SINUSOIDAL ALTERNATING WAVEFORMS

SINUSOIDAL ac VOLTAGE CHARACTERISTICS AND DEFINITIONS

THE SINUSOIDAL WAVEFORM

GENERAL FORMAT FOR THE SINUSOIDAL VOLTAGE OR CURRENT

PHASE RELATIONS

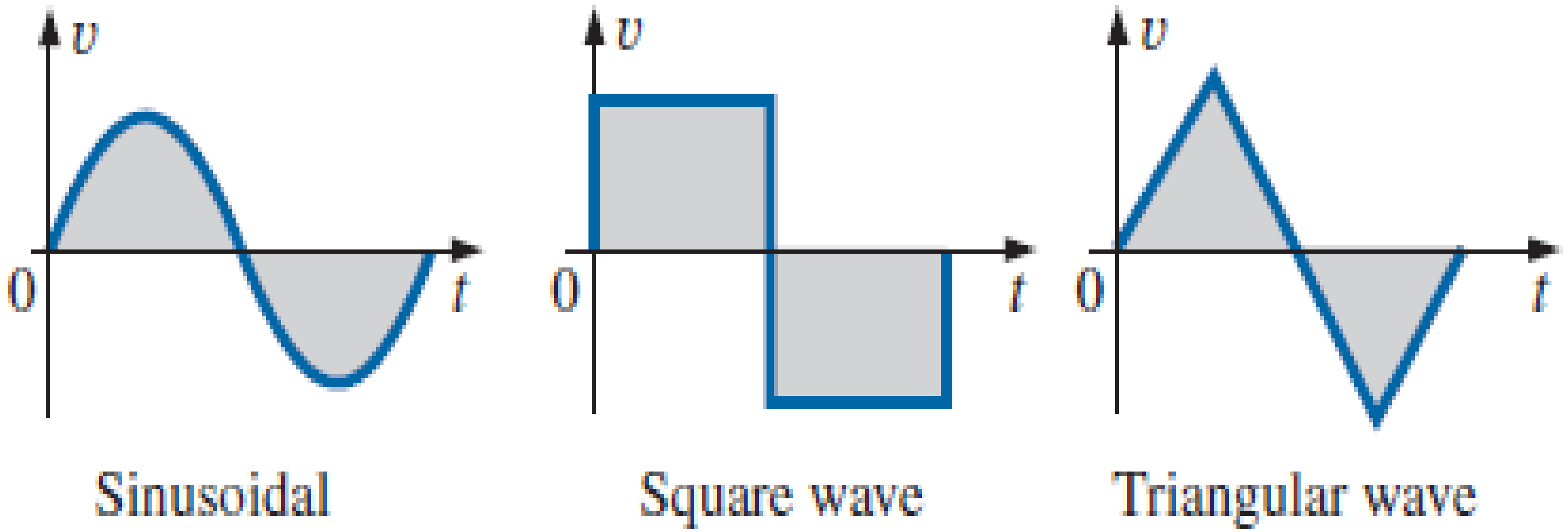
EFFECTIVE (rms) VALUES

SINUSOIDAL ALTERNATING WAVEFORMS

The time-varying voltage that is commercially available in large quantities and is commonly called the *ac voltage*

ac is an abbreviation for *alternating current*

The term *alternating* indicates only that the waveform alternates between two prescribed levels in a set time sequence. To be absolutely correct, the term *sinusoidal, square-wave,* or *triangular* must also be applied.



13.2 – Sinusoidal ac Voltage Characteristics and Definitions

- Generation
  - An **ac generator** (or *alternator*) powered by water power, gas, or nuclear fusion is the primary component in the energy-conversion process.
  - The energy source turns a rotor (constructed of alternating magnetic poles) inside a set of windings housed in the stator (the stationary part of the dynamo) and will induce voltage across the windings of the stator.

$$e = N\frac{d\phi}{dt}$$

Voltage induced in a rotating loop

- It consists of a single loop of wire rotating about a fixed axis. The rotating part of this machine is called the rotor, and the stationary part is called the stator.

- The magnetic field for the machine is supplied by the magnetic north and south poles. The loop of rotor wire lies in a slot carved in a ferromagnetic core.

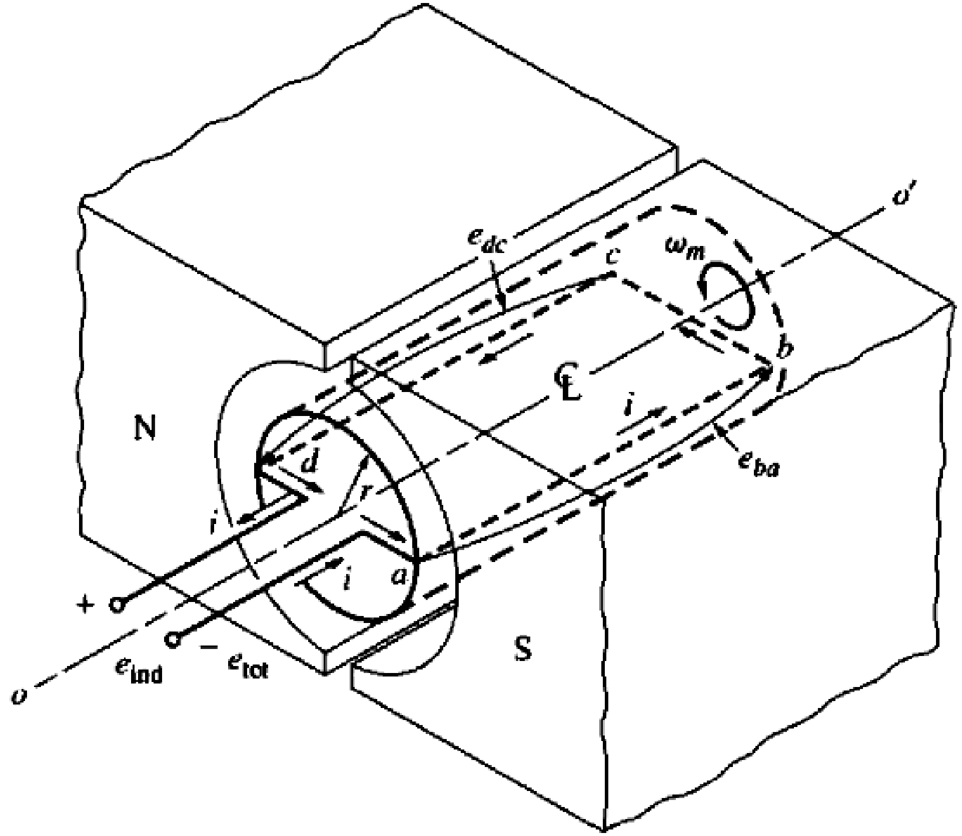
- The reluctance of air is much higher than the reluctance of the iron in the machine.

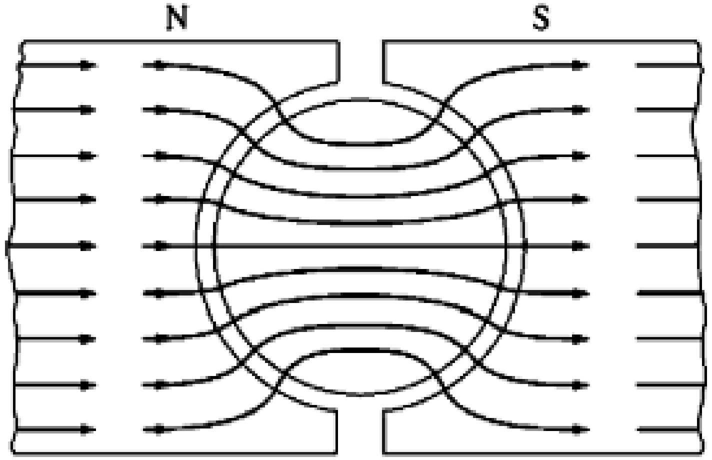
- To minimize the reluctance of the flux path through the machine, the magnetic flux must take the shortest possible path through the air between the pole face and the rotor surface.

