

COMS BC1016

Introduction to Computational Thinking and Data Science




Lecture 18: Correlation and Linear Regression

BARNARD COLLEGE OF COLUMBIA UNIVERSITY

Reminders

- Final Project Proposal Due tonight
 - 10% of your final project grade!
- HW 7 due Monday (skip Question 4 about the survey)
- Lab 9 this week
 - Last lab assignment of the semester!!
- Thanksgiving next week, then final project consultations the week after

Last Topic: Prediction

- **Today:** Correlation and Linear Regression
- **Monday, Nov 24:** Least Squares and Residuals  HW 7 due
- ***Wednesday, Nov 26: Holiday!***
- **Monday, Dec 1:** Regression Inference  HW 8 due
Progress Report due
- **Wednesday, Dec 3:** Special Topics (Data Ethics)
 - Final Project Consultations during Lab
- **Monday, Dec 8:** Special Topics (Data Privacy)  HW 9 due

Correlation

Prediction

- Guessing the future based on data
- To predict the value of a variable:
 - Identify (measurable) attributes that are associated with that variable
 - Describe the relation between the attributes and the variable you want to predict
 - Use the relation to predict the value of a variable

Correlation Coefficient r

- A metric for the strength of the **linear relationship** between two variables
 - How clustered values are in a scatter plot around a straight line
- r is always between -1 and 1 ($-1 \leq r \leq 1$)
 - $r = 1$: perfect correlation (straight line) sloping upward
 - $r = -1$: perfect correlation (straight line) sloping down
- $r = 0$: no linear association (*uncorrelated*)

Notebook demo: Correlation Plots

Notes about using r

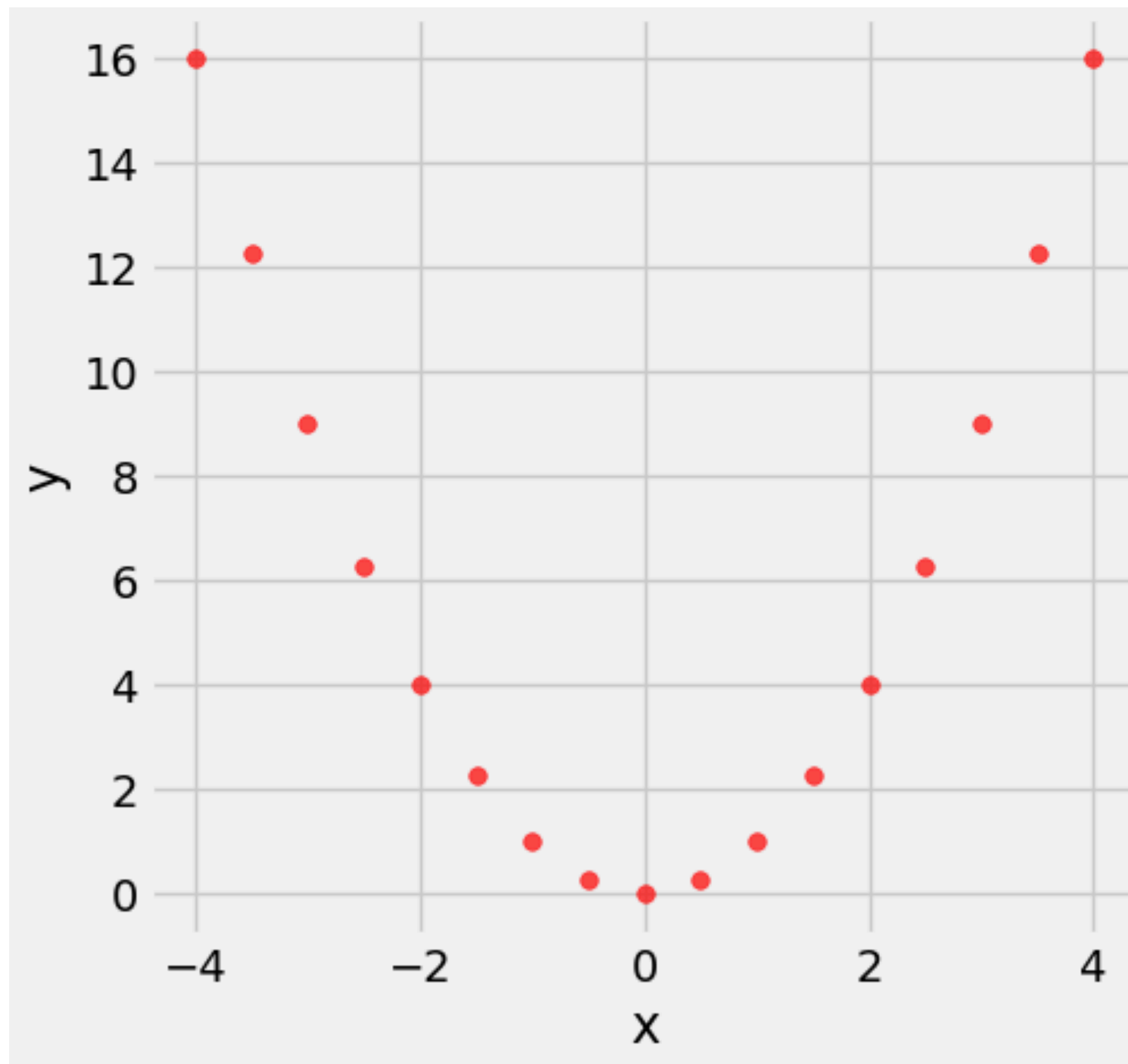
- r measures **linear** association
- Association does *not* imply causation
 - Two variables may be correlated, but one may not cause the other
 - Non-linearity and outliers can affect correlation
- Correlations based on aggregated data (**ecological correlations**) can be misleading
 - Correlations between individuals may be lower than correlation between averages of groups

