

DNN

XOR 연산 과정 시각화

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이 발표를 듣고있는 여러분들에게...

당신은 AI를 공부하면서
어떤 것이 가장 중요하다고
생각하십니까?

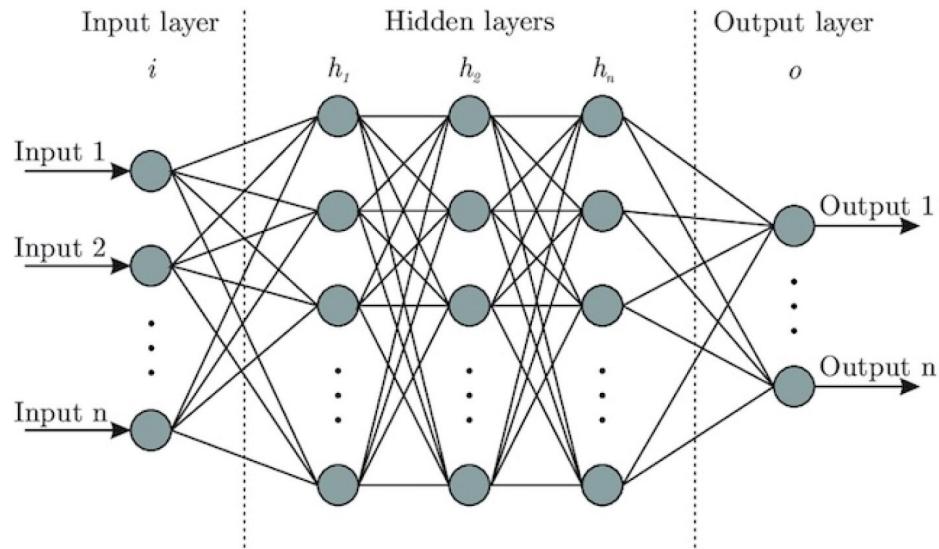


쏟아지고 있는 여러 모델들, 근본적인 개념

오늘날까지 발표된 논문들이나 모델들 - 대부분 여러 기법들을 혼합

근본적으로 빠지지 않는 한가지 개념

DNN



이번 시간에는 강력하고 중요한 DNN을 자세하게 살펴봅시다

DNN의 서사 - DNN은 ‘신’이다.

1950-60년대

로젠블롯의 퍼셉트론 - 머신이 스스로 학습할 수 있다!!

마빈 민斯基 - XOR 문제 못푼다

1980~90년대

럼멜 하트, 힌튼 등 - Backpropagation(역전파) 제시

층을 조금만 깊게 쌓으면 학습이 안 되는 **Vanishing Gradient**

DNN의 서사 - DNN은 ‘신’이다.

2006년 캐나다의 삼총사 등장 - 힌튼, 얀 르쿤, 요슈아 벤지오

Science 지에 논문 발표 - Reducing the Dimensionality of Data with Neural Networks

신경망을 사용한 데이터의 차원 축소

2009~2012년

Hinton-Deng 팀 음성 인식대회에서 DNN이 기존 GMM 모델을 크게克制(복수형)했습니다.
이미지 쪽에서는 AlexNet 같은 딥 CNN으로 ImageNet 대회를 박살

이후로 DNN이 널리 알려지며 관련 연구, 논문들이 진행되기 시작



선형변환과 그 한계

선형 분리로 classification 가능한 문제

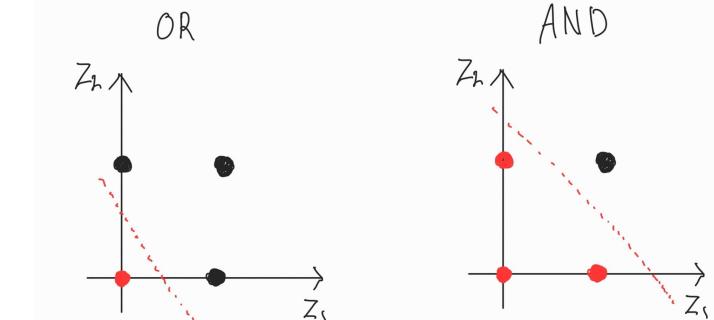
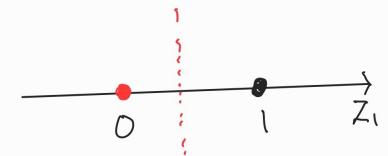
기존의 단순한 퍼셉트론 - 선형변환 $y=Wx+b$

and 게이트

or 게이트

nand 게이트

선으로 해당 게이트를 구별하여 나타낼 수 있음



선형변환을 아무리 많이, 복잡하게 수행해도 결국 직선임.

선형변환, 선형결합으로 풀 수 없는 문제는 어떤 방식으로 풀어야 할까?

XOR문제

xor문제를 선형결합, 선형변환으로 풀 수 있을까?

$$(0,0) = 0$$

Input Output
 $\begin{bmatrix} z_1^{(1)} \\ z_2^{(1)} \end{bmatrix} \longrightarrow y$

$$(0,1) = 1$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \longrightarrow 0$$

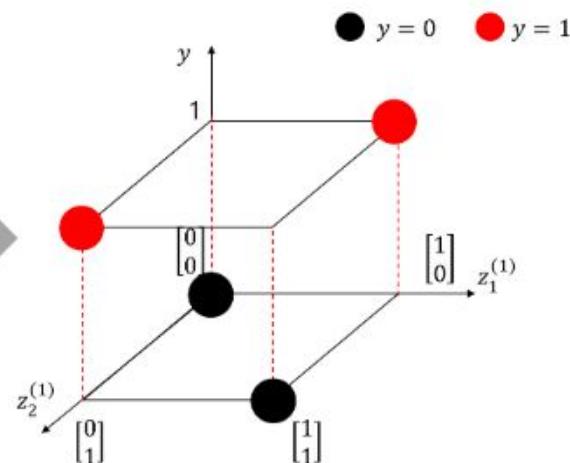
$$(1,0) = 1$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \longrightarrow 1$$

$$(1,1) = 0$$

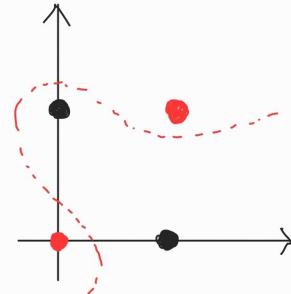
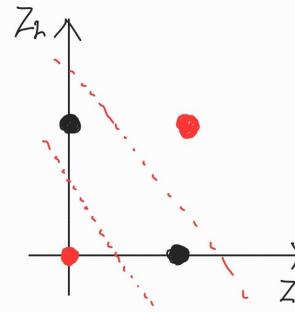
$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \longrightarrow 1$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \longrightarrow 0$$



XOR

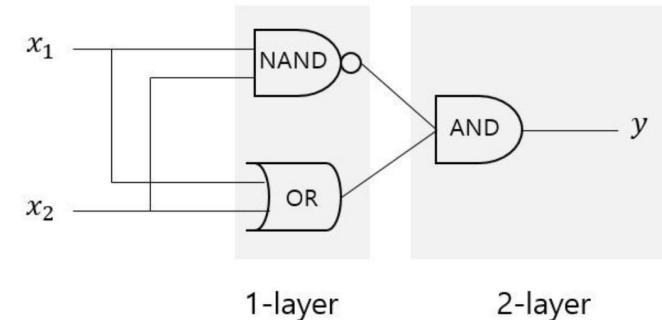
직선 하나로는 어떻게 해도 이 네 점을 구분할 수 없음



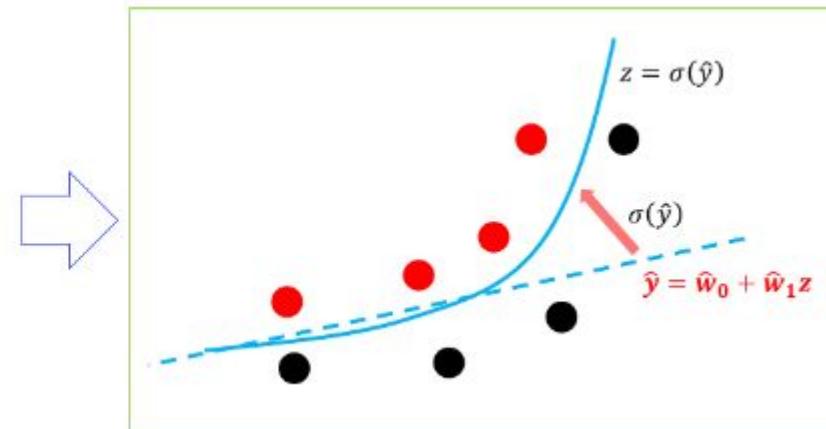
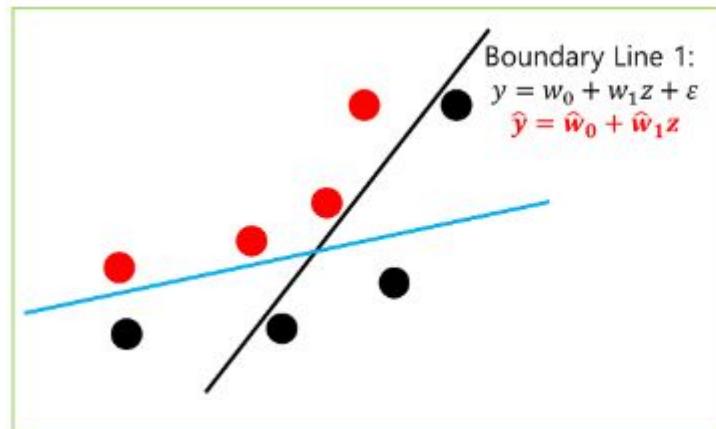
비선형변환

and, or 처럼 단층 퍼셉트론으로는 xor문제를 풀 수 없음

선형변환만으로는 이 문제를 풀 수 없음



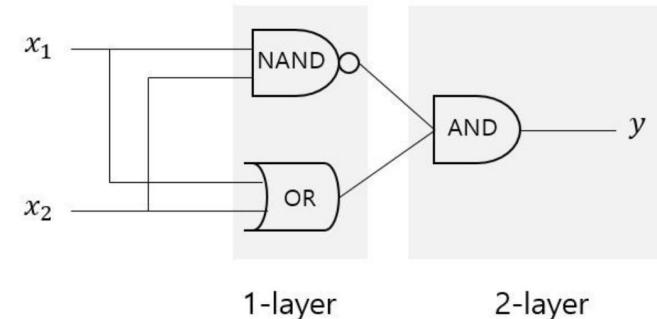
선형변환에 비선형성을 추가한다면?



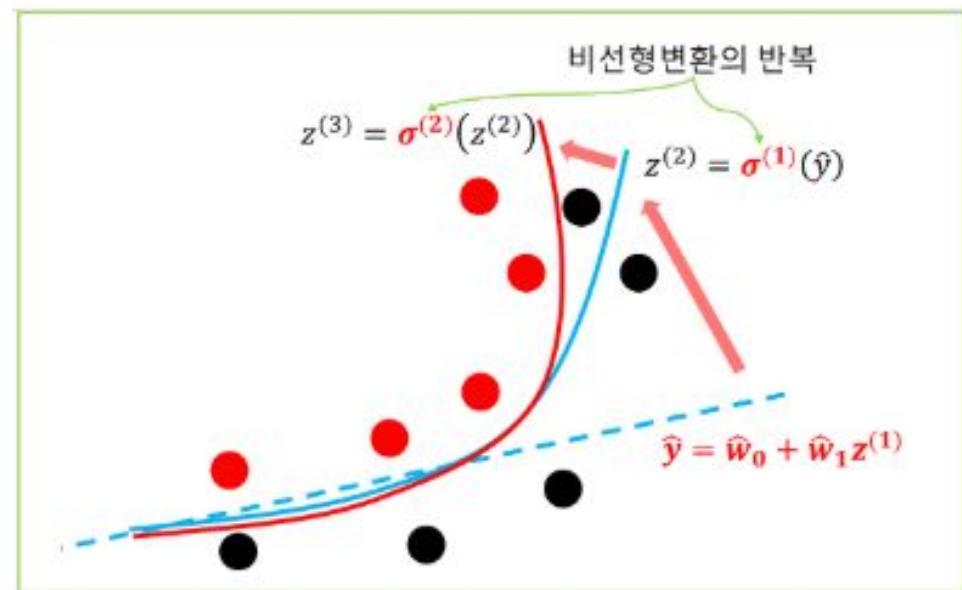
비선형변환

and, or 처럼 단층 퍼셉트론으로는 xor문제를 풀 수 없음

선형변환만으로는 이 문제를 풀 수 없음



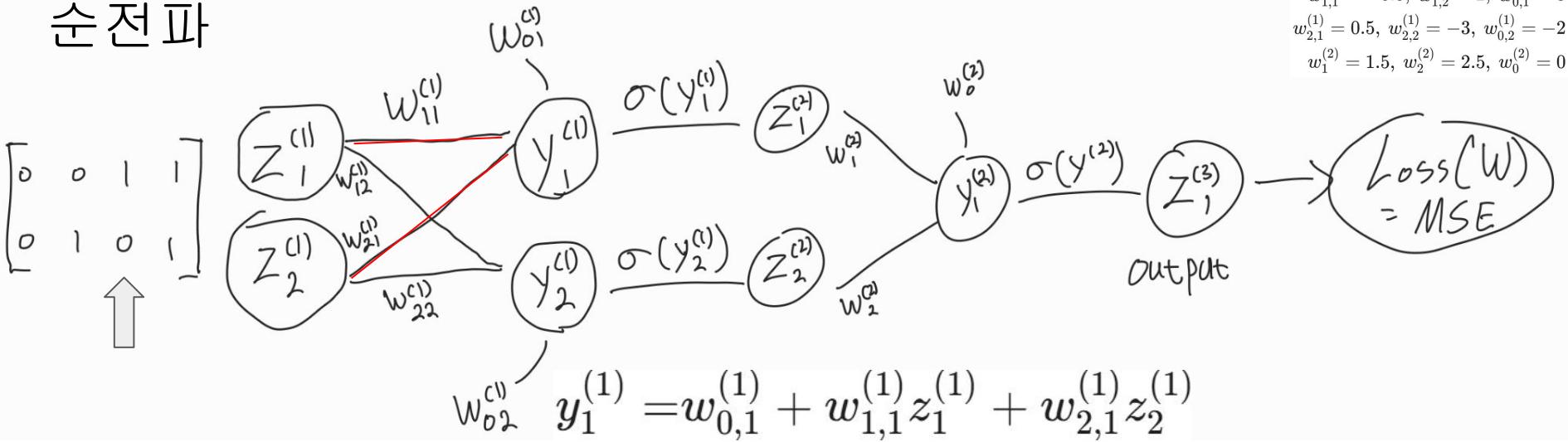
선형변환에 비선형성을 추가한다면?

