Lecture No. 22

Ex:- i) $\dfrac{\sqrt[7]{2x^3+1}\ \sin 5x}{(2x+3)(x-1)}$ (ii) $\dfrac{x\cos 2x}{e^{3x}\sqrt{1+x^2}}$

iii) $x^y = y^x$ (iv) $x^y = y^{\sin x}$ v) $y = \left(\frac{n}{x}\right)^{nx}\left(1+\log\frac{x}{n}\right)$

vi) $x^y + y^x = 0$ vii) $y = \ln\sqrt{\dfrac{1+\sin x}{1-\sin x}}$ (viii) $y = \dfrac{x\cos^{-1}x}{\sqrt{1-x^2}}$

ix) $x^y = e^{x-y}$ x) $y = (\cos x)^{(\cos x)^{\cos x \cdots \infty}}$ xi) $\sqrt{\ln x + \sqrt{\ln x + \sqrt{\ln x \cdots}}}$

xii) $y = \ln\sqrt{\dfrac{1+\cos^2 x}{1-e^{2x}}}$ xiii) $y = \ln\left[e^x\left(\dfrac{x-2}{x+2}\right)^{3/4}\right]$

xiv) $x^y + y^x = a^b$ xv) $y = \ln\dfrac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}}$

xvi) $y = e^{x+e^{x+e^{x+\cdots\infty}}}$ then prove that $\dfrac{dy}{dx} = \dfrac{y}{1-y}$.

xvii) $y = \dfrac{e^{x^2}\tan^{-1}x}{\sqrt{1+x^2}}$ xviii) $y = \dfrac{1-x}{(1-x^3)^{1/2}}$.

(c)

Sol^n (i) Let $y = \dfrac{\sqrt[7]{2x^3+1}\ \sin 5x}{(2x+3)(x-1)}$

Taking ln on both sides, we get

$$\ln y = \ln\frac{\sqrt[7]{2x^3+1}\ \sin 5x}{(2x+3)(x-1)}$$

$$\Rightarrow \ln y = \ln\left(\sqrt[7]{2x^3+1}\cdot\sin 5x\right) - \ln\{(2x+3)(x-1)\}$$

$$\Rightarrow \ln y = \ln\sqrt[7]{2x^3+1} + \ln\sin 5x - \ln(2x+3) - \ln(x-1)$$

$$\Rightarrow \ln y = \ln(2x^3+1)^{1/7} + \ln\sin 5x - \ln(2x+3) - \ln(x-1)$$

$$\Rightarrow \ln y = \frac{1}{7}\ln(2x^3+1) + \ln\sin 5x - \ln(2x+3) - \ln(x-1)$$

Differentiating w.r.to x, we get

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{7}\cdot\frac{1}{2x^3+1}(6x^2) + \frac{\cos 5x}{\sin 5x}\cdot 5 - \frac{1}{2x+3}\cdot 2 - \frac{1}{x-1}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{6x^2}{7(2x^3+1)} + \frac{5\cos 5x}{\sin 5x} - \frac{2}{2x+3} - \frac{1}{x-1}$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{6x^2}{7(2x^3+1)} + \frac{5\cos 5x}{\sin 5x} - \frac{2}{2x+3} - \frac{1}{x-1}\right]$$

$$\therefore \frac{dy}{dx} = \frac{\sqrt[7]{2x^3+1}\,\sin 5x}{(2x+3)(x-1)}\left[\frac{6x^2}{7(2x^3+1)} - \frac{5\cos 5x}{\sin 5x} - \frac{2}{2x+3} - \frac{1}{x-1}\right]$$

Ans.