- If the rotor of this machine is rotated, a voltage will be induced in the wire loop. The voltage on each segment is

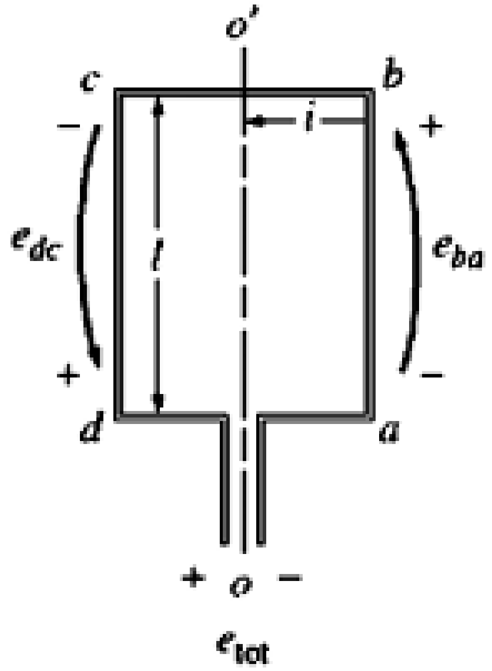
$$e_{ind} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l}$$

Voltage induced in a rotating loop





Voltage induced in a rotating loop



1. *Segment ab.* In this segment, the velocity of the wire is tangential to the path of rotation. The magnetic field **B** points out perpendicular to the rotor surface everywhere under the pole face and is zero beyond the edges of the pole face. Under the pole face, velocity **v** is perpendicular to **B**, and the quantity **v** × **B** points into the page. Therefore, the induced voltage on the segment is

$$e_{ba} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l}$$

$$= \begin{cases} vBl & \text{positive into page} \quad \text{under the pole face} \\ 0 & \text{beyond the pole edges} \end{cases} \tag{8–1}$$

2. *Segment bc.* In this segment, the quantity **v** × **B** is either into or out of the page, while length **l** is in the plane of the page, so **v** × **B** is perpendicular to **l**. Therefore the voltage in segment *bc* will be zero:

$$e_{cb} = 0 \tag{8–2}$$

Voltage induced in a rotating loop

3. *Segment cd.* In this segment, the velocity of the wire is tangential to the path of rotation. The magnetic field **B** points *in* perpendicular to the rotor surface everywhere under the pole face and is zero beyond the edges of the pole face. Under the pole face, velocity **v** is perpendicular to **B**, and the quantity **v** × **B** points out of the page. Therefore, the induced voltage on the segment is

$$
\begin{aligned}
e_{dc} &= (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \\
&= \begin{cases} vBl & \text{positive out of page} \quad \text{under the pole face} \\ 0 & \text{beyond the pole edges} \end{cases}
\end{aligned} \qquad (7\text{–}3)
$$

4. *Segment da.* Just as in segment *bc*, **v** × **B** is perpendicular to **l**. Therefore the voltage in this segment will be zero, too:

$$e_{ad} = 0 \qquad (7\text{–}4)$$

The total induced voltage on the loop e_{ind} is given by

$$e_{\text{ind}} = e_{ba} + e_{cb} + e_{dc} + e_{ad}$$

$$\boxed{e_{\text{ind}} = \begin{cases} 2vBl & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}} \qquad (7\text{–}5)$$

Voltage induced in a rotating loop

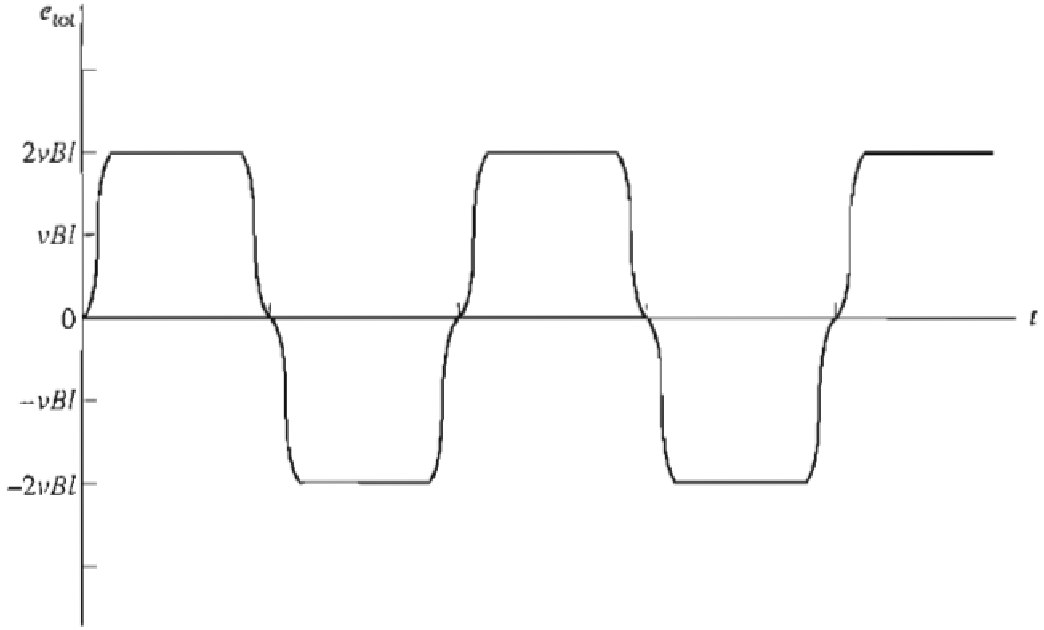


FIGURE 7–3
The output voltage of the loop.

Voltage induced in a rotating loop

$$e_{\text{ind}} = \begin{cases} 2r\omega_m Bl & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}$$

$$e_{\text{ind}} = \begin{cases} 2rlB\omega_m & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}$$

$$e_{\text{ind}} = \begin{cases} \frac{2}{\pi} A_P B\omega_m & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}$$

$$\boxed{e_{\text{ind}} = \begin{cases} \frac{2}{\pi} \phi\omega_m & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}}$$