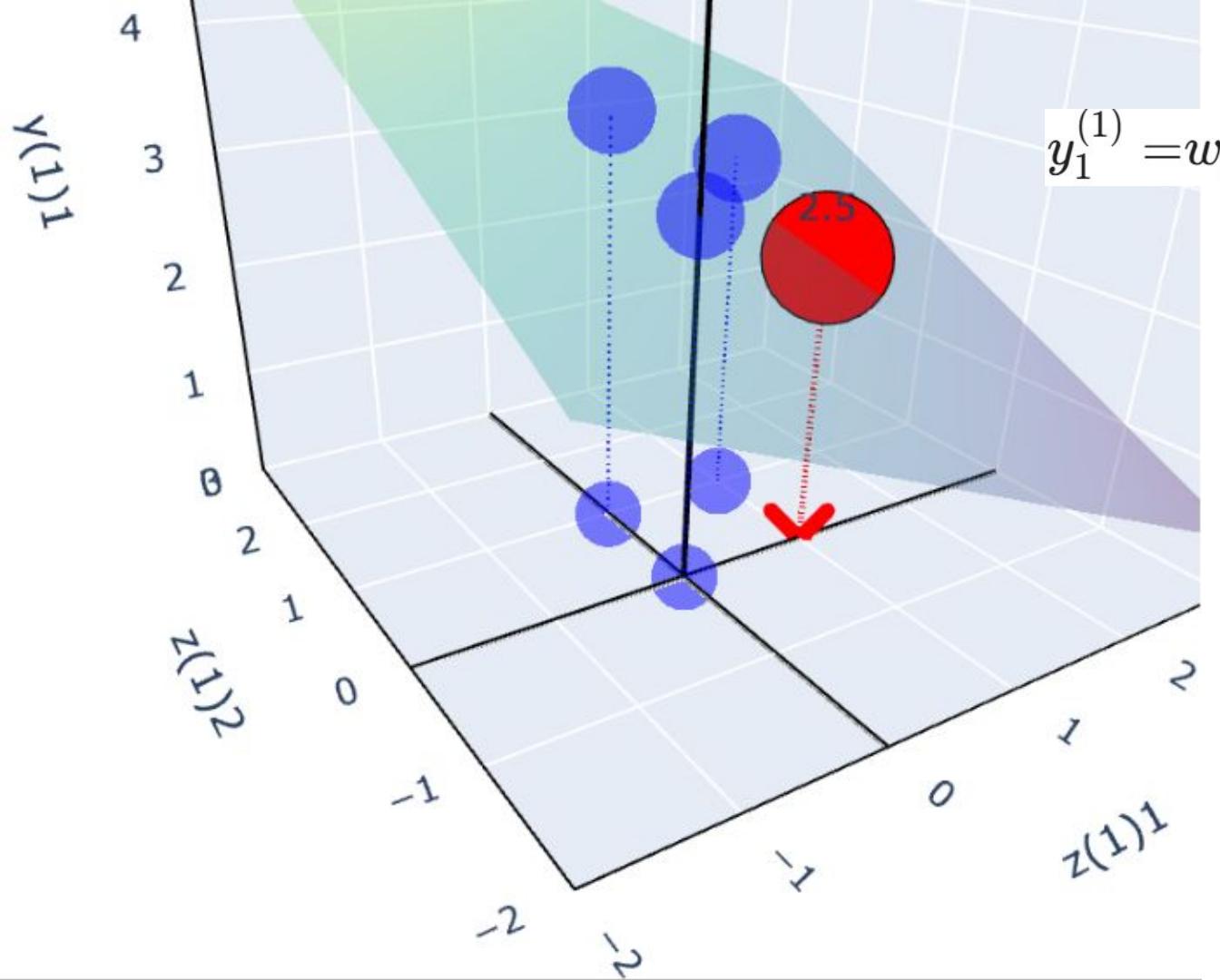


XOR - input (1,0) 을 예제로 알아봅시다

생각해보기
DNN으로 결국 알고싶은 것은 무엇일까요?

순전파

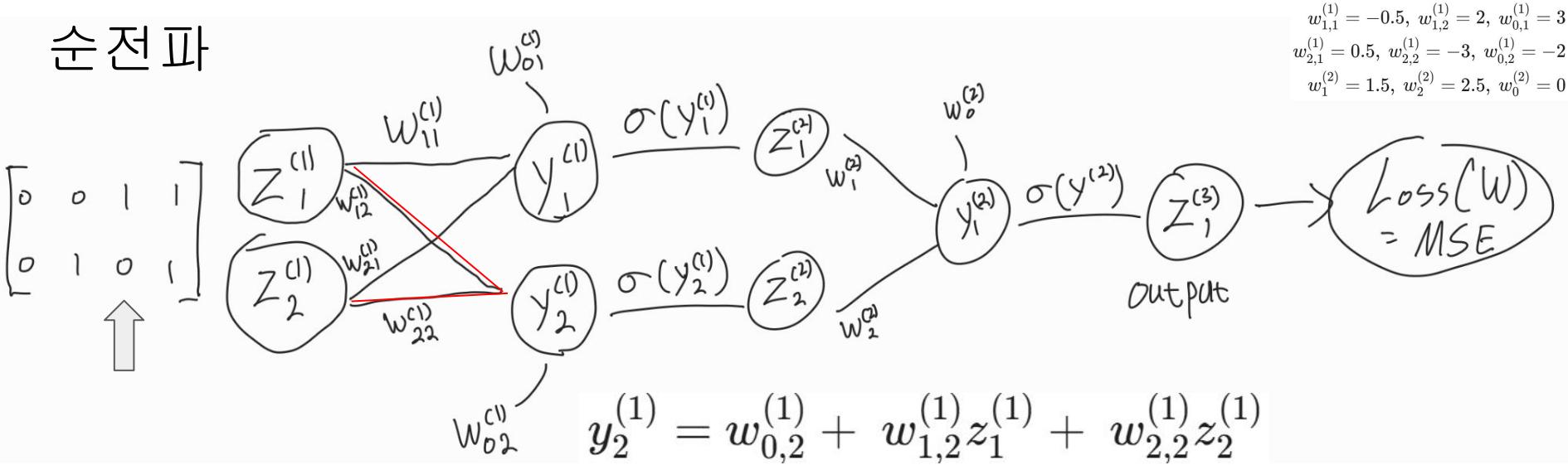




$$w_{1,1}^{(1)} = -0.5, w_{1,2}^{(1)} = 2, w_{0,1}^{(1)} = 3$$
$$w_{2,1}^{(1)} = 0.5, w_{2,2}^{(1)} = -3, w_{0,2}^{(1)} = -2$$
$$w_1^{(2)} = 1.5, w_2^{(2)} = 2.5, w_0^{(2)} = 0$$
$$y_1^{(1)} = w_{0,1}^{(1)} + w_{1,1}^{(1)}z_1^{(1)} + w_{2,1}^{(1)}z_2^{(1)}$$

$$y_1^{(1)} = 2.5$$

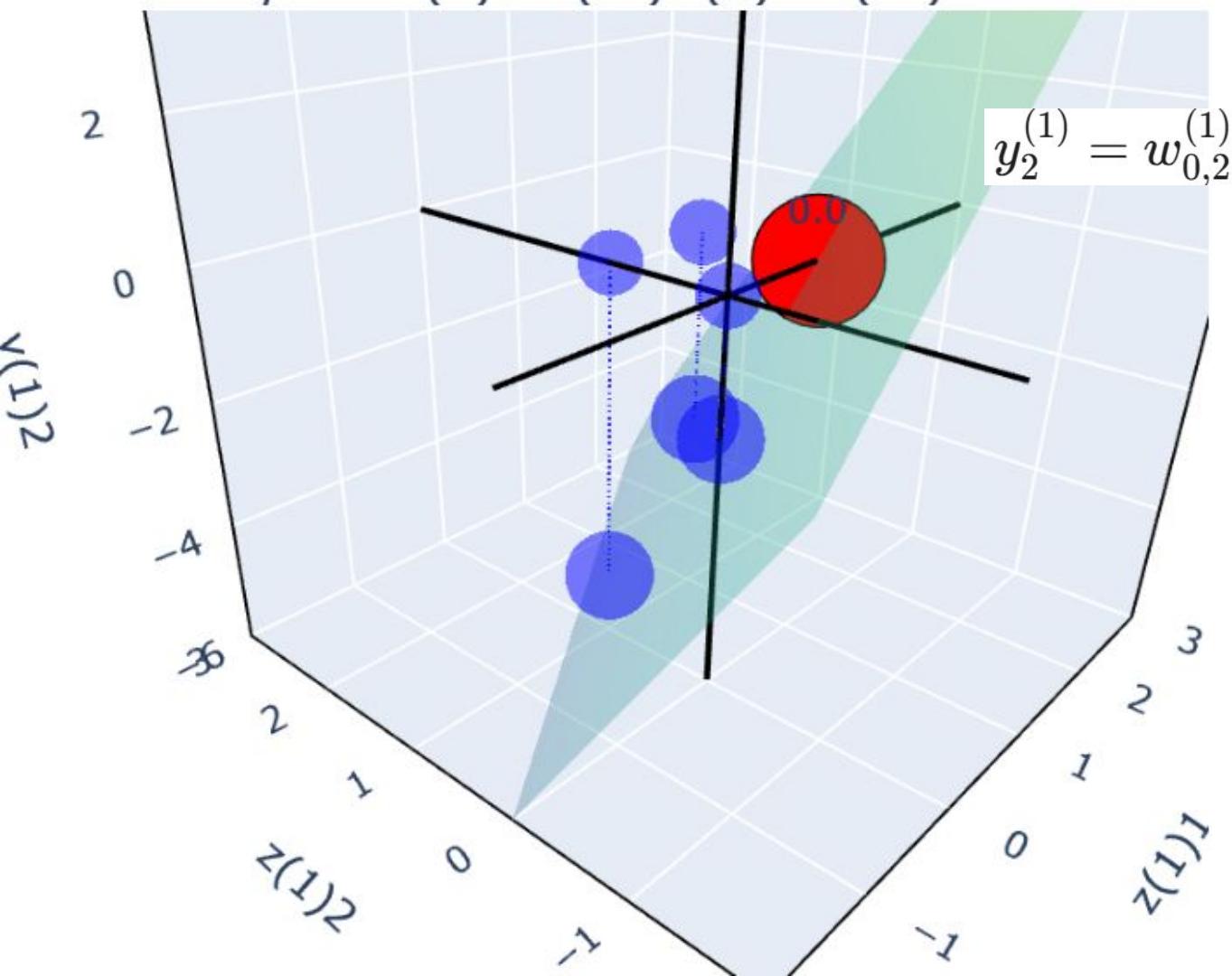
순전파



$$\begin{aligned}w_{1,1}^{(1)} &= -0.5, w_{1,2}^{(1)} = 2, w_{0,1}^{(1)} \\w_{2,1}^{(1)} &= 0.5, w_{2,2}^{(1)} = -3, w_{0,2}^{(1)} = -2 \\w_1^{(2)} &= 1.5, w_2^{(2)} = 2.5, w_0^{(2)} = 0\end{aligned}$$

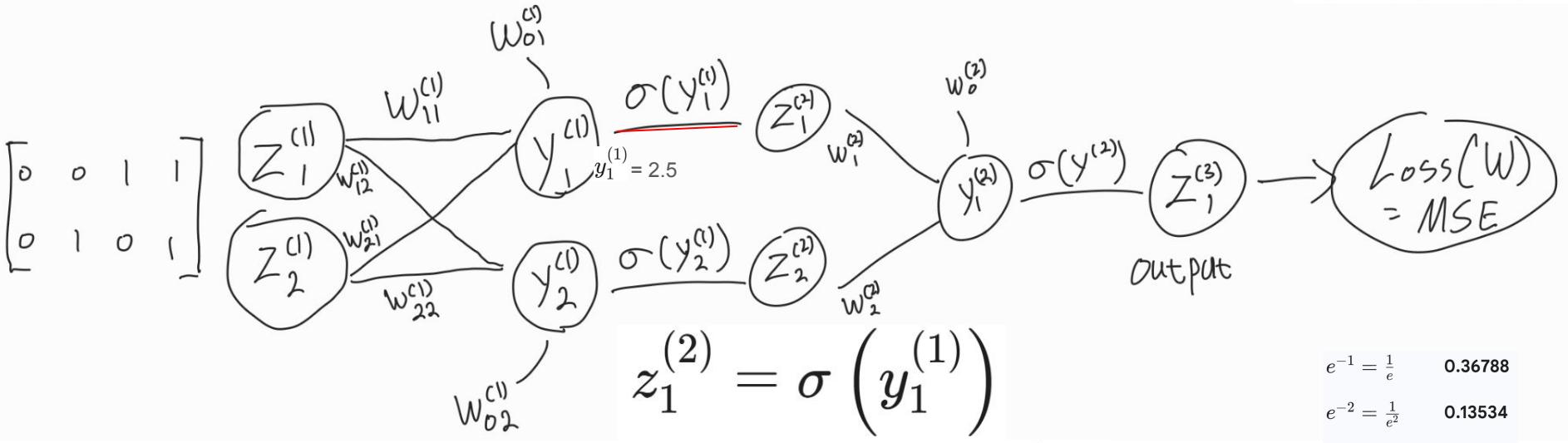
$$y_2^{(1)} = w_{0,2}^{(1)} + w_{1,2}^{(1)}z_1^{(1)} + w_{2,2}^{(1)}z_2^{(1)}$$

$$y_2^{(1)} = 0$$



순전파

$$\begin{aligned} w_{1,1}^{(1)} &= -0.5, w_{1,2}^{(1)} = 2, w_{0,1}^{(1)} \\ w_{2,1}^{(1)} &= 0.5, w_{2,2}^{(1)} = -3, w_{0,2}^{(1)} = -2 \\ w_1^{(2)} &= 1.5, w_2^{(2)} = 2.5, w_0^{(2)} = 0 \end{aligned}$$



