CS412 Machine Learning
HW 3 – Probabilities – Bayesian Learning
100pts

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1) 20 pt - Suppose that we have 3 colored boxes r (red), b (blue) and g (green).
Box r contains 8 apples, 1 oranges and 1 limes;
Box b contains 5 apples, 5 oranges and 0 limes;
Box g contains 3 apples, 3 oranges and 4 limes.

Assume a process **where we pick a box first and then pick a fruit from the selected box**. A box is chosen at random according to the following probability of being selected: p(r) = p(b) = 0.2 and p(g) = 0.6 and a piece of fruit is selected from the **chosen** box randomly.

a) 10 pt – What is the **probability of selecting an lime**?

p(r) * p(l) + p(b) * p(l) + p(g) * p(l) = probability of selecting lime

0.2 * 0.1 + 0.2 * 0 + 0.6 * 0.4 =

0.02 + 0.24 = 0.26

b) 10pt - If we **observe that the selected fruit is a lime**, what is the probability that it came from the green box?

p(g | L) = (p(L | g) * p(g)) / p (L) = 0.4 * 0.6 / 0.26 = 0.24/ 0.26 = 24/26 = 12/13

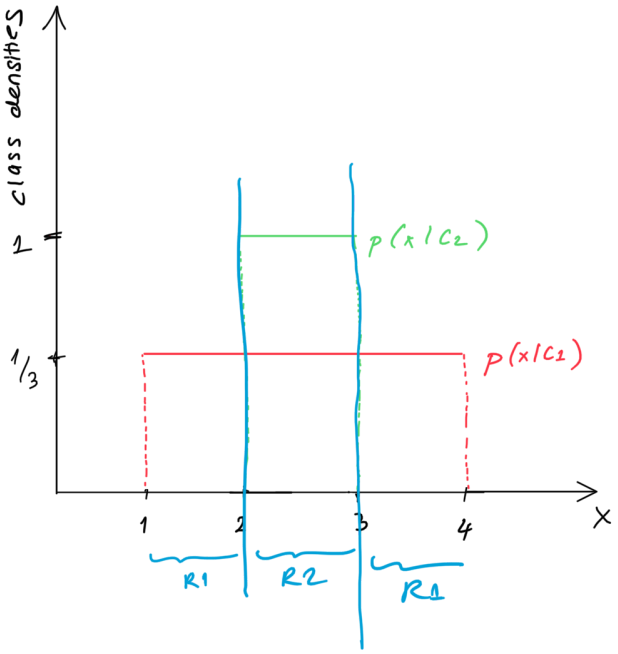
2) 40 pt - For a **1-dimensional** input x, assume we are given the following class conditional probability densities as follows:

$p(x|C_1) = 1/3$ for $1 <= x <= 4$; 0 elsewhere

$p(x|C_2) = 1$ for $2 <= x <= 3$; 0 elsewhere

Assume $P(C_1)=P(C_2)=0.5$.

a) 15pt – Draw the corresponding $p(x|C_i)$, **being <u>as precise/specific as possible</u>** (e.g. label axes and important points on the axes). You can draw by hand, take a picture and include here as image.



b) 10pts - Develop a classification strategy for a given x (<u>just looking at the graph – no formula</u>), just complete the sentence(s):

`if x is in the region **[1,2] U [3,4]** , I will classify it as **C1**;

if x is in the region **[2,3]** , I will classify it as **C2**;

c) 5pts - Draw the decision regions on the above figure.

R1 refers to the decision region of P (x | C1)* P (C1),

R2 refers to the decision region of P (x | C2)* P (C2).

d) 10pts – Give a **one line <u>qualitative</u> answer** (no precise numbers/threshold**s...)** about **how your decision changes or whether it doesn't.**

- Would your decision strategy change if $P(C_1)=0.9$ and $P(C_2)=0.1$?

  My decision strategy would change. When we have these values as prior probabilities, the result of $P(C1)^* p(x|C_1)$ will always be higher than $P(C2)^* p(x|C_2)$, which means regardless of the x's values we will classify it as C1.

- How about if it was the reverse $P(C_1)=0.1$ and $P(C_2)=0.9$?

My decision strategy wouldn't change, because the dominancy relationship between the results of P(C1)* $p(x|C_1)$ and P(C2)* $p(x|C_2)$ are the same for the same intervals.

3) 40pts – NAIVE BAYES

a) 15pts – Given that two random variables X and Y are conditionally independent given C, circle True or False (2pts for each correct answer; -1pts each wrong answer):

- P(X| Y) = P(X) True / False
- P(X | Y, C) = P(X | Y) True / False
- P(X , C | Y) = P(X | Y) True / False
- P(X ,Y | C) = P(X | C) P(Y | C) True / False
- P(X ,Y , C) = P(X | C) P(Y | C) P(C) True / False

b) 20pts - Using the PlayTennis data given below (and in the lecture slides), **how would you classify x=(Overcast, Mild, Normal, Strong), using Naive Bayes classifier *without any smoothing*. Show your work** (e.g. indicate class conditional attribute probabilities under the given table in the next page and just transfer them here).

P(Yes| x) = P(Outlook=Overcast | Yes) * P(Temperature=Mild | Yes) * P(Humidity=Normal | Yes) * P(Wind =Strong | Yes) * P(Yes)

= (4/9 * 4/9 * 6/9 * 3/9) * 9/ 14 = 0,282

P(No| x) = P(Outlook=Overcast | No) * P(Temperature=Mild | No) * P(Humidity=Normal | No) * P(Wind =Strong | No) * P(No)

= 0

Decision: P(Yes| x)

Day	Outlook	Temperature	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Write here the estimated probabilities (write those related to the question):

P(Yes) = 9/ 14

P(Outlook=Overcast | Yes) = 4 / 9

P(Temperature=Mild | Yes) = 4 / 9

P(Humidity=Normal | Yes) = 6 / 9

P(Wind =Strong | Yes) = 3 / 9

P(No) = 5/14

P(Outlook=Overcast | No) = 0

P(Temperature=Mild | No) = 2 / 5

P(Humidity=Normal | No) = 1 / 5

P(Wind =Strong | No) = 3 / 5

c) 5pts - Indicate the values **for only the following probabilities** estimated during Naive Bayes training, **using Laplace smoothing:**

P(Outlook=Overcast|No) = 0 + 1 / 5 + 3 = 1 / 8

P(Humidity=Normal|Yes) = 6 + 1 / 9 + 2 = 7 / 11