Digital Image Processing

Image Restoration:
Noise Removal

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Contents

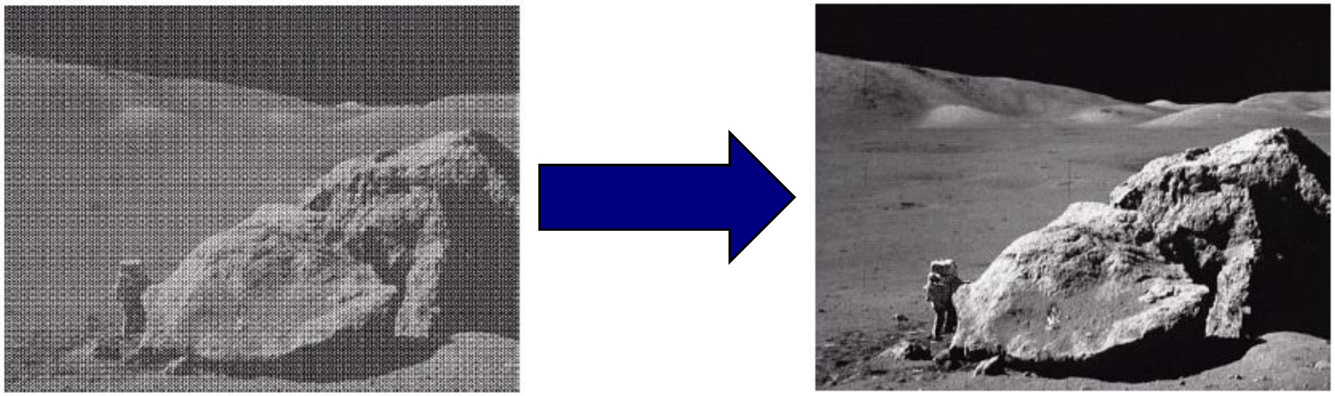
In this lecture we will look at image restoration techniques used for noise removal

- What is image restoration?
- Noise and images
- Noise models
- Noise removal using spatial domain filtering
- Periodic noise
- Noise removal using frequency domain filtering

What is Image Restoration?

Image restoration attempts to restore images that have been degraded

- Identify the degradation process and attempt to reverse it
- Similar to image enhancement, but more objective



Noise and Images

The sources of noise in digital images arise during image acquisition (digitization) and transmission

- Imaging sensors can be affected by ambient(<u>চতুরপার্শ্ববর্তী</u>) conditions
- Interference(<u>প্রতিবন্ধক</u>) can be added to an image during transmission

A model of the image degradation/restoration process

- Degradation Model

Consider an image $f(x,y)$ .Let the image undergo some degradation. and resultant image be described by $g(x,y)$.

The degradation may be of form of motion blurring,out of focus optics ,light scattering etc.

The Degradation process is modelled linear filter $h(x,y)$ and $h(x,y)$ can be called as degradation filter/operator.

Noise is modelled as additive noise and is added to output of degradation filter.

Noise could also have been added at the input of degradation filter . Since we are working with linear filter ,the noise term will be additive at the output of the filter.

Noise is modelled as sample function of random process. At the output of the filter noise process maintains its form .

Thus noise in directly included at the output of the degrdation filter.

The degraded and noisy image is represented by $g(x,y)$.

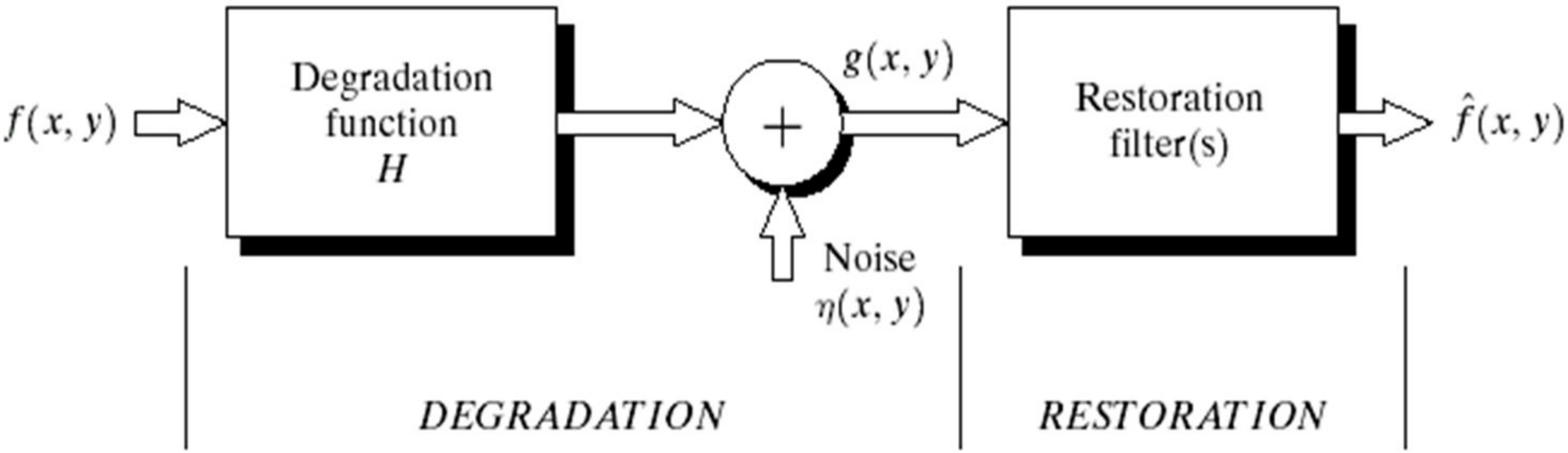
- Restoration Model

The objective is to find the estimate of orginal image $f(x,y)$. This is denoted by $fr(x,y)$.

The degraded image $g(x,y)$ is passed through the restoration filter $l(x,y)$ which produces the estimate of original signal $fr(x,y)$.

The restoration filter is designed based on the knowledge about the degradation model and noise model.

The error in estimation will be reduced the more accurately we know about the degradation and noise or if assumed degradation and noise model fits the actual scenario better.



$f(x, y)$
Degradation function H
$g(x, y)$
Noise $\eta(x, y)$
Restoration filter(s)
$\hat{f}(x, y)$
DEGRADATION
RESTORATION

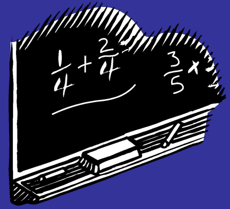
Noise Model

We can consider a noisy image to be modelled as follows:

$$g(x, y) = f(x, y) + \eta(x, y)$$

where $f(x, y)$ is the original image pixel, $\eta(x, y)$ is the noise term and $g(x, y)$ is the resulting noisy pixel

If we can estimate the model the noise in an image is based on this will help us to figure out how to restore the image



