```
    0.2093
    0.8229
    0.1041
   -0.1572
   -0.1973
    0.6288
```

Q5: Check to see if the eigenfunctions are also normalized properly: $\int_0^L \rho \cdot Y_i(x) \cdot Y_i(x) = 1$

$$\int_0^L \rho \cdot Y_i(x) \cdot Y_i(x) = \rho \cdot {B_i}^2 \cdot \int_0^L sin^2\left(\frac{i \cdot \pi}{L} \cdot x\right) dx = -\rho \cdot {B_i}^2 \cdot \frac{L \cdot (\sin(2\pi i) - 2\pi i)}{4\pi i} = -\rho \cdot {B_i}^2 \cdot -\frac{L}{2} \neq 1$$

$$unless \quad B_i = \sqrt{\frac{2}{\rho \cdot \mathrm{L}}}$$

Using the following code I will try to demonstrate that eigenfunctions are not normalized properly. For this, I considered the constant B has a value of 1.

Matlab code:

```
%% Normalization
Y1_1 = @(x) sin(pi*(x)).*sin(pi*(x));
Y2_2 = @(x) sin(2*pi*(x)).*sin(2*pi*(x));
Y3_3 = @(x) sin(3*pi*(x)).*sin(3*pi*(x));
Y4_4 = @(x) sin(4*pi*(x)).*sin(4*pi*(x));
Y5_5 = @(x) sin(5*pi*(x)).*sin(5*pi*(x));

norm1_1= integral(Y1_1,0,1);
norm2_2= integral(Y2_2,0,1);
norm3_3= integral(Y3_3,0,1);
norm4_4= integral(Y4_4,0,1);
norm5_5= integral(Y5_5,0,1);
norm =[norm1_1; norm2_2; norm3_3; norm4_4; norm5_5];
```

Output of the code:

```
norm =

    0.5000
    0.5000
    0.5000
    0.5000
    0.5000
```

To properly normalize $Y_n(x)$, it is precise to calculate the value of constant B for each case:

$$\boldsymbol{Y_n(x) = B_N \cdot \sin\left(\frac{n \cdot \pi}{L} \cdot x\right)}$$

$$\int_0^L \rho \cdot Y_N(x) \cdot Y_N(x) = \rho \cdot B_N^2 \cdot \int_0^L sin^2\left(\frac{n \cdot \pi}{L} \cdot x\right) dx = 1$$