# 组合

2019年8月9日 10:39

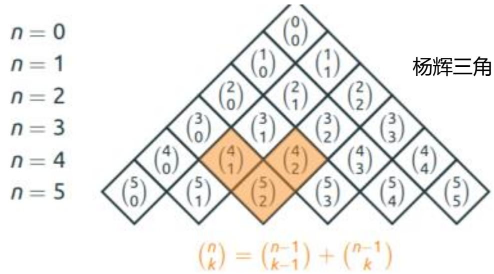
www.coursera.org/learn/combinatorics

如何组一支队伍

- Fix one of the students, call her Alice
- There are two types of teams:
  1. Teams with Alice: $\binom{n-1}{k-1}$
  2. Teams without Alice: $\binom{n-1}{k}$
- Hence,

n choose k

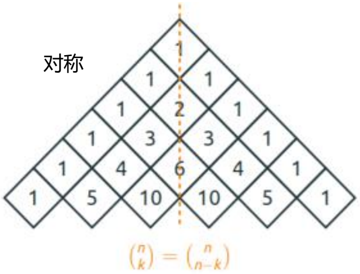
$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$




杨辉三角

屏幕剪辑的捕获时间: 2019/8/9 10:42

对称




$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} = \frac{n!}{(n-k)!\,k!} = \binom{n}{n-k}$$

行和

1 = 1

1 + 1 = 2

1 + 2 + 1 = 4

1 + 3 + 3 + 1 = 8

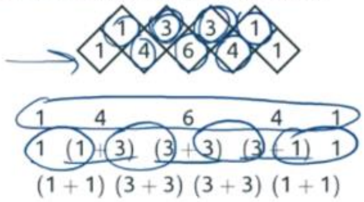
1 + 4 + 6 + 4 + 1 = 16

1 + 5 + 10 + 10 + 5 + 1 = 32

$$\binom{n}{0} + \binom{n}{1} + \cdots \binom{n}{n-1} + \binom{n}{n} = 2^n$$

2.组合证明：大小为n的集合。选or不选，2^n。

- The base case (0-th row) holds
- We'll show that the sum of each row is twice the sum of the previous row:




1.归纳法证明

交替行和

1

1 − 1 = 0

1 − 2 + 1 = 0

1 − 3 + 3 − 1 = 0

1 − 4 + 6 − 4 + 1 = 0

1 − 5 + 10 − 10 + 5 − 1 = 0

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0 \text{, for n} > 0$$

proof: $\binom{n}{0} + \binom{n}{2} + \cdots = \binom{n}{1} + \binom{n}{3} + \cdots$

偶子集 = 奇子集

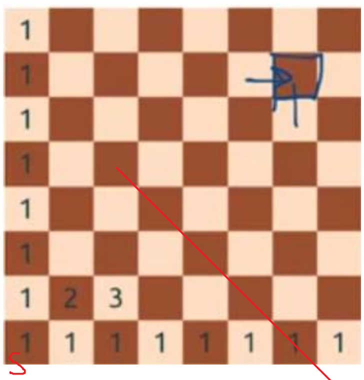
$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^n = \binom{n}{0} a^n + \binom{n}{1} a^{n-1} b + \cdots \binom{n}{k} a^{n-k} b^k + \cdots \binom{n}{n} b^n$$

$$= \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

令a=b=1,即2^n = 行和；令a=1，b=-1，即0 = 交替行和。

二项式系数。




左路径数+下路径数

C[i][j] = C[i-1][j-1] + C[i-1][j]

dp Bottom-up 填表

左下角->此点路径数:总步数6，上走4。 $\binom{6}{4}$