Noise Corruption Example

Original Image

x

54	52	57	55	56	52	51
50	49	51	50	52	53	58
51	51	52	52	56	57	60
48	50	51	49	53	59	63
49	51	52	55	58	64	67
148	154	157	160	163	167	170
151	155	159	162	165	169	172

y

Image $f(x, y)$

Noisy Image

x

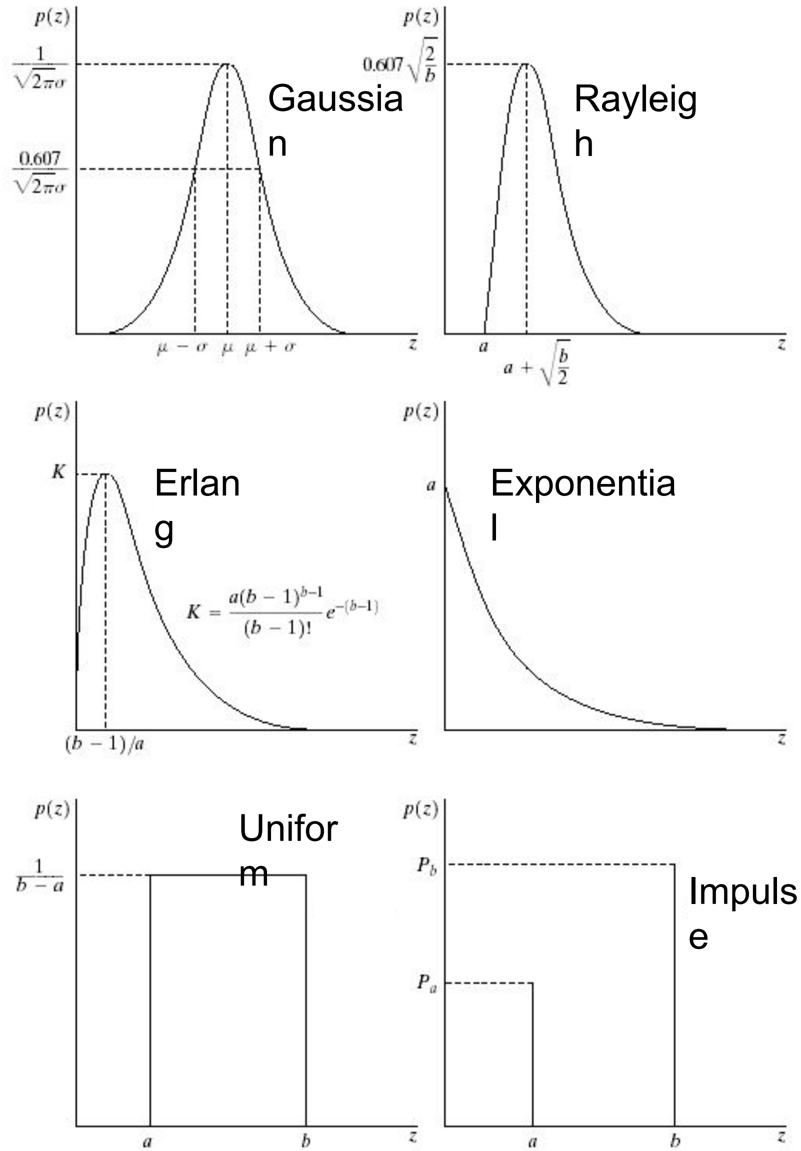
y

Image $f(x, y)$

Noise Models

There are many different models for the image noise term $\eta(x, y)$:

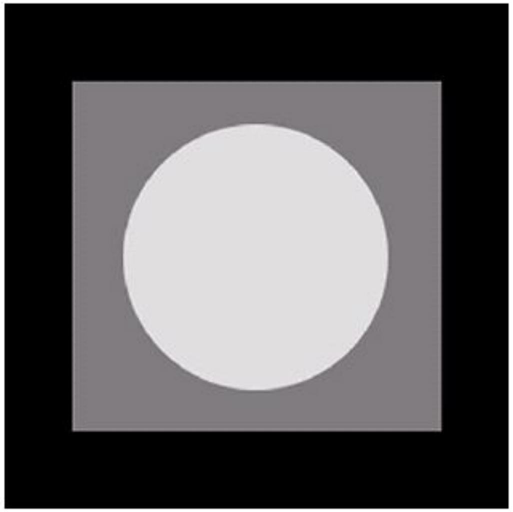
- Gaussian(normal noise)
  - Most common model
- Rayleigh
- Erlang(gamma)
- Exponential
- Uniform
- Impulse
  - *Salt and pepper* noise



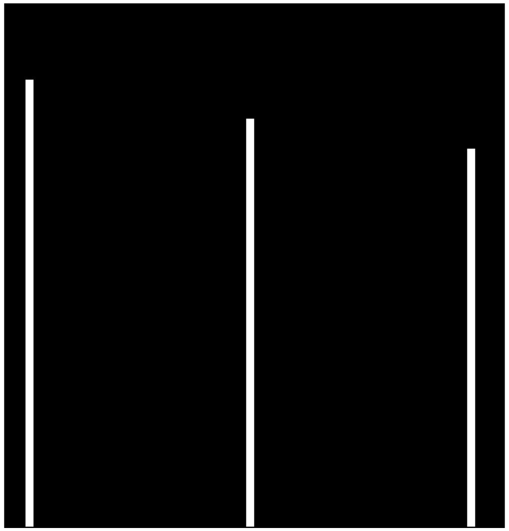
Noise Example

The test pattern to the right is ideal for demonstrating the addition of noise

The following slides will show the result of adding noise based on various models to this image