ii) Let $y = \dfrac{x\cos 2x}{e^{3x}\sqrt{1+x^2}}$

$$\therefore \ln y = \ln\frac{x\cos 2x}{e^{3x}\sqrt{1+x^2}}$$

$$\Rightarrow \ln y = \ln(x\cos 2x) - \ln\left(e^{3x}\sqrt{1+x^2}\right)$$

$$\Rightarrow \ln y = \ln x + \ln\cos 2x - \ln e^{3x} - \ln\sqrt{1+x^2}$$

$$\Rightarrow \ln y = \ln x + \ln\cos 2x - 3x\ln e - \ln(1+x^2)^{1/2}$$

$$\Rightarrow \ln y = \ln x + \ln\cos 2x - 3x - \frac{1}{2}\ln(x^2+1).$$

$$\therefore \frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{x} + \frac{1}{\cos 2x}(-\sin 2x)\cdot 2 - 3 - \frac{1}{2}\cdot\frac{1}{1+x^2}\cdot 2x$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{1}{x} - 2\tan 2x - 3 - \frac{x}{1+x^2}\right]$$

$$= \frac{x\cos 2x}{e^{3x}\sqrt{1+x^2}}\left[\frac{1}{x} - 2\tan 2x - 3 - \frac{x}{1+x^2}\right]$$ Ans.

iii) $x^y = y^x$

Taking ln on both sides we get

$\ln x^y = \ln y^x$

$\Rightarrow y \ln x = x \ln y$

Diff. w.r.to x

$\frac{d}{dx}(y \ln x) = \frac{d}{dx}(x \ln y)$

$\Rightarrow y \frac{d}{dx}(\ln x) + \ln x \frac{dy}{dx} = x \frac{d}{dx}(\ln y) + \ln y \frac{d}{dx}(x)$

$\Rightarrow y \cdot \frac{1}{x} + \ln x \frac{dy}{dx} = x \cdot \frac{1}{y} \frac{dy}{dx} + \ln y$

$\Rightarrow \ln x \frac{dy}{dx} - \frac{x}{y} \frac{dy}{dx} = \ln y - \frac{y}{x}$

$\Rightarrow \frac{dy}{dx}\left(\ln x - \frac{x}{y}\right) = \frac{x \ln y - y}{x}$

$\Rightarrow \frac{dy}{dx} \times \frac{y \ln x - x}{y} = \frac{x \ln y - y}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x} \cdot \frac{x \ln y - y}{y \ln x - x}$

$\therefore \frac{dy}{dx} = \frac{y(x \ln y - y)}{x(y \ln x - x)}$ Ans.

iv $x^y = y^{\sin x}$

$\therefore \ln x^y = \ln y^{\sin x}$

$\Rightarrow y \ln x = \sin x \ln y$

$\therefore y \frac{d}{dx}(\ln x) + \ln x \frac{dy}{dx} = \sin x \frac{d}{dx}(\ln y) + \ln y \frac{d}{dx}(\sin x)$

$\Rightarrow y \cdot \frac{1}{x} + \ln x \frac{dy}{dx} = \sin x \cdot \frac{1}{y} \frac{dy}{dx} + \ln y \cos x$

$\Rightarrow \ln x \frac{dy}{dx} - \frac{\sin x}{y} \frac{dy}{dx} = \ln y \cos x - \frac{y}{x}$

$\Rightarrow \frac{dy}{dx}\left(\ln x - \frac{\sin x}{y}\right) = \ln y \cos x - \frac{y}{x}$

$$\Rightarrow \frac{dy}{dx} \times \frac{y\ln x - \sin x}{y} = \frac{x\ln y\cos x - y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y(x\ln y\cos x - y)}{x(y\ln x - \sin x)} \quad \text{Ans.}$$

v

$$y = \left(\frac{n}{x}\right)^{nx}\left(1+\log\frac{x}{n}\right)$$

Taking log on both sides, we get

$$\log y = \log\left[\left(\frac{n}{x}\right)^{nx}\left(1+\log\frac{x}{n}\right)\right]$$

$$\Rightarrow \log y = \log\left(\frac{n}{x}\right)^{nx} + \log\left(1+\log\frac{x}{n}\right)$$

$$\Rightarrow \log y = nx\log\left(\frac{n}{x}\right) + \log(1+\log x - \log n)$$

