Lecture 5
Roots of Non-Linear Equation Using False Position Method

Roots of an Equation

- The **root of an equation** is a value of the variable that makes the equation **true**

- For the equation

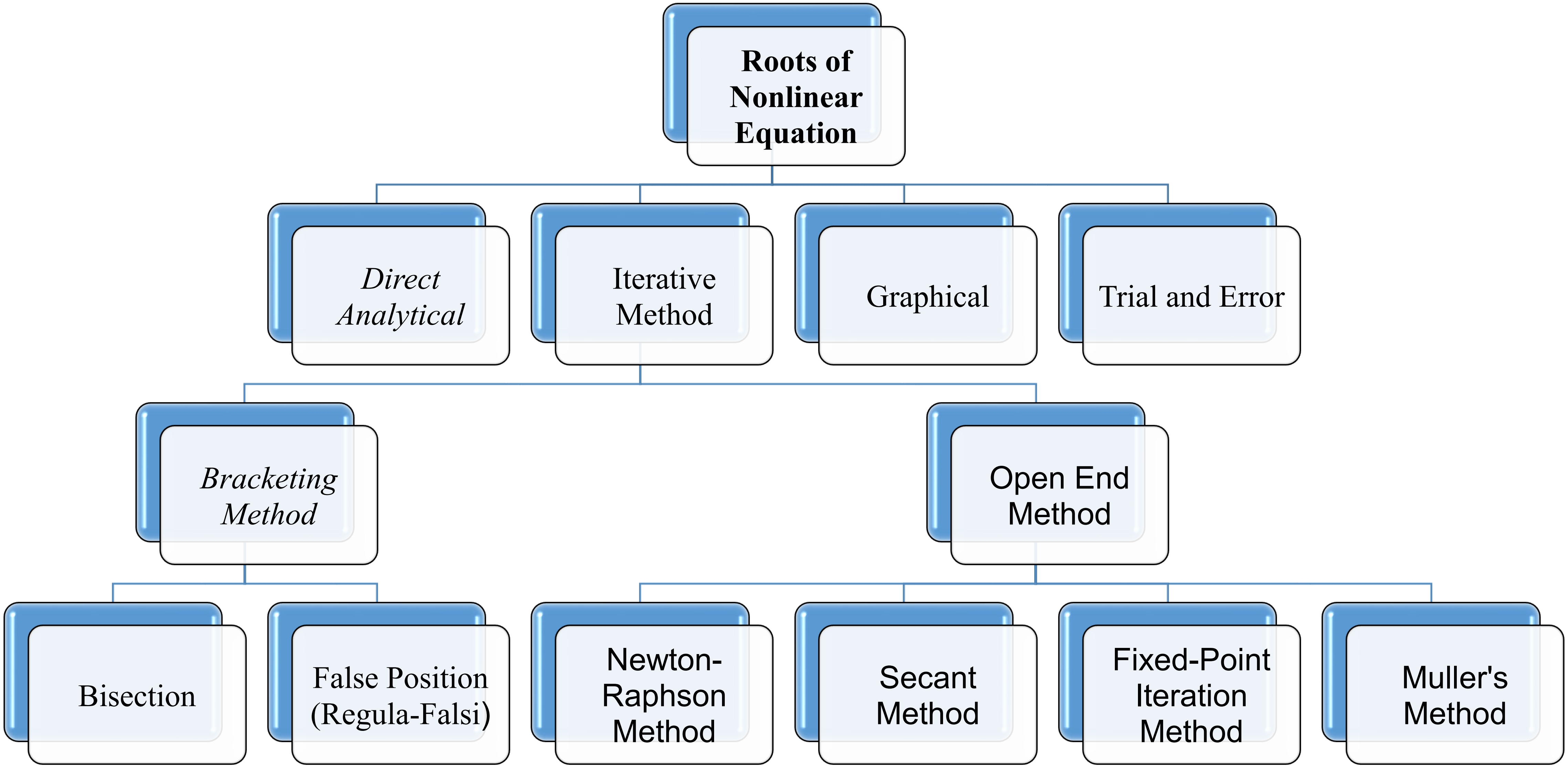
$$x^2 - 4 = 0$$

- If we solve it

$$x^2 = 4 \; => x = \pm 2$$

So, the roots are **x = 2** and **x = -2**

Objectives


Roots of Nonlinear Equation
Direct Analytical
Iterative Method
Graphical
Trial and Error
Bracketing Method
Open End Method
Bisection
False Position (Regula-Falsi)
Newton-Raphson Method
Secant Method
Fixed-Point Iteration Method
Muller's Method