Questions

True or false?

1. If the correlation coefficient of x and y is 0, then knowing one cannot help us predict the other

Non-linear correlations have $r \neq 0$



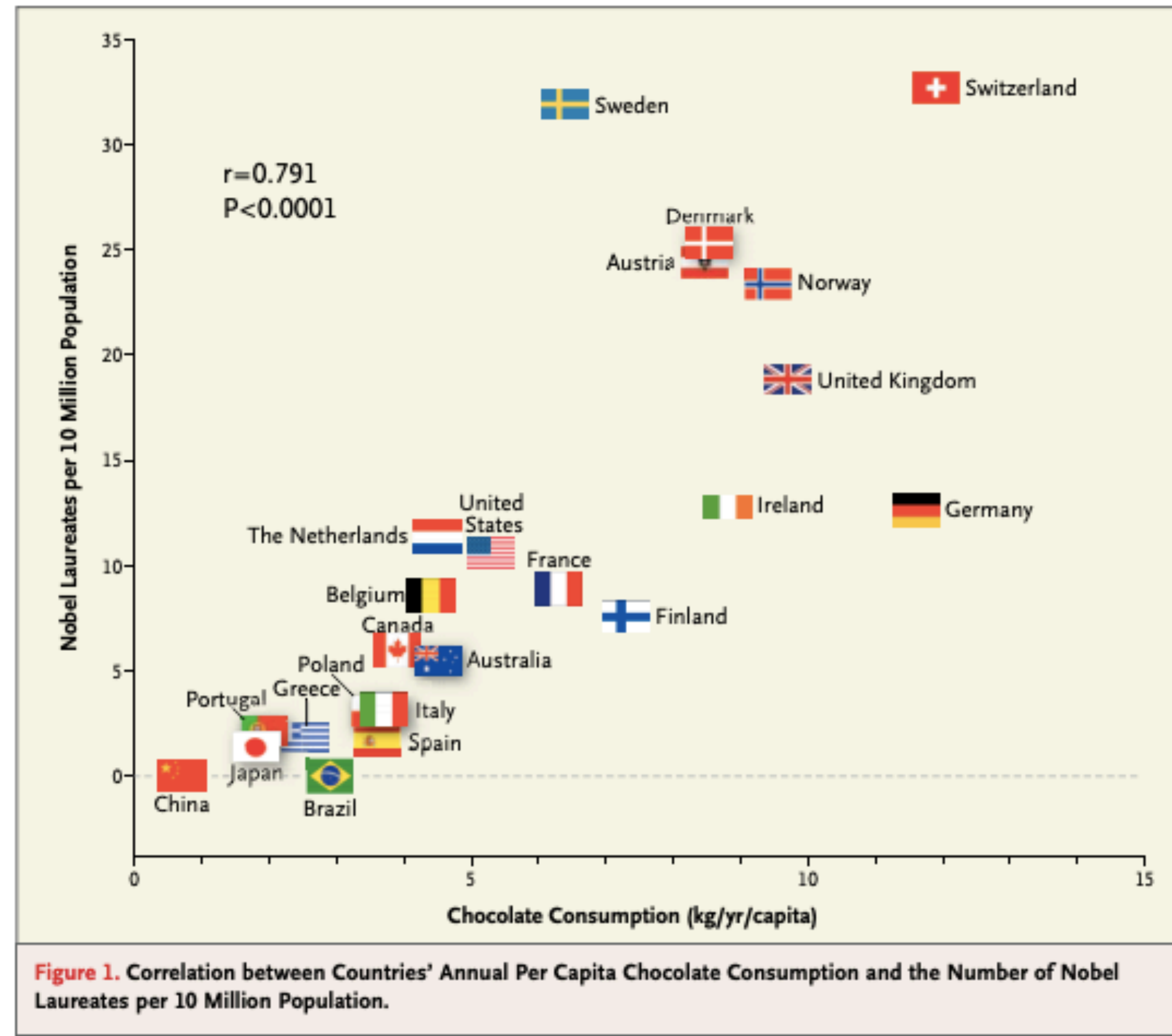
x and y are highly correlated,
but not linearly so $r \neq 0$