$$z_1^{(2)} = \sigma(y_1^{(1)})$$

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

$$e^{-1} = \frac{1}{e} \quad 0.36788$$

$$e^{-2} = \frac{1}{e^2} \quad 0.13534$$

$$e^{-3} = \frac{1}{e^3} \quad 0.04979$$

$$e^{-4} = \frac{1}{e^4} \quad 0.01832$$

$$e^{-5} = \frac{1}{e^5} \quad 0.00674$$

$$e^{-6} = \frac{1}{e^6} \quad 0.00248$$

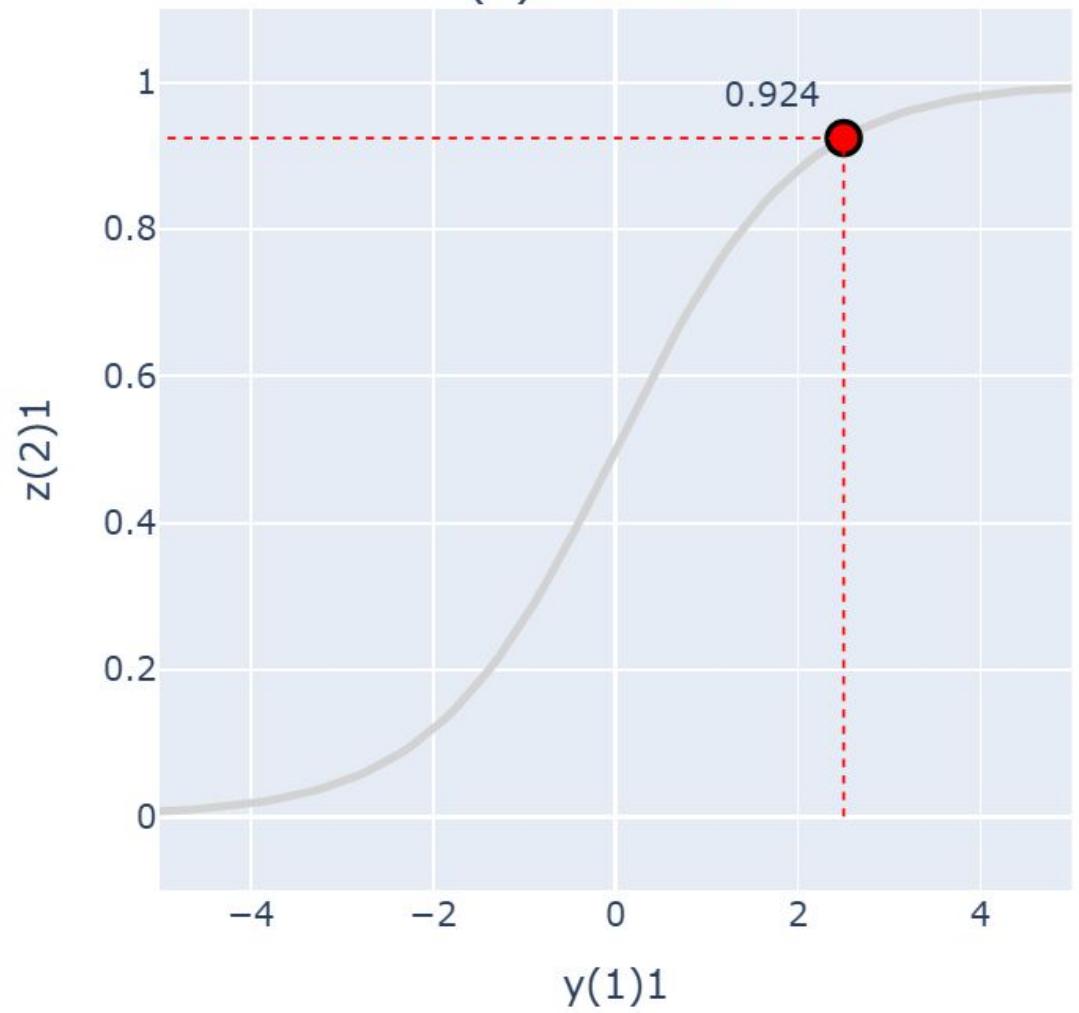
$$e^{-7} = \frac{1}{e^7} \quad 0.00091$$

$$e^{-8} = \frac{1}{e^8} \quad 0.00034$$

$$e^{-9} = \frac{1}{e^9} \quad 0.00012$$

$$e^{-10} = \frac{1}{e^{10}} \quad 0.00005$$

$\text{Y}(1)1$ Activation



sigmoid
Input (1,0)

$$\begin{aligned} w_{1,1}^{(1)} &= -0.5, w_{1,2}^{(1)} = 2, w_{0,1}^{(1)} = 3 \\ w_{2,1}^{(1)} &= 0.5, w_{2,2}^{(1)} = -3, w_{0,2}^{(1)} = -2 \\ w_1^{(2)} &= 1.5, w_2^{(2)} = 2.5, w_0^{(2)} = 0 \end{aligned}$$

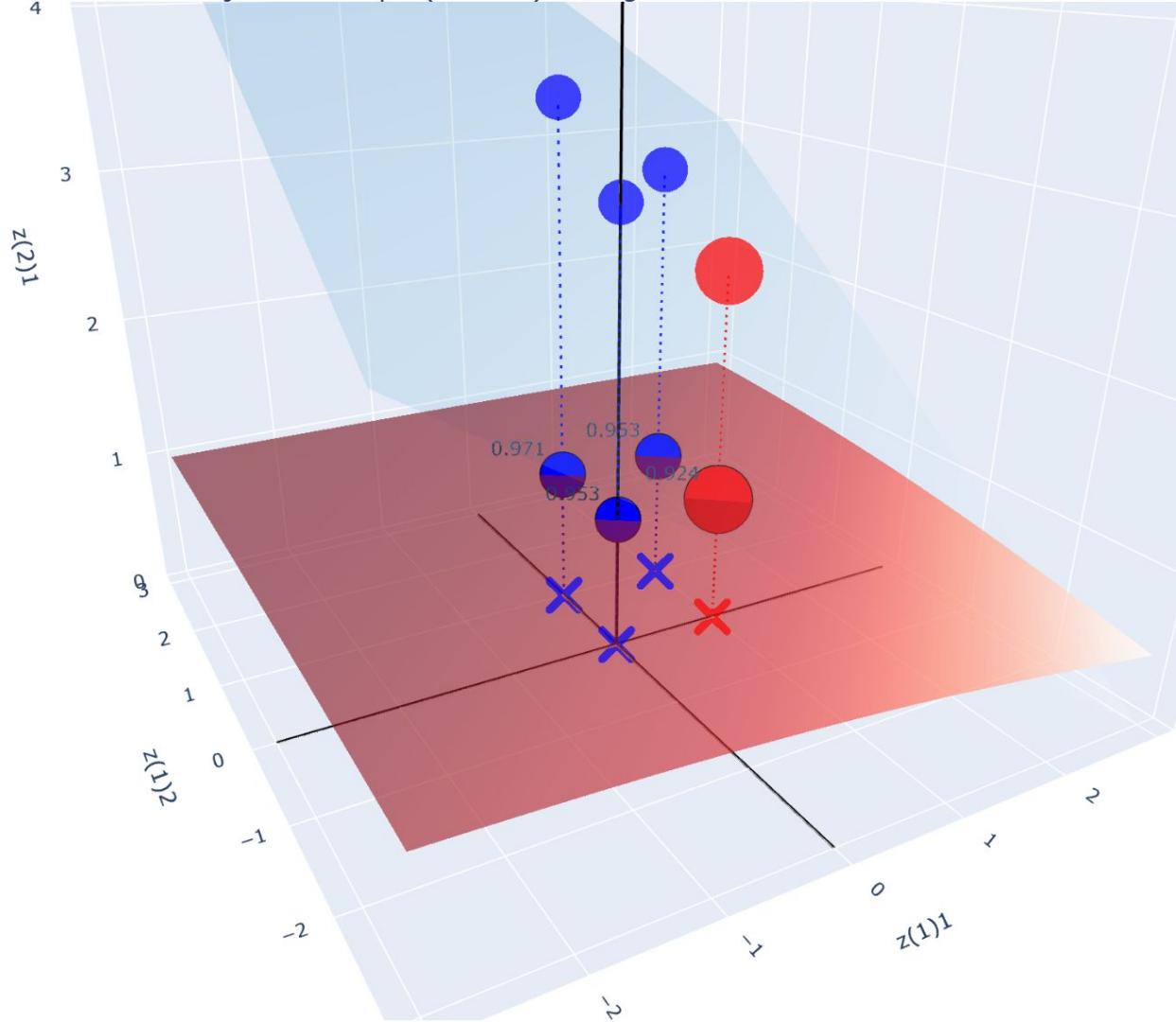
$$z_1^{(2)} = \sigma \left(y_1^{(1)} \right)$$
$$y_1^{(1)} = 2.5$$
$$z_1^{(2)} \approx 0.924$$



$$z_1^{(2)} = \sigma^{(1)} \left(y_1^{(1)} \right),$$

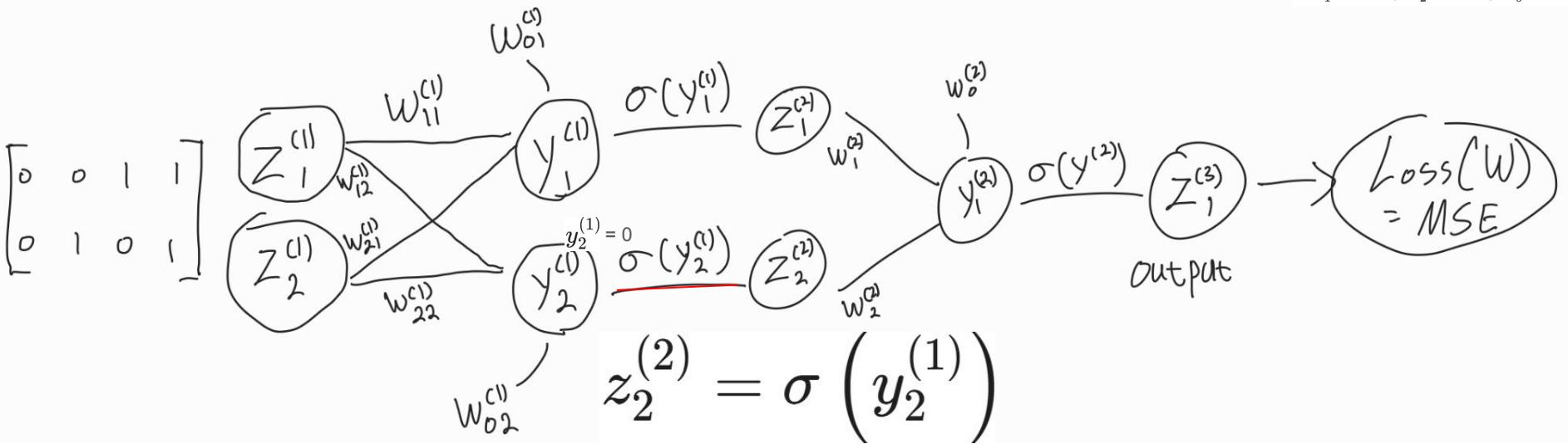
$$y_1^{(1)} = w_{0,1}^{(1)} + w_{1,1}^{(1)} z_1^{(1)} + w_{2,1}^{(1)} z_2^{(1)}$$

$$\rightarrow z_1^{(2)} = \sigma^{(1)} \left(w_{0,1}^{(1)} + w_{1,1}^{(1)} z_1^{(1)} + w_{2,1}^{(1)} z_2^{(1)} \right)$$

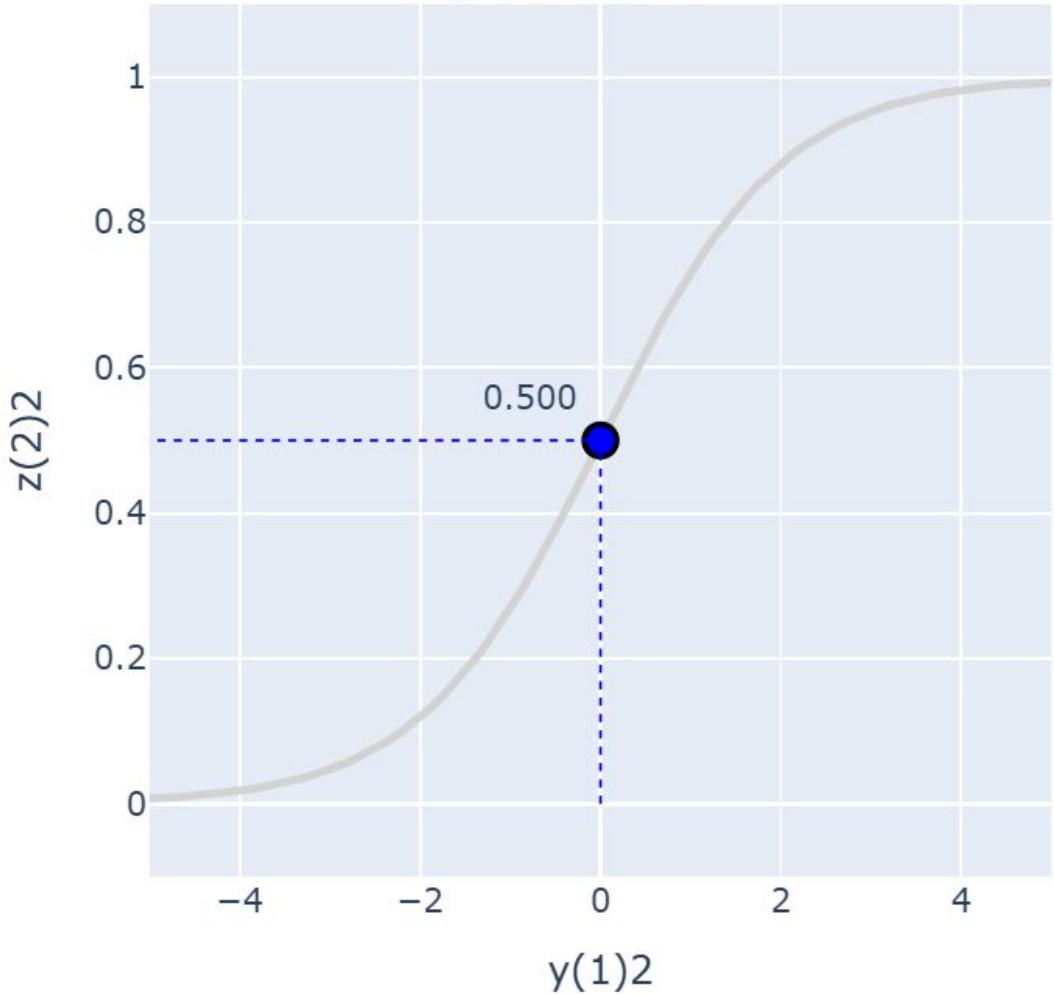


순전파

$$\begin{aligned}
 w_{1,1}^{(1)} &= -0.5, w_{1,2}^{(1)} = 2, w_{0,1}^{(1)} = 3 \\
 w_{2,1}^{(1)} &= 0.5, w_{2,2}^{(1)} = -3, w_{0,2}^{(1)} = -2 \\
 w_1^{(2)} &= 1.5, w_2^{(2)} = 2.5, w_0^{(2)} = 0
 \end{aligned}$$