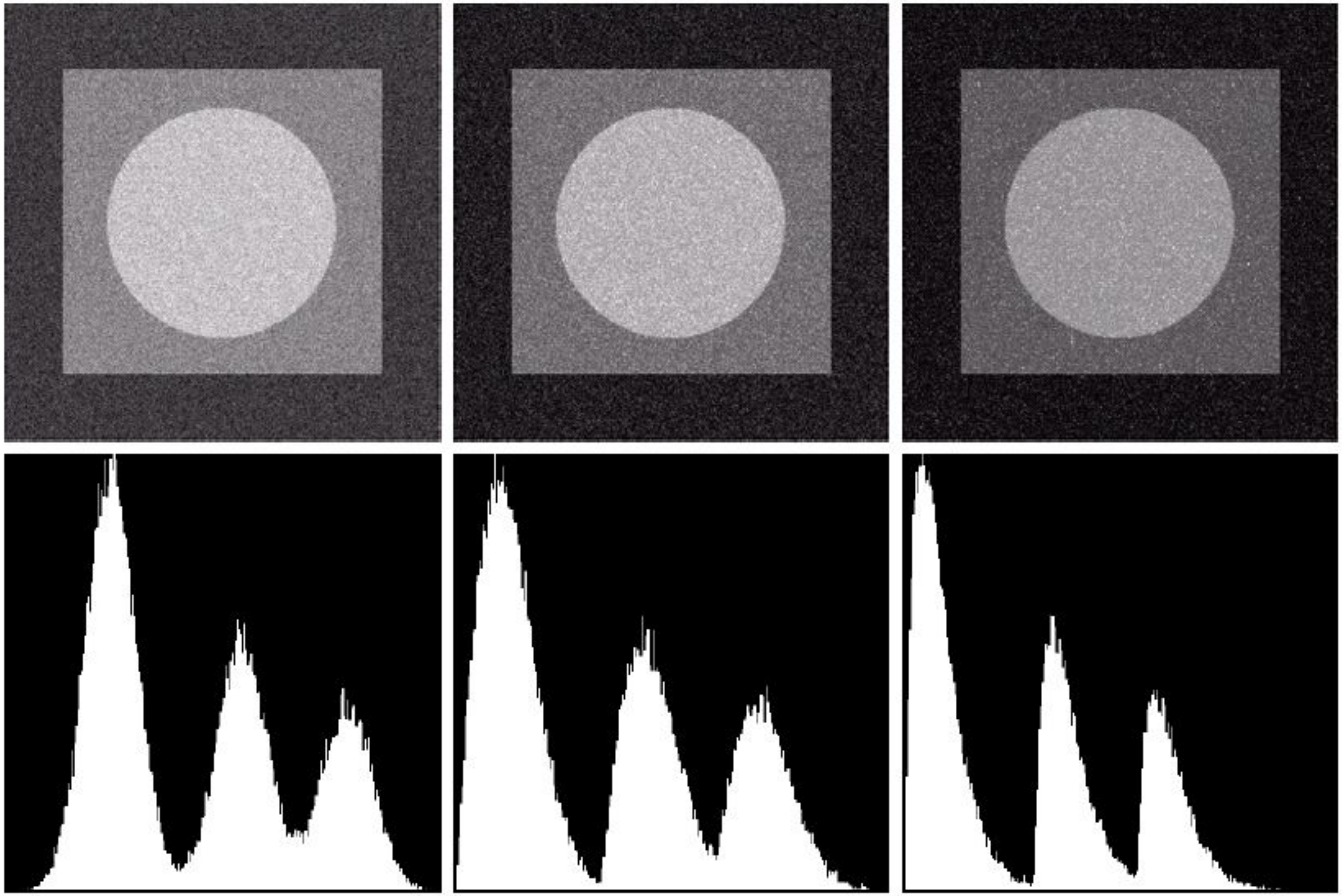


Image



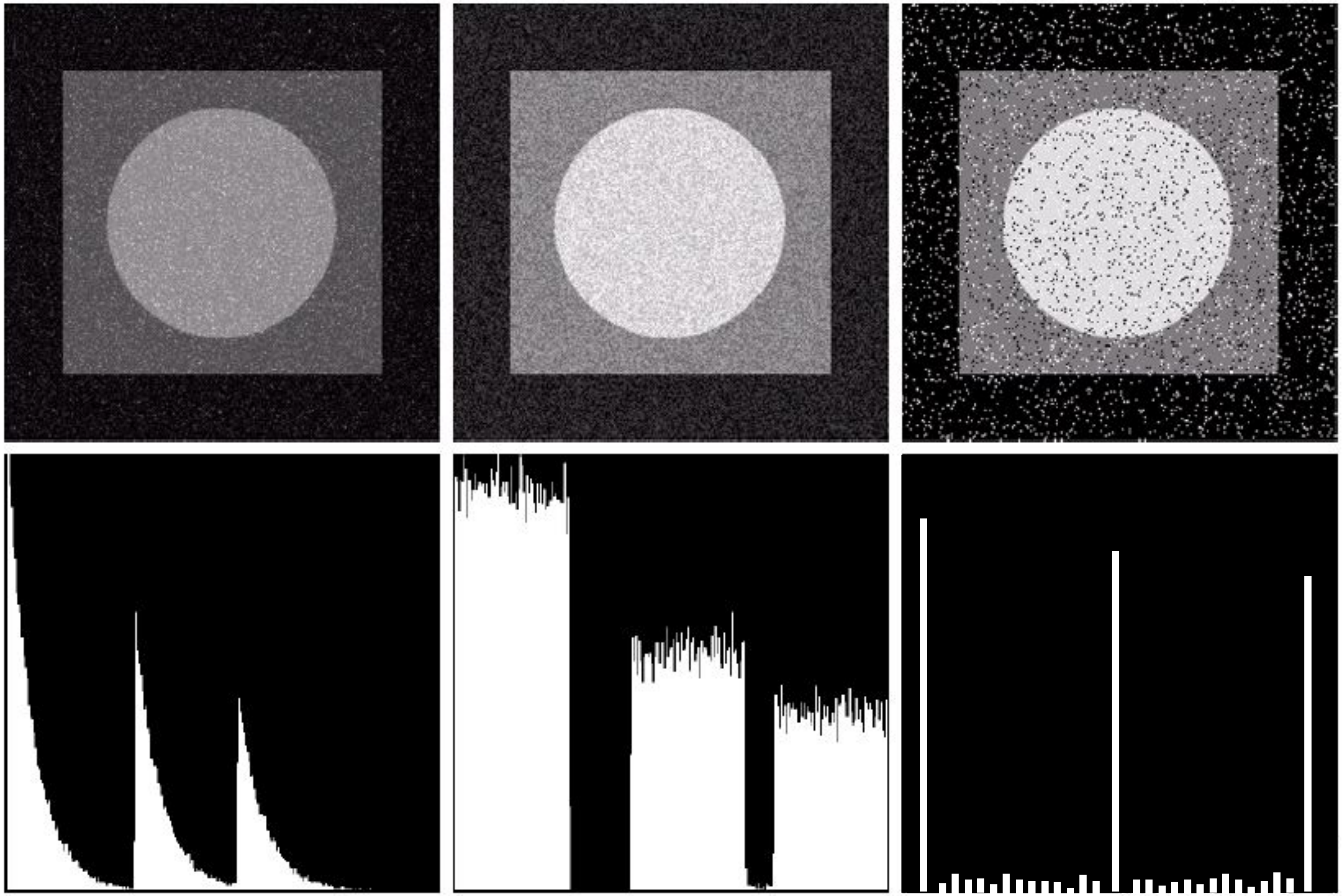
Histogram

Noise Example (cont…)



Gaussian Rayleigh Erlang

Noise Example (cont…)



Exponential Uniform Impulse

Filtering to Remove Noise

We can use spatial filters of different kinds to remove different kinds of noise

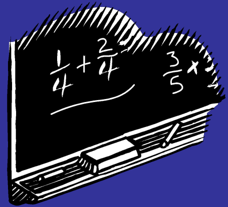
The *arithmetic mean* filter is a very simple one and is calculated as follows:

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t)$$

$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$

This is implemented as the simple smoothing filter

Blurs the image to remove noise



Noise Removal Example

Original Image

x

54	52	57	55	56	52	51
50	49	51	50	52	53	58
51	204	52	52	0	57	60
48	50	51	49	53	59	63
49	51	52	55	58	64	67
148	154	157	160	163	167	170
151	155	159	162	165	169	172

y

Image f (x, y)

Filtered Image

x

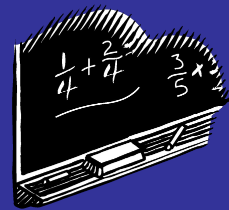
y

Image f (x, y)

Other Means

There are different kinds of mean filters all of which exhibit slightly different behaviour:

- Geometric Mean
- Harmonic Mean
- Contraharmonic Mean



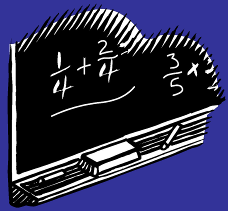
Other Means (cont…)

There are other variants on the mean which can give different performance

Geometric Mean:

$$\hat{f}(x, y) = \left[\prod_{(s,t) \in S_{xy}} g(s, t) \right]^{\frac{1}{mn}}$$

Achieves similar smoothing to the arithmetic mean, but tends to lose less image detail



Noise Removal Example

Original Image

x

54	52	57	55	56	52	51
50	49	51	50	52	53	58
51	204	52	52	0	57	60
48	50	51	49	53	59	63
49	51	52	55	58	64	67
148	154	157	160	163	167	170
151	155	159	162	165	169	172

y

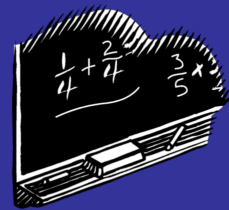
Image f (x, y)

Filtered Image

x

y

Image f (x, y)



Other Means (cont…)

Harmonic Mean:

$$\hat{f}(x, y) = \frac{mn}{\displaystyle\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

Works well for salt noise, but fails for pepper noise

Also does well for other kinds of noise such as Gaussian noise

Noise Corruption Example

Original Image

54	52	57	55	56	52	51
50	49	51	50	52	53	58
51	204	52	52	0	57	60
48	50	51	49	53	59	63
49	51	52	55	58	64	67
50	54	57	60	63	67	70
51	55	59	62	65	69	72

x

y

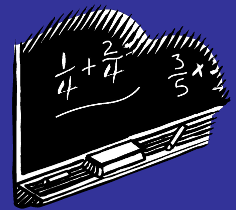
Image $f(x, y)$

Filtered Image

x

y

Image $f(x, y)$



Other Means (cont…)