Differentiating both sides w.r.to x, we get

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{d}{dx}\left[x(\log n - \log x) + \log(1+\log x - \log n)\right]$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(x\log n) - \frac{d}{dx}(x\log x) + \frac{d}{dx}\left[\log(1+\log x - \log n)\right]$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \log n\cdot 1 - \left[x\cdot\frac{1}{x} + \log x\cdot 1\right] + \frac{1}{(1+\log x - \log n)}\frac{d}{dx}(1+\log x - \log n)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \log n - 1 - \log x + \frac{1}{1+\log x - \log n}\cdot\frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\log n - 1 - \log x + \frac{1}{x(1+\log x - \log n)}\right\}$$

$$\therefore \frac{dy}{dx} = \left(\frac{n}{x}\right)^{nx}\left(1+\log\frac{x}{n}\right)\left[\log n - 1 - \log x + \frac{1}{x(1+\log x - \log n)}\right] \quad \text{Ans.}$$

vi) $x^y + y^x = 0$

$\Rightarrow x^y = -y^x$

$\therefore \log x^y = \log(-y)^x$

$\Rightarrow y \log x = x \log(-y)$

$\therefore y \cdot \frac{1}{x} + \log x \frac{dy}{dx} = x \cdot \frac{1}{-y} \cdot \frac{d}{dx}(-y) + \log(-y) \cdot 1$

$\Rightarrow \frac{y}{x} + \log x \frac{dy}{dx} = + \frac{x}{y} \frac{dy}{dx} + \log(-y)$

$\Rightarrow \left(\log x - \frac{x}{y}\right)\frac{dy}{dx} = -\frac{y}{x} + \log(-y)$

$\Rightarrow \frac{y \log x - x}{y} \frac{dy}{dx} = \frac{-y + x\log(-y)}{x}$

$\therefore \frac{dy}{dx} = \frac{y\{-y + x\log(-y)\}}{x(y\log x - x)}$ Ans.

vii) $y = \ln\sqrt{\frac{1+\sin x}{1-\sin x}}$

$\Rightarrow y = \ln\left(\frac{1+\sin x}{1-\sin x}\right)^{1/2}$

$\Rightarrow y = \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x}$

$\Rightarrow y = \frac{1}{2}\{\ln(1+\sin x) - \ln(1-\sin x)\}$

$\Rightarrow y = \frac{1}{2}\ln(1+\sin x) - \frac{1}{2}\ln(1-\sin x)$

$\therefore \frac{dy}{dx} = \frac{1}{2}\cdot\frac{1}{1+\sin x}\cdot\cos x - \frac{1}{2}\cdot\frac{1}{1-\sin x}(-\cos x)$

$= \frac{\cos x - \cos x\sin x + \cos x + \cos x \sin x}{2(1+\sin x)(1-\sin x)}$

$= \frac{2\cos x}{2(1-\sin^2 x)} = \frac{\cancel{2}\cos x}{\cancel{2}\cdot\cos^2 x} = \frac{1}{\cos x} = \sec x$ Ans.

viii)

$$y = \frac{x\cos^{-1}x}{\sqrt{1-x^2}}$$

Taking ln on both sides we have

$$\ln y = \ln \frac{x\cos^{-1}x}{\sqrt{1-x^2}}$$

$$\cancel{\therefore \frac{1}{y}\frac{dy}{dx} =} \Rightarrow \ln y = \ln\left(x\cos^{-1}x\right) - \ln\left(\sqrt{1-x^2}\right)$$

$$\Rightarrow \ln y = \ln x + \ln\cos^{-1}x - \frac{1}{2}\ln\left(1-x^2\right)$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{1}{\cos^{-1}x}\frac{d}{dx}\left(\cos^{-1}x\right) - \frac{1}{2}\cdot\frac{1}{1-x^2}\frac{d}{dx}\left(1-x^2\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{1}{\cos^{-1}x}\cdot\frac{-1}{\sqrt{1-x^2}} - \frac{1}{\cancel{2}(1-x^2)}\left(-\cancel{2}x\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x} - \frac{1}{\sqrt{1-x^2}\,\cos^{-1}x} + \frac{x}{1-x^2}$$