Iterative methods

- Based on the number of guesses they use, can he grouped into two categories:

    - Bracketing methods

    - Open end methods

Iterative methods

- Bracketing methods
  - Enclose the root within a specific interval $[a,b]$
  - Iteratively reduces the interval size until the root is found with sufficient accuracy
  - Examples:
    - Bisection Method
    - False Position

False Position Method

False Position method

$$f(x) = 0 \qquad (1)$$

- In the Bisection method

$$f(x_L) * f(x_U) < 0 \qquad (2)$$

$$x_r = \frac{x_L + x_U}{2} \qquad (3)$$

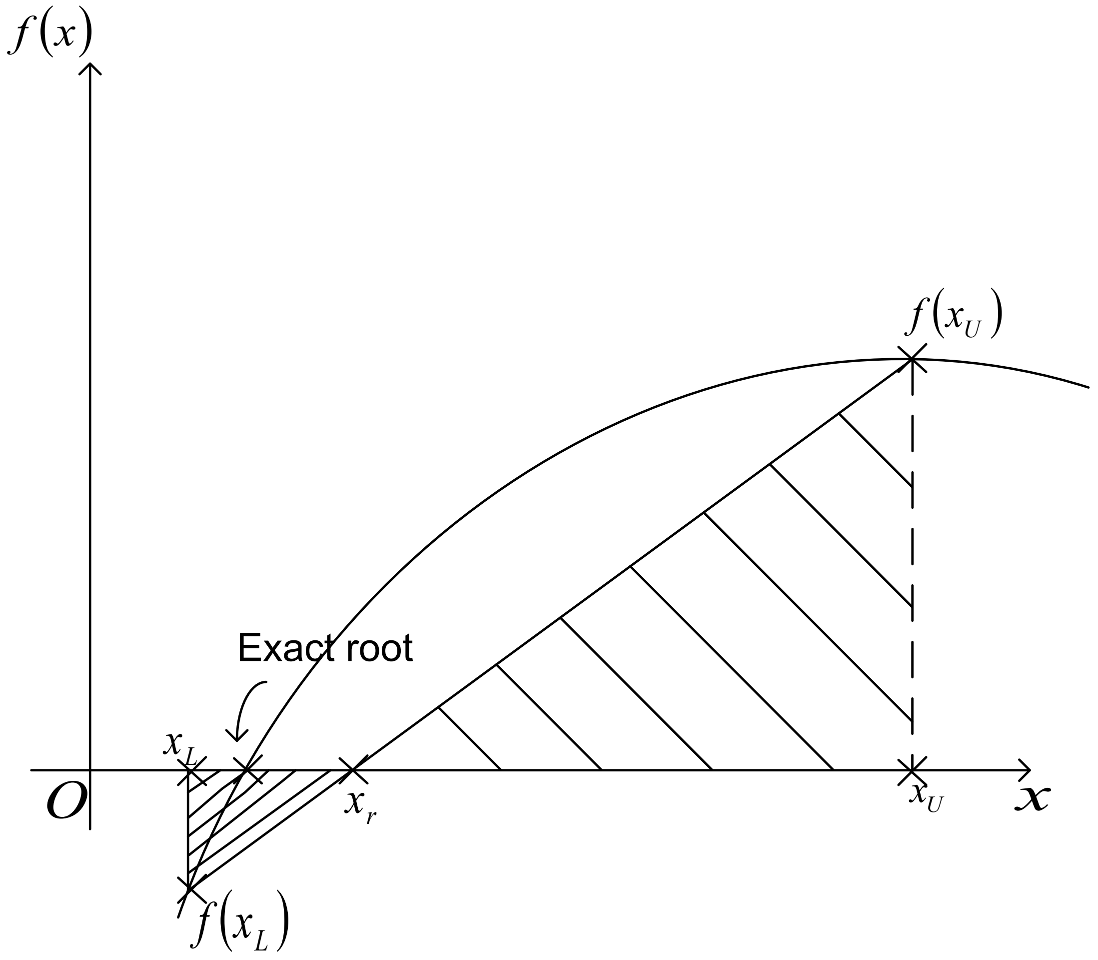


Figure 1 False-Position Method

False Position method

- Based on two similar triangles, shown in Figure 1, one gets:

$$\frac{f(x_L)}{x_r - x_L} = \frac{f(x_U)}{x_r - x_U} \qquad (4)$$

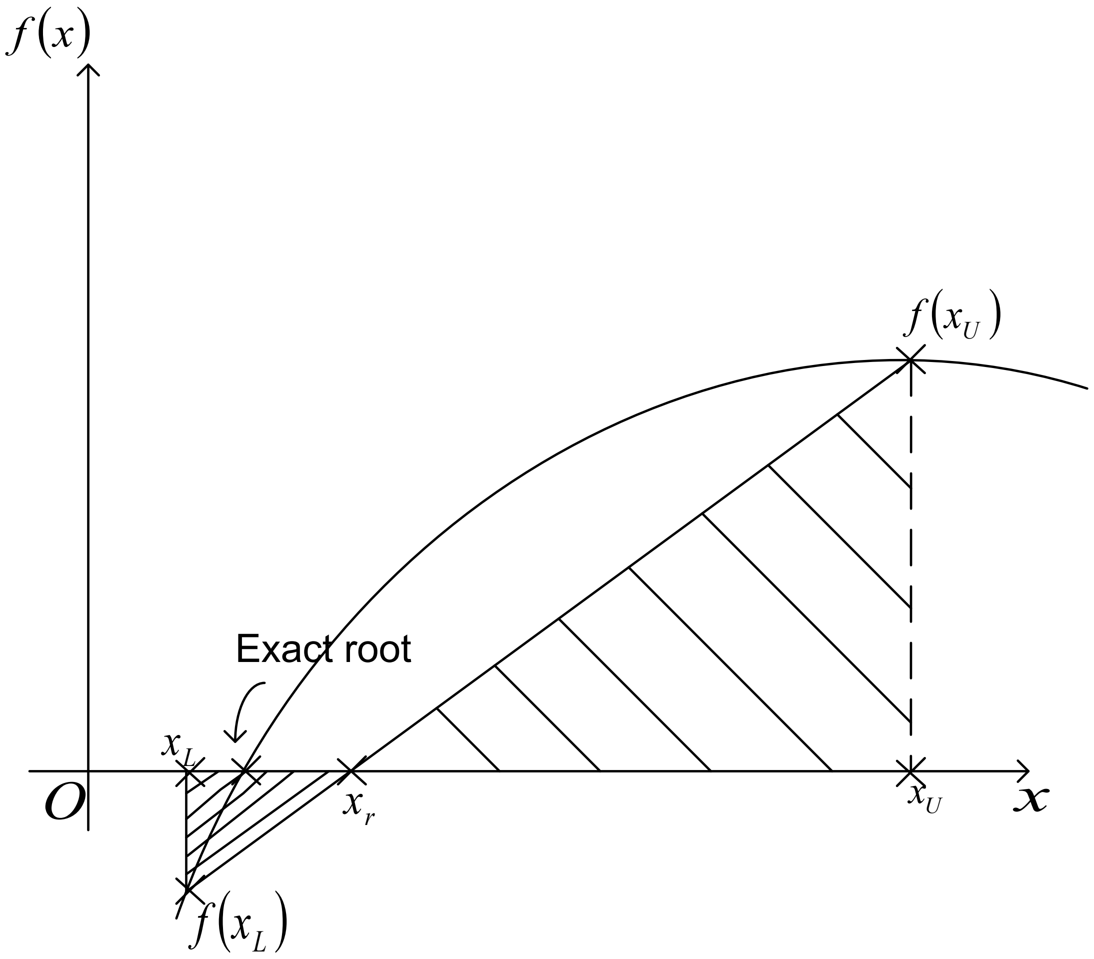


Figure 1 False-Position Method

False Position method

- From Eq. (4), one obtains

$$(x_r - x_L)f(x_U) = (x_r - x_U)f(x_L)$$

$$x_U f(x_L) - x_L f(x_U) = x_r\{f(x_L) - f(x_U)\}$$

- The above equation can be solved to obtain the next predicted root x_r

$$x_r = \frac{x_U f(x_L) - x_L f(x_U)}{f(x_L) - f(x_U)} \qquad (5)$$

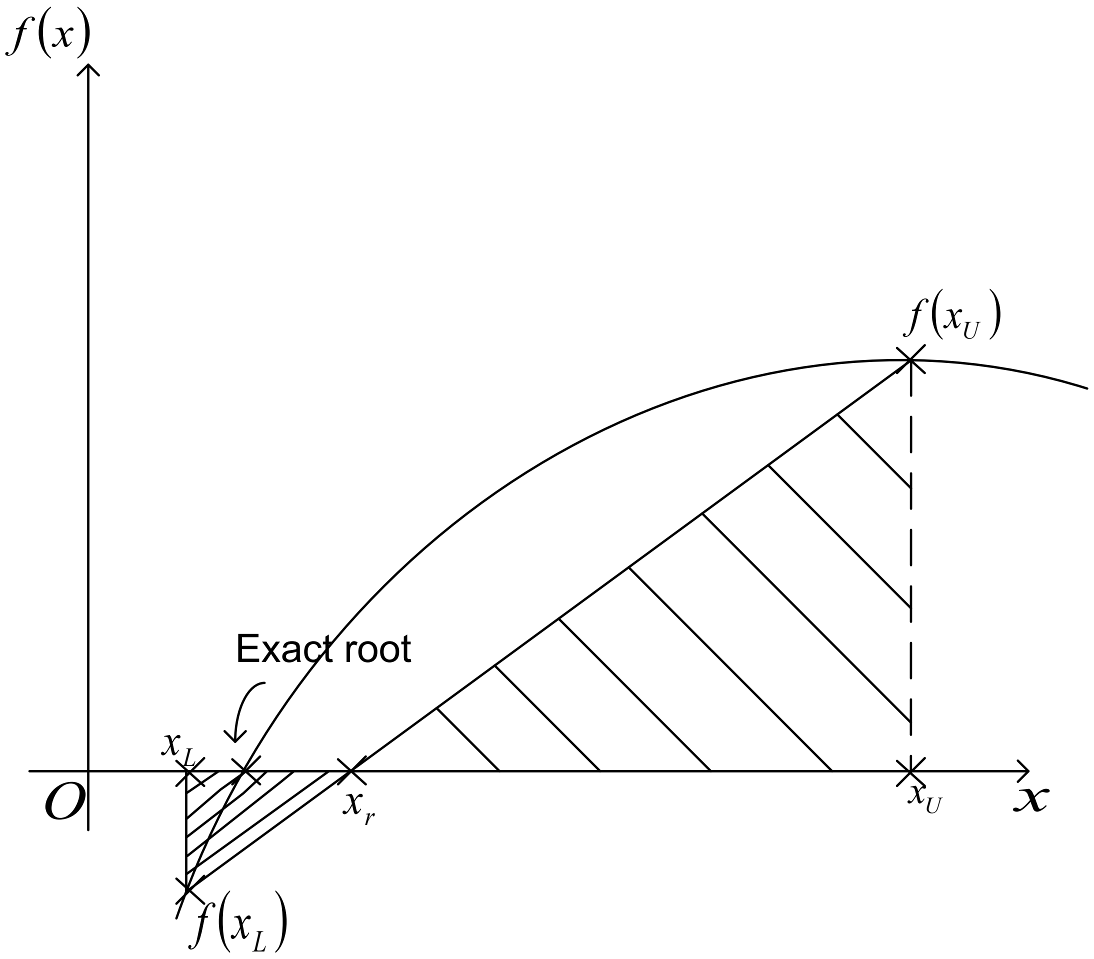


Figure 1 False-Position Method

Step-By-Step False-Position Algorithm

1. Choose X_L and X_U as two guesses for the root such that

$$f(x_L)f(x_U) < 0$$

2. Estimate the root, $$x_m = \frac{x_U f(x_L) - x_L f(x_U)}{f(x_L) - f(x_U)}$$

3. Now check the following

   a) If $f(x_L)f(x_m) < 0$, then the root lies between X_L and X_m; then $X_L = X_L$ and $X_U = X_m$

