Use Python and Matlab for all questions below (unless some derivation is required which you can do better by hand). If using Python, use Jupyter notebook to format properly, comment when necessary, and include all required and necessary outputs (Jupyter allows LATEX embedding which would be great if you could use it). Convert the notebook to pdf and upload to blackboard against the assignment. If using MATLAB, use MATLAB publish to convert to pdf. Solution documents should be well commented.

Load the file dataHW6.mat in your matlab workspace. This will load variables named f,x,X,Y,Z.

Q1. x is a 1000 element vector with elements $x_i, i=1,2\ldots1000$. f is a corresponding 1000 element vector which is a function of x and with values $f(x_i), i=1,2\ldots1000$.

a. Create a scatter plot of f vs. x
b. Use the central difference formula to find the first derivative $f'=df/dx$ at all x_i values except the first and the last ones (since you cannot calculate central difference at these points.) You should do it in a loop to automate the process. This will give you a new vector $f'(x_i), i=2,3,\ldots999$. Plot $f'(x_i)$ from central difference against a new x vector which is derived from the original x vector by leaving out only the first and last terms.
c. Use the central difference formula to find the **second** derivative $f''=d^2f/dx^2$ at all x_i values except the first and the last ones (since you cannot calculate central difference at these points.) You should do it in a loop to automate the process. This will give you a new vector $f''(x_i), i=2,3,\ldots999$. Plot $f''(x_i)$ from central difference against a new x vector which is derived from the original x vector by leaving out only the first and last terms.

Q2. The central span of Golden Gate bridge is 4200 ft long. The shape of the main suspension cables is approximately given by the equation:

$$y=f(x)=C\left(\frac{e^{x/C}+e^{-x/C}}{2}-1\right); -2100\le x\le 2100\,ft, C=4491$$

a. Plot the equation for the suspension cable as a function of span x within the range specified above.
b. The length of the cable is given by $L=\int_a^b \sqrt{1+(dy/dx)^2}\,dx$ where a,b are the left and right limits of x. Find the total length of the bridge by using the trapz function.

Q3. The force per unit length f that is exerted by the wind on a sail is a function of the height z:

$$f=160\frac{z}{z+4}e^{-z/8}\,lb/ft$$

What is the total wind force on a sail of height 24 ft? You will need to integrate $f(z)$ between $z=0,24$.

Q2. Use ode45/odeint/ivp_solve for this problem. Consider the equation of the pendulum as derived in the first class

$$L\frac{d^2\theta}{dt^2}+g\sin\theta=0\,;\theta(0)=A\,;\frac{d\theta}{dt}(0)=B$$

where L is the length of the pendulum and g is the acceleration due to gravity.

a. Write the above second order ODE as a system of two first order ODEs. Clearly specify the resulting initial conditions as well.

b. Consider the initial conditions $A=\frac{\pi}{100}, B=0$ and length of the pendulum $L=g/(2\pi)^2$ meters. For these parameters plot the solution $\theta(t)$ for $0\le t\le 2$. For such a small initial angle, it is possible that the differential equation can be linearized as we did in the first class. In this case the solution may be approximated by $\theta(t)=A\cos\left(\sqrt{\frac{g}{L}}t\right)$. Plot this linearized solution on the same plot as in c and for the same time period. Is there a good match?

c. Consider the initial conditions $A=\frac{\pi}{10}, B=0$ and length of the pendulum $L=g/(2\pi)^2$ meters. For these parameters plot the solution $\theta(t)$ for $0\le t\le 2$. Can you say whether the linearized formulation still applies to this case or not?

d. Consider the initial conditions $A=\frac{\pi}{2}, B=0$ and length of the pendulum $L=g/(2\pi)^2$ meters. For these parameters plot the solution $\theta(t)$ for $0\le t\le 2$. Can you say whether the linearized formulation still applies to this case or not?

e. Consider the initial conditions $A=\pi, B=0$ and length of the pendulum $L=g/(2\pi)^2$ meters. For these parameters plot the solution $\theta(t)$ for $0\le t\le 2$. How do you explain this result physically?

f. Consider the initial conditions $A=\pi, B=.1$ and length of the pendulum $L=g/(2\pi)^2$ meters. For these parameters plot the solution $\theta(t)$ for $0\le t\le 2$. How do you explain this result physically?