Voltage induced in a rotating loop

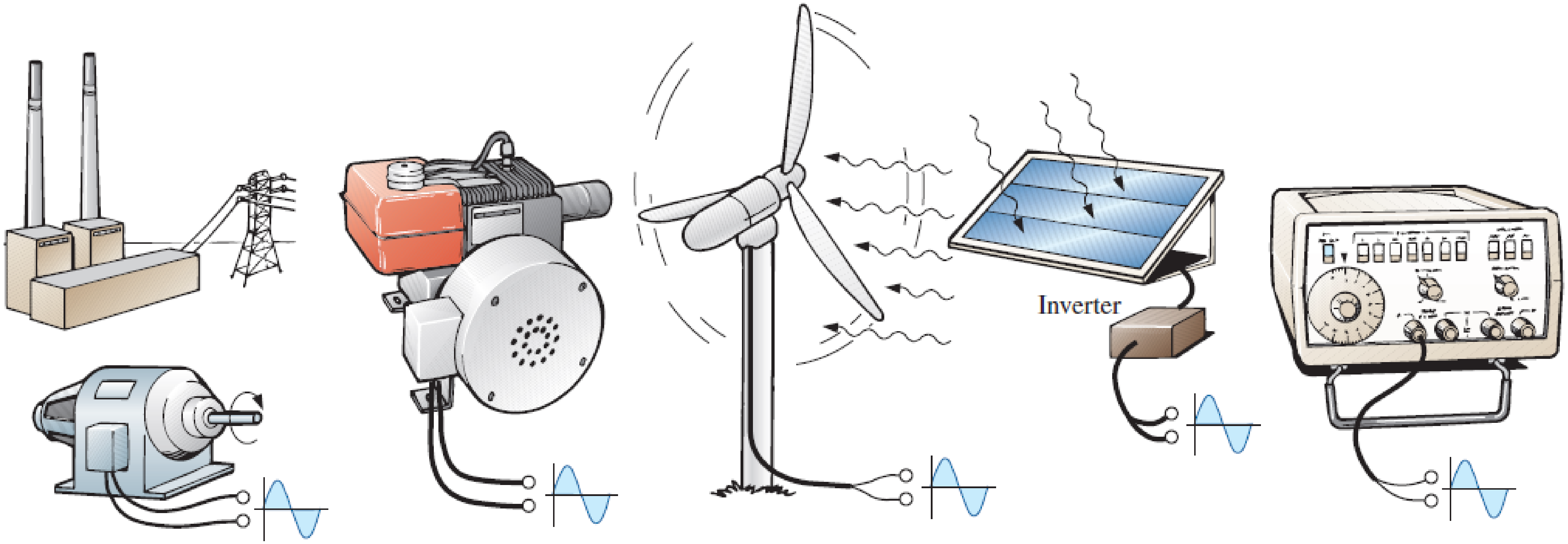
In general, the voltage in any real machine will depend on the same three factors:

- The flux in the machine
- The speed of rotation
- A constant representing the construction of the machine

Sinusoidal ac Voltage Characteristics and Definitions

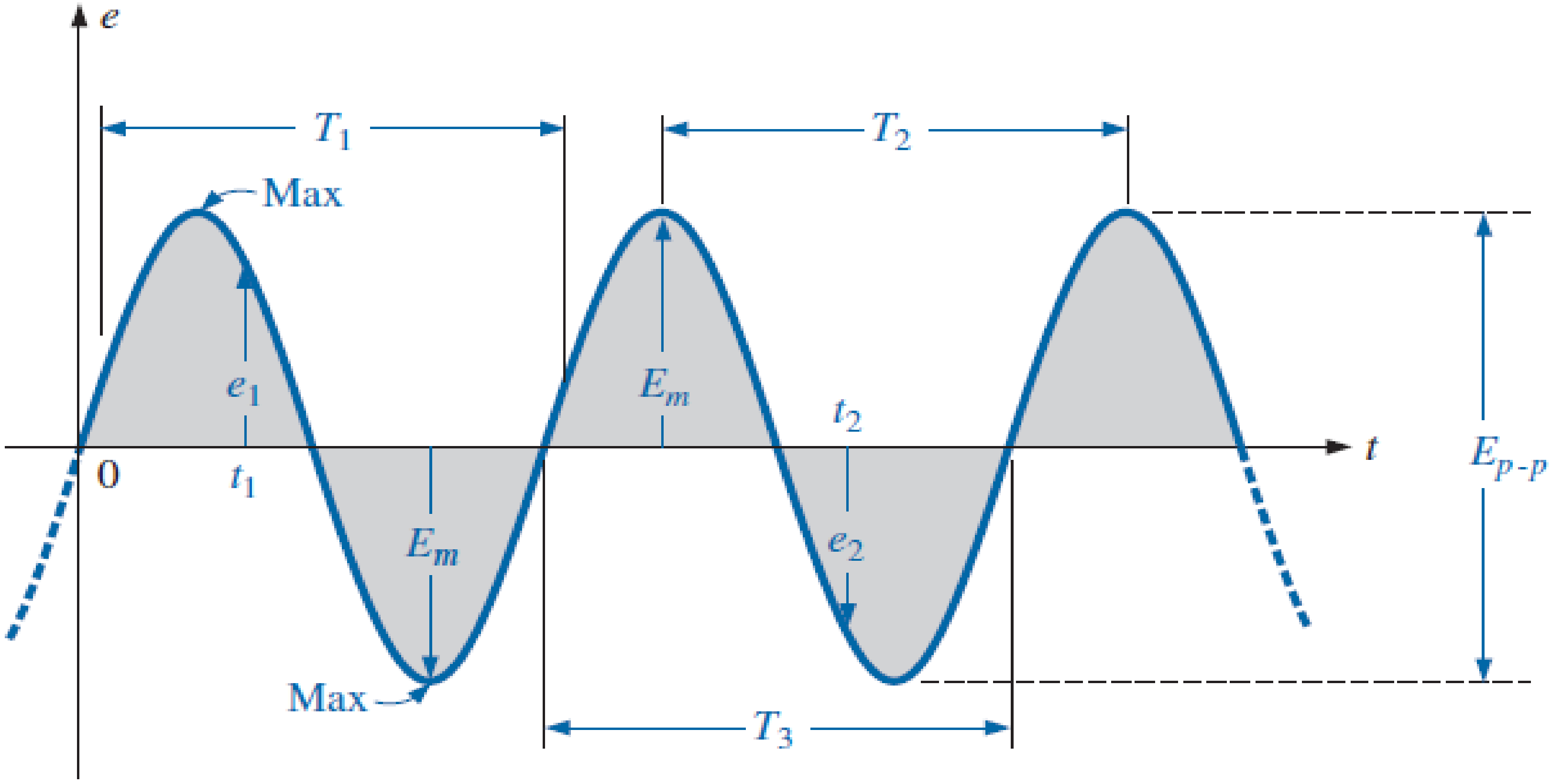
- Generation
  - Wind power and solar power energy are receiving increased interest from various districts of the world.
    - The turning propellers of the wind-power station are connected directly to the shaft of an ac generator.
  - Light energy in the form of photons can be absorbed by solar cells. Solar cells produce dc, which can be electronically converted to ac with an *inverter.*
  - A *function generator*, as used in the lab, can generate and control alternating waveforms.

SINUSOIDAL AC VOLTAGE CHARACTERISTICS AND DEFINITIONS



DEFINITIONS

Waveform: The path traced by a quantity, plotted as a function of some variable such as time, position, degrees, radians, temperature.

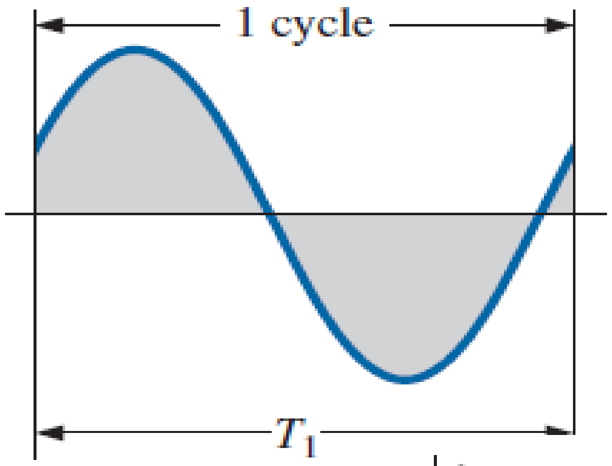


Instantaneous value: The magnitude of a waveform at any instant of time; denoted by lowercase letters (e_1, e_2).