$\gamma(1)2$ Activation

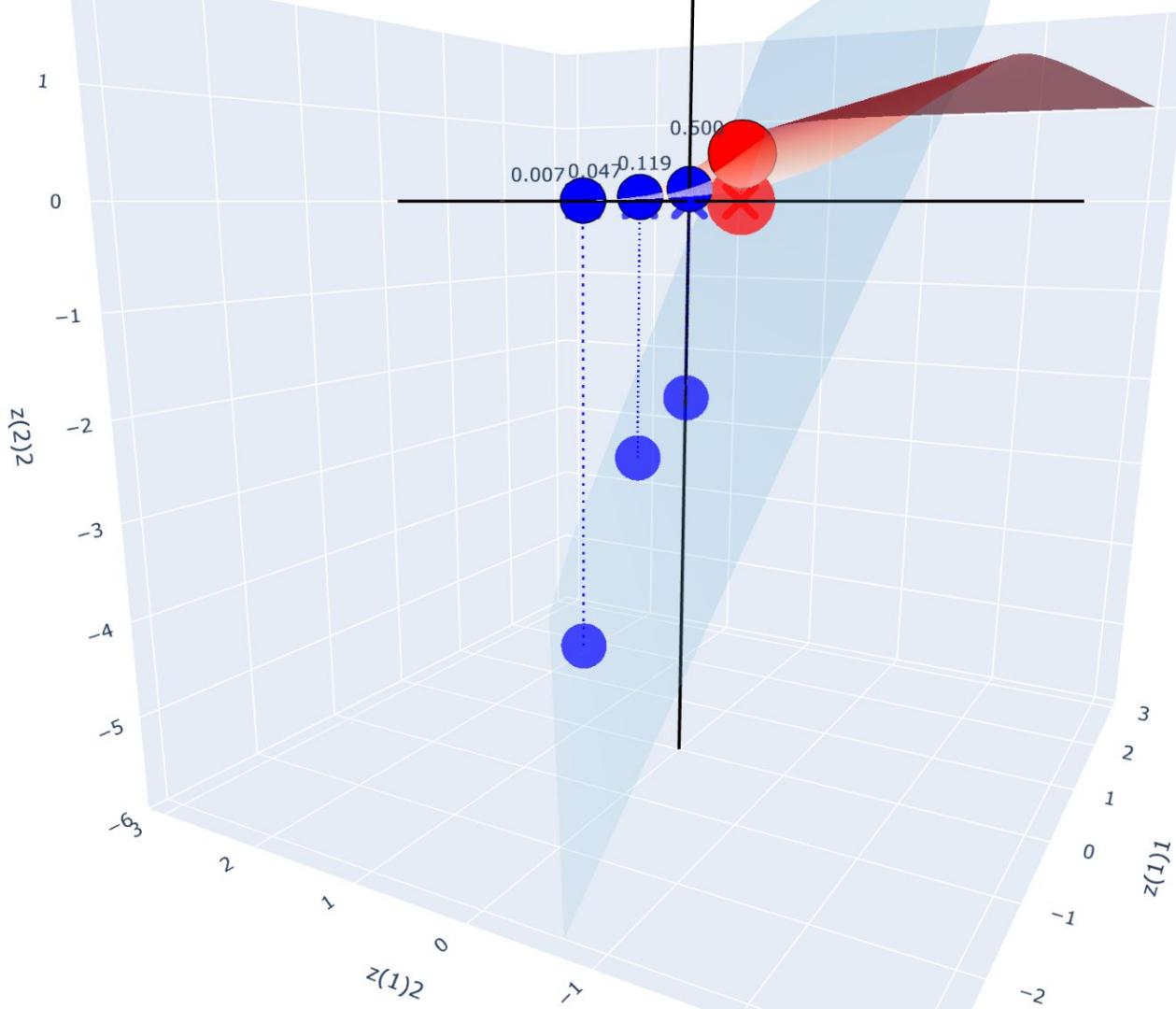


$$\begin{aligned} w_{1,1}^{(1)} &= -0.5, & w_{1,2}^{(1)} &= 2, & w_{0,1}^{(1)} &= 3 \\ w_{2,1}^{(1)} &= 0.5, & w_{2,2}^{(1)} &= -3, & w_{0,2}^{(1)} &= -2 \\ w_1^{(2)} &= 1.5, & w_2^{(2)} &= 2.5, & w_0^{(2)} &= 0 \end{aligned}$$

$$z_2^{(2)} = \sigma \left(y_2^{(1)} \right)$$

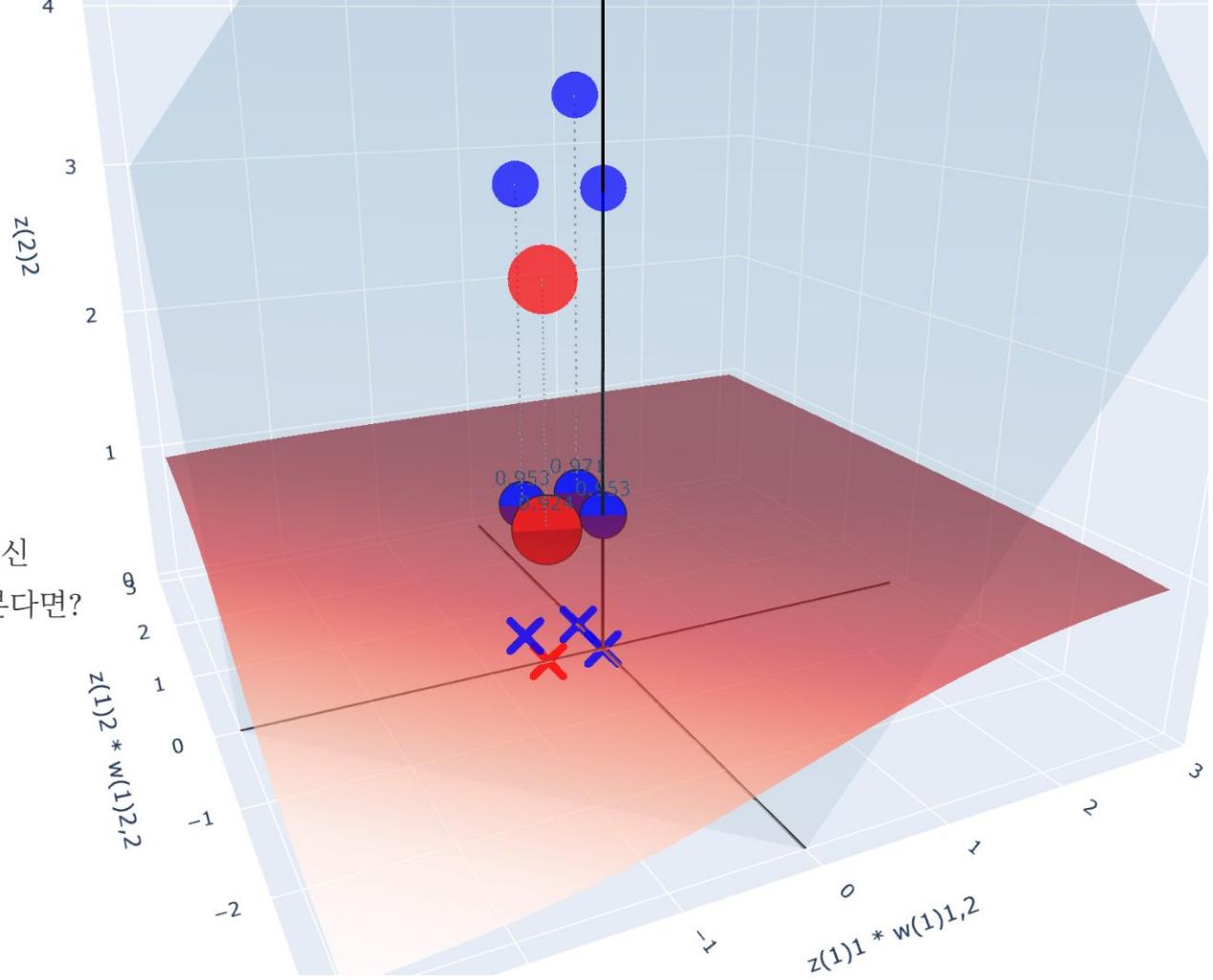
$$y_2^{(1)} = 0$$

$$z_2^{(2)} = 0.5$$

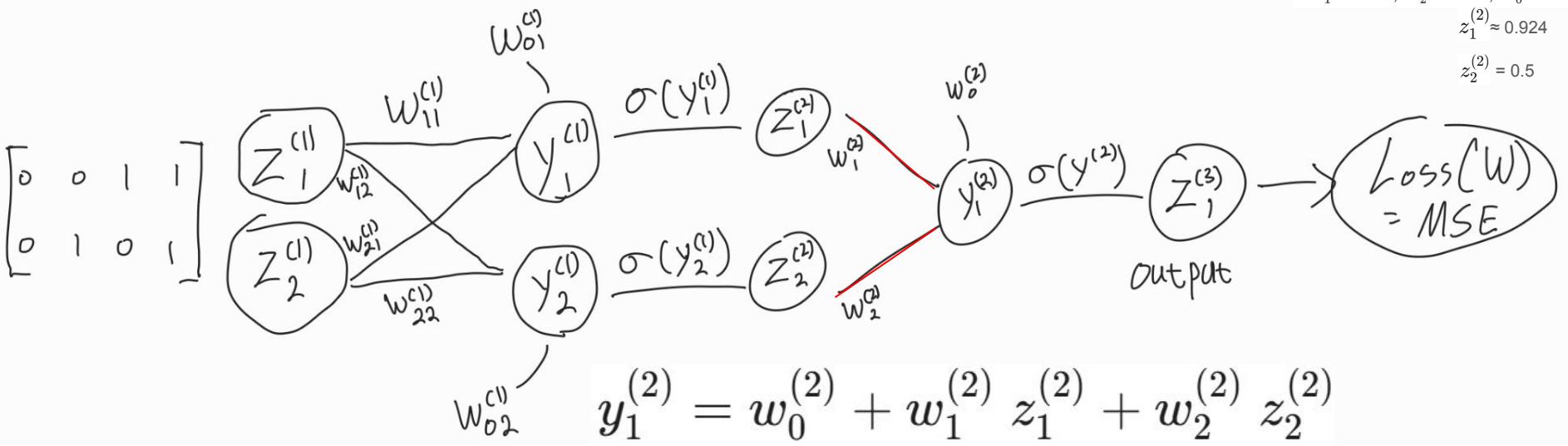


$$\begin{aligned}
 z_2^{(2)} &= \sigma^{(1)} \left(y_2^{(1)} \right), \\
 y_2^{(1)} &= w_{0,2}^{(1)} + w_{1,2}^{(1)} z_1^{(1)} + w_{2,2}^{(1)} z_2^{(1)} \\
 \rightarrow z_2^{(2)} &= \sigma^{(1)} \left(w_{0,2}^{(1)} + w_{1,2}^{(1)} z_1^{(1)} + w_{2,2}^{(1)} z_2^{(1)} \right)
 \end{aligned}$$

x 축과 y 축에 input으로 들어오는 $z_1^{(1)}, z_2^{(1)}$ 대신
 $w_{1,2}^{(1)} * z_1^{(1)}, w_{2,2}^{(1)} * z_2^{(1)}$ 이 들어온 상황을 생각해본다면?



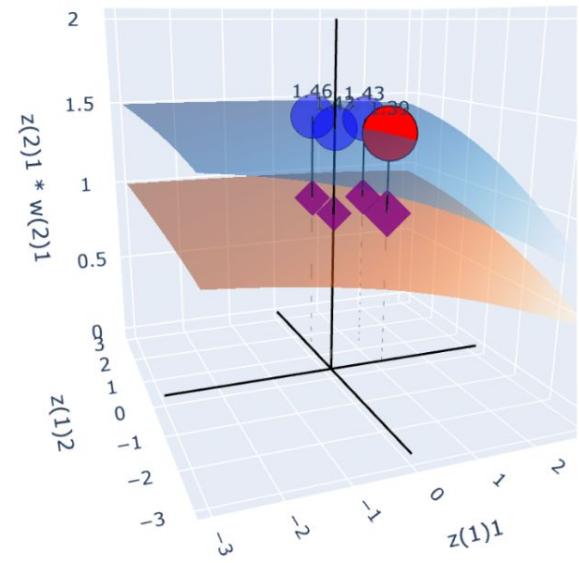
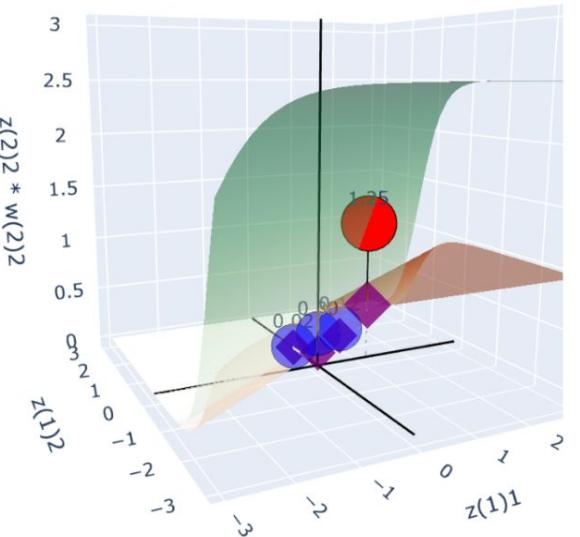
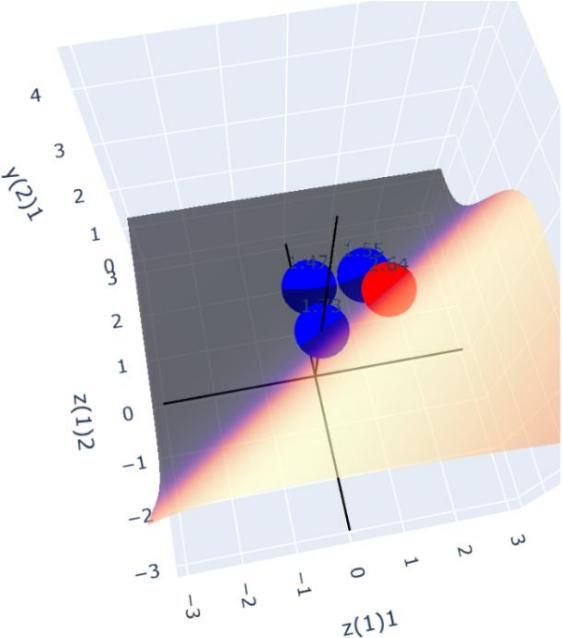
순전파