Step-By-Step False-Position Algorithm

3. Now check the following
   a) If $f(x_L)f(x_m) < 0$, then the root lies between X_L and X_m; then $X_L = X_L$ and $X_U = X_m$

   b) If $f(x_L)f(x_m) > 0$, then the root lies between X_U and X_m; then $X_L = X_m$ and $X_U = X_U$

   c) If $f(x_L)f(x_m) = 0$, then the root is X_m; stop the algorithm if this is true

Step-By-Step False-Position Algorithm

4. Find the new estimate of the root

$$x_m = \frac{x_U f(x_L) - x_L f(x_U)}{f(x_L) - f(x_U)}$$

Find the relative approximate error as

$$\left|\in_a\right| = \left|\frac{x_m^{new} - x_m^{old}}{x_m^{new}}\right| \times 100$$

- **Example 1**
  - The floating ball has a specific gravity of 0.6 and has a radius of 5.5cm. You are asked to find the depth to which the ball is submerged when floating in water.
  - The equation that gives the depth x to which the ball is submerged under water is given by

$$x^3 - 0.165x^2 + 3.993\times10^{-4} = 0$$

  - Use the false-position method of finding roots of equations to find the depth to which the ball is submerged under water.

- **Solution**

  - From the physics of the problem

$$0 \le x \le 2R$$
$$0 \le x \le 2(0.055)$$
$$0 \le x \le 0.11$$

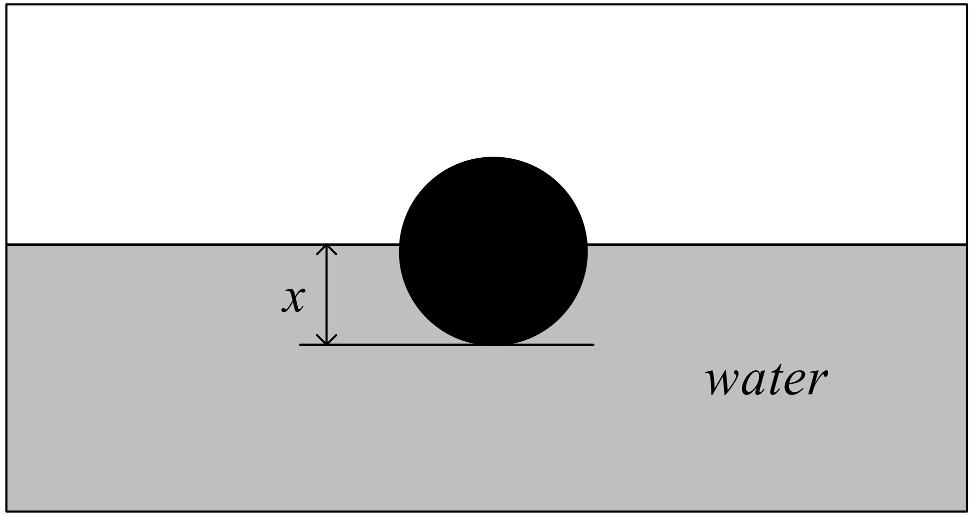


Figure 2 : Floating ball problem

Step-By-Step False-Position Algorithm

$$x^3 - 0.165x^2 + 3.993\times10^{-4} = 0$$

- **Solution**

  - Let us assume $x_L = 0, x_U = 0.11$

$$f(x_L) = f(0) = (0)^3 - 0.165(0)^2 + 3.993\times10^{-4} = 3.993\times10^{-4}$$
$$f(x_U) = f(0.11) = (0.11)^3 - 0.165(0.11)^2 + 3.993\times10^{-4} = -2.662\times10^{-4}$$