Peak amplitude: The maximum value of a waveform as measured from its *average,* or *mean,* value, denoted by uppercase letters [such as E_m for sources of voltage and V_m for the voltage drop across a load]. the average value is zero volts.

Peak value: The maximum instantaneous value of a function as measured from the zero volt level. The peak amplitude and peak value are the same, since the average value of the function is zero volts.

Peak-to-peak value: Denoted by $E_{p\text{-}p}$ or $V_{p\text{-}p}$, the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.

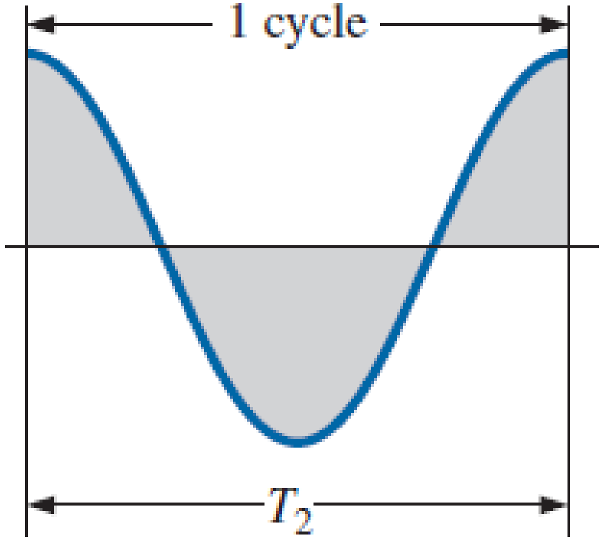


Periodic waveform: A waveform that continually repeats itself after the same time interval.

Period (T): The time of a periodic waveform.

Cycle: The portion of a waveform contained in one period of time.

The cycles within T_1, T_2,

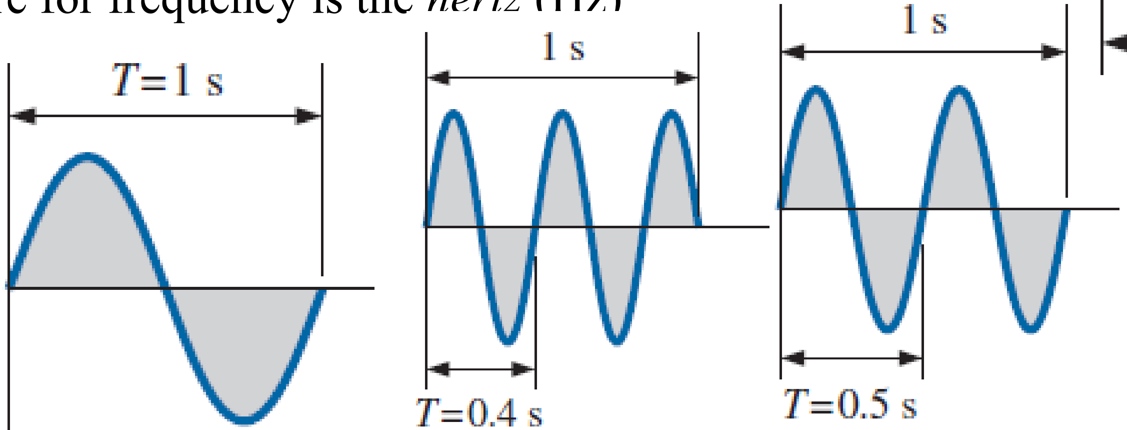


Frequency (f): The number of cycles that occur in 1 s. The frequency of the waveform is 1 cycle per second, 2½ cycles per second. If a waveform of similar shape had a period of 0.5 s, the frequency would be 2 cycles per second.

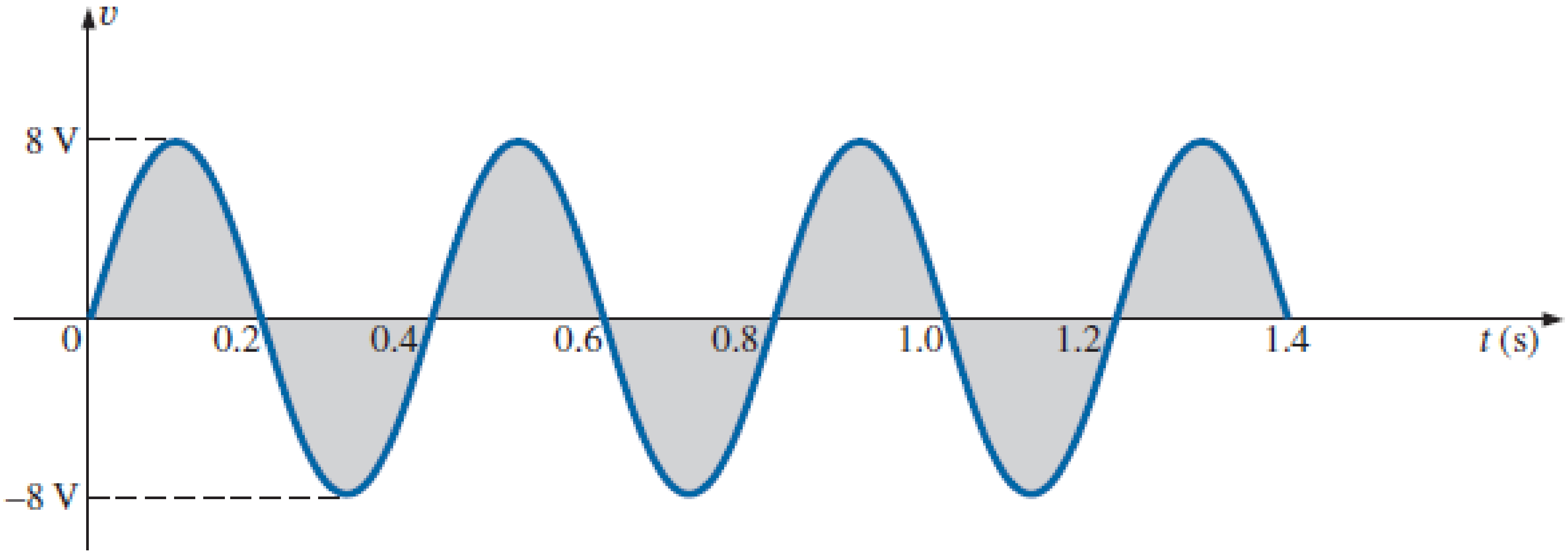
The unit of measure for frequency is the *hertz* (Hz)

$$\boxed{f = \frac{1}{T}}$$

$$\begin{aligned} f &= \text{Hz} \\ T &= \text{seconds (s)} \end{aligned}$$



EXAMPLE 1 For the sinusoidal waveform



a. What is the peak value?

b. What is the instantaneous value at 0.3 s and 0.6 s?

c. What is the peak-to-peak value of the waveform?

d. What is the period of the waveform?

e. How many cycles are shown?

f. What is the frequency of the waveform?