Contraharmonic Mean:

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Q is the *order* of the filter and adjusting its value changes the filter's behaviour

Positive values of Q eliminate pepper noise

Negative values of Q eliminate salt noise

Noise Corruption Example

Original Image

x

54	52	57	55	56	52	51
50	49	51	50	52	53	58
51	204	52	52	0	57	60
48	50	51	49	53	59	63
49	51	52	55	58	64	67
50	54	57	60	63	67	70
51	55	59	62	65	69	72

Image f (x, y)

y

Filtered Image

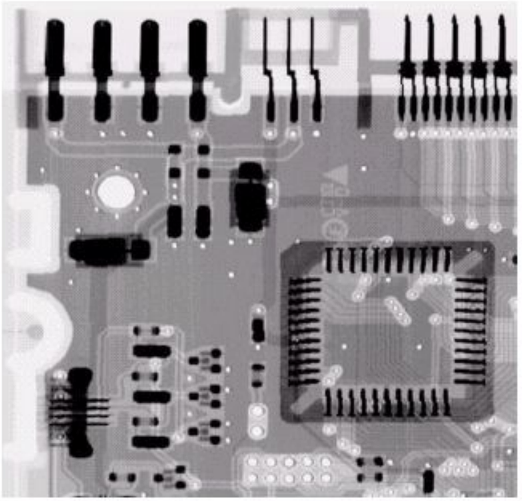
x

Image f (x, y)

y

Noise Removal Examples

Original Image



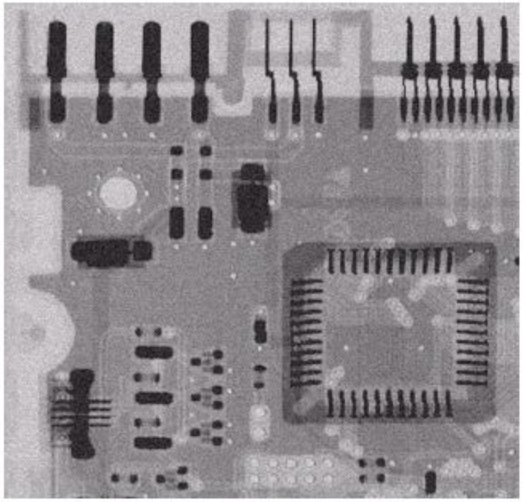
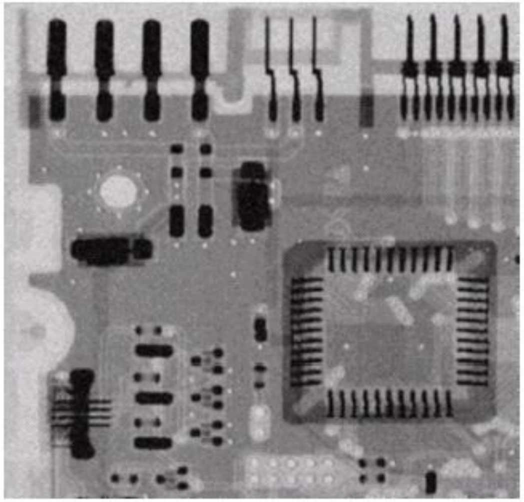
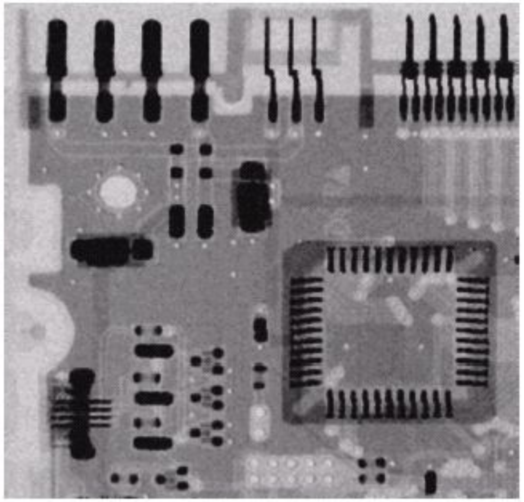


Image Corrupted By Gaussian Noise

After A 3*3 Arithmetic Mean Filter

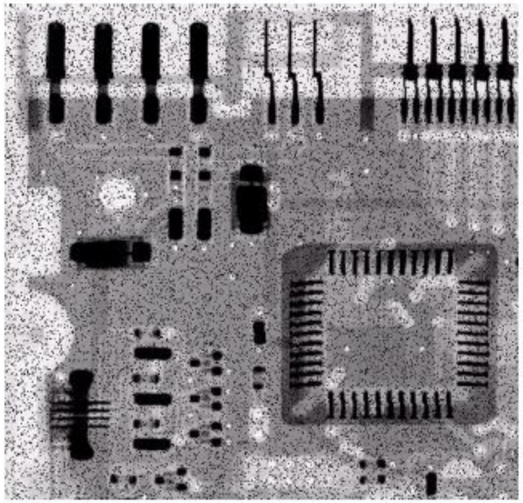




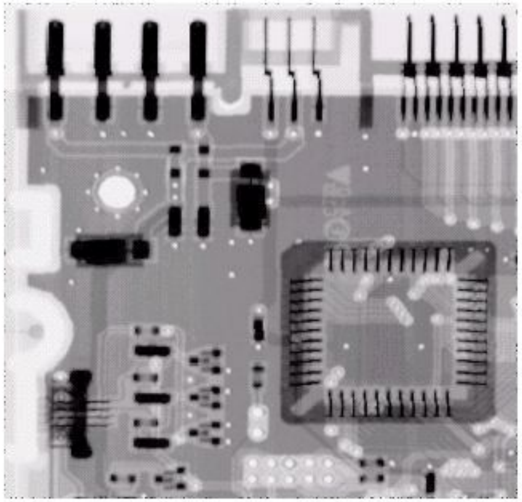
After A 3*3 Geometric Mean Filter

Noise Removal Examples (cont…)

Image Corrupted By Pepper Noise



Result of Filtering Above With 3*3 Contraharmonic Q=1.5



Noise Removal Examples (cont…)

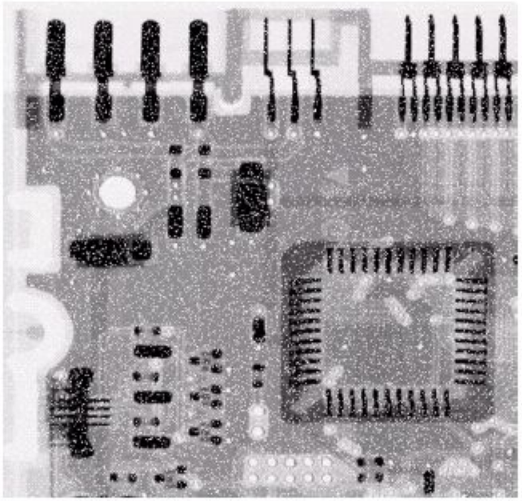
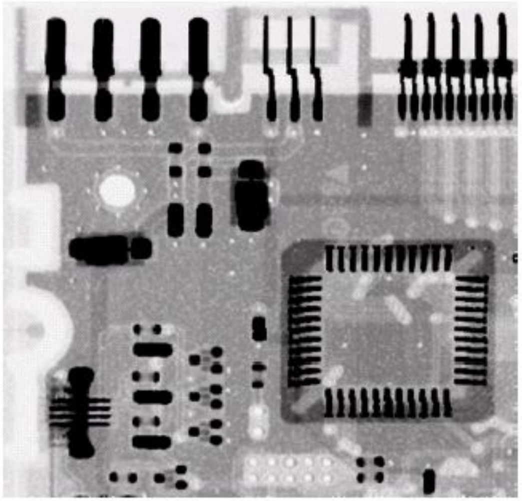


