

$$1a. \quad k_0 + k_0' P = -v \left(\frac{\partial P}{\partial v} \right)$$

$$= - \frac{1}{\frac{1}{v} \left(\frac{\partial v}{\partial P} \right)} = - \frac{1}{\frac{\partial \ln v}{\partial P}} = - \left(\frac{\partial P}{\partial \ln v} \right)$$

$$k_0 + k_0' P = - \frac{\partial P}{\partial \ln v}$$

$$- \frac{1}{k_0 + k_0' P} = \frac{\partial \ln v}{\partial P}$$

$$- \int \frac{1}{k_0 + k_0' P} dP = \ln v$$

$$- \frac{\ln(k_0 + k_0' P)}{k_0'} + C = \ln v$$

$$\text{At } P=0$$

$$C = \ln v_0 + \frac{\ln(k_0)}{k_0'}$$

$$- \frac{\ln(k_0 + k_0' P)}{k_0'} + \ln v_0 + \frac{\ln(k_0)}{k_0'} = \ln v$$

$$\ln \left(\frac{v_0}{v} \right) + \frac{\ln(k_0)}{k_0'} = \frac{\ln(k_0 + k_0' P)}{k_0'}$$

$$k_0' \ln \left(\frac{v_0}{v} \right) + \ln(k_0) = \ln(k_0 + k_0' P)$$

$$\ln \left(\frac{v_0^{k_0'}}{v} \right) + \ln(k_0) = \ln(k_0 + k_0' P)$$

$$k_0 \left(\frac{v_0}{v} \right)^{k_0'} = k_0 + k_0' P$$

$$\frac{k_0 \left(\frac{v_0}{v} \right)^{k_0'} - k_0}{k_0'} = P$$

$$\frac{k_0}{k_0'} \left[\left(\frac{v_0}{v} \right)^{k_0'} - 1 \right] = P$$

$$b. \quad P = - \left(\frac{\partial U}{\partial V} \right)_{T, n}$$

$$\frac{k_0}{k_0'} \left[\left(\frac{V}{V_0} \right)^{-k_0'} - 1 \right] = - \left(\frac{\partial U}{\partial V} \right)_{T, N}$$

$$\frac{k_0}{k_0'} \cdot \left(\frac{V}{V_0} \right)^{-k_0'} - \frac{k_0}{k_0'} = - \left(\frac{\partial U}{\partial V} \right)_{T, N}$$

$$\frac{k_0 V_0^{k_0'}}{k_0'} \int \frac{1}{V^{k_0'}} - \frac{k_0 \cdot V}{k_0'} + C = U(V)$$

$$\frac{k_0 V_0^{k_0'}}{k_0'} \left(\frac{V^{k_0'-1}}{1-k_0'} \right) - \frac{k_0}{k_0'} \cdot V + C = U(V)$$

$$\text{At } V = V_0$$

$$\frac{k_0 V_0^{k_0'}}{k_0'} \left(\frac{V_0^{k_0'-1}}{1-k_0'} \right) - \frac{k_0 V_0}{k_0'} + C = U(V_0)$$

$$C - U(V_0) = \frac{k_0 V_0}{k_0'} - \frac{k_0 V_0^{k_0'}}{k_0'} \left(\frac{V_0^{k_0'-1}}{1-k_0'} \right)$$

$$= \frac{k_0}{k_0'} \left(V_0 - \frac{V_0^{k_0'} V_0^{k_0'-1}}{1-k_0'} \right)$$

$$C = \frac{k_0}{k_0'} \left(V_0 - \frac{V_0^{2k_0'-1}}{1-k_0'} \right) + U(V_0)$$

$$U(V) = \frac{k_0 V_0^{k_0'}}{k_0'} \left(\frac{V^{k_0'-1}}{1-k_0'} \right) - \frac{k_0 V}{k_0'} + \frac{k_0}{k_0'} \left(V_0 - \frac{V_0^{2k_0'-1}}{1-k_0'} \right) + U(V_0)$$