Solutions:

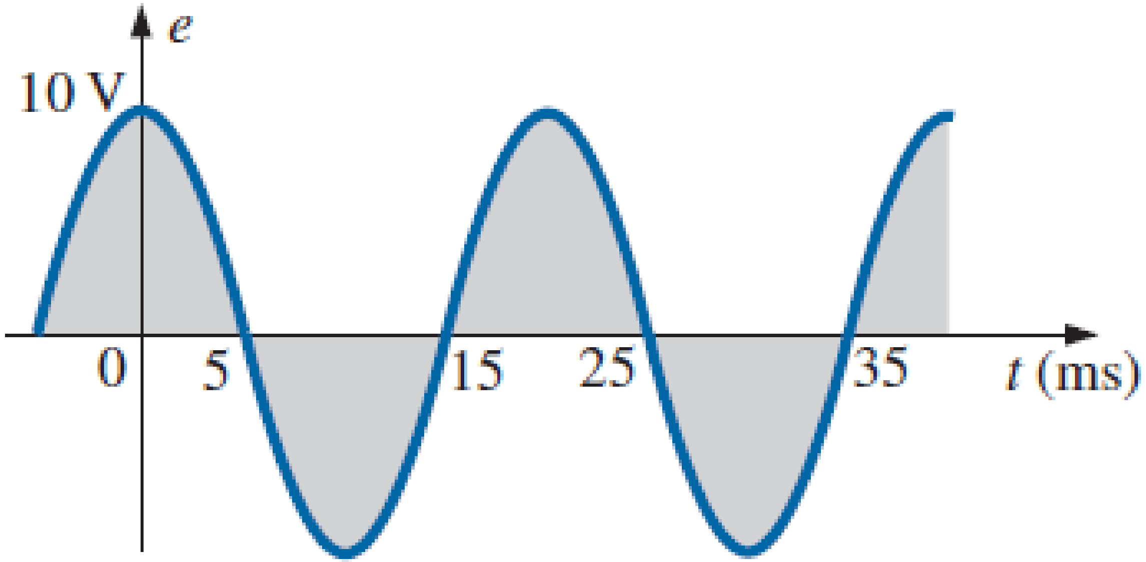
a. **8 V.**
b. At 0.3 s, **−8 V**; at 0.6 s, **0 V.**
c. **16 V.**
d. **0.4 s.**
e. **3.5 cycles.**
f. **2.5 cps, or 2.5 Hz.**

EXAMPLE 2 Find the period of periodic waveform with a frequency of

a. 60 Hz.

b. 1000 Hz.

EXAMPLE 3 Determine the frequency of the waveform



Solutions:

a. $T = \dfrac{1}{f} = \dfrac{1}{60\ \text{Hz}} \cong 0.01667\ \text{s}$ or $\mathbf{16.67\ ms}$

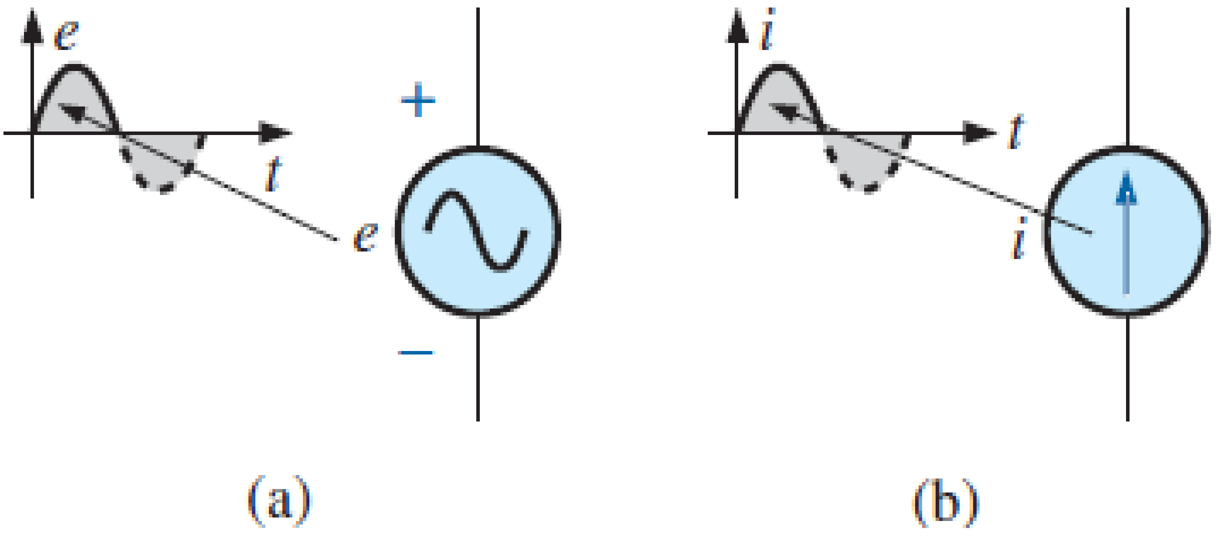
(a recurring value since 60 Hz is so prevalent)

b. $T = \dfrac{1}{f} = \dfrac{1}{1000\ \text{Hz}} = 10^{-3}\ \text{s} = \mathbf{1\ ms}$

Solution: From the figure, $T = (25\ \text{ms} - 5\ \text{ms})$ or $(35\ \text{ms} - 15\ \text{ms}) =$ 20 ms, and

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}\ \text{s}} = \mathbf{50\ Hz}$$

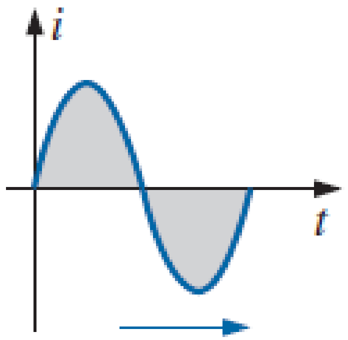
Polarities and Direction



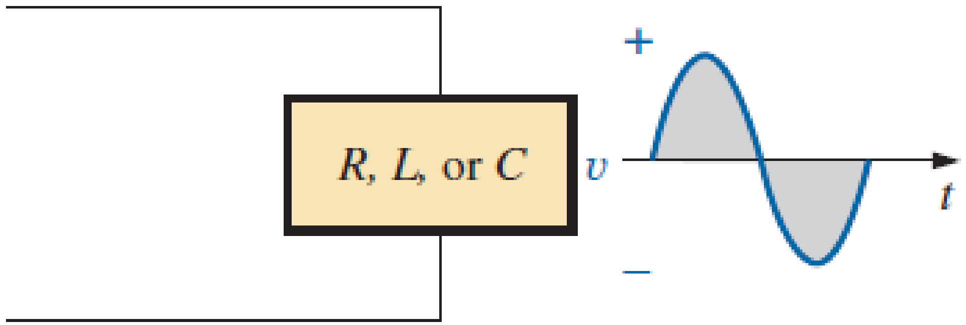
A positive sign is applied if the voltage is above the axis, For a current source, the direction in the symbol corresponds with the positive region of the waveform.

For any quantity that will not change with time, an uppercase letter such as V or I is used. For expressions that are time dependent or that represent a particular instant of time, a lowercase letter such as e or i is used.

