton

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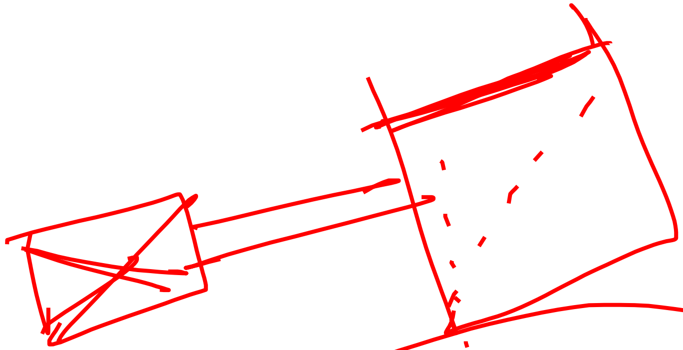
Basic system models:

$\mu P \rightarrow$ Motor

0 rpm — 500 rpm → changes w.r.t time

rpm

Hydraulic system



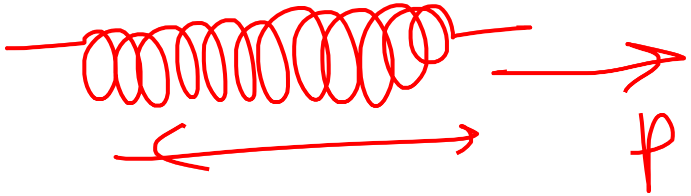
Equations

Equations

→ Relationship

→ input

→ output



Spring:

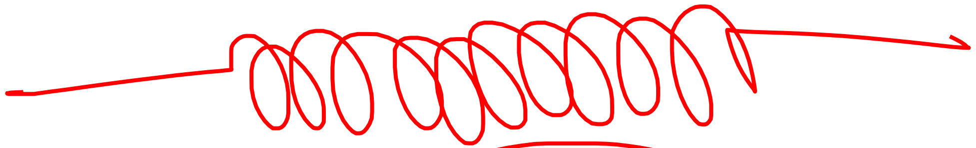
extension x

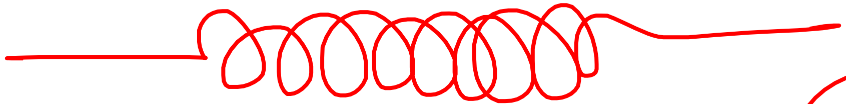
$F \propto x$

$F = Kx$

spring constant

K = High





K = 2

K = 10

Rigid

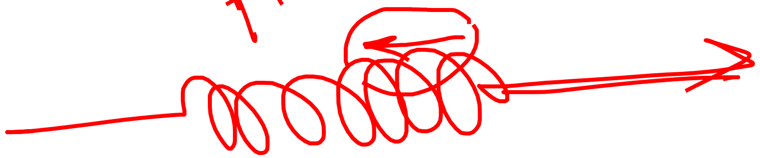
F

F = 10N

$$F = Kx$$

$$x = \frac{10}{10} = 1\,m$$

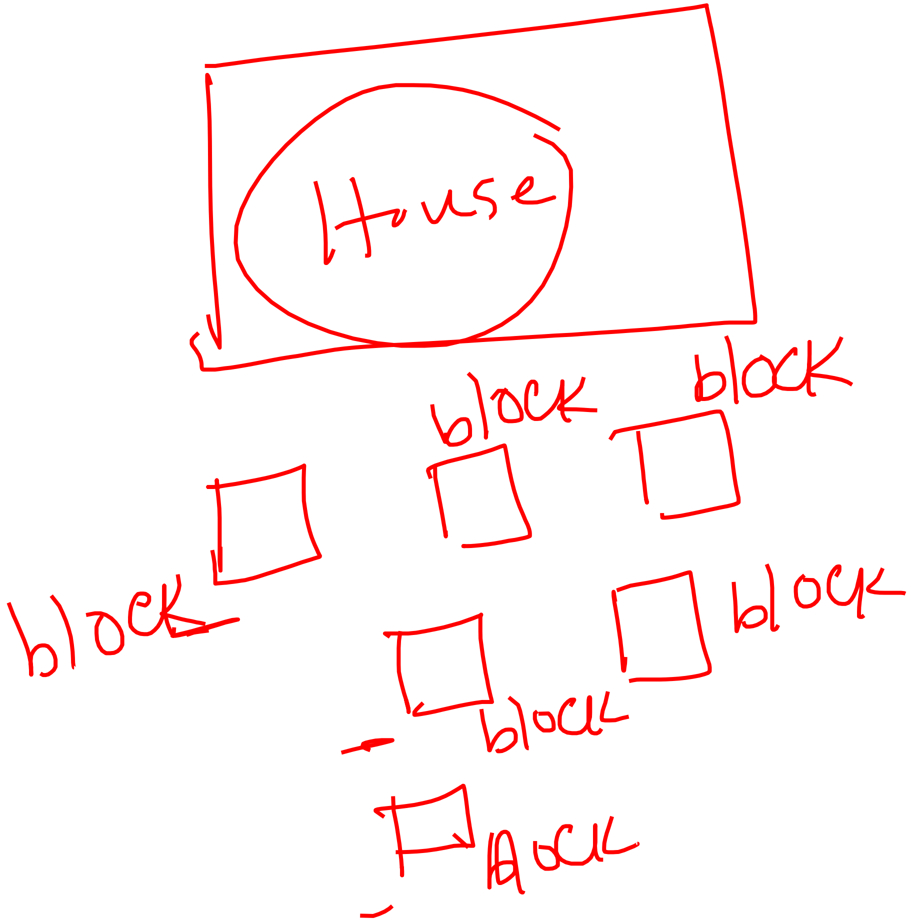
Flexible



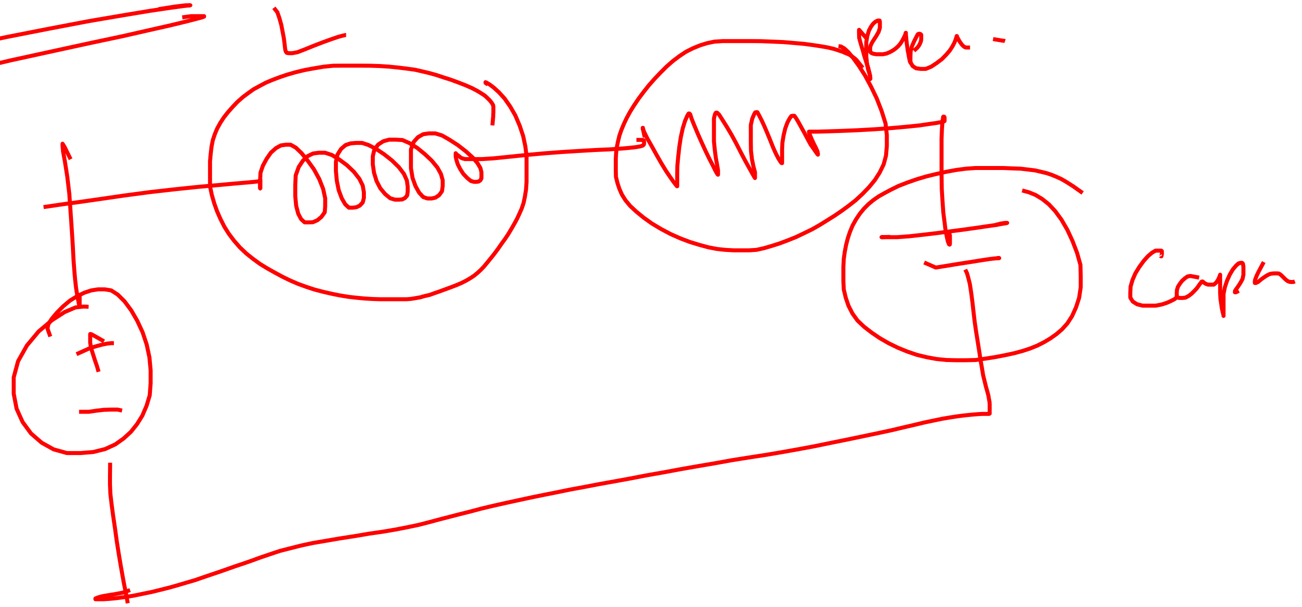
$$x = \frac{F}{K} = \frac{10}{2} = 5\,m$$

Mathematical Modeling:

- Mechanical
- electrical
- thermal
- fluid

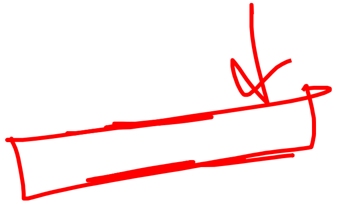


electrical:



Mechanical system building blocks.