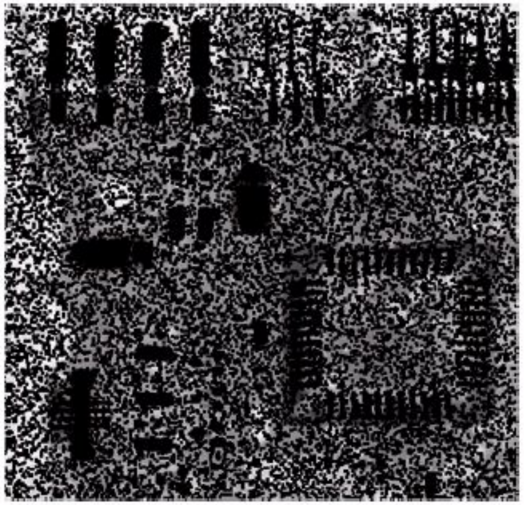
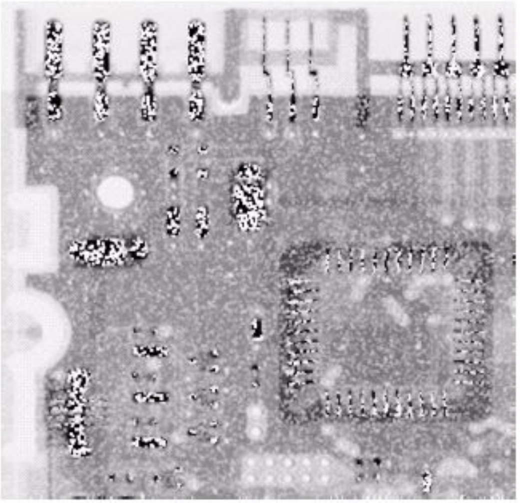
Image Corrupted By Salt Noise



Result of Filtering Above With 3*3 Contraharmonic Q=-1.5

Contraharmonic Filter: Here Be Dragons

Choosing the wrong value for Q when using the contraharmonic filter can have drastic results

Order Statistics Filters

Spatial filters that are based on ordering the pixel values that make up the neighbourhood operated on by the filter

Useful spatial filters include

– Median filter

– Max and min filter

– Midpoint filter

– Alpha trimmed(ছাঁটা) mean filter

Median Filter

Median Filter:

$$\hat{f}(x, y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

Excellent at noise removal, without the smoothing effects that can occur with other smoothing filters

Particularly good when salt and pepper noise is present

Noise Corruption Example

Original Image

x

54	52	57	55	56	52	51
50	49	51	50	52	53	58
51	204	52	52	0	57	60
48	50	51	49	53	59	63
49	51	52	55	58	64	67
50	54	57	60	63	67	70
51	55	59	62	65	69	72

Image f (x, y)

y

Filtered Image

x

Image f (x, y)

y

Max and Min Filter

Max Filter:

$$\hat{f}(x, y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\}$$

Min Filter:

$$\hat{f}(x, y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$

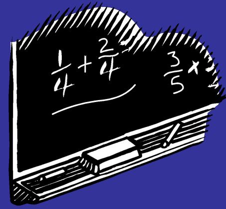
Max filter is good for pepper noise and min is good for salt noise

- Max filter→ Replace the value of a pixel by the maximum of the gray levels (the brightest point) in the neighborhood of that pixel
- Min filter→ Replace the value of a pixel by the minimum of the gray levels (the darkest point) in the neighborhood of that pixel

$$\hat{f}(x, y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\}$$

$$\hat{f}(x, y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$

- – Max filter also known as 100th percentile filter
- – Min filter also known as zeroth percentile filter
- – Max filter helps in removing pepper noise
- – Min filter helps in removing salt noise



Noise Corruption Example

Original Image

x

54	52	57	55	56	52	51
50	49	51	50	52	53	58
51	204	52	52	0	57	60
48	50	51	49	53	59	63
49	51	52	55	58	64	67
50	54	57	60	63	67	70
51	55	59	62	65	69	72

Image f (x, y)

y

Filtered Image

x

Image f (x, y)

y

Midpoint Filter

Midpoint Filter:

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

Good for random Gaussian and uniform noise

Noise Corruption Example

Original Image

x

54	52	57	55	56	52	51
50	49	51	50	52	53	58
51	204	52	52	0	57	60
48	50	51	49	53	59	63
49	51	52	55	58	64	67
50	54	57	60	63	67	70
51	55	59	62	65	69	72

Image f (x, y)

y

Filtered Image

x

Image f (x, y)

y

Alpha-Trimmed Mean Filter

Alpha-Trimmed Mean Filter:

$$\hat{f}(x, y) = \frac{1}{mn - d} \sum_{(s,t) \in S_{xy}} g_r(s, t)$$

We can delete the $d/2$ lowest and $d/2$ highest grey levels

So $g_r(s, t)$ represents the remaining $mn - d$ pixels

- Filter's output→ average of gray levels of the remaining (mn–d) pixels ($g_r(s,t)$) in the mask after removing the d/2 lowest and the d/2 highest gray levels in S_{xy}
- $0 \leq d \leq (mn-1)$
- d=0 → arithmetic mean filter
- d=(mn-1)/2 → median filter
- Alpha-trimmed filter can be used to solve the multi-type noise problem (e.g., combination of salt-and pepper and Gaussian)

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

- Uses another combination of order statistics and averaging

- Average of the pixel values closest to the median, after the D lowest and the D highest values in an ordered set have been excluded.

Noise Corruption Example

Original Image

x

54	52	57	55	56	52	51
50	49	51	50	52	53	58
51	204	52	52	0	57	60
48	50	51	49	53	59	63
49	51	52	55	58	64	67
50	54	57	60	63	67	70
51	55	59	62	65	69	72

y

Image f (x, y)

Filtered Image

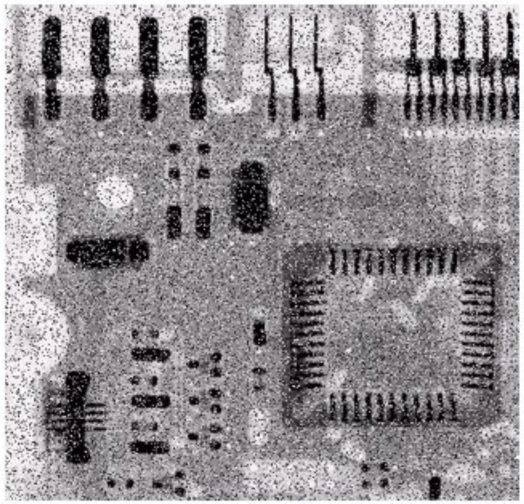
x

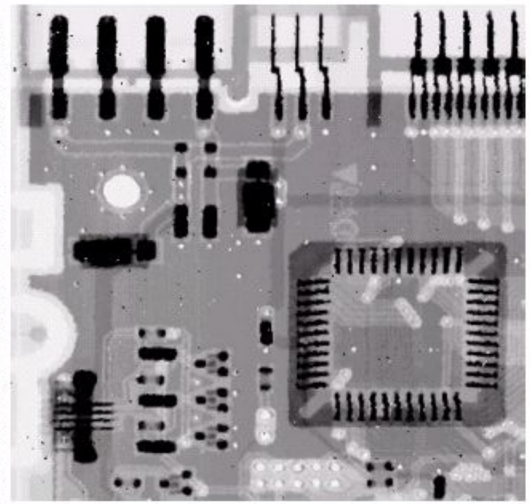
y

Image f (x, y)