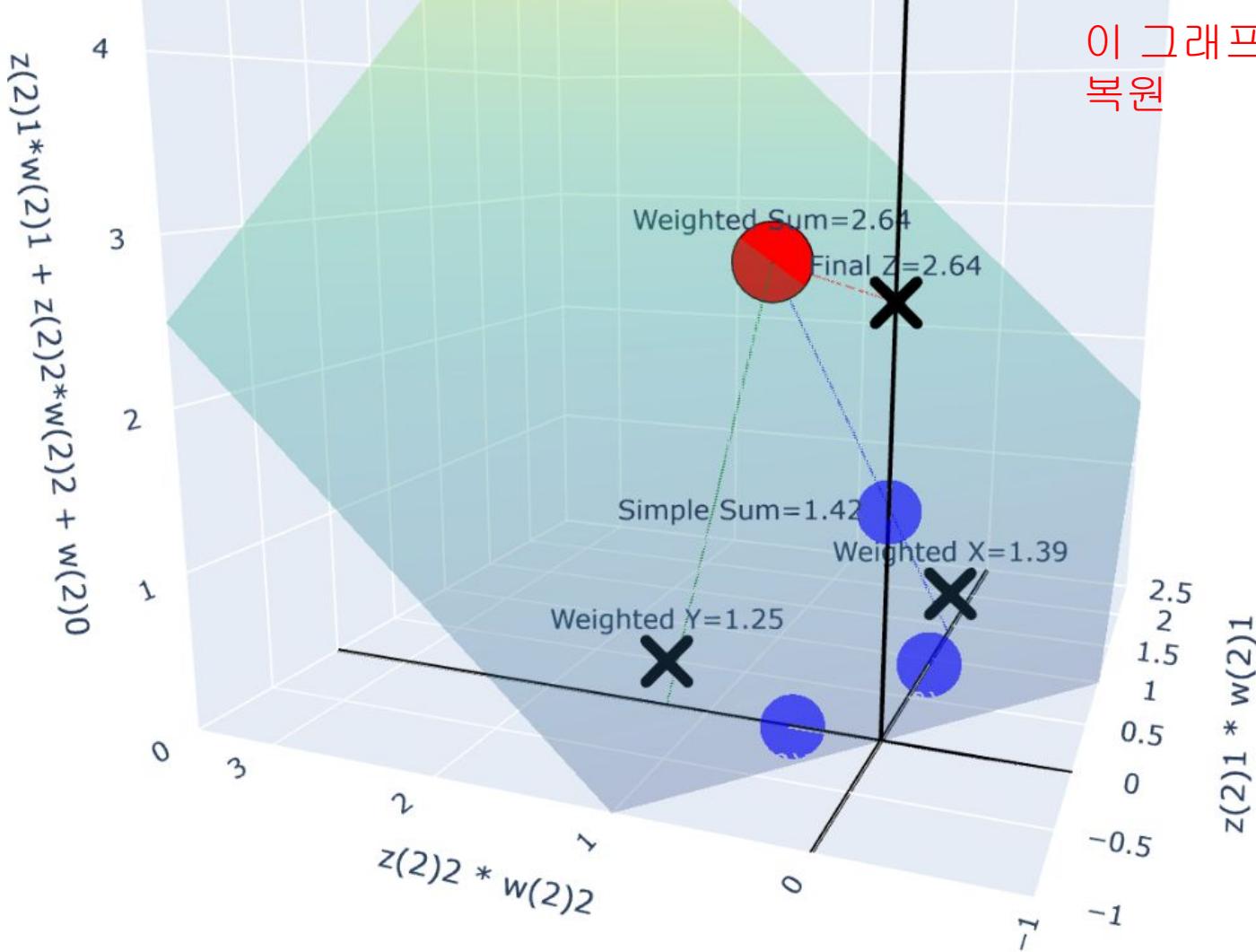
$$\begin{aligned}
w_{1,1}^{(1)} &= -0.5, w_{1,2}^{(1)} = 2, w_{0,1}^{(1)} = 3 \\
w_{2,1}^{(1)} &= 0.5, w_{2,2}^{(1)} = -3, w_{0,2}^{(1)} = -2 \\
w_1^{(2)} &= 1.5, w_2^{(2)} = 2.5, w_0^{(2)} = 0 \\
z_1^{(2)} &\approx 0.924
\end{aligned}$$

$$y_1^{(2)} \approx 2.64$$

$$y_1^{(2)} = w_0^{(2)} + w_1^{(2)} z_1^{(2)} + w_2^{(2)} z_2^{(2)}$$

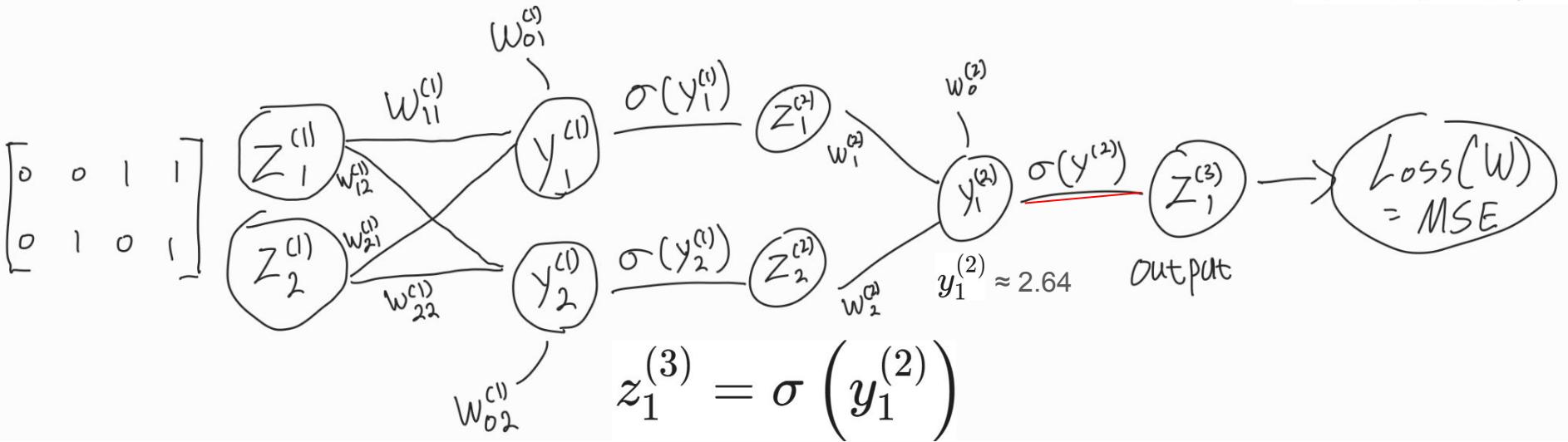


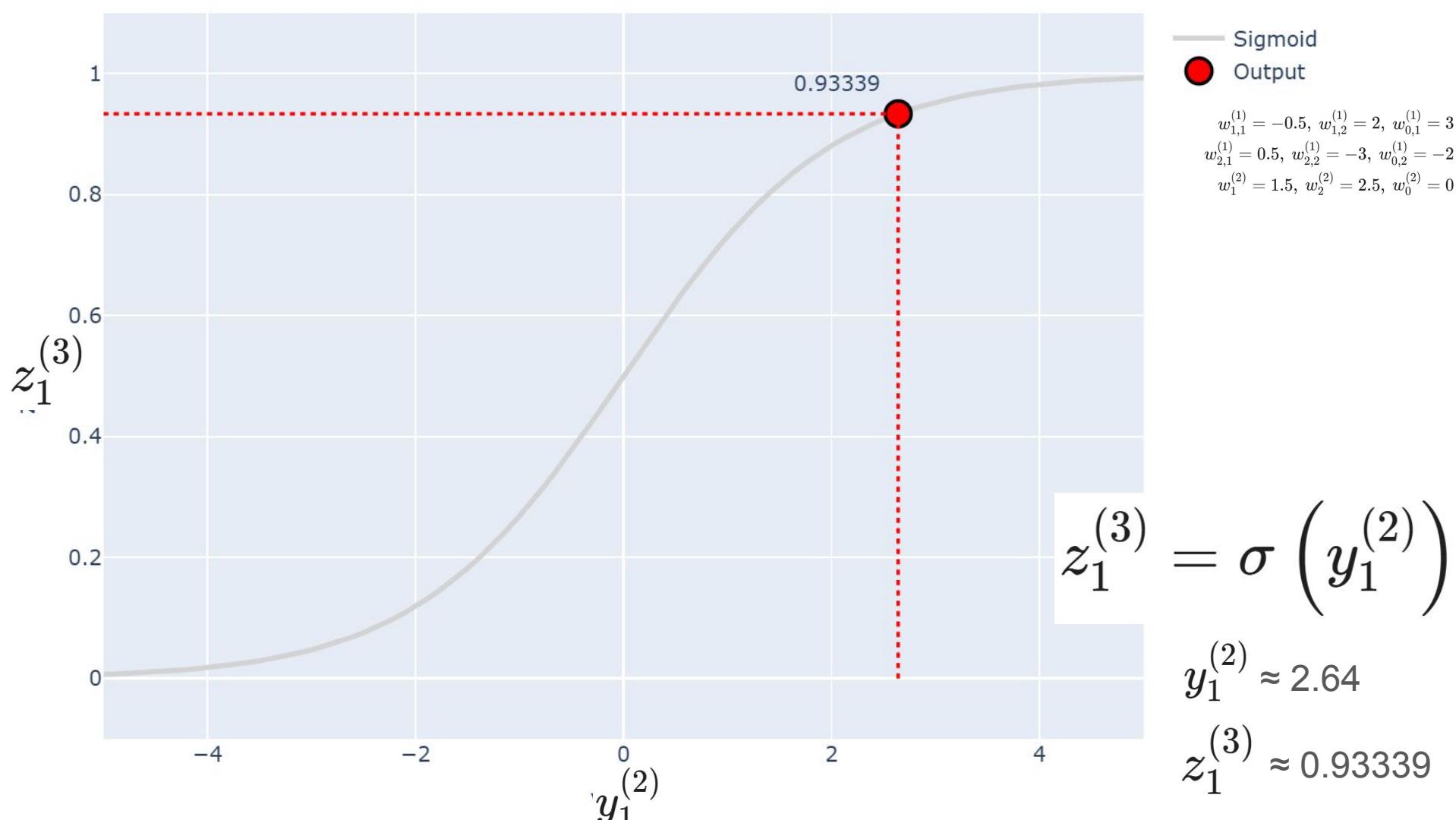
이 그래프 코드로
만들기

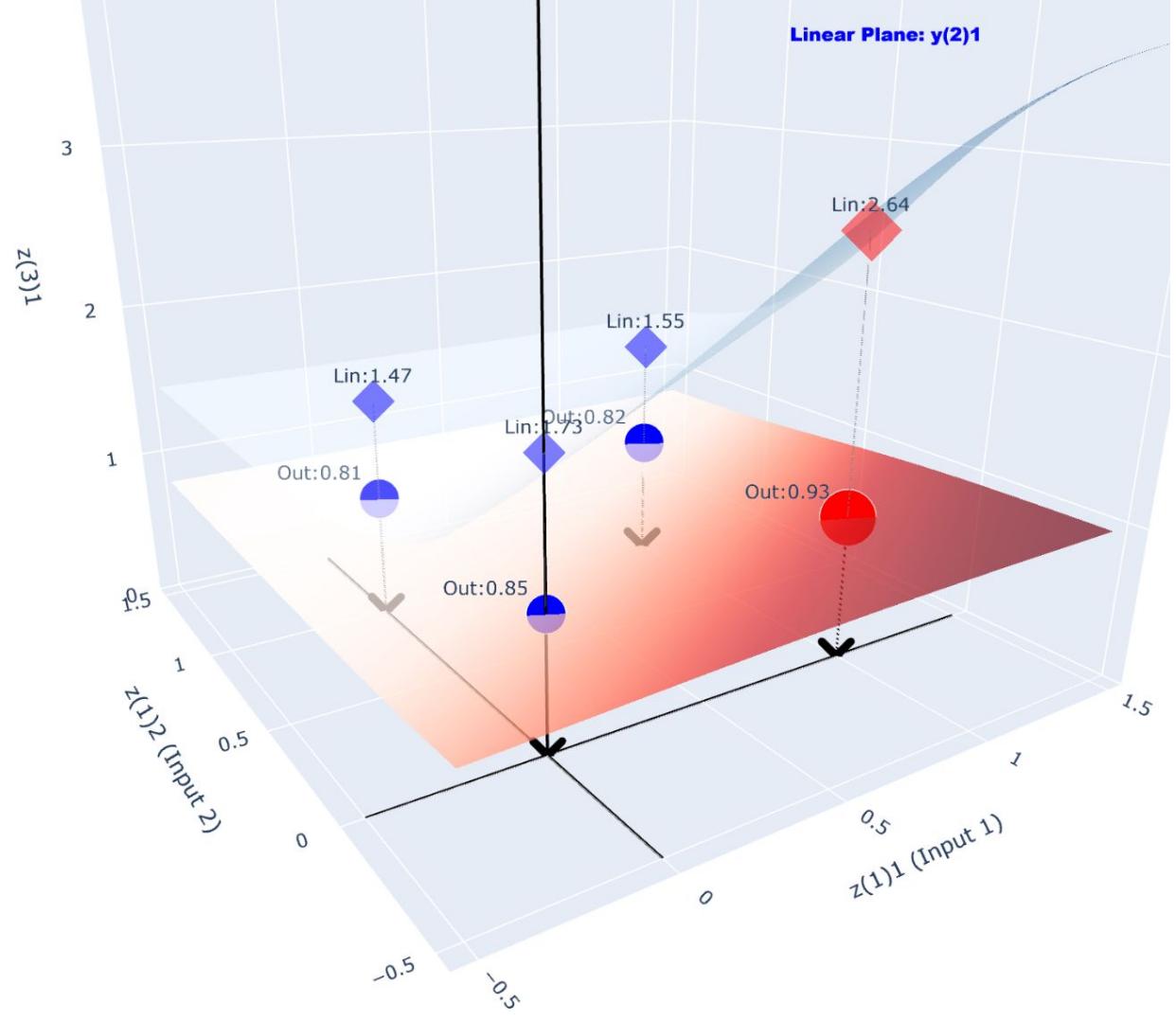


순전파

$$\begin{aligned}
 w_{1,1}^{(1)} &= -0.5, w_{1,2}^{(1)} = 2, w_{0,1}^{(1)} = 3 \\
 w_{2,1}^{(1)} &= 0.5, w_{2,2}^{(1)} = -3, w_{0,2}^{(1)} = -2 \\
 w_1^{(2)} &= 1.5, w_2^{(2)} = 2.5, w_0^{(2)} = 0
 \end{aligned}$$







