



WELCOME

Lecture# 4

Date: 27 March,2022

Rational functions:

If $P(x)$ and $Q(x)$ be two polynomials, the ratio $y = \frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ have no Common factor, is said to be a rational function

$$f(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \dots + b_2 x^2 + b_1 x + b_0}$$

Examples: $f(x) = \frac{2x^2+x+1}{x-3}$ $f(x) = x^{-1}$

$$f(x) = \frac{x^3+4x+1}{x^3+5x^2+6x+8}$$

Irrational functions:

A function which is an algebraic function of x when it involves root Extraction of terms involving x .

Examples: $f(x) = \sqrt{x+2}$ $f(x) = \sqrt{x^2+4}+3$

Power functions:

If n is any non-zero real number, then the function $f(x) = x^n$ is called a power function with exponent n .

Here n may be positive integer, negative integer, rational number.

$$f(x) = x^5, f(x) = x^{-5} \text{ and } f(x) = x^{\frac{2}{3}}$$

Transcendental functions:

Functions which are not algebraic are said to be transcendental

The following are transcendental functions

1. Trigonometric function

2. Exponential function

3. Logarithmic function

4. Inverse trigonometric function

5. Hyperbolic function

6. Inverse hyperbolic function

Trigonometric function

The Trigonometric functions are real functions to ratios of two side lengths

$$y = \sin x, y = \cos x \text{ etc.}$$

Exponential function

$y = e^x, 2^x, 2^x$ are all exponential functions of x .

Logarithmic function

$y = \log_e x, \log_{10} x, \log_a x$ are all logarithmic functions of x .

Inverse trigonometric function

$y = \sin^{-1} x, \cos^{-1} x, \tan^{-1} x \text{ etc}$ are all inverse trigonometric functions of x .

Hyperbolic function

$y = \sinh x, \cosh x, \operatorname{sech} x \text{ etc.}$ are all hyperbolic function.

$$\text{Here, } \sinh x = \frac{e^x - e^{-x}}{2}, \cosh x = \frac{e^x + e^{-x}}{2}$$

Explicit function:

A function which is directly expressed in terms of the independent variable is called an explicit function.

e.g. $y = x \sin x$, $y = a \cos \theta$, $y = e^{ax} \cos bx$, $y = x^3 + 4x^2 + 6x + 2$ etc. are called explicit functions.

Implicit functions:

A function which is not expressed directly in terms of the independent variable or can be expressed in terms of the independent variable but not done is called implicit function.

e.g. $x^2 + y^2 = a^2$, $ax^2 + 2hxy + by^2 = 0$, $3x^2y + 2xy^2 + 4xy + 5x + 7y + 3 = 0$

Odd functions:

A function $f(x)$ is said to be odd if $f(-x) = -f(x)$

e.g. $f(x) = \sin x$

Even functions:

A function $f(x)$ is said to be even if $f(-x) = f(x)$

e.g. $f(x) = \cos x$

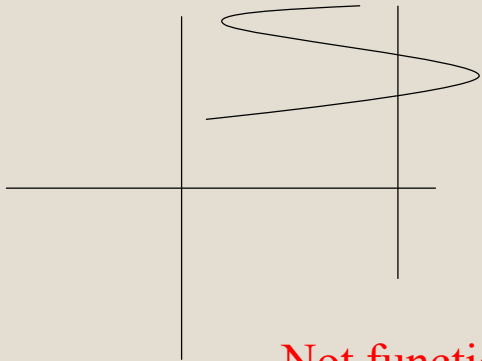
Piece-wise function:

A piecewise function is a function defined by multiple sub-functions, each sub-function Applying to a certain interval of the main function's domain, a sub-domain

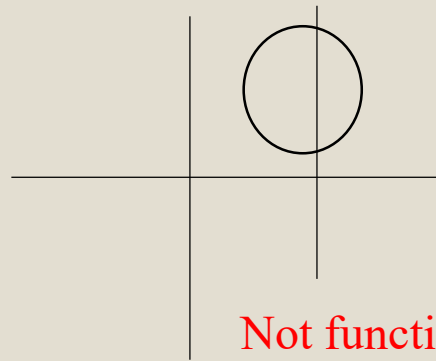
$$\text{e.g. } f(x) = \begin{cases} x^2 + 1, & \text{when } x < 0 \\ x, & 0 \leq x < 1 \\ \frac{1}{x} & x > 1 \end{cases}$$

The vertical line test for the function:

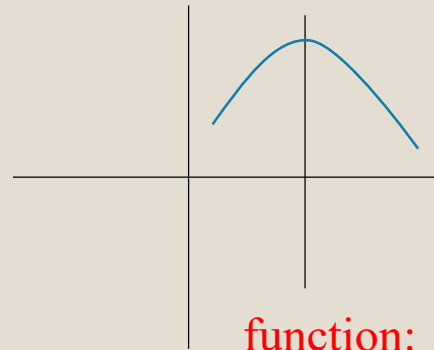
A curve in the xy-plane is the graph of some function f if and only if no vertical line intersects the curve more than once.



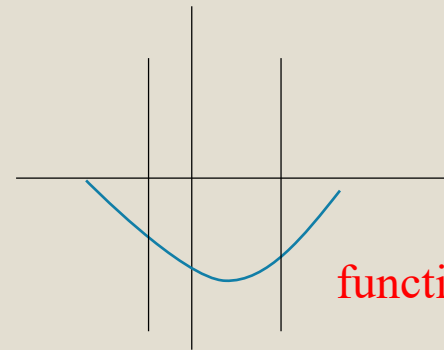
Not function:



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