$$\rho \cdot B_N^2 \cdot \frac{L}{2} = 1$$

$$B_N = \sqrt{\frac{2}{\rho \cdot L}}$$

The properly normalized eigenvectors have the following expression:

$$\boldsymbol{\overline{Y_n}(x) = \sqrt{\frac{2}{\rho \cdot L}} \cdot \sin\left(\frac{n \cdot \pi}{L} \cdot x\right) \qquad n = 1, 2 \ldots.}$$

<u>Q6: Show that the second orthogonality holds:</u> $\int_0^L \mathrm{T} \cdot \overline{Y_i'(x)} \cdot \overline{Y_j'(x)}\, dx = \omega_i^2 \cdot \delta_{ij}$

$$\overline{Y_i'(x)} = \sqrt{\frac{2}{\rho \cdot L}} \cdot \frac{i \cdot \pi}{L} \cdot \cos\left(\frac{i \cdot \pi}{L} \cdot x\right)$$

$$\int_0^L \mathrm{T} \cdot \overline{Y_i'(x)} \cdot \overline{Y_j'(x)} = T \cdot \frac{2}{\rho \cdot L} \cdot \left(\frac{\pi}{L}\right)^2 \cdot i \cdot j \cdot \int_0^L \cos\left(\frac{i \cdot \pi}{L} \cdot x\right) \cdot \cos\left(\frac{j \cdot \pi}{L} \cdot x\right) dx$$

Considering that:

$$\omega_i = \frac{i\pi}{L} \sqrt{\frac{T}{\rho}}$$

For i=j:

$$\int_0^L \mathrm{T} \cdot \overline{Y_i'(x)} \cdot \overline{Y_j'(x)} = T \cdot \frac{2}{\rho \cdot L} \cdot \left(\frac{\pi}{L}\right)^2 \cdot i^2 \cdot \int_0^L cos^2\left(\frac{i \cdot \pi}{L} \cdot x\right) dx = \frac{2}{L} \omega_i^2 \cdot \frac{L sin(2\pi i) + 2\pi L i}{4\pi i} = \omega_i^2$$

For i =/j:

$$\int_0^L \mathrm{T} \cdot \overline{Y_i'(x)} \cdot \overline{Y_j'(x)} = T \cdot \frac{2}{\rho \cdot L} \cdot \left(\frac{\pi}{L}\right)^2 \cdot i \cdot j \cdot \int_0^L \cos\left(\frac{i \cdot \pi}{L} \cdot x\right) \cdot \cos\left(\frac{j \cdot \pi}{L} \cdot x\right) dx =$$

$$T \cdot \frac{2}{\rho \cdot L} \cdot \left(\frac{\pi}{L}\right)^2 \cdot i \cdot j \cdot \frac{L \cdot \big((i-j) \cdot \sin(\pi i + \pi j) + (i+j) \cdot \sin(\pi i - \pi j)\big)}{2\pi \cdot (i^2 - j^2)} = 0$$

Because $\sin(\pi i + \pi j) = \sin(\pi i - \pi j) = 0 \ \forall i, j \in N$