MECH 307 Numerical Methods for Mechanical Engineering
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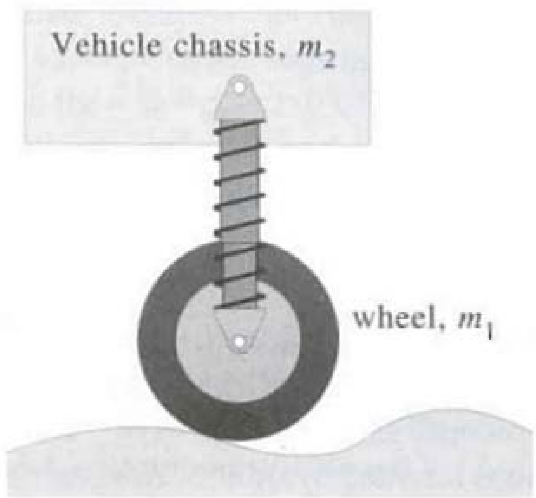
Design Project #3

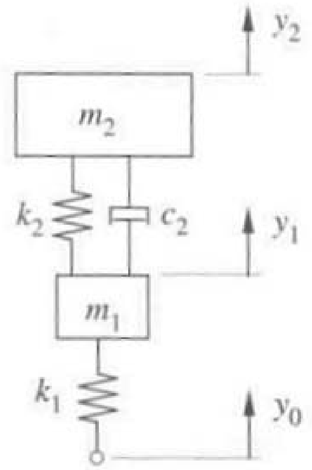
FORMAT of the file to be submitted:

1. **All the m-files should be named as pr1.m, pr2.m and so on.**
2. **The results (figure, table, or individual result such as x = 2.653, …, and any comment) should be placed in a WORD file named as yourlastname_DP_03.doc**
3. **All the m-files should be inserted at the end of the WORD file using COURIER 9 font.**
4. **The WORD file and all the m-files should be ZIPPED together, and the file should be named as yourlastname_DP_03.zip or (or alternatively as yourlastname_DP_03.rar).**
5. **Place the file to the following folder: F:\COURSES\UGRADS\MECH\MECH307\HOMEWORK\...**

Problem 1.

The following is a simplified model of the suspension system of one wheel of an automobile:





The input to the system is the time-varying displacement y_0 (t) corresponding to changes in the road elevation.
The shock absorber is characterized by its spring coefficient k_2 and damping coefficient c_2.
Damping in the tire is neglected. (There is no c_1 term.)
Applying Newton's law of motion and force balances to the wheel and vehicle chassis yields the following system of equations:

$$m_1\ddot{y}_1 + c_2(\dot{y}_1 - \dot{y}_2) + k_2(y_1 - y_2) + k_1 y_1 = k_1 y_0(t)$$

$$m_2\ddot{y}_2 - c_2(\dot{y}_1 - \dot{y}_2) - k_2(y_1 - y_2) = 0$$

a) Convert these two second-order equations into an equivalent system of first-order equations.
b) Use m_1 = 14 kg, k_1 = 110 N/m, m_2 = 380 kg, k_2 = 520 N/m, c_2 = 25 Ns/m. Forcing function is dependent on the road's surface and how fast the car is being driven. Use $y_0(t)$ = 0.12 sin(4 t). Plot y_1(t) and y_2(t) versus time t in $0 \le t \le 10$ seconds.
c) Animate the vibration of the system (two masses, springs and the ground position) clearly describing the vibration along with the y values printed as the title of the figure. (Recall the similar animation that we studied in the class for one mass-spring-damper system.) (Include the final frame of the animation into your report.)
d) Repeat the solution of (c) such that the car is driven with a speed which is doubled now. Hint: the forcing function will be affected such that the frequency is different now. Do you expect the amplitude of y_0 to be different as well or not?
e) Repeat the solution of (c) with c_2 increased by a factor of 20.
f) Describe the change in behavior of the system caused by changing the car speed and damping coefficient c_2.

Note:

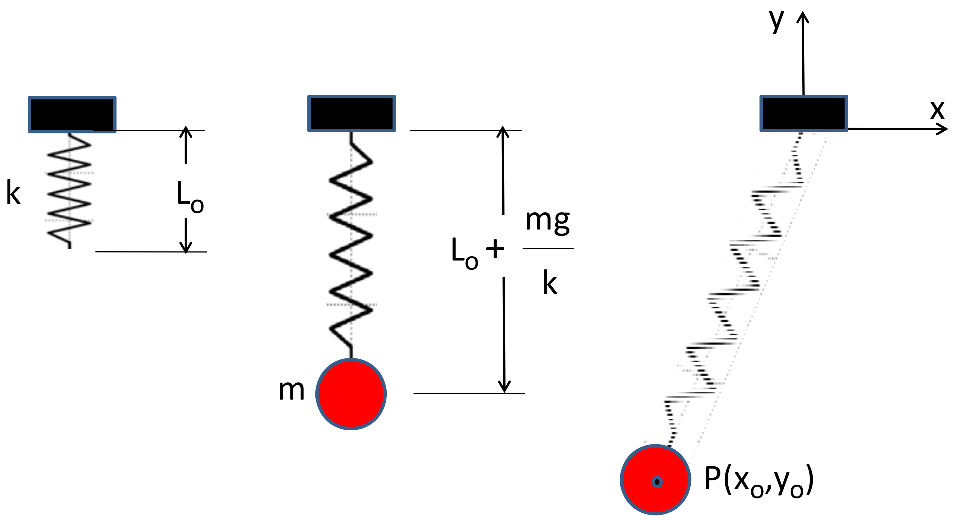
This is a forced-vibration case, and the initial conditions of y_1(t) and y_2(t) are not important. You can take the initial displacement and the initial velocities (the derivatives of y_1(t) and y_2(t)) to be **zero**.

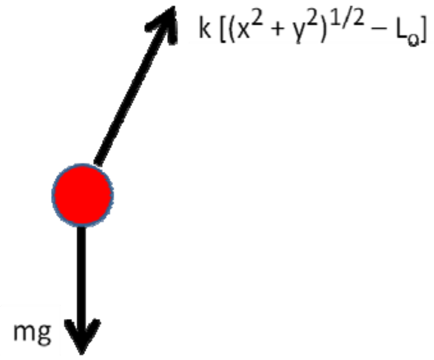
Problem 2.

A mass m is suspended from the upper horizontal wall by using a spring. The spring has a stiffness coefficient of k, and an undisturbed length of L_O. Initially, we move the mass to point $P(x_O, y_O)$, and then let it vibrate freely. Applying Newton's law of motion and force balances, the components of the inertial forces are written as follows:

$$ma_x = m\frac{d^2x}{dt^2} = -k\left(\sqrt{x^2+y^2} - L_0\right)\frac{x}{\sqrt{x^2+y^2}}$$

$$ma_y = m\frac{d^2y}{dt^2} = -mg - k\left(\sqrt{x^2+y^2} - L_0\right)\frac{y}{\sqrt{x^2+y^2}}$$





a) Convert these two second-order equations into an equivalent system of first-order equations.
b) Use m = 1 kg, k = 12 N/m, L_O = 1 m, $P(x_O, y_O)$ = (–1.2,–2.6) m. Plot x(t) and y(t) versus time in $0 \le t \le 10$ seconds.
c) Animate the vibration of the system (mass and the spring) clearly describing the vibration along with the x and y values printed as the title of the figure. You may represent the spring as a single straight line.
d) If $P(x_O, y_O)$ = (0,–1.8175) m, what do you expect to see? Does your code simulate what you expect?