$$\therefore \frac{dy}{dx} = y\left(\frac{1}{x} - \frac{1}{\sqrt{1-x^2}\,\cos^{-1}x} + \frac{x}{1-x^2}\right)$$

$$= \frac{x\cos^{-1}x}{\sqrt{1-x^2}}\left(\frac{1}{x} - \frac{1}{\sqrt{1-x^2}\,\cos^{-1}x} + \frac{x}{1-x^2}\right) \quad \text{Ans.}$$

ix)

$$x^y = e^{x-y}$$

Taking ln on both sides, we get

$$\ln x^y = \ln e^{x-y}$$

$$\Rightarrow y\ln x = (x-y)\ln e$$

$$\Rightarrow y\ln x = x-y$$

Differentiating w.r.to x we have

$$y\cdot\frac{1}{x} + \ln x\cdot\frac{dy}{dx} = 1 - \frac{dy}{dx}$$

$$\Rightarrow \frac{y}{x} + \ln x\frac{dy}{dx} = 1 - \frac{dy}{dx}$$

$\Rightarrow \ln x \frac{dy}{dx} + \frac{dy}{dx} = 1 - \frac{y}{x}$

$\Rightarrow \frac{dy}{dx}(\ln x + 1) = \frac{x-y}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{x-y}{x(1+\ln x)}$ Ans.

x) $y = (\cos x)^{(\cos x)^{(\cos x)^{\cdots \infty}}}$

Taking ln we get

$\ln y = \ln (\cos x)^{(\cos x)^{(\cos x)^{\cdots \infty}}}$

$\Rightarrow \ln y = (\cos x)^{(\cos x)^{(\cos x)^{\cdots \infty}}} \ln \cos x$

$\Rightarrow \ln y = y \ln \cos x$

Differentiating w. r. to x we get

$\frac{1}{y} \cdot \frac{dy}{dx} = y \cdot \frac{1}{\cos x} \cdot (-\sin x) + \ln \cos x \frac{dy}{dx}$

$\Rightarrow \frac{1}{y} \frac{dy}{dx} - \ln \cos x \frac{dy}{dx} = -y \tan x$

$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y} - \ln \cos x\right) = -y \tan x$

$\Rightarrow \frac{dy}{dx} \times \frac{1 - \ln \cos x}{y} = -y \tan x$

$\Rightarrow \frac{dy}{dx} = \frac{-y^2 \tan x}{1 - \ln \cos x}$ Ans.

xi) let $y = \sqrt{\ln x + \sqrt{\ln x + \sqrt{\ln x + \cdots \infty}}}$

$\Rightarrow y = \sqrt{\ln x + y}$

$\Rightarrow y^2 = \ln x + y$

$\therefore 2y \frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx}$

$\Rightarrow 2y \frac{dy}{dx} - \frac{dy}{dx} = \frac{1}{x}$

$\Rightarrow \frac{dy}{dx}(2y - 1) = \frac{1}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x(2y-1)}$ Ans.

xii) $y = \ln\sqrt{\dfrac{1+\cos^2 x}{1-e^{2x}}}$

$$\Rightarrow y = \ln\left(\frac{1+\cos^2 x}{1-e^{2x}}\right)^{1/2}$$

$$\Rightarrow y = \frac{1}{2}\ln\frac{1+\cos^2 x}{1-e^{2x}}$$

$$\Rightarrow y = \frac{1}{2}\ln(1+\cos^2 x) - \frac{1}{2}\ln(1-e^{2x})$$

$$\therefore \frac{dy}{dx} = \frac{1}{\cancel{2}}\cdot\frac{1}{1+\cos^2 x}\cdot \cancel{2}\cos x(-\sin x) - \frac{1}{\cancel{2}}\frac{1}{1-e^{2x}}\cdot -e^{2x}\cdot \cancel{2}$$

$$= \frac{-\cos x\sin x}{1+\cos^2 x} + \frac{e^{2x}}{1-e^{2x}}$$

$$\therefore \frac{dy}{dx} = \frac{e^{2x}}{1-e^{2x}} - \frac{\cos x\sin x}{1+\cos^2 x} \quad \underline{\underline{\text{Ans.}}}$$