$$z_1^{(3)} = \sigma \left(y_1^{(2)} \right)$$

$$y_1^{(2)} \approx 2.64$$

$$z_1^{(3)} \approx 0.93339$$

LOSS

w 파라미터들을 무작위로 설정했을 때

$$w_{1,1}^{(1)} = -0.5, w_{1,2}^{(1)} = 2, w_{0,1}^{(1)} = 3$$

$$w_{2,1}^{(1)} = 0.5, w_{2,2}^{(1)} = -3, w_{0,2}^{(1)} = -2$$

$$w_1^{(2)} = 1.5, w_2^{(2)} = 2.5, w_0^{(2)} = 0$$

input (1,0) 의 예측 y값 = $z_1^{(3)} \approx 0.93339$

실제 정답 (1) 과의 차이 ≈ 0.07

하지만 다른 값들은 정답 - 예측 간의 차이가 심하게 발생

입력 A, B에 대해서 Y를 잘 예측하게 하는 W 파라미터들을

어떻게 수정해야 하는지 알아봅시다

입력		출력
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

입력(Input)	정답(Target)	예측값(Pred)	손실(Loss)
[0 0]	0	0.84901	1.89055
[0 1]	1	0.81348	0.20644
[1 0]	1	0.93316	0.06918
[1 1]	0	0.82454	1.74035

전체 평균 손실 (Mean Loss): 0.97663

LOSS

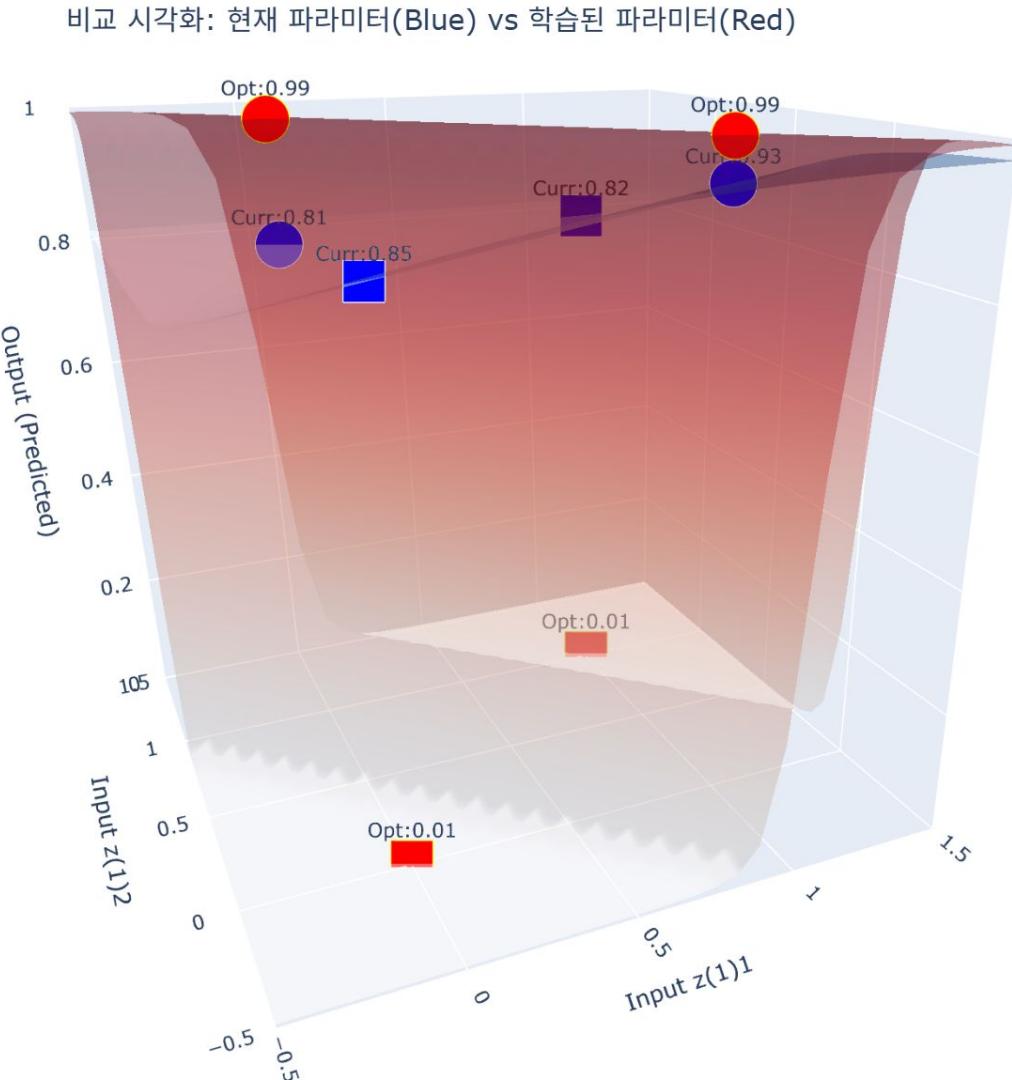
이 오차를 어떻게 줄여야 할까요?

학습 대상인 w파라미터들을 수정

얼마나? 양수로? 음수로?

입력(Input)	정답(Target)	예측값(Pred)	손실(Loss)
[0 0]	0	0.84901	1.89055
[0 1]	1	0.81348	0.20644
[1 0]	1	0.93316	0.06918
[1 1]	0	0.82454	1.74035

전체 평균 손실 (Mean Loss): 0.97663



LOSS

유의미한 연관성이 보이시나요?

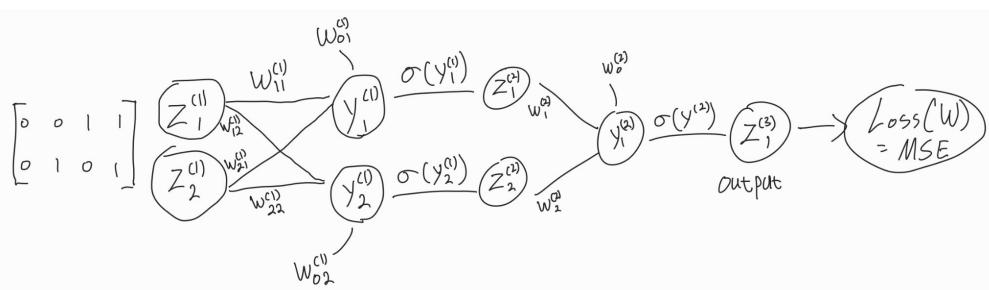
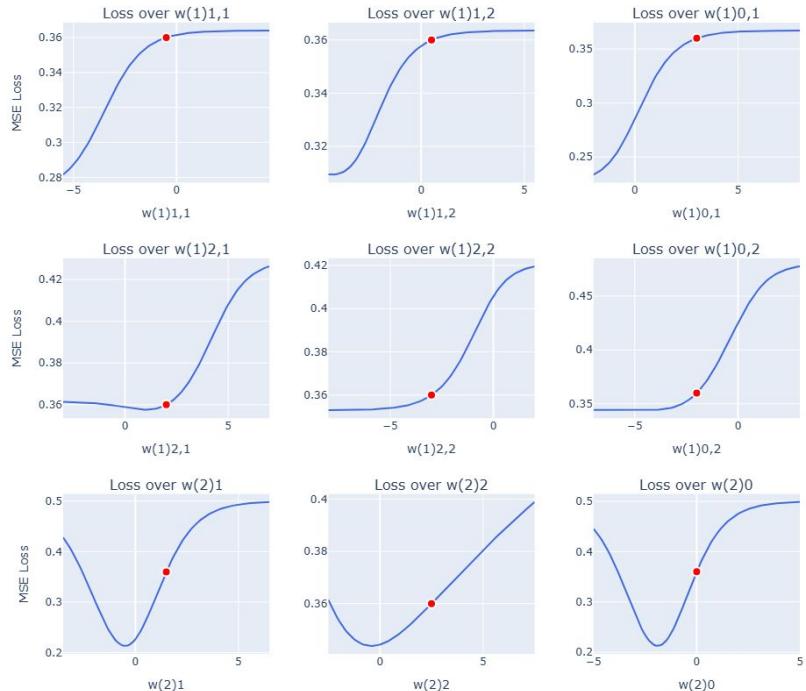
동일한 y노드로 가는 w들이 비슷한 경향

편향 w 는 고정하고, 선형결합되는 y 에 대한
 $w_{0,1}^{(1)}, w_{0,2}^{(1)}, w_0^{(2)}$ $y_1^{(1)}, y_2^{(1)}, y_1^{(2)}$

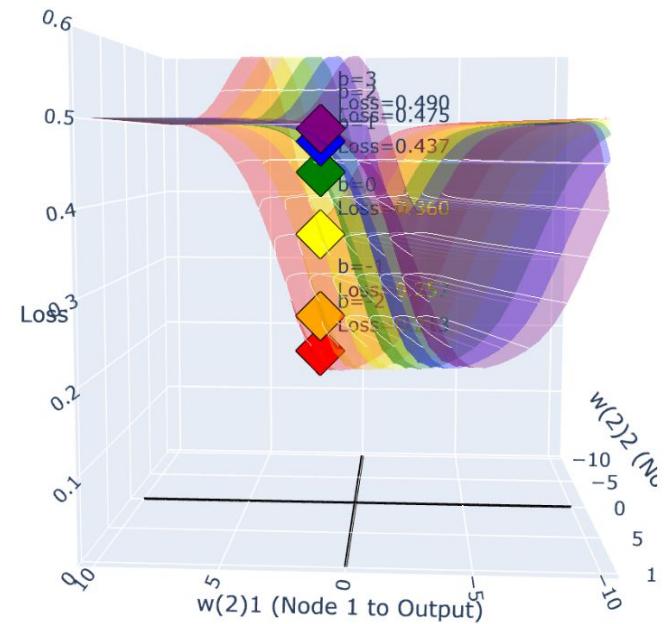
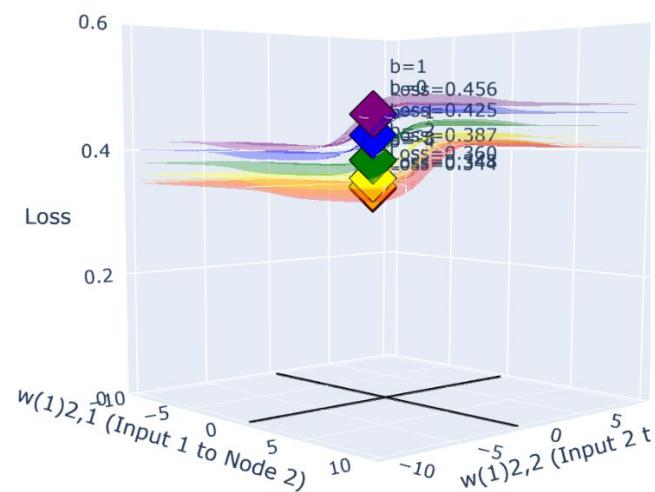
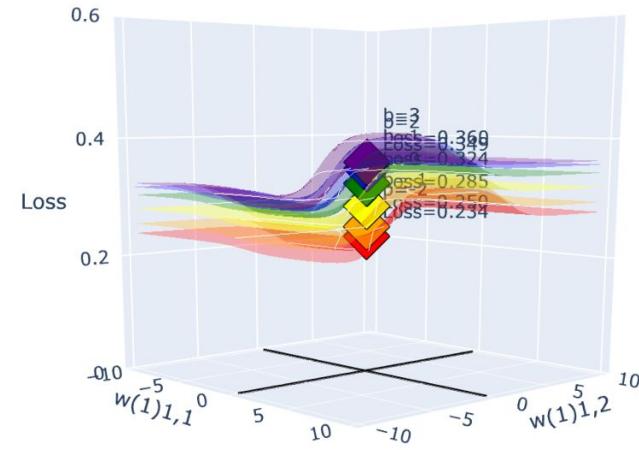
w 들을 x, y 로 갖는 3차원 Loss함수를 시각화

$$\begin{bmatrix} w_{1,1}^{(1)}, & w_{1,2}^{(1)} \\ w_{2,1}^{(1)}, & w_{2,2}^{(1)} \\ w_1^{(2)}, & w_2^{(2)} \end{bmatrix}$$

9개 파라미터별 Loss Function 단면 시각화 (빨간 점: 현재 상태)



