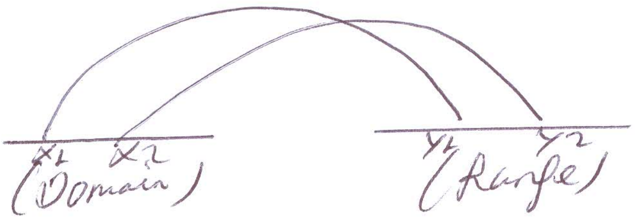
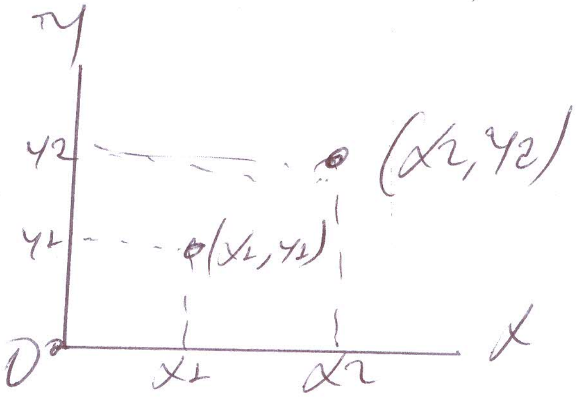
Functions

- y is said to be a function of x, and this denotes by $y = f(x)$
- A function is therefore a set of ordered pairs with the property that any "x" value uniquely determines a "y" value.
- A functions is also called a mapping, or transformation : ~~both words~~
- In the statement $y = f(x)$, the functional notation f may thus be interpreted to mean a rule by which the set "X" is "mapped" ("transformed") into the set "Y"

$$f : X \to Y$$





- In the function $y = f(x)$, x is referred to as the argument of the function, and y is called value
- alternatively referred to x as the independent variable and y as dependent variable
- all the set of x value domain
- all the set of y value range.

Types of functions (Read ~~memorize~~ by yourself)

The basics of Matrix algebra

A matrix is a two dimensional rectangular array of numbers:

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

- There are n columns each with m elements or m rows each with n elements.
- We say that the size of A is $m \times n$
  - If $m = n$, then A is a square matrix
  - A $m \times 1$ matrix is called a column vector and
  - A $1 \times n$ matrix is ~~just a number~~ a row vector
  - A 1×1 matrix is just a number, called a scalar number.

Matrix Operations

- Addition and Subtraction is possible when size (A) = size (B)

e.g. $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$A + B = \begin{pmatrix} 2 & 2 \\ 3 & 5 \end{pmatrix}$$

$$A - B = \begin{pmatrix} 0 & 2 \\ 3 & 3 \end{pmatrix}$$

$$4A = \begin{pmatrix} 4 & 8 \\ 12 & 16 \end{pmatrix}$$

Read Others by Ur Self (Reading 2)

Matrix representation of a linear Simultaneous equation System

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cancel{\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}} = A \cdot B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$

$$B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 5+14 & 6+16 \\ 15+28 & 18+32 \end{pmatrix}$$

$$AB = \begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix}$$

A linear simultaneous equation system

$$a_{11}X_1 + \cdots + a_{1n}X_n = b_1$$

$$\vdots$$

$$a_{n1}X_1 + \cdots + a_{nn}X_n = b_n$$

Define $A \equiv \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$, $X = \begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix}$ and $b \equiv \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$

Then the equation $Ax = b$ is equivalent to simultaneous equation system

Example: Income determination model:

$$\begin{aligned} C &= a + bY \\ I &= I(r) \\ Y &= C + I \end{aligned} \Rightarrow \begin{pmatrix} 1 & 0 & -b \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{pmatrix}\begin{pmatrix} C \\ I \\ Y \end{pmatrix} = \begin{pmatrix} a \\ I(r) \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} C & 0 & -bY & = a \\ 0 & I & 0 & = I(r) \\ C & I & -Y & = 0 \end{pmatrix} \Rightarrow \begin{pmatrix} C - bY = a \\ I = I(r) \\ C + I - Y = 0 \end{pmatrix}$$

Economic applications of matrix.

⇒ Linear 2. market model:

$$\begin{pmatrix} a_1 - b_1 & a_2 - b_2 \\ \alpha_1 - \beta_1 & \alpha_2 - \beta_2 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} b_0 - a_0 \\ \beta_0 - \alpha_0 \end{pmatrix}$$

$$p_1 = \frac{\begin{vmatrix} b_0 - a_0 & a_2 - b_2 \\ \beta_0 - \alpha_0 & \alpha_2 - \beta_2 \end{vmatrix}}{\begin{vmatrix} a_1 - b_1 & a_2 - b_2 \\ \alpha_1 - \beta_1 & \alpha_2 - \beta_2 \end{vmatrix}}$$

$$p_2 = \frac{\begin{vmatrix} a_1 - b_1 & b_0 - a_0 \\ \alpha_1 - \beta_1 & \beta_0 - \alpha_0 \end{vmatrix}}{\begin{vmatrix} a_1 - b_1 & a_2 - b_2 \\ \alpha_1 - \beta_1 & \alpha_2 - \beta_2 \end{vmatrix}}$$

⇒ Income Determination model:

$$\begin{pmatrix} 1 & 0 & -b \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} C \\ I \\ Y \end{pmatrix} = \begin{pmatrix} a \\ I(r) \\ 0 \end{pmatrix}$$

$$C = \frac{\begin{vmatrix} a & 0 & -b \\ I(r) & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{vmatrix}}, \quad I = \frac{\begin{vmatrix} 1 & a & -b \\ 0 & I(r) & 0 \\ 1 & 0 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & 0 \\ 1 & 1 & -b \end{vmatrix}}$$

$$Y = \frac{\begin{vmatrix} 1 & 0 & a \\ 0 & 1 & I(r) \\ 1 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{vmatrix}}$$

⇒ IS – LM Model

Good market:

$C = a + bY$

$I = I_0 - iR$

$C + I + \bar{G} = Y$

Money market:

$L = KY - IR$

$M = \bar{M}$

$M = L$

then,

Good market:

$a + bY + I_0 - iR + G = Y$

Money market

$KY - hR = \bar{M}$

Endogenous Variables:

Y, R

$$\begin{pmatrix} 1-b & i \\ K & -h \end{pmatrix} \begin{pmatrix} Y \\ R \end{pmatrix} = \begin{pmatrix} a + I_0 + \bar{G} \\ \bar{M} \end{pmatrix}$$

$$Y^* = \frac{\begin{vmatrix} a + I_0 + \bar{G} & i \\ M & h \end{vmatrix}}{\begin{vmatrix} 1-b & i \\ K & -h \end{vmatrix}}$$

$$R^* = \frac{\begin{vmatrix} 1-b & a + I_0 + \bar{G} \\ K & M \end{vmatrix}}{\begin{vmatrix} 1-b & i \\ K & h \end{vmatrix}}$$