2

xiii) $y = \ln\left[e^x\left(\dfrac{x-2}{x+2}\right)^{3/4}\right]$

$$\Rightarrow y = \ln e^x + \ln\left(\frac{x-2}{x+2}\right)^{3/4}$$

$$\Rightarrow y = x\ln e + \frac{3}{4}\ln\frac{x-2}{x+2}$$

$$\Rightarrow y = x + \frac{3}{4}\ln(x-2) - \frac{3}{4}\ln(x+2)$$

$$\therefore \frac{dy}{dx} = 1 + \frac{3}{4}\cdot\frac{1}{x-2}\cdot 1 - \frac{3}{4}\cdot\frac{1}{x+2}\cdot 1$$

$$\Rightarrow \frac{dy}{dx} = 1 + \frac{\cancel{3x}+6-\cancel{3x}+6}{4(x-2)(x+2)}$$

$$\Rightarrow \frac{dy}{dx} = 1 + \frac{\cancel{12}\,3}{\cancel{4}(x^2-4)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2-4+3}{x^2-4} = \frac{x^2-1}{x^2-4}$$

$$\therefore \frac{dy}{dx} = \frac{x^2-1}{x^2-1} \quad \underline{\underline{\text{Ans.}}}$$

XIV. $x^y + y^x = a^b$

$\therefore e^{\ln x^y} + e^{\ln y^x} = a^b$

$\Rightarrow e^{y\ln x} + e^{x\ln y} = a^b$

$\therefore \frac{d}{dx}\left(e^{y\ln x}\right) + \frac{d}{dx}\left(e^{x\ln y}\right) = \frac{d}{dx}(a^b)$

$\Rightarrow e^{y\ln x}\frac{d}{dx}(y\ln x) + e^{x\ln y}\frac{d}{dx}(x\ln y) = 0.$

$\Rightarrow x^y\left[y\cdot\frac{1}{x} + \ln x\cdot\frac{dy}{dx}\right] + y^x\left[x\cdot\frac{1}{y}\frac{dy}{dx} + \ln y\right] = 0$

$\Rightarrow x^y\cdot\frac{y}{x} + x^y\ln x\frac{dy}{dx} + y^x\frac{x}{y}\frac{dy}{dx} + y^x\ln y = 0$

$\Rightarrow \frac{dy}{dx}\left(x^y\ln x + y^x\frac{x}{y}\right) = -y^x\ln y - x^y\cdot\frac{y}{x}$

$\Rightarrow \cancel{\frac{dy}{dx} \times \frac{yx^y\ln x + x}{y}}$

$\Rightarrow \frac{dy}{dx}\times\left(x^y\ln x + xy^{x-1}\right) = -\left(y^x\ln y + yx^{y-1}\right)$

$$\therefore \frac{dy}{dx} = \frac{-(y^x \ln y + y x^{y-1})}{x^y \ln x + x y^{x-1}} \quad \text{An.}$$

xv. $y = \ln \dfrac{\sqrt{a+x} + \sqrt{a-x}}{\sqrt{a+x} - \sqrt{a-x}}$

$$= \ln\left[\sqrt{a+x} + \sqrt{a-x}\right] - \ln\left[\sqrt{a+x} - \sqrt{a-x}\right]$$

$$= \frac{1}{\sqrt{a+x}+\sqrt{a-x}}\left[\frac{1}{2\sqrt{a+x}} + \frac{1}{2\sqrt{a-x}}(-1)\right] - \frac{1}{\sqrt{a+x}-\sqrt{a-x}}\left[\frac{1}{2\sqrt{a+x}} - \frac{1}{2\sqrt{a-x}}(-1)\right]$$