Noise Removal Examples

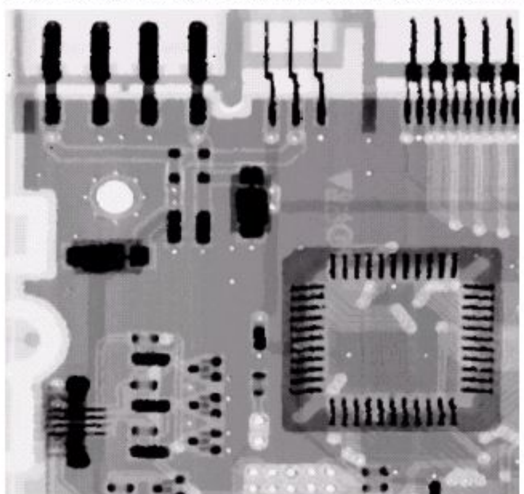
Image Corrupted By Salt And Pepper Noise

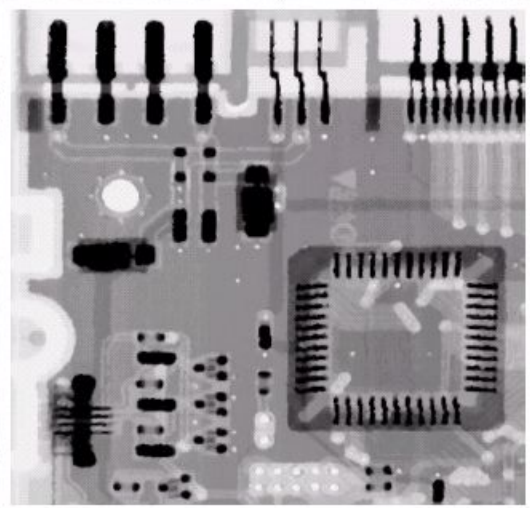




Result of 1 Pass With A 3*3 Median Filter

Result of 2 Passes With A 3*3 Median Filter

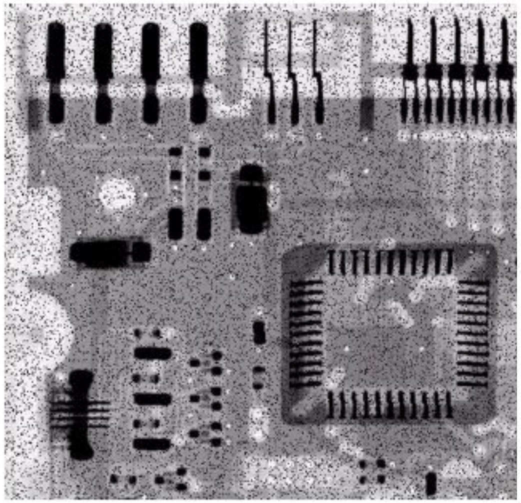




Result of 3 Passes With A 3*3 Median Filter

Noise Removal Examples (cont…)

Image Corrupted By Pepper Noise



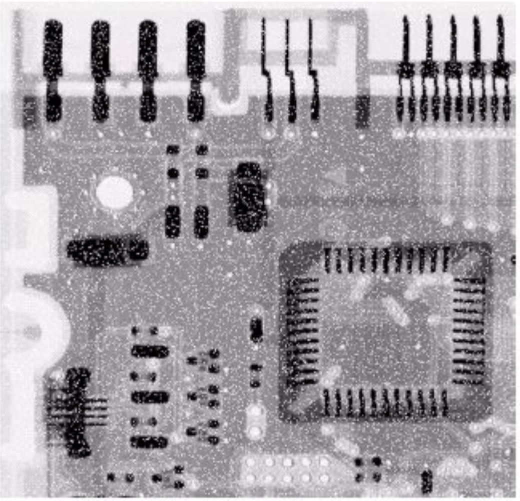
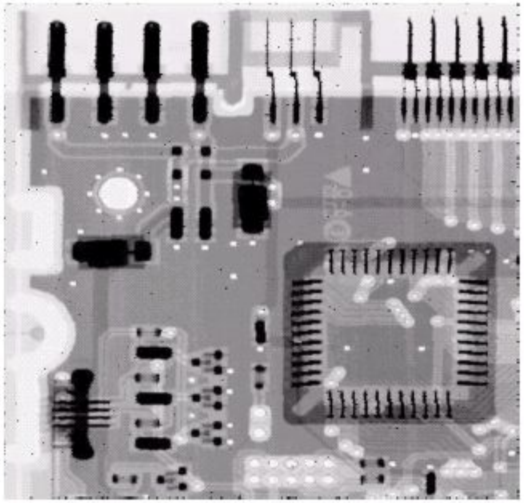
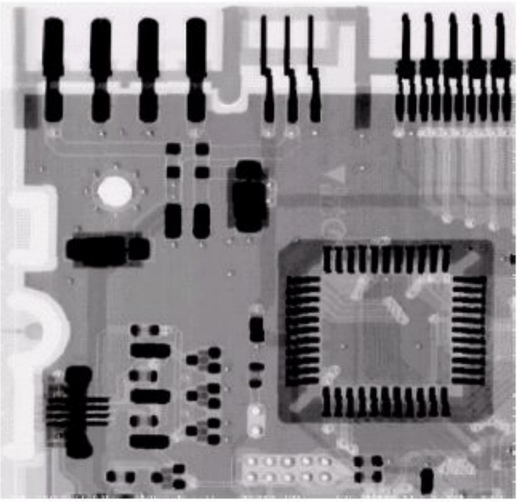


Image Corrupted By Salt Noise

Result Of Filtering Above With A 3*3 Max Filter

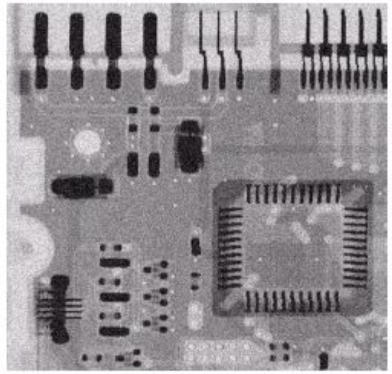




Result Of Filtering Above With A 3*3 Min Filter

Noise Removal Examples (cont…)

Image Corrupted By Uniform Noise



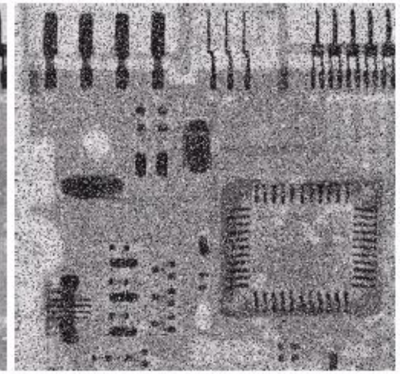
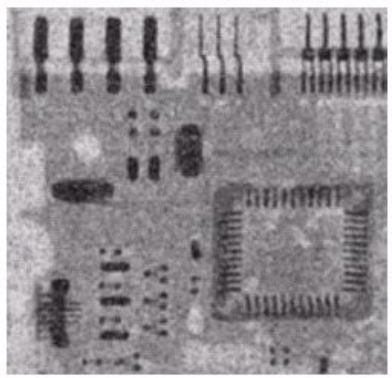
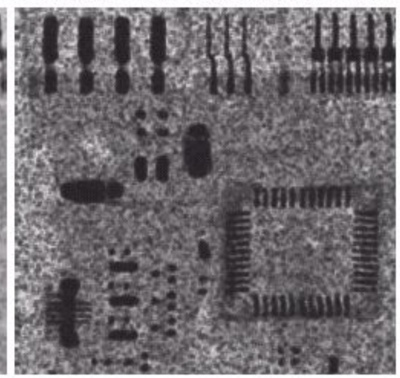