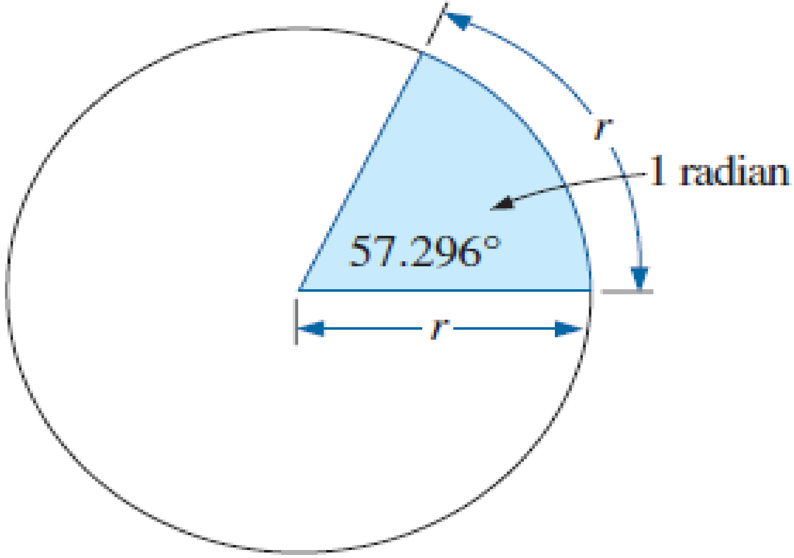
THE SINUSOIDAL WAVEFORM



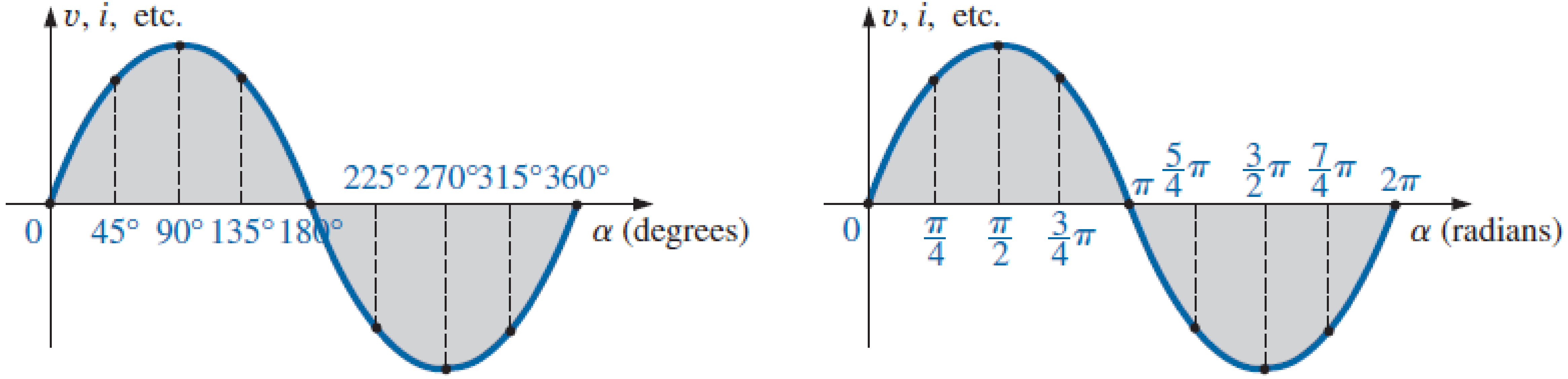
The sinusoidal waveform is the only alternating waveform whose shape is unaffected by the response characteristics of R, L, and C elements.



If the voltage across (or current through) a resistor, inductor, or capacitor is sinusoidal in nature, the resulting current (or voltage, respectively) for each will also have sinusoidal characteristics



The unit of measurement for the horizontal axis can be **time,** **degrees**, or **radians.**



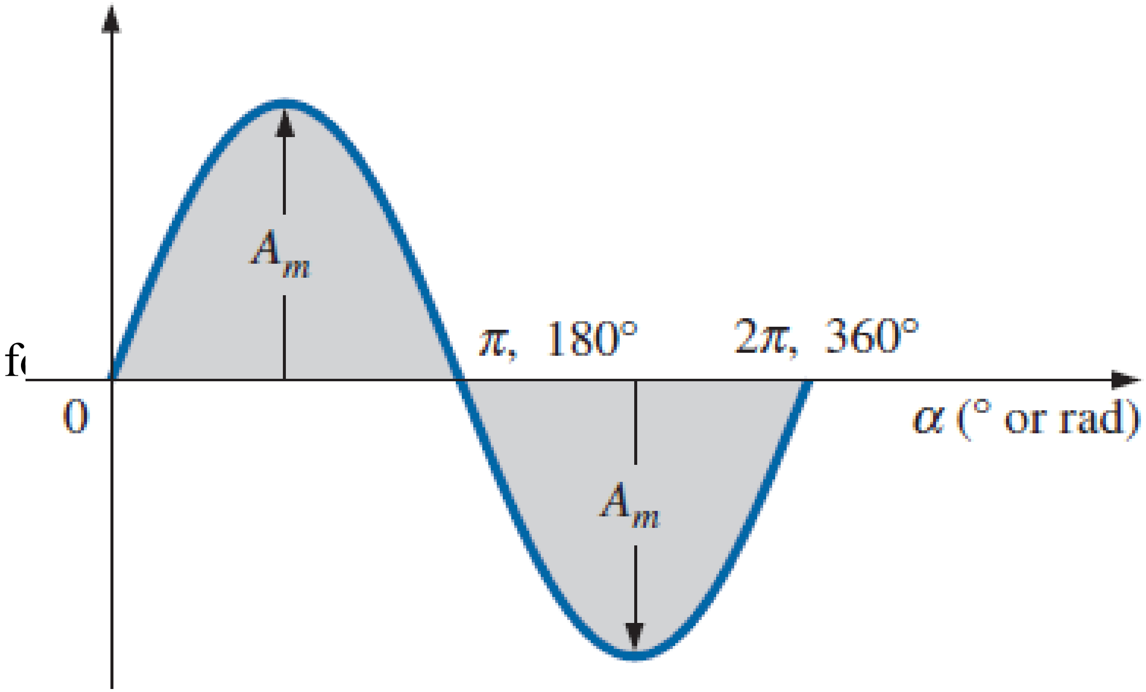
For comparison purposes, two sinusoidal voltages are plotted using degrees and radians as the units of measurement for the horizontal axis.

The sinusoidal waveform can be derived from the length of the *vertical projection* of a radius vector rotating in a uniform circular motion about a fixed point

GENERAL FORMAT FOR THE SINUSOIDAL VOLTAGE OR CURRENT

The basic mathematical format for the sinusoidal waveform is

$$\boxed{A_m \sin \omega t}$$



where A_m is the peak value of the waveform and a is the unit of measure for the horizontal axis

For electrical quantities such as current and voltage, the general format is

$$i = I_m \sin \omega t = I_m \sin \alpha$$
$$e = E_m \sin \omega t = E_m \sin \alpha$$

where the capital letters with the subscript *m* represent the amplitude, and the lowercase letters *i* and *e* represent the instantaneous value of current and voltage, respectively, at any time *t*.

The angle at which a particular voltage level is attained can be determined by rearranging the equation

$$e = E_m \sin \alpha \qquad \sin \alpha = \frac{e}{E_m}$$

$$\boxed{\alpha = \sin^{-1} \frac{e}{E_m}}$$

For a particular current level

$$\boxed{\alpha = \sin^{-1} \frac{i}{I_m}}$$

EXAMPLE 4 Given e=5sinα, determine e at α =40° and α =0.8π.

