Runtime Analysis

Part A:

k		i	
1	2	2^1	$2^{2^k} < n$
2	4	2^2	
3	16	2^4	$k < \log(\log(n))$
4	256	2^8	

$$T(n) = O(\log(\log(n))) = \Theta(\log(\log(n)))$$

Part B:

$$T(n) = \sum_{i=1}^{n} \left(\Theta(1) + O\left(\sum_{k=0}^{i^{3}-1} \Theta(1)\right)\right) \Rightarrow \sum_{i=1}^{n} \Theta(1) + \sum_{i} \sum_{k=0}^{n-1} \Theta(1)$$

Outer loop

Inner loop

k	1	2	3	j
i	$\sqrt{n}$	$2\sqrt{n}$	$3\sqrt{n}$	$j\sqrt{n}$

Stop when $k = ?$

Stop when $i = n$, $i = j\sqrt{n}$, $j\sqrt{n} = n$, $j = \sqrt{n}$

$$= \boxed{\Theta\left(n^{\frac{7}{2}}\right)}$$

$$T(n) = \Theta(n) + \sum_{j=1}^{\sqrt{n}} \sum_{k=0}^{i^{3}-1} \Theta(1) = \Theta(n) + \sum_{j=1}^{\sqrt{n}} \Theta(n^3) = \Theta(n) + \Theta(n^3 \cdot \sqrt{n})$$

c)

Data - dependent

- worst-case scenario, if statement true $O(n)$ times

for (int m = 1; m ≤ n; m = m+m)

k	1	2	3	4	5
i	2^0	2^1	2^2	2^3	2^4

$$2^{k-1} \le n$$

$$k-1 \le \log n$$

$$k \le \log(n)+1$$

$$\Theta(n \log n)$$

$$T(n) = \Theta(n^2) + \Theta(n \log n) = \boxed{\Theta(n^2)}$$

d)

$$T(n) = \sum_{i=0}^{n-1} \le n + \sum_{i=0}^{\log n} \frac{3}{2}^i = n + \frac{3}{2}n - 1 = \frac{5}{2}n - 1$$

k	0	1	2	3
i	10	15	22	33

$$10 \cdot \left(\frac{3}{2}\right)^k = n$$

$$k \log\left(\frac{3}{2}\right) = \log\left(\frac{n}{10}\right)$$

$$k = \log_{3/2}\left(\frac{n}{10}\right)$$

$$\Theta(n) + \Theta\left(10 \sum_{j=0}^{k} \left(\frac{3}{2}\right)^j\right)$$

$$= \Theta(n) + \Theta\left(10 \sum_{j=0}^{\log_{\frac{3}{2}}\left(\frac{n}{10}\right)} \left(\frac{3}{2}\right)^j\right)$$

$$10\left(\Theta\left(\frac{n}{10}\right)\right) = \Theta(n)$$

$$T(n) = \Theta(n)$$