Hence,

$$f(x_L)f(x_U) = f(0)f(0.11) = \left(3.993\times10^{-4}\right)\left(-2.662\times10^{-4}\right) < 0$$

Step-By-Step False-Position Algorithm

- **Iteration 1**

$$x^3 - 0.165x^2 + 3.993\times10^{-4} = 0$$

$$x_m = \frac{x_U f(x_L) - x_L f(x_U)}{f(x_L) - f(x_U)}$$

$$= \frac{0.11\times3.993\times10^{-4} - 0\times\left(-2.662\times10^{-4}\right)}{3.993\times10^{-4} - \left(-2.662\times10^{-4}\right)}$$

$$= 0.0660$$

$$f(x_m) = f(0.0660) = (0.0660)^3 - 0.165(0.0660)^2 + \left(3.993\times10^{-4}\right)$$

$$= -3.1944\times10^{-5}$$

$$f(x_L)f(x_m) = f(0)f(0.0660) = (+)(-) < 0$$

$$x_L = 0,\ x_U = 0.0660$$

Step-By-Step False-Position Algorithm

- **Iteration 2**

$$x^3 - 0.165x^2 + 3.993\times10^{-4} = 0$$

$$x_m = \frac{x_U f(x_L) - x_L f(x_U)}{f(x_L) - f(x_U)}$$

$$= \frac{0.0660\times3.993\times10^{-4} - 0\times\left(-3.1944\times10^{-5}\right)}{3.993\times10^{-4} - \left(-3.1944\times10^{-5}\right)}$$

$$= 0.0611$$

$$f(x_m) = f(0.0611) = (0.0611)^3 - 0.165(0.0611)^2 + \left(3.993\times10^{-4}\right)$$

$$= 1.1320\times10^{-5}$$

$$f(x_L)f(x_m) = f(0)f(0.0611) = (+)(+) > 0$$

Hence, $x_L = 0.0611, x_U = 0.0660$

Step-By-Step False-Position Algorithm

- **Iteration 2**

$$x^3 - 0.165x^2 + 3.993\times10^{-4} = 0$$

For, $x_L = 0.0611, x_U = 0.0660$

$$\in_a = \left|\frac{0.0611 - 0.0660}{0.0611}\right| \times 100 \cong 8\%$$

Step-By-Step False-Position Algorithm

- **Iteration 3** $x^3 - 0.165x^2 + 3.993\times10^{-4} = 0$

$$x_m = \frac{x_U f(x_L) - x_L f(x_U)}{f(x_L) - f(x_U)}$$

$$= \frac{0.0660\times1.132\times10^{-5} - 0.0611\times(-3.1944\times10^{-5})}{1.132\times10^{-5} - (-3.1944\times10^{-5})}$$

$$= 0.0624$$

$$f(x_m) = -1.1313\times10^{-7}$$

$$f(x_L)f(x_m) = f(0.0611)f(0.0624) = (+)(-) < 0$$

Step-By-Step False-Position Algorithm

- **Iteration 3** $$x^3 - 0.165x^2 + 3.993\times 10^{-4} = 0$$

Hence,

$$x_L = 0.0611,\ x_U = 0.0624$$

$$\in_a = \left|\frac{0.0624 - 0.0611}{0.0624}\right| \times 100 \cong 2.05\%$$

Step-By-Step False-Position Algorithm

Table 1: Root of $f(x) = x^3 - 0.165x^2 + 3.993\times10^{-4} = 0$ for False-Position Method.

Iteration					
1	0.0000	0.1100	0.0660	N/A	$-3.1944\text{x}10^{-5}$
2	0.0000	0.0660	0.0611	8.00	$1.1320\text{x}10^{-5}$
3	0.0611	0.0660	0.0624	2.05	$-1.1313\text{x}10^{-7}$
4	0.0611	0.0624	0.0632377619	0.02	$-3.3471\text{x}10^{-10}$

Thank you

Question and Suggestion