Questions

True or false?

1. If the correlation coefficient of x and y is 0, then knowing one cannot help us predict the other
2. If x and y have a correlation coefficient of 1, then one must cause the other

Association \neq causation



Questions

True or false?

1. If the correlation coefficient of x and y is 0, then knowing one cannot help us predict the other
2. If x and y have a correlation coefficient of 1, then one must cause the other
3. If x and y have a correlation coefficient of -0.8, they have a negative association

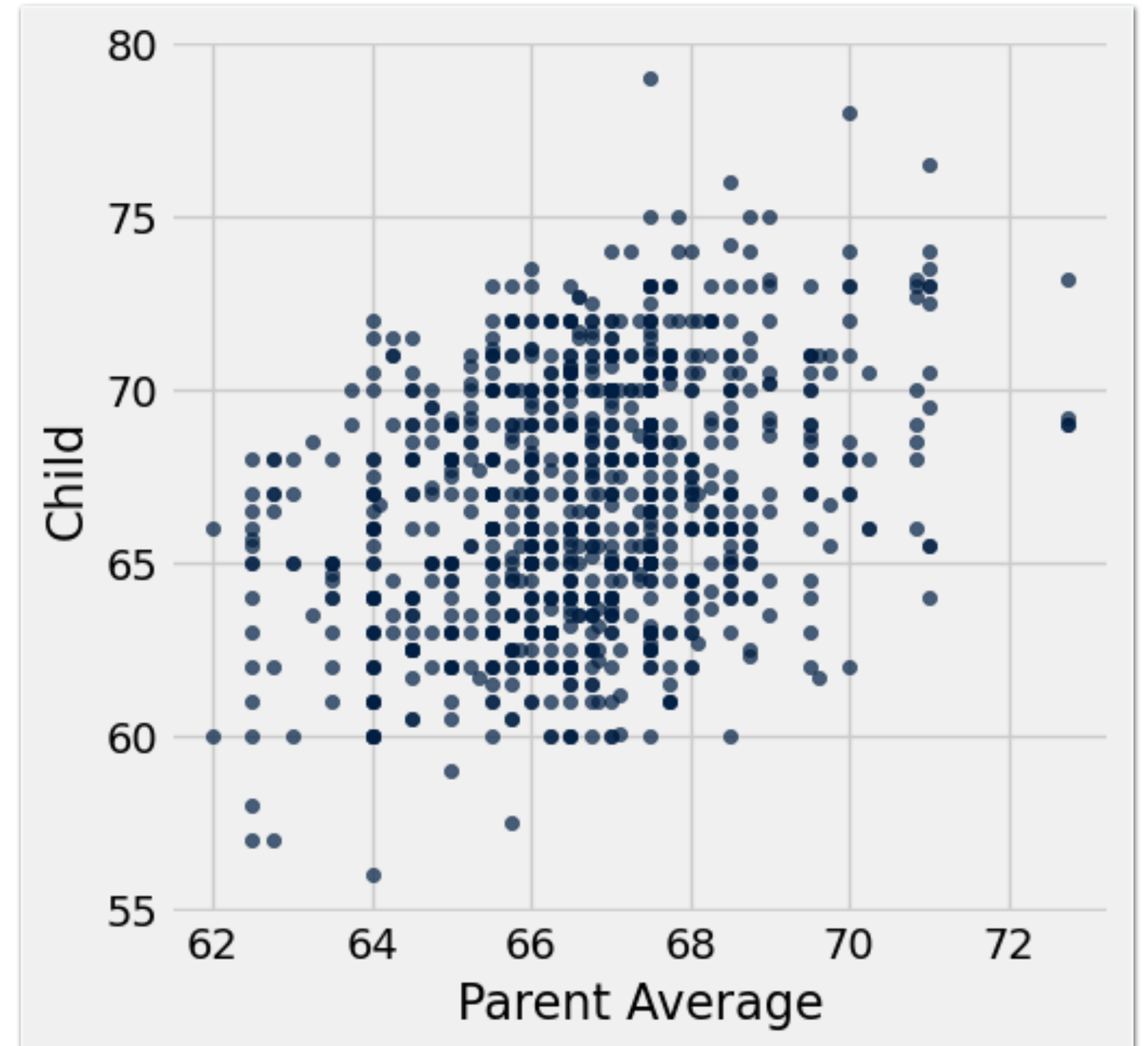
Prediction

Guess the future

- Based on incomplete information
- One way of making predictions:
 - To predict the outcome for an individual, find others who are like that individual and whose outcomes you know. Use those outcomes as the basis of your prediction

Example: Galton's Heights

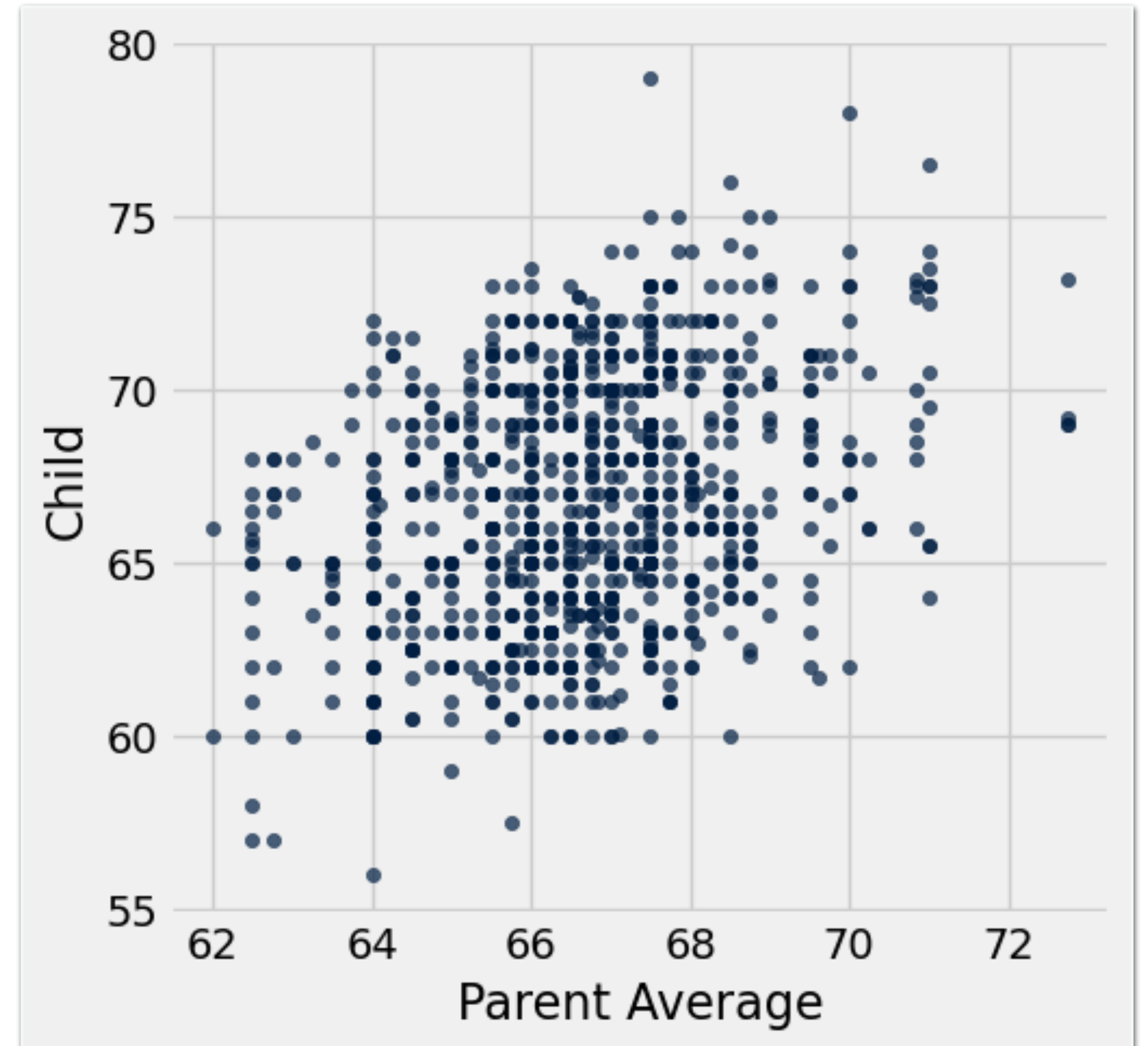
Goal: Predict the height of a new child based on that child's midparent height (average parent height)