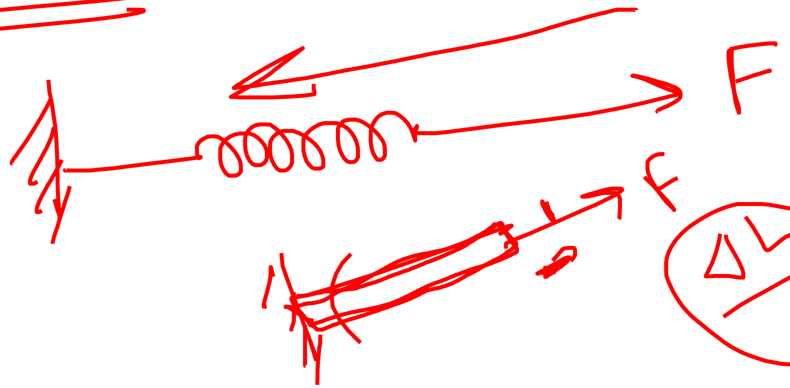
1) Springs → stiffness



2) dashpots ← opposing motion

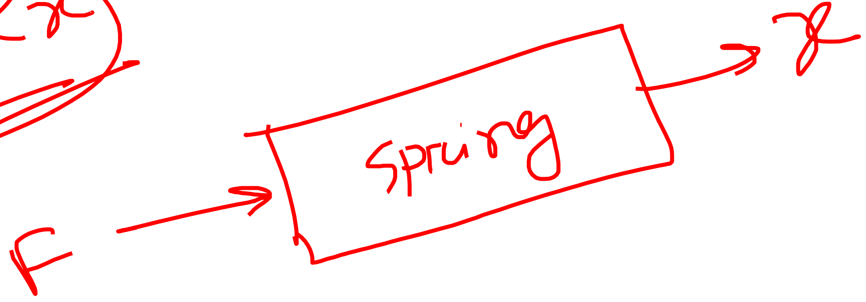
3) Masses. → interia/Resistance to acceleration

1) Springs:

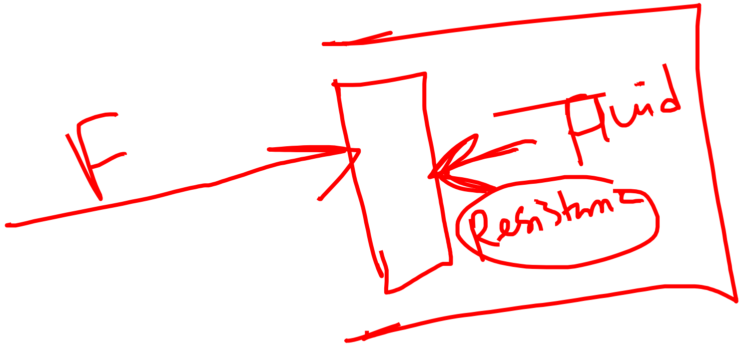


ΔL

$F = Kx$

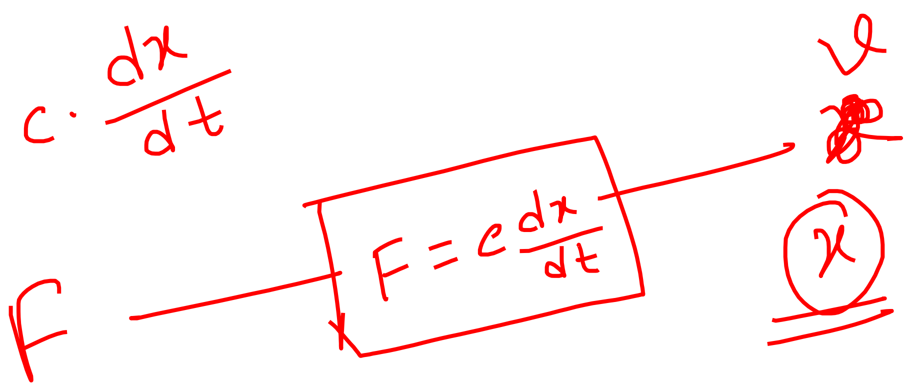


2) Dashpots:

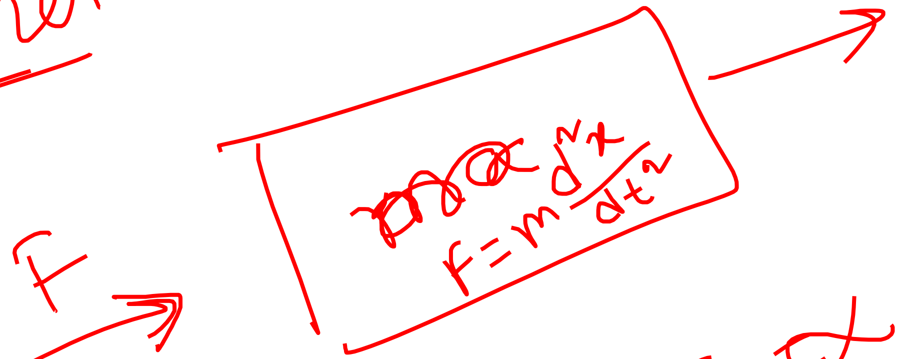


$$F \propto v$$

$$F = cv = c \cdot \frac{dx}{dt}$$



3) Masses:



acceleration (a)

$$F \propto a$$

$$F = ma$$

$$F = m \cdot \frac{dv}{dt} = m \cdot \frac{d}{dt}\left(\frac{dx}{dt}\right) = m\frac{d^2x}{dt^2}$$

Energy:

Spring:

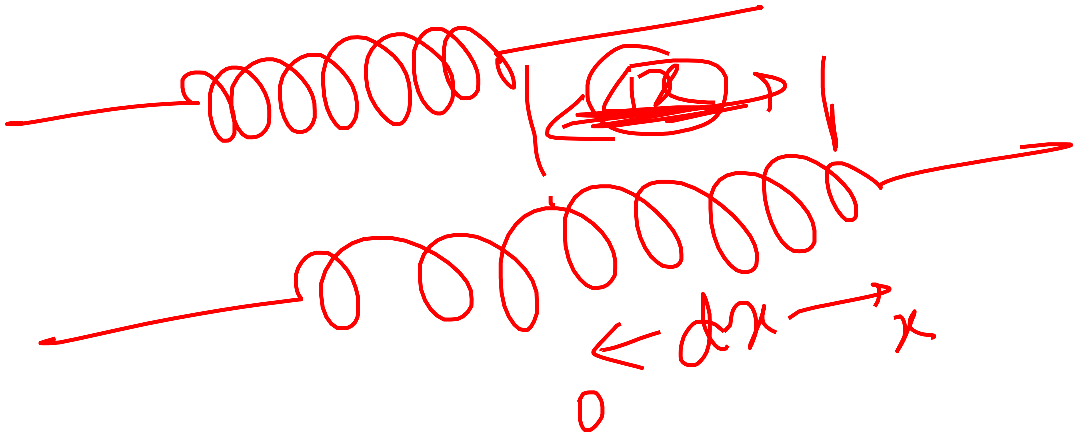
$$F = -kx$$

$$W = \int_0^x F \cdot dx$$

$$= \int_0^x -kx \, dx$$

$$= -k \int_0 \frac{x^2}{2}$$

$$\boxed{W = \frac{1}{2}kx^2}$$



Dashpot:

$$F \propto -v$$

$$F = -cv$$

$$W = F \cdot x$$

$$P = \frac{W}{t} = \frac{F \cdot x}{t} = F \cdot \frac{x}{t} = Fv$$

$$P =$$

$$P = Fv$$

$$P = -cv^2$$

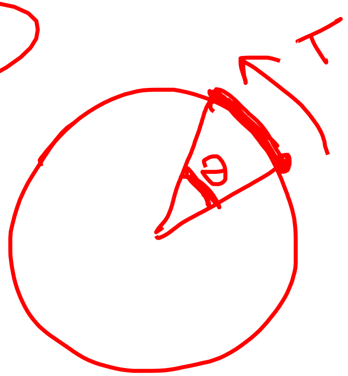
Rotational

Torsional spring

Rotary Damp

Moment of inertia

Wasted



$F = Kx$

$\boxed{T = K\theta}$ ✓

$E = \frac{1}{2}\frac{T^2}{K}$

$F = cv$

$T = c\omega$

$\boxed{T = c\frac{d\theta}{dt}}$ ✓

$E = c\omega^2$

$F = ma$

$T = I\alpha$

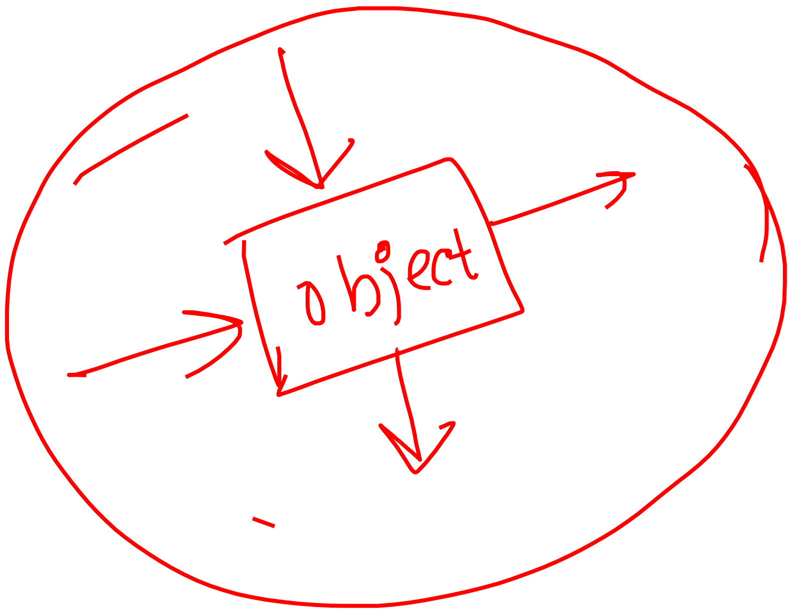
$= I.\frac{d\omega}{dt}$

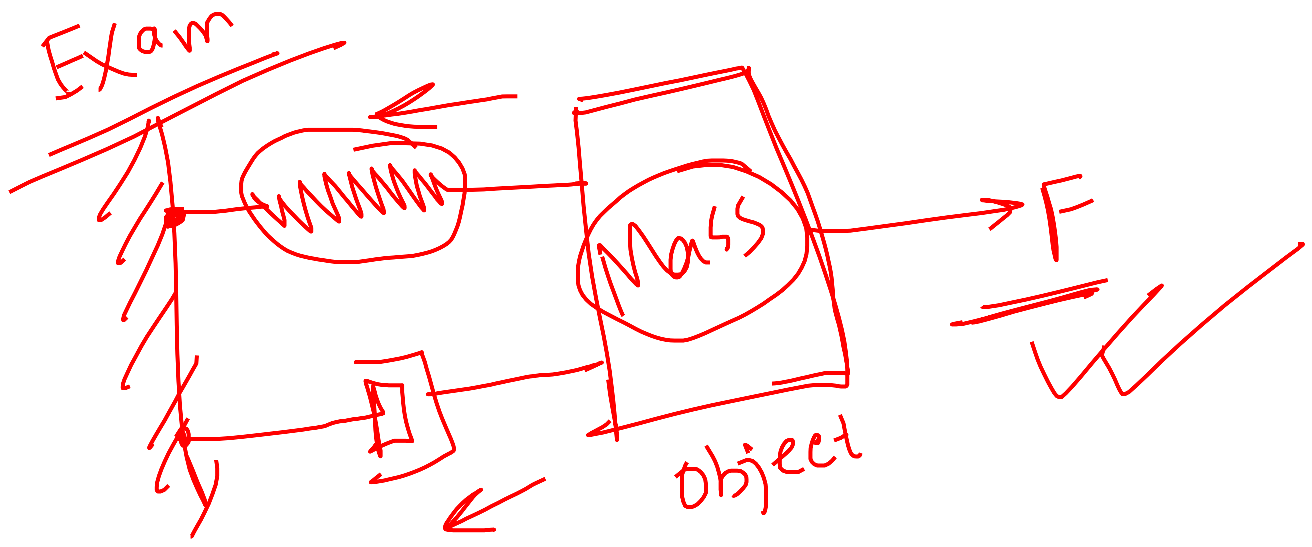
$= I.\frac{d}{dt}\left(\frac{d\theta}{dt}\right)$

