Leveraging Knowledge Profiles and Generative AI for Realistic Student Response Generation

1 Errors & Mistakes Per Rubric

See Table 1.

2 Prompt Example

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"You are a text generator that generates realistic
   student responses for a proof by contraposition
   question. Your role is to simulate student answers
   based on provided knowledge profiles and specific
   mistakes, which will be explained in more detail
   along with examples. Avoid including any
   evaluative language or comments that could bias
   the responses, as the generated data is used to
   train classifiers. Maintain a neutral,
   student-like tone and focus on coherence and
   clarity in your responses.

TASK DESCRIPTION:
Students are expected to answer this task:
"Let x be a real number. Prove that if x^2 is
   irrational, then x is irrational using a proof by
   contraposition.

Reminders:
- A real number x is irrational if it cannot be
   expressed as the quotient of two integers - that
   is, p/q where p and q (both integers) are not zero.
- A real number x is rational if it can be expressed
   as the quotient of two integers - that is, p/q
   where p and q (both integers) are not zero.

SOLUTION:
As a reference, here is the correct solution for this
   problem:
A proof by contraposition of an implication consists
   in showing that if x is rational, then x^2 is
   rational.
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1. x is rational [Assumption]
2. x = a/b, where a and b ̸= 0 are integers with no
   common factors. [Definition of a rational number]
3. Therefore, x^2 = a^2/b^2. [Squaring]
4. Hence, x^2 is rational. [Definition of a rational
   number]
5. By contraposition, if x^2 is irrational, then x is
   irrational.

GRADING RUBRICS:

Students are graded using 4 binary rubrics:

1. Statement of what should be proven: A proof by
   contraposition of an implication consists in
   showing that if x is rational, then x^2 is
   rational.
2. Correct assumption: x is rational
3. Correct proof reasoning
4. Proof conclusion: By contraposition, if x^2 is
   irrational, then x is irrational.

It is acceptable if students do not write the exact
   same answer as the solution. If the response
   satisfies the rubrics more or less, the student
   receives the point.

KNOWLEDGE PROFILES:

We identify a student's knowledge profile based on
   their score in these 4 rubrics. For example:

- A student with all rubrics incorrect will have the
   profile 0-0-0-0.
- A student who has rubric 1 and 2 correct but 3 and 4
   incorrect will have the profile 1-1-0-0.

TARGET KNOWLEDGE PROFILE AND MISTAKE:

I want you to generate answers for 2 students with the
   knowledge profile 0000. This knowledge profile
   represents student answers where the student fails
   to correctly state what should be proven (rubric
   1), does not make the correct assumption (rubric
   2), lacks correct proof reasoning (rubric 3), and
   does not conclude the proof correctly (rubric 4).
   A common mistake students with this profile make
   is: The student incorrectly assumes x^2 is
   rational instead of assuming x is rational, which
   leads them to misapply the proof technique from
   the beginning.

You will generate student answers with this profile
   and this mistake.
Please ensure that each answer demonstrates the
   specified knowledge profile characteristics in
   addition to the specific mistake. Do not just
   generate an answer with the mistake and a correct
   proof otherwise; the answer should reflect both
   the knowledge profile and the mistake.
The generated answers have to be different and
   diverse, do not just paraphrase the given examples.

Example answers: - 1. CONTRAPOSITION
  (x POWER 2) is irrational -> x is irrational
  (x POWER 2) is rational) -> x is rational:
  (x POWER 2) is rational -. (x POWER 2) = a/b with a
     and b integers and b different than 0.
     definition
  x = a/b

     asumption
  (x POWER 2) = (a POWER 2)/(b POWER 2)
                     squaring
  a integer ->. (a POWER 2) integer
  b integer different from 0 -> (b POWER 2) integer
     different from 0
  so (x POWER 2) is rational.

- 1. CONTRAPOSITION
  1) the opposite statement would be that if x^2 is
     rational then x is rational
  2) x^2 = a^2/b^2
  3) x^2 is rational so is x

- 1. CONTRAPOSITION
  Supposer that (x power 2) is not irrational
  If (x power 2) si not irrational, then x is also not
     irrational.

- 1. CONTRAPOSITION
  1) assume that x^2 is rational [premise]
  2)

#	Rubric	Description	Correct Solution, Errors and Mistakes
R1	Statement of what should be proven	A proof by contraposition of an implication requires showing that if x is rational, then x^2 is rational	Correct solution: A proof by contraposition of an implication consists of showing that if x is rational, then x^2 is rational. Errors: − Contraposition statement to be proven is missing − Contradiction statement instead of contraposition − if x^2 is rational, then x is rational − if x is irrational, then x^2 is irrational
R2	Correct assumption	The proof should correctly assume x is rational	Correct solution: x is rational. [Assumption] Errors: − Assumption is missing − Assuming x^2 is irrational − Assuming x^2 is rational − Assuming x is irrational
R3	Correct proof reasoning	The reasoning in the proof must be logically sound and consistent with the contraposition method.	Correct Solution: Then we have $x = \frac{a}{b}$, where a and $b \neq 0$ are integers with no common factors. [Definition of a rational number] Therefore, $x^2 = \frac{a^2}{b^2}$. [Squaring] Hence, x^2 is rational. [Definition of a rational number] Errors: − Proof is missing − Proof is by contradiction − Proof is incomplete Mistakes: − Assumption that a and b are integers is missing − Assumption that a and b have no common factors is missing − Expressing $a^2 = c$ and $b^2 = d$ so $x^2 = \frac{c}{d}$
R4	Proof conclusion	The proof should conclude that, by contraposition, if x^2 is irrational, then x is irrational.	Correct Solution: By contraposition, if x^2 is irrational, then x is irrational. Errors: − Conclusion is missing − Contraposition statement as conclusion − x irrational $\rightarrow x^2$ irrational

Table 1. Rubric-based assessment for student proofs