$$= \frac{1}{\sqrt{a+x}+\sqrt{a-x}} \times \frac{\sqrt{a-x}-\sqrt{a+x}}{2\sqrt{a+x}\,\sqrt{a-x}} - \frac{1}{\sqrt{a+x}-\sqrt{a-x}} \times \frac{\sqrt{a-x}+\sqrt{a+x}}{2\sqrt{a+x}\,\sqrt{a-x}}$$

$$= \frac{-(\sqrt{a+x}-\sqrt{a-x})^2 - (\sqrt{a+x}+\sqrt{a-x})^2}{2\sqrt{a+x}\,\sqrt{a-x}\,(\sqrt{a+x}+\sqrt{a-x})(\sqrt{a+x}-\sqrt{a-x})}$$

$$= \frac{-\{a+x-2\sqrt{a+x}\sqrt{a-x}+a-x\} - \{a+x+2\sqrt{a+x}\sqrt{a-x}+a-x\}}{2\sqrt{a^2-x^2}\;\{(\sqrt{a+x})^2-(\sqrt{a-x})^2\}}$$

$$= \frac{-2a+2\sqrt{a^2-x^2}-2a-2\sqrt{a^2-x^2}}{2\sqrt{a^2-x^2}\,(a+x-a+x)}$$

$$= \frac{-4a}{2\sqrt{a^2-x^2}\cdot 2x}$$

$$= \frac{-a}{x\sqrt{a^2-x^2}} \quad \text{An.}$$

xvi $y = e^{x+e^{x+e^{x+\cdots\infty}}}$

$\therefore \ln y = \ln e^{x+e^{x+e^{x+\cdots\infty}}}$

$\Rightarrow \ln y = (x+e^{x+e^{x+\cdots\infty}})(\ln e)$

(10)

$\Rightarrow \ln y = x + y$

$\therefore \frac{1}{y} \cdot \frac{dy}{dx} = 1 + \frac{dy}{dx}$

$\Rightarrow \frac{1}{y} \frac{dy}{dx} - \frac{dy}{dx} = 1$

$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y} - 1\right) = 1$

$\Rightarrow \frac{dy}{dx} \times \frac{1-y}{y} = 1 \quad \therefore \frac{dy}{dx} = \frac{y}{1-y}$ (proved)

xvii

$$y = \frac{e^{x^2} \tan^{-1}x}{\sqrt{1+x^2}}$$

Taking ln on both sides then we get

$$\ln y = \ln \frac{e^{x^2} \tan^{-1}x}{\sqrt{1+x^2}}$$

$\therefore \ln y = \ln (e^{x^2} \tan^{-1}x) - \ln \sqrt{1+x^2}$

$\Rightarrow \ln y = \ln e^{x^2} + \ln(\tan^{-1}x) - \ln(1+x^2)^{1/2}$

$\Rightarrow \ln y = x^2 \ln e + \ln(\tan^{-1}x) - \frac{1}{2}\ln(1+x^2)$

$\Rightarrow \ln y = x^2 + \ln(\tan^{-1}x) - \frac{1}{2}\ln(1+x^2)$

Differentiating w.r.to x then we get

$$\frac{1}{y} \cdot \frac{dy}{dx} = 2x + \frac{1}{\tan^{-1}x} \cdot \frac{1}{1+x^2} - \frac{1}{\cancel{2}} \cdot \frac{1}{1+x^2} \cdot \cancel{2}x$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = 2x + \frac{1}{(1+x^2)\tan^{-1}x} - \frac{x}{1+x^2}$$

$$\Rightarrow \frac{dy}{dx} = y\left\{2x + \frac{1}{(1+x^2)\tan^{-1}x} - \frac{x}{1+x^2}\right\}$$

$$\therefore \frac{dy}{dx} = \frac{e^{x^2}\tan^{-1}x}{\sqrt{1+x^2}}\left\{2x + \frac{1}{(1+x^2)\tan^{-1}x} - \frac{x}{1+x^2}\right\}$$ Ans.