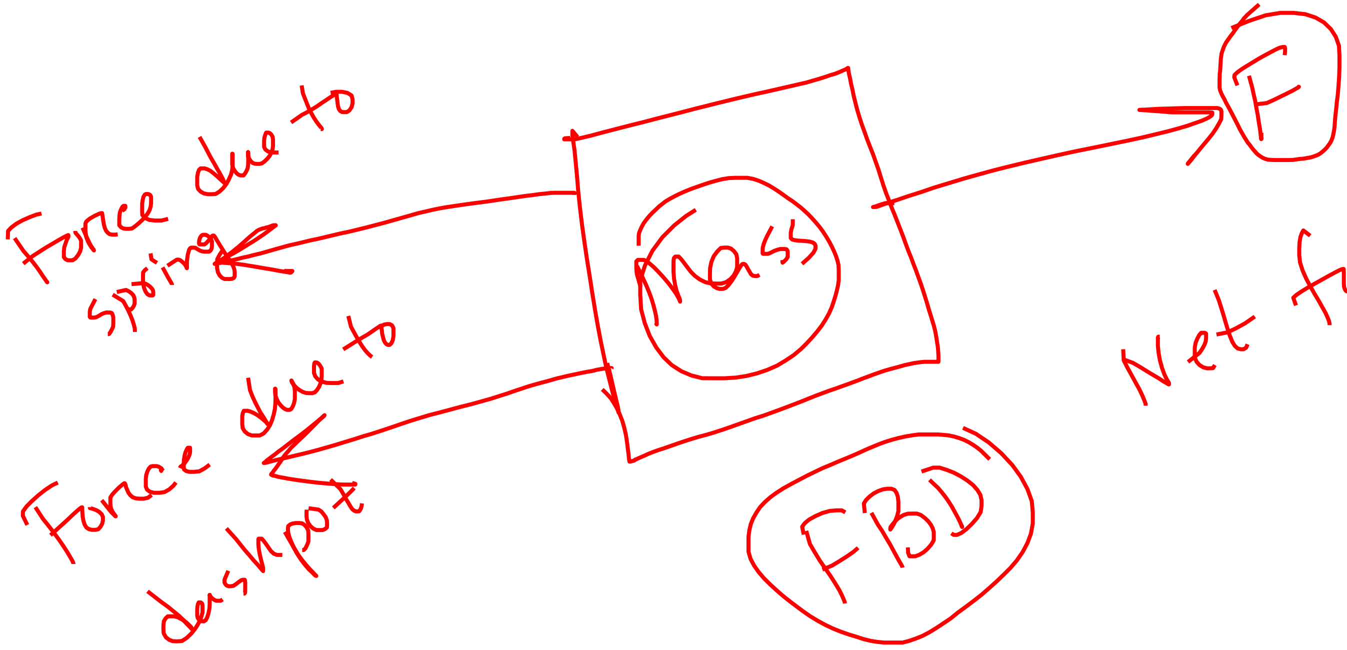
✓ $\boxed{T = I\frac{d^2\theta}{dt^2}}$

Building up a mechanical system:

Free-body Diagram:







Net force applied to mass $= F - kx - cv$

$ma = F - kx - cv$

$\cancel{ma = kx - cv = F}$

$ma = F - kx - cv$

$F - kx - cv = ma$

$F = ma + kx + cv$

$F = m\frac{d^2x}{dt^2} + kx + c.\frac{dx}{dt}$

$\Rightarrow \boxed{F = m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx}$ ✓