Image Further Corrupted By Salt and Pepper Noise

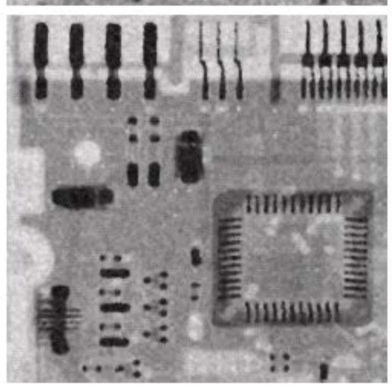
Filtered By 5*5 Arithmetic Mean Filter

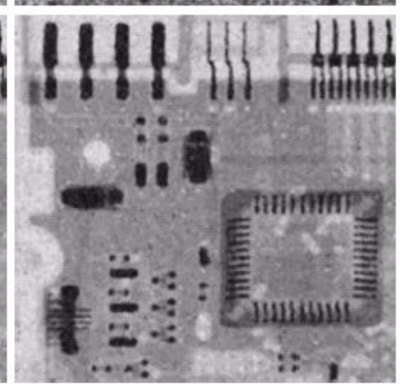




Filtered By 5*5 Geometric Mean Filter

Filtered By 5*5 Median Filter

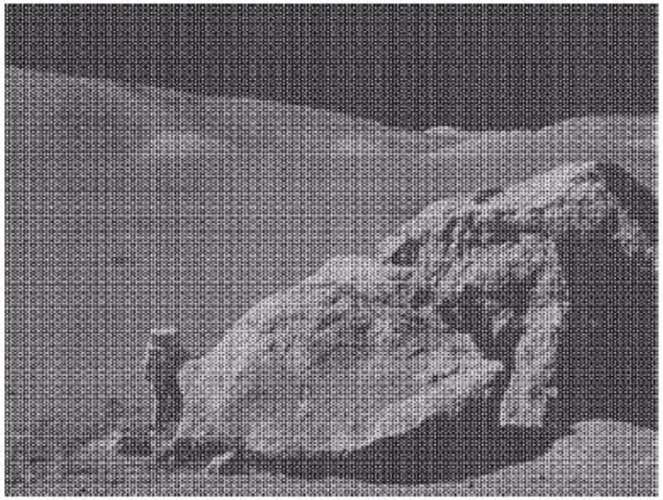




Filtered By 5*5 Alpha-Trimmed Mean Filter

Periodic Noise

- Typically arises due to electrical or electromagnetic interference
- Gives rise to regular noise patterns in an image
- Frequency domain techniques in the Fourier domain are most effective at removing periodic noise



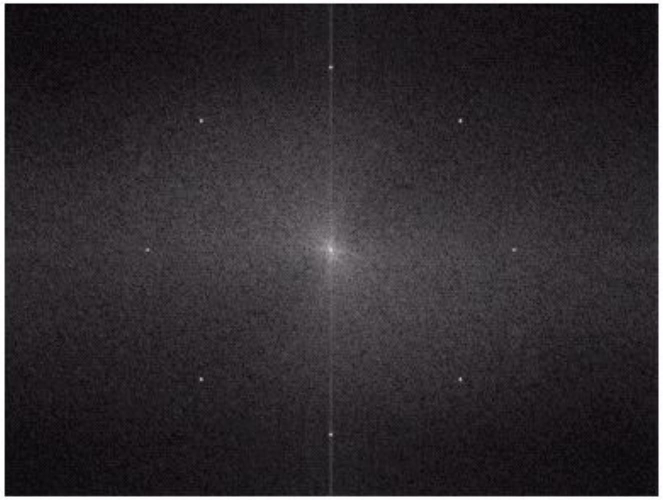


Image enhancement vs. Image Restoration

- Image restoration assumes a degradation model that is known or can be estimated.
- Image Enhancement is subjective, whereas image restoration is objective process.
- Image restoration try to recover original image from degraded with prior knowledge of degradation process.
- Restoration involves modeling of degradation and applying the inverse process in order to recover the original image.
- Although the restore image is not the original image,its approximation of actual image.

Band Reject Filters

Removing periodic noise form an image involves removing a particular range of frequencies from that image

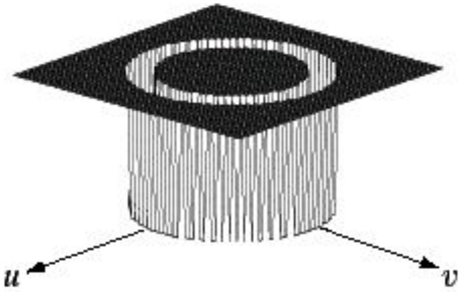
Band reject filters can be used for this purpose

An ideal band reject filter is given as follows:

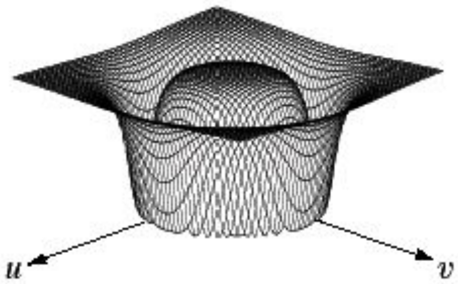
$$H(u,v)=\begin{cases} 1 & if\ D(u,v) < D_0 - \dfrac{W}{2} \\ 0 & if\ D_0 - \dfrac{W}{2} \le D(u,v) \le D_0 + \dfrac{W}{2} \\ 1 & if\ D(u,v) > D_0 + \dfrac{W}{2} \end{cases}$$

Band Reject Filters (cont…)

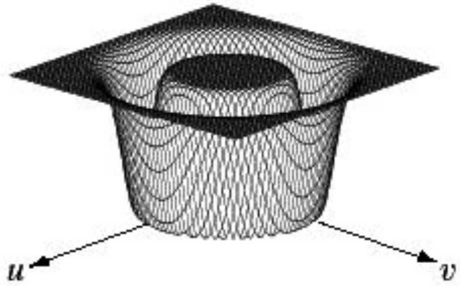
The ideal band reject filter is shown below, along with Butterworth and Gaussian versions of the filter



Ideal Band Reject Filter



Butterworth Band Reject Filter (of order 1)