Example: Galton's Heights

Goal: Predict the height of a new child based on that child's midparent height (average parent height)

- How could we predict the child's height given a mid parent height of 68 inches?
- Idea: Use the average height of the children of families whose midparent height is close to 68 inches



Notebook Demo: Child's Height

Graph of Average

- For each x value, the prediction is the average of the y values in its nearby group
- The graph of these predictions is the **graph of averages**
- If the association between x and y is linear, then points in the graph of averages tend to fall on a line
- This line is called the **regression line**

Nearest Neighbor Regression

- A method for predicting a numerical y given a value of x :
 - Identify the group of points where the values of x are close to the given value
 - The prediction is the average of the y values for the group

Calculating r

Formula for r

- The correlation coefficient (r) is the average product of x in standard units and y in standard units
- To determine r , we first convert our values in x & y to standard units
- We'll denote x_{su} and y_{su} , respectively

$$y_{\text{su}} = r \times x_{\text{su}}$$

This is known as the **linear regression line**

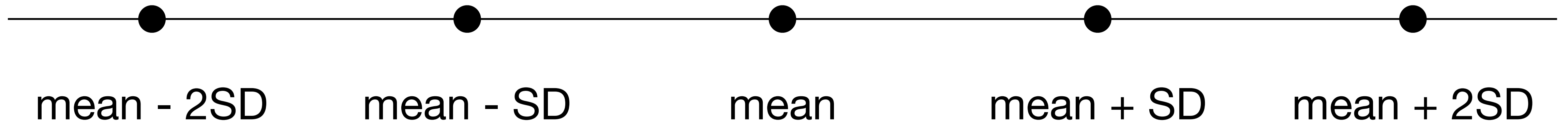
Standard Units

- The quantity z (from “average $\pm z$ SDs” in Chebychev’s inequality) measures **standard units**
- **Standard units** is the number of standard deviations away from the average
- To convert a value (v) to standard units, compare the deviation from the average (μ) with the standard deviation (SD):

