Unit 2: Mathematical Foundations for Economics

Overview

Mathematics is the language of economics.

It helps economists;

Represent relationships between variables, analyze changes, and derive optimal decisions.

This unit introduces the mathematical tools most frequently used in economic theory and analysis.

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1. Algebraic Functions and Equations

1.1 Concept of a Function

A **function** expresses the relationship between two variables — typically one dependent variable and one or more independent variables.

Definition:
A function *f* assigns each value of *x* a unique value of *y*:

$y=f(x)$

Example (Economic):
Let the **demand function** be:

$Qd=100-5P$

Here,

- Qd: Quantity demanded (dependent variable)
- P: Price (independent variable)

If $P=10$

$Qd=100-5(10)=50$

1.2 Types of Functions in Economics

Function Type	General Form	Example	Economic Interpretation
Linear	**(y = a + bx)**	**(C = 50 + 0.8Y)**	**Consumption function**
Quadratic	**($y = ax^2 + bx + c$)**	**(pi = $-2Q^2 + 40Q - 100$)**	**Profit function with diminishing returns**
Cubic	**($y = ax^3 + bx^2 + cx + d$)**	**(Q_s = $0.2P^3 - 2P^2 + 10P + 5$)**	**Complex supply response**
Exponential	**(y = Ae^{bx})**	**(GDP = 100e^{0.05t})**	**Growth models**
Logarithmic	**(y = a + b\ln x)**	**(U = \ln X + \ln Y)**	**Utility functions (Cobb-Douglas form)**

1.3 Solving Algebraic Equations

Linear Equation Example:

$5x+10=30 \Rightarrow 5x=20 \Rightarrow x=4$

Quadratic Equation Example:

$x^2-5x+6=0$

$(x-2)(x-3)=0 \Rightarrow x=2 \text{ or } x=3$

Economic Example:
Equating demand and supply:

$Qd=100-5P, Qs=20+3P$

At equilibrium, $Qd=Qs$

$100-5P=20+3P \Rightarrow 8P=80 \Rightarrow P=10$

Then $Q=100-5(10)=50$

→ **Equilibrium Price = 10, Quantity = 50**

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2. Linear and Quadratic Functions in Economics

2.1 Linear Functions

A **linear function** has a constant slope and shows proportional relationships.

General Form:

$y=a+bx$

where:

- a = intercept
- b = slope (rate of change)

Example (Consumption Function):

$C=50+0.8Y$

If income $Y=200$

$C=50+0.8(200)=210$

Slope Interpretation:
For each 1 unit increase in income, consumption rises by 0.8.

2.2 Quadratic Functions

A **quadratic function** shows non-linear relationships — often used for cost, revenue, or profit analysis.

General Form:

$y=ax^2+bx+c$

Economic Example (Profit Function):

$\pi=-2Q^2+40Q-100$

To find the output that maximizes profit:

- This is a downward-opening parabola since $a=-2<0$
- Vertex (maximum) at:

$Q*=-b/2a=-40/2(-2)=10$

Maximum Profit:

$\pi = -2(10)^2 + 40(10) - 100 = -200 + 400 - 100 = 100$ $= 100\pi = -2(10)2 + 40(10) - 100 = -200 + 400 - 100 = 100$

→ Optimal Output = 10 units; Maximum Profit = 100

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3. Systems of Equations and Matrices

3.1 Systems of Linear Equations

Systems appear frequently in economic models (e.g., equilibrium in multi-market systems).

Example:

$Qd = 20 - 2P$
$Qs = -4 + 3P$

At equilibrium $Qd = Qs$

$20 - 2P = -4 + 3P \Rightarrow 24 = 5P \Rightarrow P = 4.8$
$Q = 20 - 2(4.8) = 10.4$

3.2 Matrix Representation

A system can be written as:

$AX = B$

Example:

$2x + 3y = 8$
$x - y = 1$

Matrix form:

Solve the above using Cramer's Rule

Economic Application:
Can represent market equilibrium in two interrelated goods or input-output models.

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4. Calculus Basics: Differentiation and Optimization

4.1 Differentiation

Differentiation measures **rate of change** — how one variable changes in response to another.

Basic Rules:

Note 6

4.2 Marginal Analysis in Economics

Marginal values (like marginal cost, marginal revenue) are derivatives.

Example:
If total cost function $C=100+5Q+Q^2$
then marginal cost:

$$MC=dC/dQ=5+2Q$$

If $Q=10$
$MC=25$

4.3 Optimization (Maximization/Minimization)

To find the value of *x* that maximizes or minimizes a function:

1. Differentiate: $f'(x)=0 \rightarrow$ first-order condition
2. Use $f''(x)$ to check:
   - $f''(x)<0$: maximum
   - $f''(x)>0$: minimum

Example (Profit Maximization):

$\pi=-Q^2+20Q-50$
$\pi'=-2Q+20=0 \Rightarrow Q=10$
$\pi''=-2<0 \Rightarrow$ Maximum at $Q=10$

Maximum Profit:

$\pi=-(10)2+20(10)-50=50$

4.4 Economic Interpretation of Derivatives

Concept	Mathematical Form	Interpretation
Marginal Cost (MC)	$MC=dC/dQ$	Additional cost per unit
Marginal Revenue (MR)	$MR=dR/dQ$	Additional revenue per unit
Elasticity	$E=dQ/dP\times P/Q$	Responsiveness of demand/supply
Growth Rate	$g=1/Y*dY/dt$	Rate of change in income/output

Summary Table

Concept	Mathematical Tool	Economic Application
Functions	Express relationships	Demand, supply, cost, utility
Equations	Determine equilibrium	Market clearing
Matrices	Solve systems	Multi-market or input-output models
Derivatives	Measure change	Marginal analysis, elasticity
Optimization	Find maxima/minima	Profit maximization, cost minimization

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Discussion / Practice Questions

1. Derive equilibrium price and quantity for:
   $Qd=50-2P, Qs=10+3P$
2. If $C=40+0.75Y$, find consumption when $Y=200$.
3. Given $\pi=-3Q^2+24Q-36$, find Q that maximizes profit and compute the maximum profit.
4. Represent the following system as a matrix and solve:
   $3x+2y=12, \; x-y=1$
5. Explain how differentiation helps in identifying marginal cost and marginal revenue.