xviii)
$$y = \frac{1-x}{(1-x^3)^{1/2}}$$

$$\therefore \ln y = \ln \frac{1-x}{(1-x^3)^{1/2}}$$

$$\Rightarrow \ln y = \ln(1-x) - \ln(1-x^3)^{1/2}$$

$$\Rightarrow \ln y = \ln(1-x) - \frac{1}{2}\ln(1-x^3)$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = \frac{1}{1-x}(-1) - \frac{1}{2}\cdot\frac{1}{1-x^3}\cdot(-3x^2)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -\frac{1}{1-x} + \frac{3x^2}{2(1-x^3)}$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\frac{3x^2}{2(1-x^3)} - \frac{1}{1-x}\right\}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1-x}{(1-x^3)^{1/2}}\left\{\frac{3x^2}{2(1-x^3)} - \frac{1}{1-x}\right\}$$

$$= \frac{1-x}{(1-x^3)^{1/2}}\left\{\frac{3x^2}{2(1-x)(1+x+x^2)} - \frac{1}{1-x}\right\}$$

$$= \frac{1-x}{(1-x^3)^{1/2}} \times \frac{3x^2-2-2x-2x^2}{2(1-x)(1+x+x^2)}$$

$$= \frac{(1-x)(x^2-2x-2)}{(1-x^3)^{1/2}\,2(1-x^3)} = \frac{(1-x)(x^2-2x-2)}{2(1-x^3)^{3/2}}$$ **Ans**

Ex. If $f(x) = \left(\frac{a+x}{b+x}\right)^{a+b+2x}$ then prove that

$$f'(0) = \left(2\ln\frac{a}{b} + \frac{b^2-a^2}{ab}\right)\left(\frac{a}{b}\right)^{a+b}$$

Sol^n Given that $f(x) = \left(\frac{a+x}{b+x}\right)^{a+b+2x}$

Taking ln on both sides then we get

$$\ln f(x) = \ln\left(\frac{a+x}{b+x}\right)^{a+b+2x}$$

$$\Rightarrow \ln f(x) = (a+b+2x) \ln \frac{a+x}{b+x}$$

$$\Rightarrow \ln f(x) = (a+b+2x)\left[\ln(a+x) - \ln(b+x)\right]$$

$$\therefore \frac{1}{f(x)} \cdot f'(x) = (a+b+2x)\left[\frac{1}{a+x} - \frac{1}{b+x}\right] + \ln\frac{a+x}{b+x} \cdot (0+0+2)$$

$$\Rightarrow f'(x) = f(x)\left[(a+b+2x)\frac{b+x-a-x}{(a+x)(b+x)} + 2\ln\frac{a+x}{b+x}\right]$$

$$\Rightarrow f'(x) = \left(\frac{a+x}{b+x}\right)^{a+b+2x}\left[(a+b+2x)\frac{b-a}{(a+x)(b+x)} + 2\ln\frac{a+x}{b+x}\right]$$

$$\therefore f'(0) = \left(\frac{a}{b}\right)^{a+b}\left[\frac{(a+b)(b-a)}{ab} + 2\ln\frac{a}{b}\right]$$

$$\therefore f'(0) = \left[2\ln\frac{a}{b} + \frac{b^2-a^2}{ab}\right]\left(\frac{a}{b}\right)^{a+b}$$

(proved)

Ex. (i) $y = x^3\sqrt{\frac{x^2+4}{x^2+3}}$

Ans. $x^3\sqrt{\frac{x^2+4}{x^2+3}}\left[\frac{3}{x} - \frac{x}{(x^2+4)(x^2+3)}\right]$

ii) $y = a^{3x}\ln(\sin\sqrt{x})$

Ans: $a^{3x}\left[\frac{1}{2\sqrt{x}}\cot\sqrt{x} + 3(\ln a)\ln(\sin\sqrt{x})\right]$

iii) $x^y + y^x = (x+y)^{x+y}$

iv) $\log\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$

v) $(\sin x)^{\log x} + \cot\{e^x(a+bx)\} + a^b$

vi) $x^{\frac{2}{3}}\sqrt{\frac{x-1}{x+1}}$