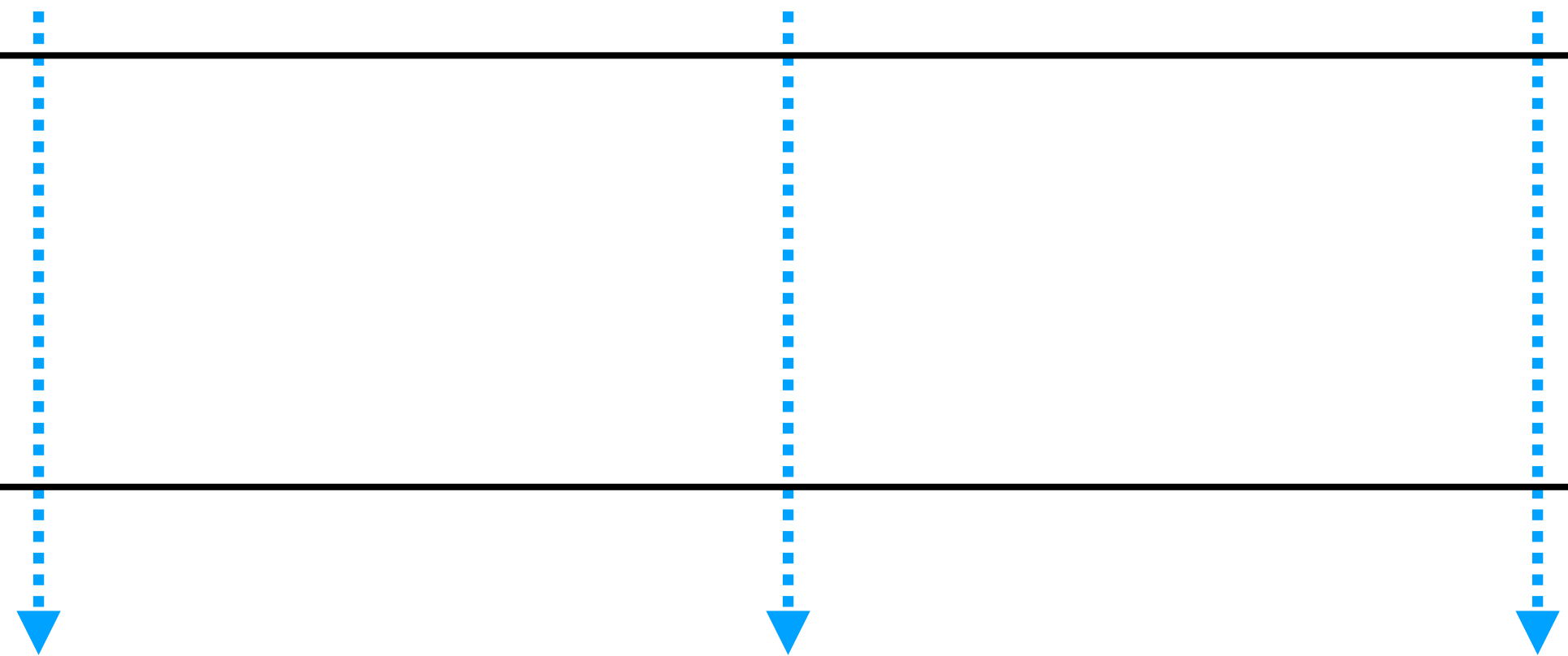
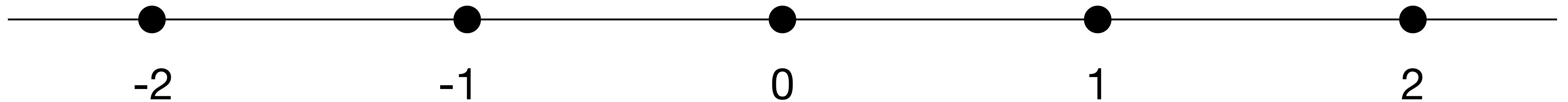
$$z = \frac{v - \mu}{\text{SD}}$$

Converting to Standard Units

Original Units



Standard Units



Correlation Coefficient r

- r is the average of the product of two variables, when both variables are measured in standard units
- What this means for us:
 - r is not affected by changing the units of the measurement of the data
 - r will be the same regardless of which variable is plotted on the x- and y- axes