is 2nd order diffn equation

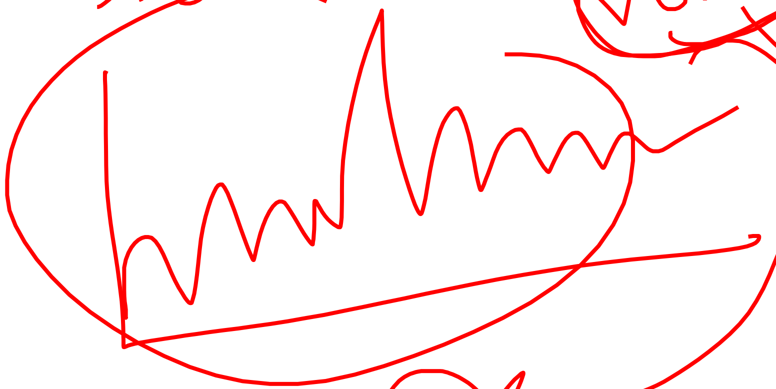
$x = ?$

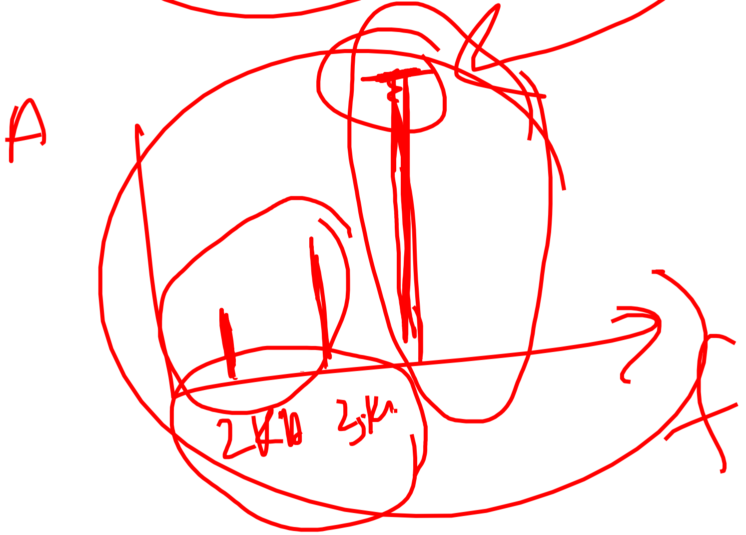
Laplace Transform

Fourier Transform

Signal processing

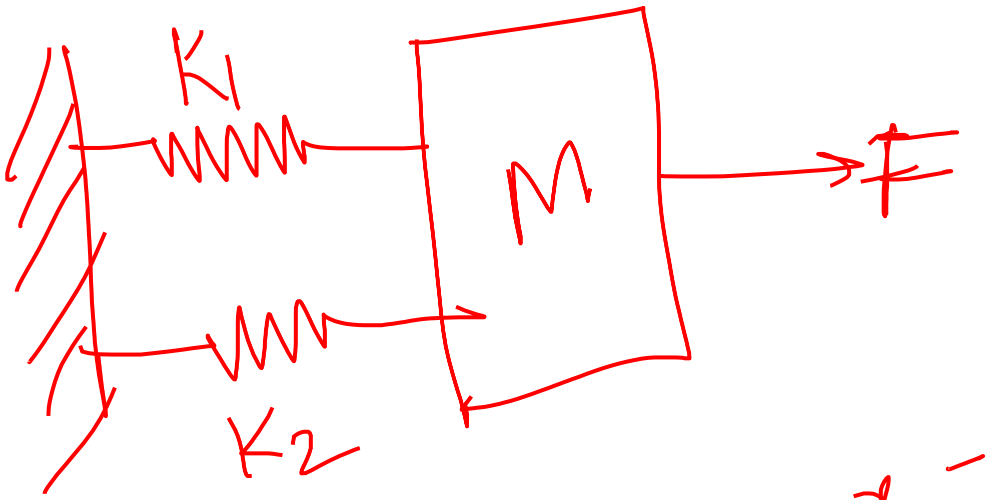
Voice + noise





Differential equation
to liner equati

x

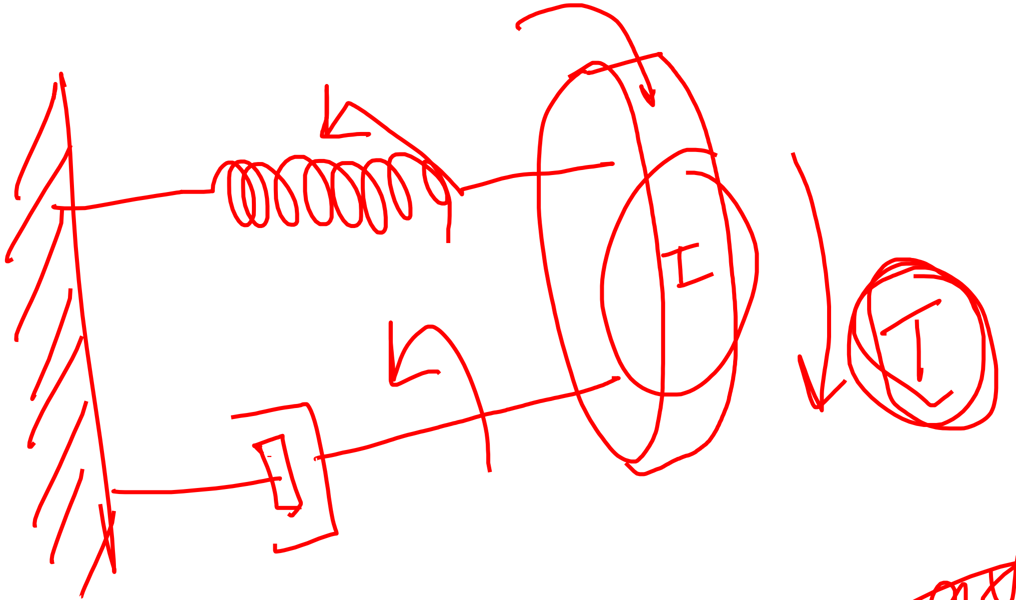


$$ma = F - K_1 x - K_2 x$$

$$ma + K_1 x + K_2 x = F$$

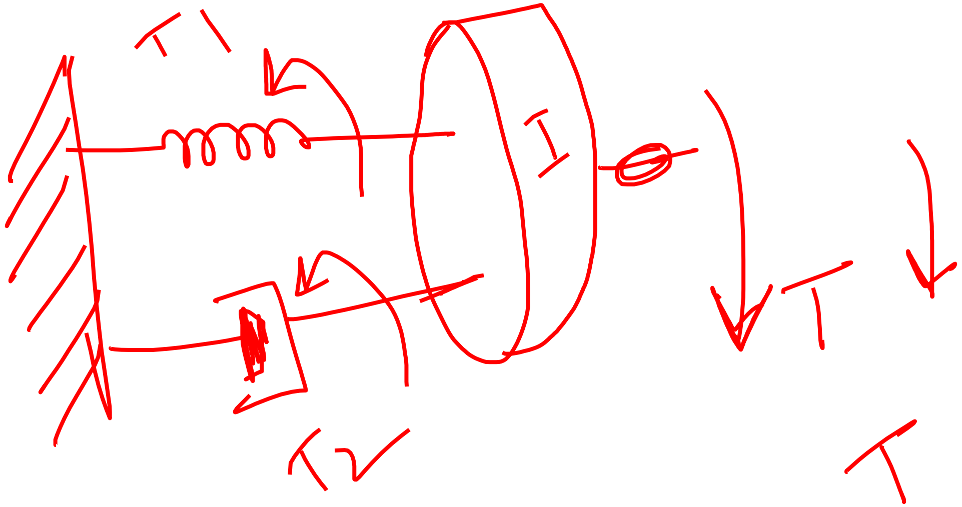
$$m\frac{d^2x}{dt^2} + K_1 x + K_2 x = F$$

$$\boxed{m\frac{d^2x}{dt^2} + (K_1 + K_2)x = F}$$



Net Torque =

$$\boxed{I\frac{d^2\theta}{dt^2} + c\frac{d\theta}{dt} + K\theta = T}$$



$$+\ \text{Net Torque} = T - T_1 - T_2$$

$$I\alpha = T - T_1 - T_2$$

$$I\alpha + T_1 + T_2 = T$$

$$I\frac{d\omega}{dt} + K\theta + c\omega = T$$

$$I\frac{d^2\theta}{dt^2} + K\theta + C\cdot\frac{d\theta}{dt} = T$$

$$I\frac{d^2\theta}{dt^2} + c\frac{d\theta}{dt} + K\theta = T$$

100 N·m