LOSS



LOSS

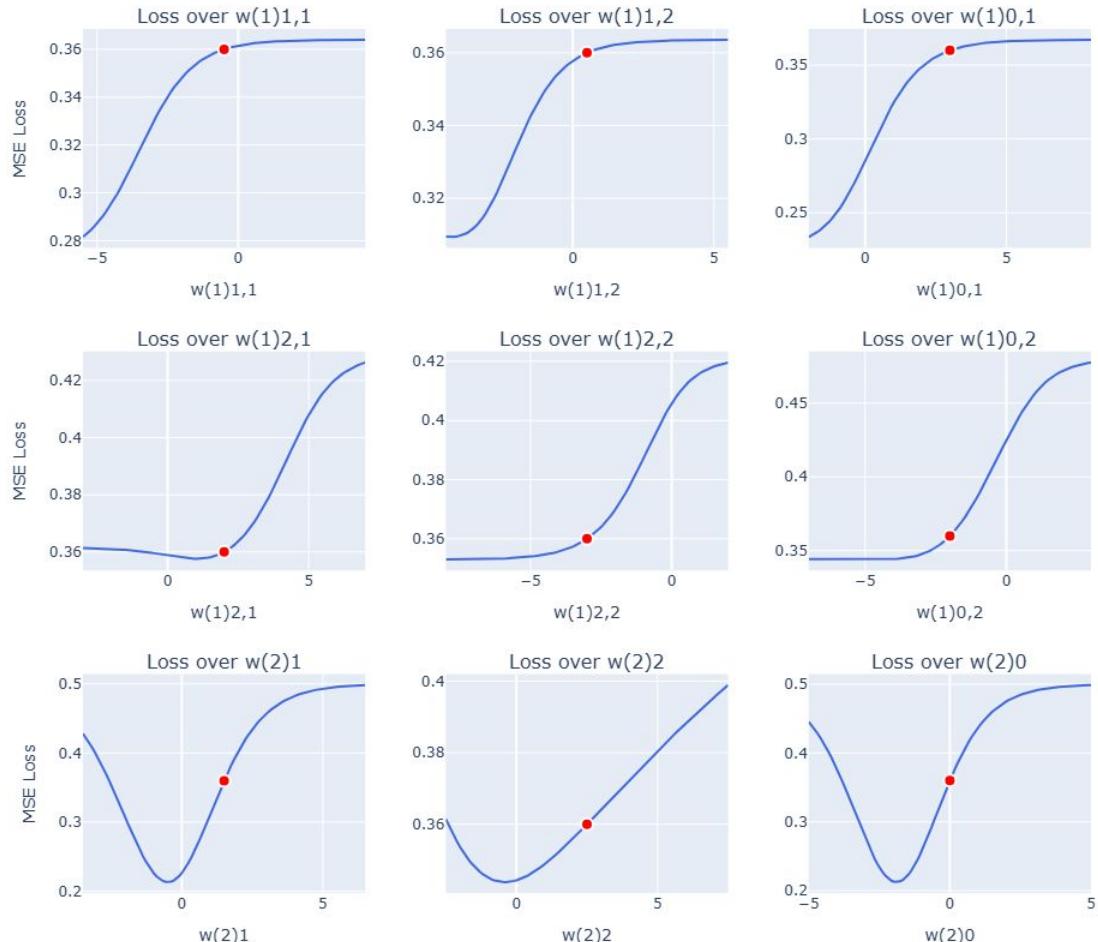
w별 gradient 확인

각 w에 대한 Loss를 측정

$$w_{new} = w_{old} - \eta \cdot \frac{\partial L}{\partial w}$$

$$\begin{aligned} w_{1,1}^{(1)} &= -0.5, \quad w_{1,2}^{(1)} = 2, \quad w_{0,1}^{(1)} = 3 \\ w_{2,1}^{(1)} &= 0.5, \quad w_{2,2}^{(1)} = -3, \quad w_{0,2}^{(1)} = -2 \\ w_1^{(2)} &= 1.5, \quad w_2^{(2)} = 2.5, \quad w_0^{(2)} = 0 \end{aligned}$$

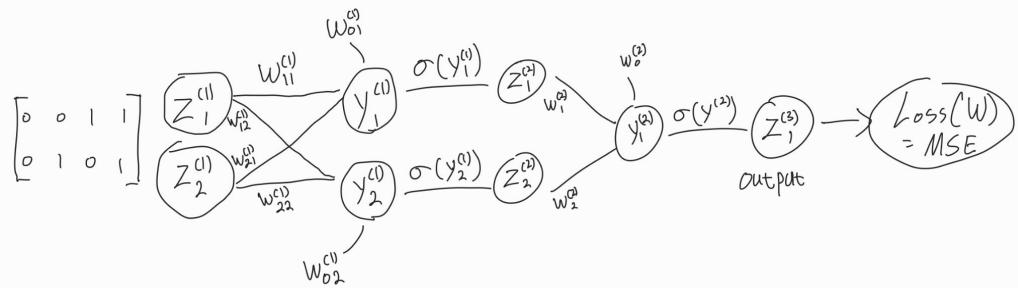
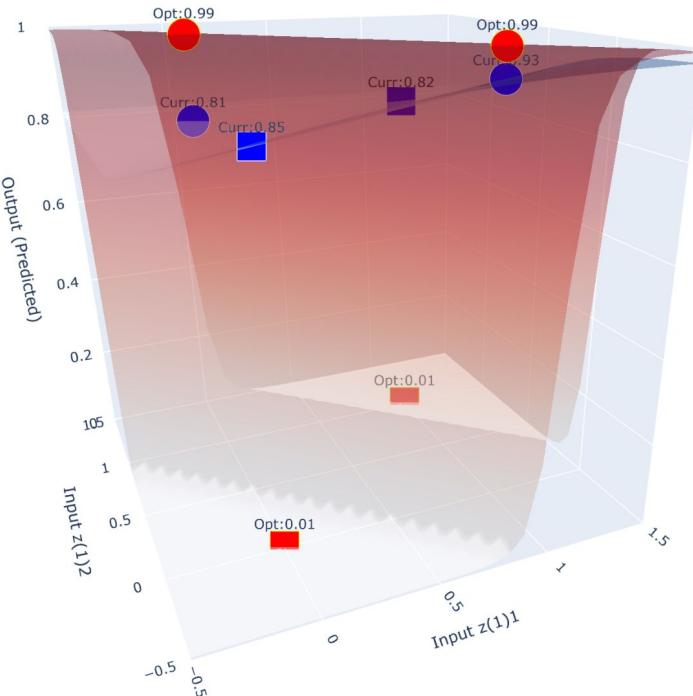
9개 파라미터별 Loss Function 단면 시각화 (빨간 점: 현재 상태)



역전파

$w_{1,1}^{(1)}$ 을 타겟으로 설정하고 역전파

비교 시각화: 현재 파라미터(Blue) vs 학습된 파라미터(Red)



$$\frac{\partial Loss}{\partial w_{1,1}^{(1)}} = \frac{\partial Loss}{\partial z_1^{(3)}} * \frac{\partial z_1^{(3)}}{\partial y_1^{(2)}} * \frac{\partial y_1^{(2)}}{\partial z_1^{(2)}} * \frac{\partial z_1^{(2)}}{\partial y_1^{(1)}} * \frac{\partial y_1^{(1)}}{\partial w_{1,1}^{(1)}}$$

입력 (Input)	정답 (Target)	예측값 (Pred)	손실 (Loss)
[0 0]	0	0.84901	1.89055
[0 1]	1	0.81348	0.20644
[1 0]	1	0.93316	0.06918
[1 1]	0	0.82454	1.74035

전체 평균 손실 (Mean Loss): 0.97663

역전파

$$(0.9331) * (1 - 0.9331) \approx 0.0624$$

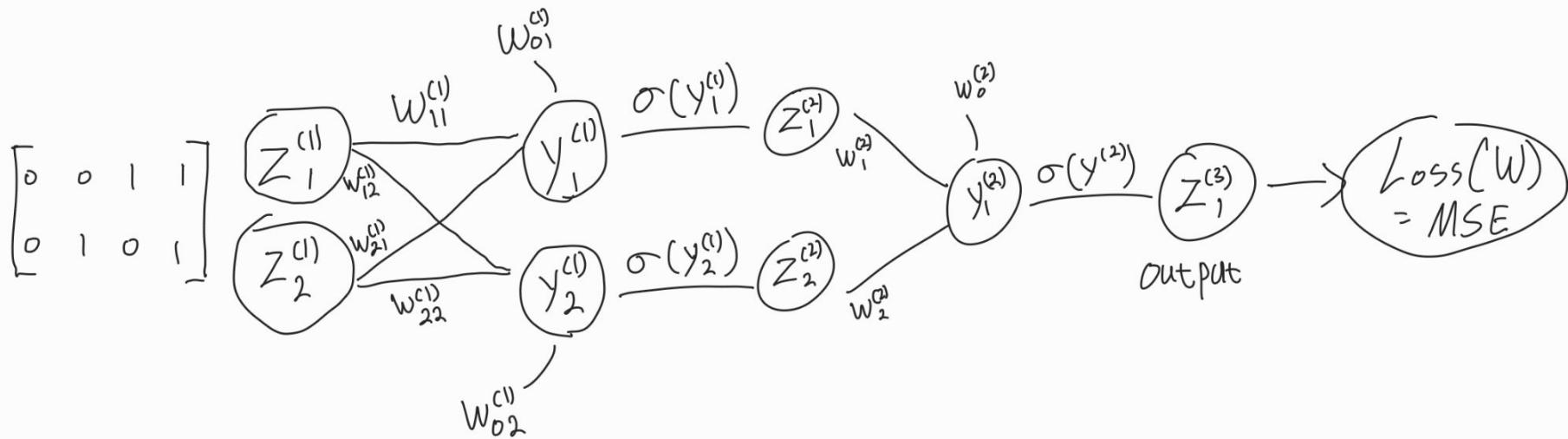
$$(0.9241) * (1 - 0.9241) \approx 0.0701$$

$$\frac{\partial Loss}{\partial w_{1,1}^{(1)}} = \frac{\partial Loss}{\partial z_1^{(3)}} * \frac{\partial z_1^{(3)}}{\partial y_1^{(2)}} * \frac{\partial y_1^{(2)}}{\partial z_1^{(2)}} * \frac{\partial z_1^{(2)}}{\partial y_1^{(1)}} * \frac{\partial y_1^{(1)}}{\partial \mathbf{w}_{1,1}^{(1)}}$$

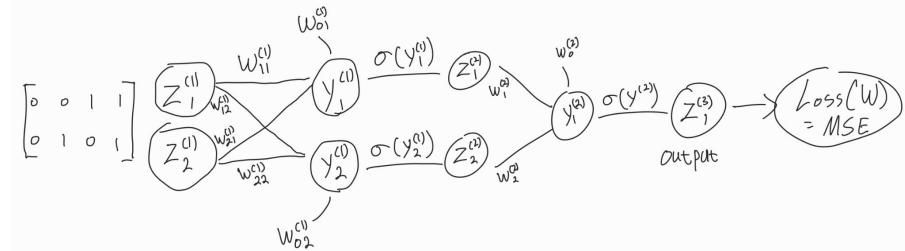
$-1(1 - 0.9331) = -0.0669$

$w_1^{(2)} = 1.5$

$z_1^{(1)} = 1$



역전파



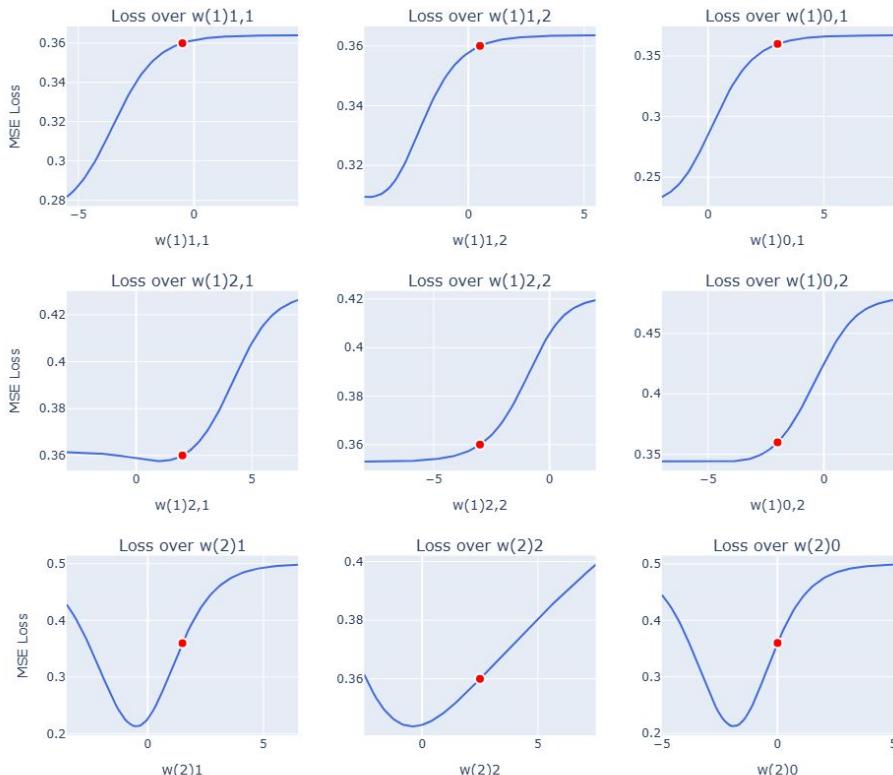
$$\begin{aligned}
 \text{Gradient} &= (-0.0669) \times (0.0624) \times (1.5) \times (0.0701) \times (1) \\
 &= (-0.00417) \times (1.5) \times (0.0701) \\
 &= -0.00625 \times 0.0701 \\
 &\approx \mathbf{-0.000438}
 \end{aligned}$$

파라미터 업데이트

어느 방향으로 얼마나 수정해야 하는지 알 수 있는 Loss함수

각 w 에 대해서 모두 역전파

9개 파라미터별 Loss Function 단면 시각화 (빨간 점: 현재 상태)

