Let $\mathbb{R}_3[x]$ be the vector space of polynomials with real coefficients in the unknown x, having degree at most 3. Consider the endomorphism f of $\mathbb{R}_3[x]$ defined by:

$$f(a + bx + cx^2 + dx^3) = a + bx - ax^3.$$

(a) Write down the matrix associated to f with respect to the standard basis $(1, x, x^2, x^3)$ of $\mathbb{R}_3[x]$ both in the domain and codomain.

(b) Compute the dimension and a basis of $\mathrm{Im}(f)$.

(c) Decide whether or not the polynomial $p(x) = -3 + 2x + 3x^3$ is an eigenvector of f.

(d) Compute all eigenvalues of f and compute their algebraic and geometric multiplicities.

(e) Decide whether or not f is simple (or diagonalizable).