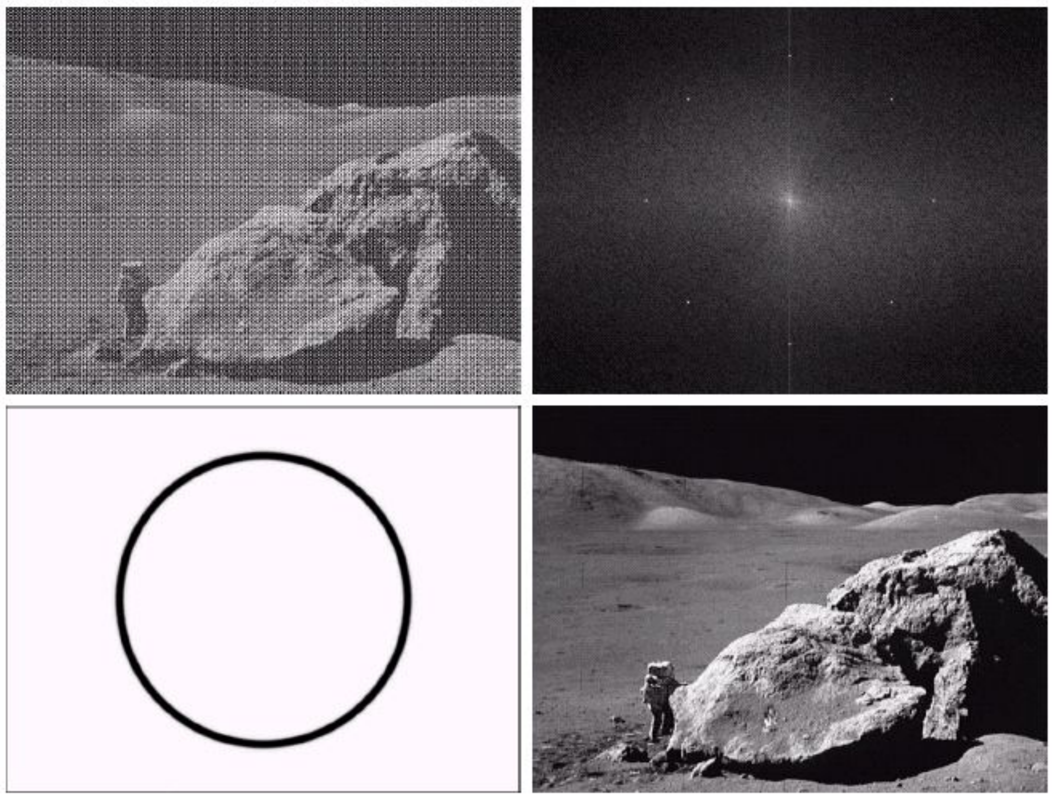


Gaussian Band Reject Filter

Band Reject Filter Example

Image corrupted by sinusoidal noise

Fourier spectrum of corrupted image



Butterworth band reject filter

Filtered image

Summary

In this lecture we will look at image restoration for noise removal

Restoration is slightly more objective than enhancement

Spatial domain techniques are particularly useful for removing random noise

Frequency domain techniques are particularly useful for removing periodic noise

Adaptive Filters

The filters discussed so far are applied to an entire image without any regard(বিবেচনা) for how image characteristics vary from one point to another

The behaviour of **adaptive filters** changes depending on the characteristics of the image inside the filter region

We will take a look at the **adaptive median filter**

Adaptive Median Filtering

The median filter performs relatively well on impulse noise as long as the spatial density of the impulse noise is not large

The adaptive median filter can handle much more spatially dense impulse noise, and also performs some smoothing for non-impulse noise

The key insight in the adaptive median filter is that the filter size changes depending on the characteristics of the image

Adaptive Median Filtering (cont…) (2013)

Remember that filtering looks at each original pixel image in turn and generates a new filtered pixel.

First examine the following notation: Suppose

- z_{min} = minimum grey level in S_{xy}
- z_{max} = maximum grey level in S_{xy}
- z_{med} = median of grey levels in S_{xy}
- z_{xy} = grey level at coordinates (x, y)
- S_{max} =maximum allowed size of S_{xy}

Adaptive Median Filtering (cont…)

Algorithm:

Level A: $A1 = z_{med} - z_{min}$

$A2 = z_{med} - z_{max}$

If $A1$ > 0 and $A2$ < 0, Go to level B

Else increase the window size

If window size ≤ repeat S_{max} level A

Else output z_{med}

Level B: $B1 = z_{xy} - z_{min}$

$B2 = z_{xy} - z_{max}$

If $B1$ > 0 and $B2$ < 0, output z_{xy}

Else output z_{med}

Adaptive Median Filtering (cont…)

The key to understanding the algorithm is to remember that the adaptive median filter has three purposes:

- Remove impulse noise(salt and pepper noise)
- Provide smoothing of other noise
- Reduce distortion

Minimum loss of information

Adaptive Filtering Example

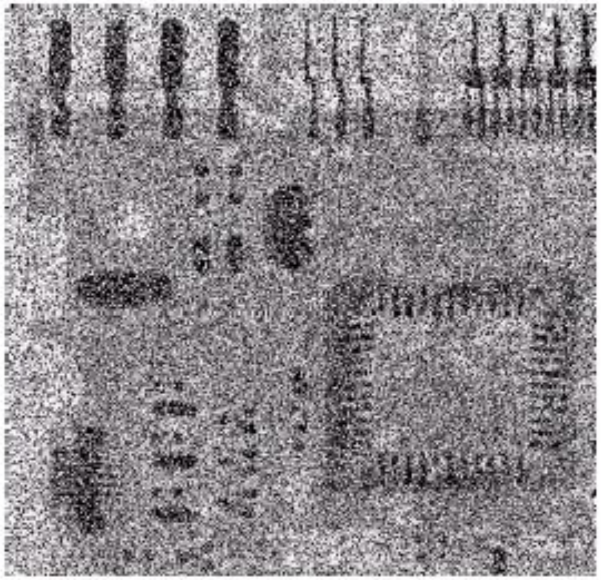
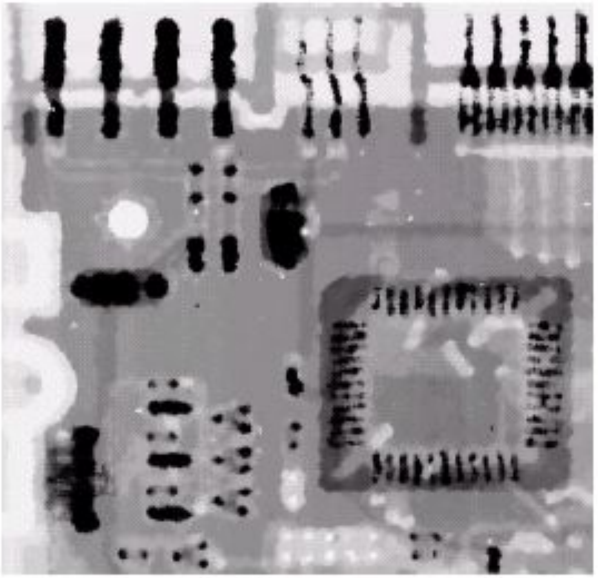
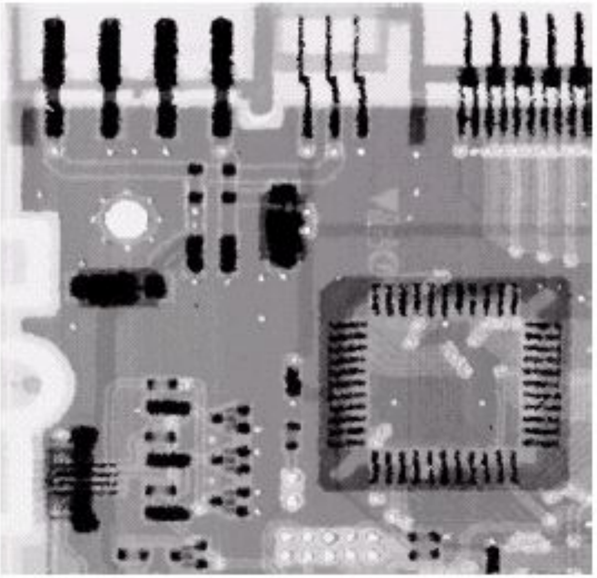


Image corrupted by salt and pepper noise with probabilities $P_a = P_b = 0.25$



Result of filtering with a 7 * 7 median filter



Result of adaptive median filtering with i = 7