Solution: For $\alpha = 40°$,

$$e = 5 \sin 40° = 5(0.6428) = \mathbf{3.21\ V}$$

For $\alpha = 0.8\pi$,

$$\alpha\,(°) = \frac{180°}{\pi}(0.8\pi) = 144°$$

and

$$e = 5 \sin 144° = 5(0.5878) = \mathbf{2.94\ V}$$

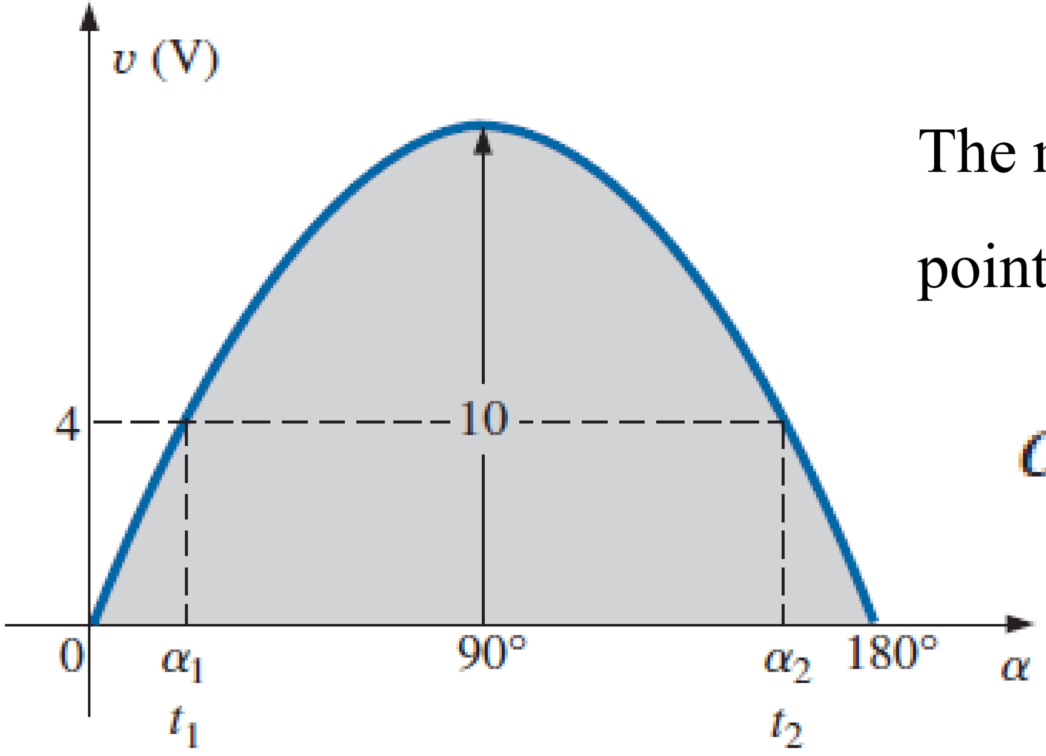
EXAMPLE 5

a. Determine the angle at which the magnitude of the sinusoidal function y=10sin377t is 4 V.

b. Determine the time at which the magnitude is attained.

Solutions:

(a) $$\alpha_1 = \sin^{-1}\frac{v}{E_m} = \sin^{-1}\frac{4\text{ V}}{10\text{ V}} = \sin^{-1} 0.4 = \mathbf{23.58^\circ}$$



The magnitude of 4 V (positive) will be attained at two points between 0° and 180°.

$$\alpha_2 = 180^\circ - 23.578^\circ = \mathbf{156.42^\circ}$$

$\alpha = \omega t$, and so $t = \alpha/\omega$

$$\alpha\,(\text{rad}) = \frac{\pi}{180^\circ}(23.578^\circ) = 0.412 \text{ rad} \qquad \alpha\,(\text{rad}) = \frac{\pi}{180^\circ}(156.422^\circ) = 2.73 \text{ rad}$$

$$t_1 = \frac{\alpha}{\omega} = \frac{0.412 \text{ rad}}{377 \text{ rad/s}} = \mathbf{1.09\ ms} \qquad t_2 = \frac{\alpha}{\omega} = \frac{2.73 \text{ rad}}{377 \text{ rad/s}} = \mathbf{7.24\ ms}$$

EXAMPLE 6 Given $i = 6\text{x}10^{-3}\sin 1000t$, determine i at t=2 ms.

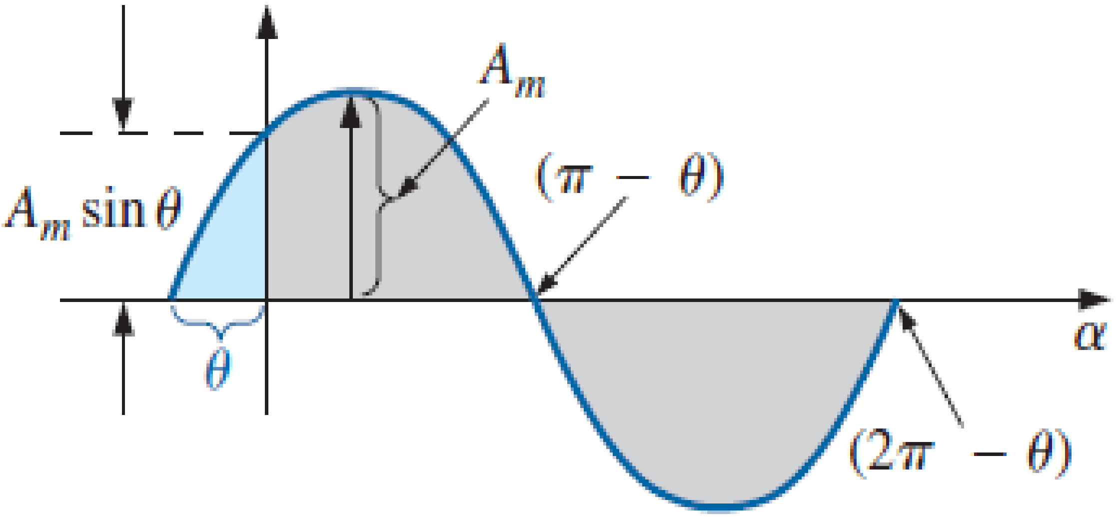
Solution:

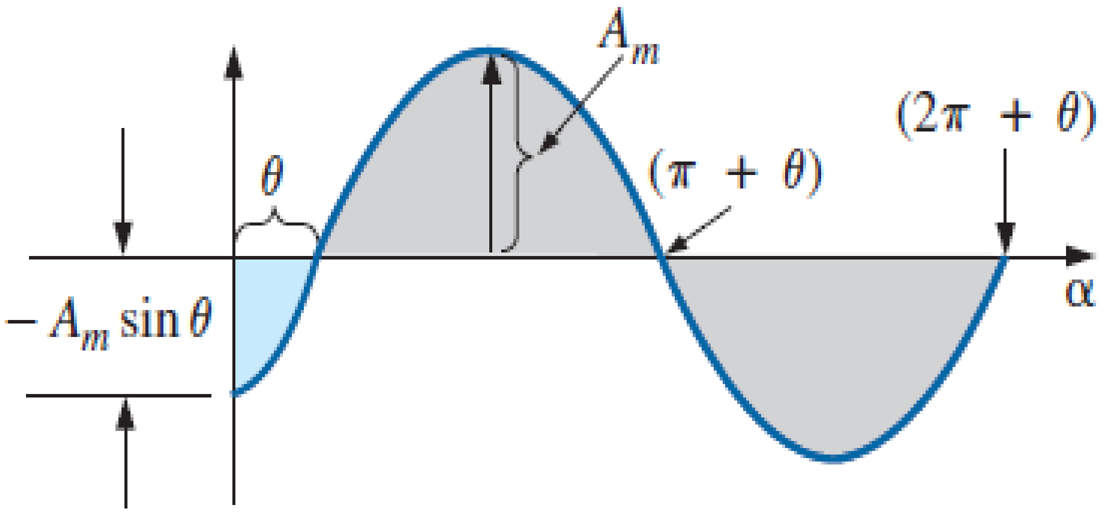
$$\alpha = \omega t = 1000t = (1000 \text{ rad/s})(2 \times 10^{-3} \text{ s}) = 2 \text{ rad}$$

$$\alpha\,(^\circ) = \frac{180^\circ}{\pi \text{ rad}}(2 \text{ rad}) = 114.59^\circ$$

$$i = (6 \times 10^{-3})(\sin 114.59^\circ) = (6 \text{ mA})(0.9093) = \mathbf{5.46\ mA}$$

PHASE RELATIONS





If the waveform is shifted to the right or left of 0°, the expression becomes

$$\boxed{A_m \sin(\omega t \pm \theta)}$$

where θ is the angle in degrees or radians that the waveform has been shifted.

