Lecture 3
Measuring Errors

Objectives

- Differentiate among different types of Errors
- Measure different Errors
- Reduce numerical Errors
- Modify numerical technique to reduce numerical Errors
- Compare different Errors

Why Measure Error?

- To determine the accuracy of numerical results
- To develop stopping criteria for iterative algorithms

True Error

- Defined as the difference between the true value in a calculation and the approximate value found using a numerical method etc

True Error = True Value – Approximate Value

Example of True Error

The derivative, $f'(x)$ of a function $f(x)$ can be approximated by the equation,

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

If $f(x) = 7e^{0.5x}$ and $h = 0.3$

a) Find the approximate value of $f'(2)$

b) True value of $f'(2)$

c) True error for part (a)

Example Cont..

Solution:

a) For $x = 2$ and $h = 0.3$

$$f'(2) \approx \frac{f(2+0.3) - f(2)}{0.3}$$

$$= \frac{f(2.3) - f(2)}{0.3}$$

$$= \frac{7e^{0.5(2.3)} - 7e^{0.5(2)}}{0.3}$$

$$= \frac{22.107 - 19.028}{0.3} = 10.263$$

Example Cont..

Solution:

b) The exact value of $f'(2)$ can be found by using our knowledge of differential calculus.

$$f(x) = 7e^{0.5x}$$

$$f'(x) = 7 \times 0.5 \times e^{0.5x}$$

$$= 3.5e^{0.5x}$$

So the true value of $f'(2)$ is

$$f'(2) = 3.5e^{0.5(2)}$$

$$= 9.5140$$

True error is calculated as

$$E_t = \text{True Value} - \text{Approximate Value}$$

$$= 9.5140 - 10.263 = -0.722$$

Relative True Error

- Defined as the ratio between the true error, and the true value.

$$\text{Relative True Error } (\in_t) = \frac{\text{True Error}}{\text{True Value}}$$

Example of Relative True Error

Following from the previous example for true error, find the relative true error for $f(x) = 7e^{0.5x}$ at $f'(2)$ with $h = 0.3$

From the previous example,

$$E_t = -0.722$$

Relative True Error is defined as

$$\in_t = \frac{\text{True Error}}{\text{True Value}}$$

$$= \frac{-0.722}{9.5140} = -0.075888$$

as a percentage,

$$\in_t = -0.075888 \times 100\% = -7.5888\%$$

Approximate Error

- What can be done if true values are not known or are very difficult to obtain?
- Approximate error is defined as the difference between the present approximation and the previous approximation

$$\text{Approximate Error } (E_a) = \text{Present Approximation} - \text{Previous Approximation}$$

Example of Approximate Error

For $f(x) = 7e^{0.5x}$ at $x = 2$ find the following,

a) $f'(2)$ using $h = 0.3$

b) $f'(2)$ using $h = 0.15$

c) approximate error for the value of $f'(2)$ for part b)

Solution:

a) For $x = 2$ and $h = 0.3$

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

$$f'(2) \approx \frac{f(2+0.3) - f(2)}{0.3}$$

Example Cont..

Solution: (cont.)

$$f'(2) = \frac{f(2.3) - f(2)}{0.3}$$

$$= \frac{7e^{0.5(2.3)} - 7e^{0.5(2)}}{0.3}$$

$$f'(2) = \frac{22.107 - 19.028}{0.3} = 10.263$$

b) For $x = 2$ and $h = 0.15$

$$f'(2) \approx \frac{f(2+0.15) - f(2)}{0.15}$$

$$= \frac{f(2.15) - f(2)}{0.15}$$

Example Cont….

Solution: (cont.)

$$= \frac{7e^{0.5(2.15)} - 7e^{0.5(2)}}{0.15}$$

$$= \frac{20.50 - 19.028}{0.15} = 9.8800$$

c) So the approximate error, E_a is

$$E_a = \text{Present Approximation} - \text{Previous Approximation}$$

$$= 9.8800 - 10.263$$

$$= -0.38300$$

Relative Approximate Error

- Defined as the ratio between the approximate error and the present approximation.

$$\text{Relative Approximate Error } (\in_a) = \frac{\text{Approximate Error}}{\text{Present Approximation}}$$

Example of Relative Approximate Error

For $f(x)=7e^{0.5x}$ at $x=2$, find the relative approximate error using values from $h=0.3$ and $h=0.15$

Solution:

From Example 3, the approximate value of $f'(2)=10.263$ using $h=0.3$ and $f'(2)=9.8800$ using $h=0.15$

$$E_a = \text{Present Approximation} - \text{Previous Approximation}$$

$$= 9.8800 - 10.263$$

$$= -0.38300$$

Example Cont…

Solution: (cont.)

$$\in_a = \frac{\text{Approximate Error}}{\text{Present Approximation}}$$

$$= \frac{-0.38300}{9.8800} = -0.038765$$

as a percentage,

$$\in_a = -0.038765 \times 100\,\% = -3.8765\,\%$$

Absolute relative approximate errors may also need to be calculated,

$$|\in_a| = |-0.038765| = 0.038765 \text{ or } 3.8765\%$$

Table of Values

For $f(x) = 7e^{0.5x}$ at $x = 2$ with varying step size, h

h	$f'(2)$	$\lvert \in_a \rvert$	m
0.3	10.263	N/A	0
0.15	9.8800	3.877%	1
0.10	9.7558	1.273%	1
0.01	9.5378	2.285%	1
0.001	9.5164	0.2249%	2

Thank you

Question and Suggestion