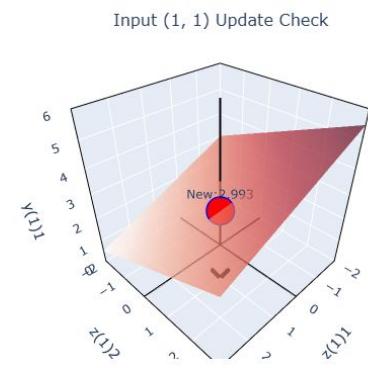
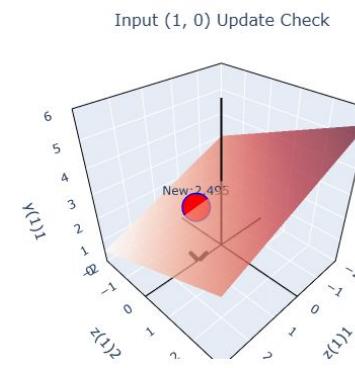
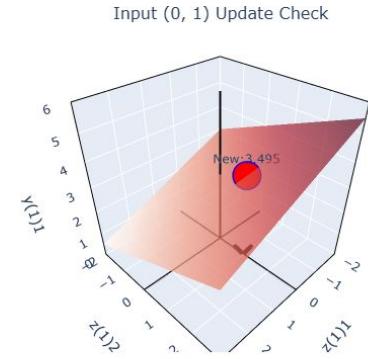
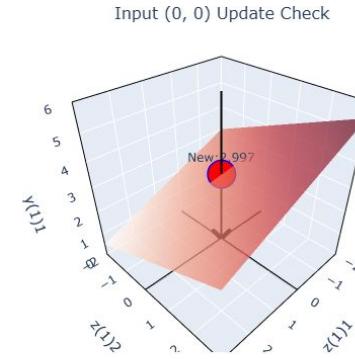
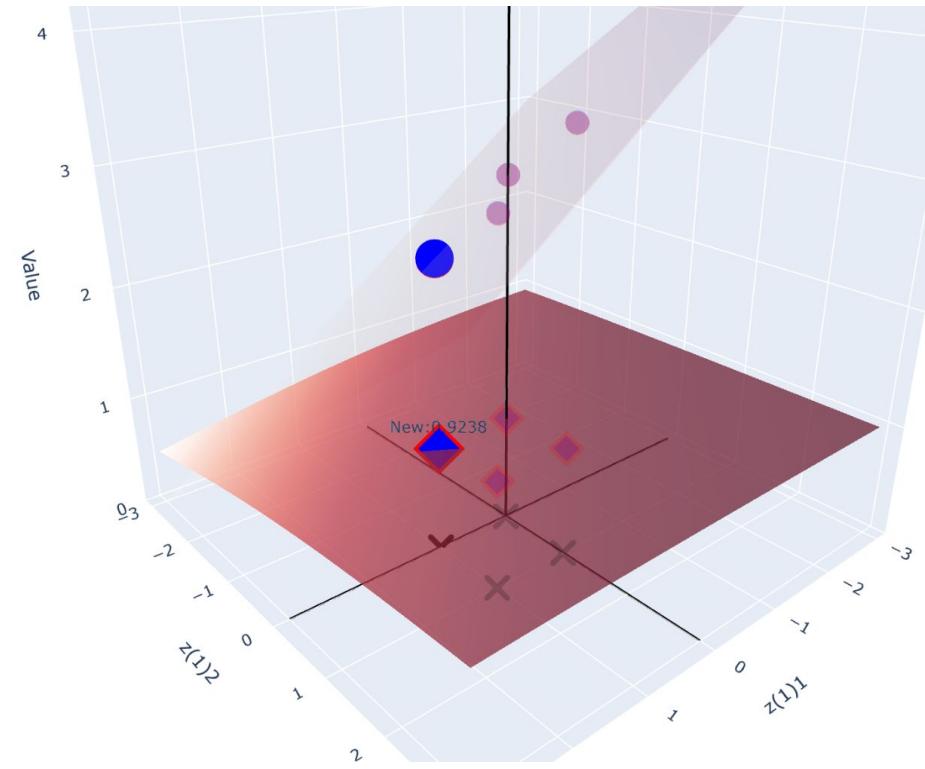


파라미터 업데이트

lr = 1.0

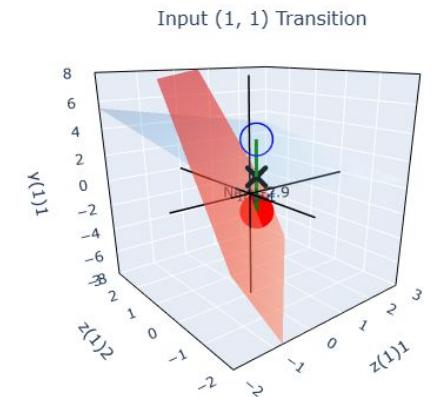
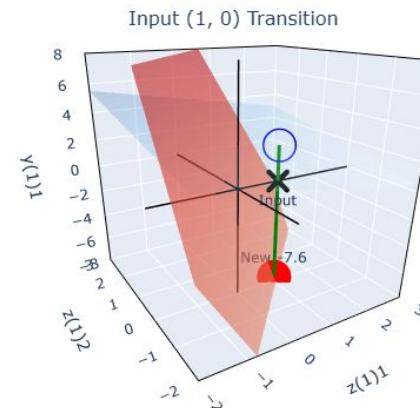
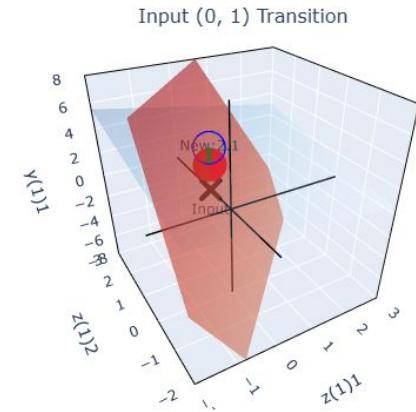
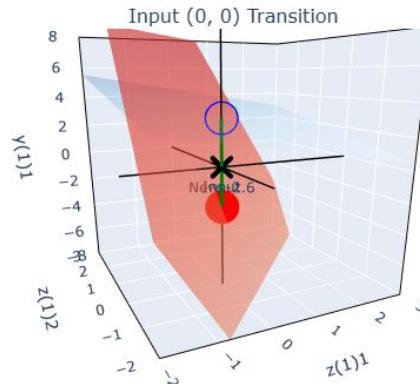
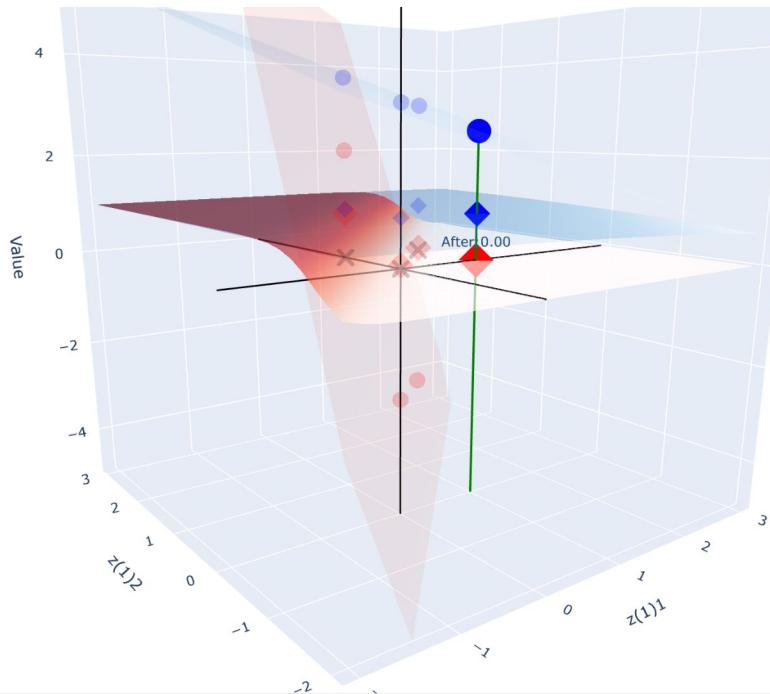
Parameter	Old Value	Gradient	New Value	Change
w11	-0.500000	0.001911	-0.501911	-0.001911
w21	0.500000	0.001719	0.498281	-0.001719
b1	3.000000	0.003453	2.996547	-0.003453
w12	2.000000	0.002717	1.997283	-0.002717
w22	-3.000000	0.003251	-3.003251	-0.003251
b2	-2.000000	0.009741	-2.009741	-0.009741
w_out1	1.500000	0.046495	1.453505	-0.046495
w_out2	2.500000	0.004089	2.495911	-0.004089
b_out	0.000000	0.048913	-0.048913	-0.048913

순전파

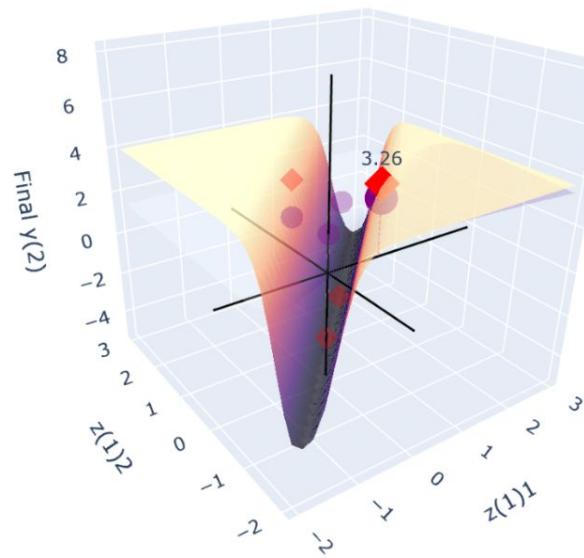
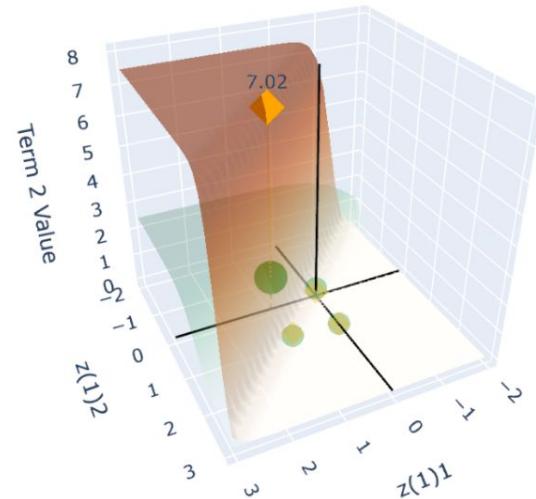
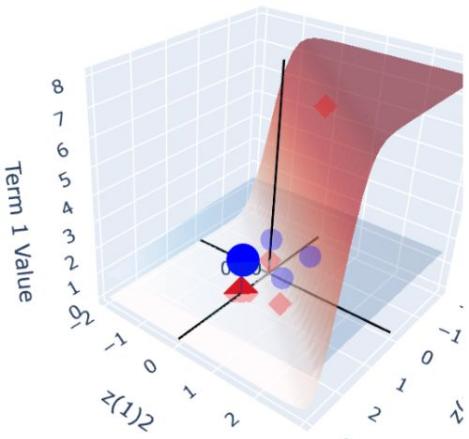


순전파

훈련을 잘 시킨 w파라미터로 순전파 시작하



순전파



코드로 이해

pytorch

PyTorch 예측 결과:

[[0.01603173]]

[0.9808172]

[0.9807922]

[0.03328492]]

```
import torch
import torch.nn as nn
import torch.optim as optim

X = torch.tensor([[0,0], [0,1], [1,0], [1,1]], dtype=torch.float32)
y = torch.tensor([[0], [1], [1], [0]], dtype=torch.float32)

class XORModel(nn.Module):
    def __init__(self):
        super().__init__()
        self.layer1 = nn.Linear(2, 2)

        # [수정됨] 활성화 함수: ReLU -> Sigmoid
        # 아까 수식에서 몇 가지 옵션을 적용합니다.
        self.activation = nn.Sigmoid()

        self.layer2 = nn.Linear(2, 1)
        self.sigmoid = nn.Sigmoid()

    def forward(self, x):
        x = self.layer1(x)
        x = self.activation(x) # 여기서 부드럽게 흰 공간이 만들어집니다.
        x = self.layer2(x)
        return self.sigmoid(x)

model = XORModel()

criterion = nn.BCELoss()
optimizer = optim.Adam(model.parameters(), lr=0.01)

for epoch in range(2000):
    prediction = model(X)
    loss = criterion(prediction, y)
    optimizer.zero_grad()
    loss.backward()
    optimizer.step()

print("PyTorch (Sigmoid) 예측 결과:\n", model(X).detach().numpy())
```

코드로 이해

Keras

Keras 예측 결과:

[0.02846328]

[0.96662706]

[0.97150636]

[0.0256856]]

```
import numpy as np
import tensorflow as tf
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import Dense
from tensorflow.keras.optimizers import Adam

# 1. 데이터 준비
X = np.array([[0,0], [0,1], [1,0], [1,1]])
y = np.array([[0], [1], [1], [0]])

model = Sequential()

# [중요] 은닉층 2개 (Sigmoid)
# 이론 설명대로 뉴런 2개를 유지합니다.
model.add(Dense(units=2, input_dim=2, activation='sigmoid'))

# 출력층 1개 (Sigmoid)
model.add(Dense(units=1, activation='sigmoid'))

optimizer = Adam(learning_rate=0.01)

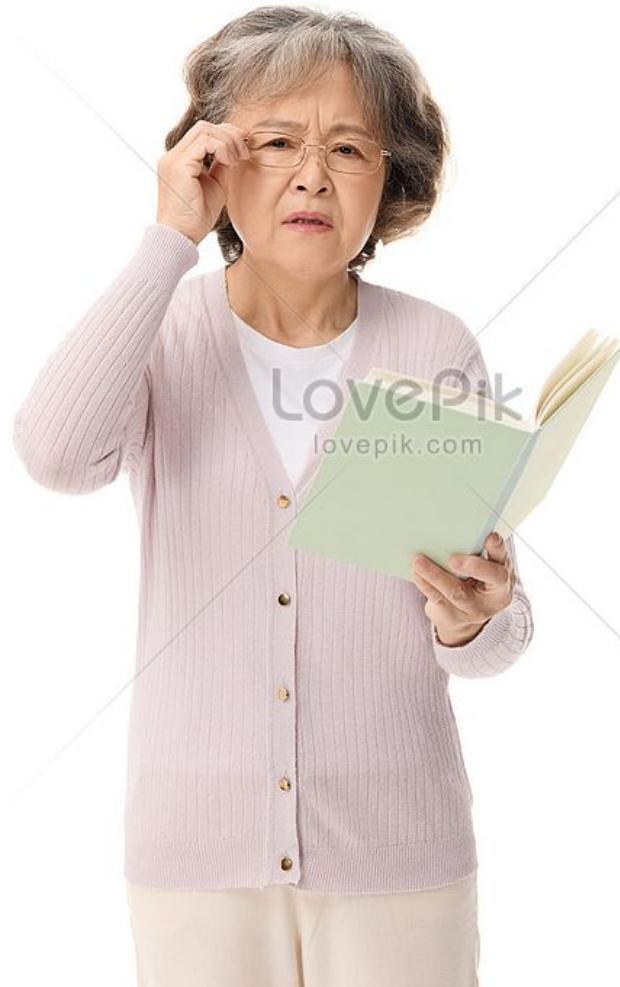
model.compile(loss='binary_crossentropy', optimizer=optimizer, metrics=['accuracy'])

# 학습
model.fit(X, y, epochs=1000, verbose=0)

# 결과 출력 (소수점 3자리까지만 깔끔하게)
pred = model.predict(X)
print("Keras 예측 결과:\n", pred)
print(np.round(pred)) # 반올림해서 0과 1로 변환
```

QnA

질문 받겠습니다



감사합니다.

참고 링크 (출처)

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and, or 게이트 사진 - <http://www.ktword.co.kr/test/view/view.php?no=4557>

nand 게이트 사진 - <http://www.ktword.co.kr/test/view/view.php?no=4560>

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