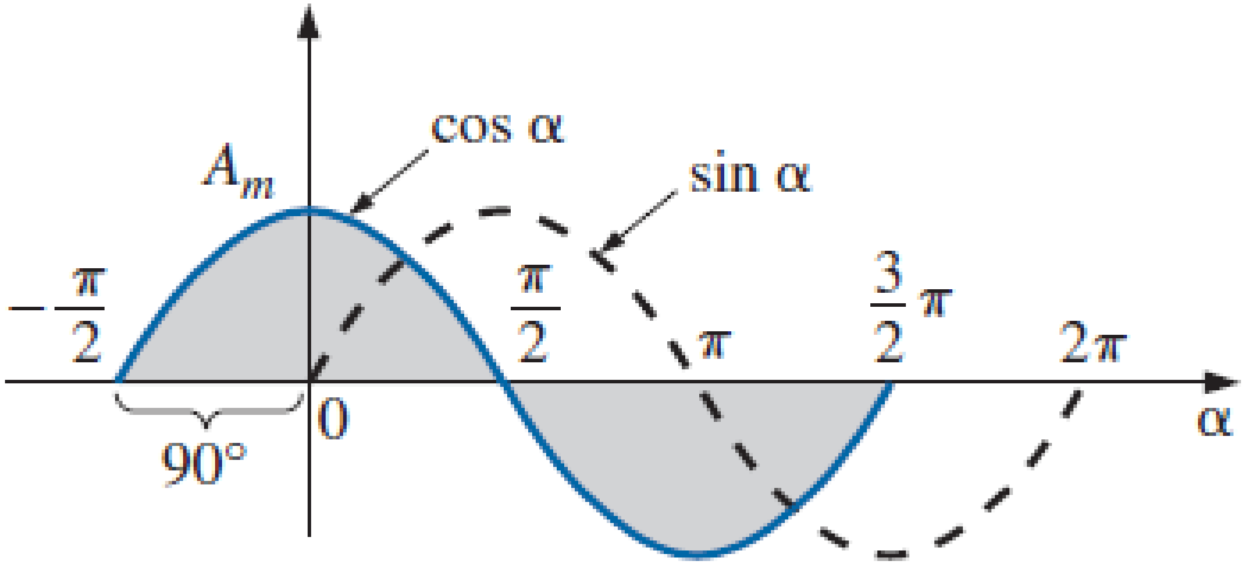
If the waveform passes through the horizontal axis with a *positive going* (increasing with time) slope *before* 0°, the expression is

$$\boxed{A_m \sin(\omega t + \theta)}$$

If the waveform passes through the horizontal axis with a positive-going slope *after* 0°, the expression is

$$\boxed{A_m \sin(\omega t - \theta)}$$

The terms **leading** and **lagging** are used to indicate the relationship between two sinusoidal waveforms of the *same frequency* plotted on the same set of axes



The **cosine curve is said to *lead* the sine curve by 90°,** and the **sine curve is said to *lag* the cosine curve by 90°.** The 90° is referred to as the phase angle between the two waveforms.

The phase angle between the two waveforms is measured between those two points on the horizontal axis through which each passes with the *same slope.*

If both waveforms cross the axis at the same point with the same slope, they are *in phase.*

The **phase relationship** between two waveforms indicates which one leads or lags the other, and by how many degrees or radians.

EXAMPLE 7 What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. $v = 10\sin(\omega t + 30^\circ)$

$i = 5\sin(\omega t + 70^\circ)$

b. $i = 15\sin(\omega t + 60^\circ)$

$v = 10\sin(\omega t - 20^\circ)$

c. $i = 2\cos(\omega t + 10^\circ)$

$v = 3\sin(\omega t - 10^\circ)$

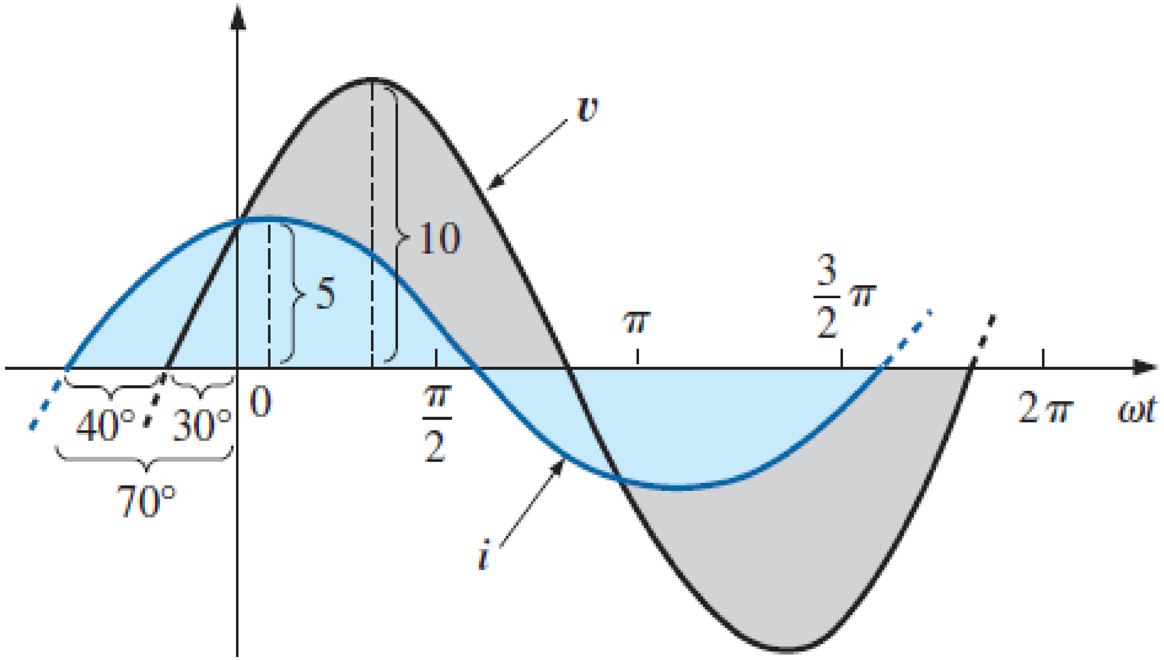
d. $i = -\sin(\omega t + 30^\circ)$

$v = 2\sin(\omega t + 10^\circ)$

e. $i = -2\cos(\omega t - 60^\circ)$

$v = 3\sin(\omega t - 150^\circ)$

(a) *i* leads *v* by 40°, or *v* lags *i* by 40°



(b) *i* leads *v* by 80°, or *v* lags *i* by 80°.