Unit 2: Mathematical Foundations for Economics

- Mathematics is the language of economics. It helps economists represent relationships between variables, analyze changes, and derive optimal decisions.

Overview

- This unit introduces mathematical tools used in economic theory and analysis:
- - Algebraic functions and equations
- - Linear and quadratic functions
- - Systems of equations and matrices
- - Calculus basics: differentiation and optimization

Algebraic Functions and Equations

- Concept of a Function
- A function expresses the relationship between two variables: $y = f(x)$
- Example: Demand function: $Qd = 100 - 5P$
- If $P = 10$, then $Qd = 50$

Types of Functions in Economics

- Linear: $y = a + bx \rightarrow C = 50 + 0.8Y$ (Consumption)
- Quadratic: $y = ax^2 + bx + c \rightarrow \pi = -2Q^2 + 40Q - 100$ (Profit)
- Cubic: $y = ax^3 + bx^2 + cx + d \rightarrow Qs = 0.2P^3 - 2P^2 + 10P + 5$ (Supply)
- Exponential: $y = Ae^{bx} \rightarrow GDP = 100e^{0.05t}$ (Growth)
- Logarithmic: $y = a + b \ln(x) \rightarrow U = \ln X + \ln Y$ (Utility)

Solving Algebraic Equations

- Linear Example: $5x + 10 = 30 \rightarrow x = 4$
- Quadratic Example: $x^2 - 5x + 6 = 0 \rightarrow x = 2$ or 3
- Economic Example: $Qd = 100 - 5P$, $Qs = 20 + 3P \rightarrow P = 10$, $Q = 50$

Linear Functions

- A linear function shows a proportional relationship:
- $y = a + bx$
- Example: $C = 50 + 0.8Y$
- If $Y = 200 \rightarrow C = 210$
- Interpretation: For each 1 unit increase in income, consumption rises by 0.8.

Quadratic Functions

- Quadratic functions show non-linear relationships, often for cost or profit.
- $y = ax^2 + bx + c$
- Example: $\pi = -2Q^2 + 40Q - 100$
- Optimal Output = 10; Maximum Profit = 100

Systems of Equations

- Systems appear in equilibrium models:
- $Q_d = 20 - 2P$, $Q_s = -4 + 3P$
- At equilibrium: $P = 4.8$, $Q = 10.4$

Matrix Representation

- A system can be represented as $AX = B$
- Example:
- $2x + 3y = 8$
- $x - y = 1$
- Used in market equilibrium and input-output analysis.

Calculus Basics: Differentiation

- Differentiation measures rate of change.
- Example: If $C = 100 + 5Q + Q^2$, then $MC = dC/dQ = 5 + 2Q$
- If $Q = 10 \rightarrow MC = 25$

Optimization

- Used to find maxima/minima.
- Example: $\pi = -Q^2 + 20Q - 50$
- $\pi' = -2Q + 20 = 0 \rightarrow Q = 10$
- Maximum Profit = 50

Economic Interpretation of Derivatives

- Marginal Cost ($MC = dC/dQ$): Additional cost per unit
- Marginal Revenue ($MR = dR/dQ$): Additional revenue per unit
- Elasticity ($E = (dQ/dP)\times(P/Q)$): Responsiveness of demand/supply
- Growth Rate ($g = (1/Y)(dY/dt)$): Rate of income/output change

Summary Table

- Functions → Express relationships (Demand, supply, cost)
- Equations → Determine equilibrium (Market clearing)
- Matrices → Solve systems (Input-output models)
- Derivatives → Measure change (Marginal analysis)
- Optimization → Find maxima/minima (Profit or cost analysis)

Discussion / Practice Questions

- 1. Derive equilibrium for $Qd = 50 - 2P$, $Qs = 10 + 3P$
- 2. If $C = 40 + 0.75Y$, find C when $Y = 200$
- 3. For $\pi = -3Q^2 + 24Q - 36$, find Q maximizing profit
- 4. Represent $3x + 2y = 12$, $x - y = 1$ as matrix
- 5. Explain differentiation in marginal analysis.