Regression Line: Slope & Intercept

Regression Line

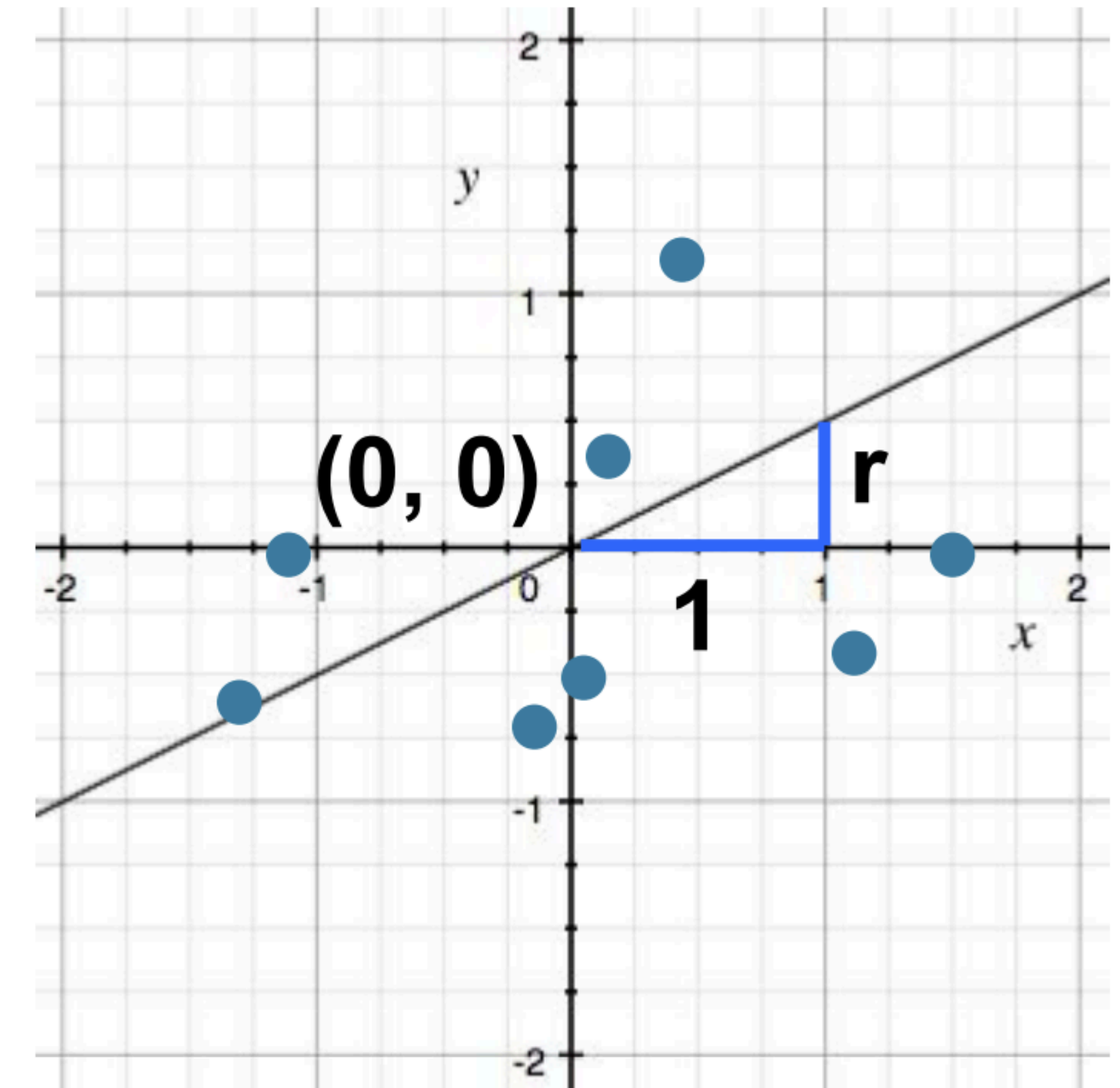
Lines generally can be expressed by

$$y = \text{slope} \times x + \text{intercept}$$

When in standard units, this is equivalent to

$$y_{\text{su}} = r \times x_{\text{su}}$$

Standard Units



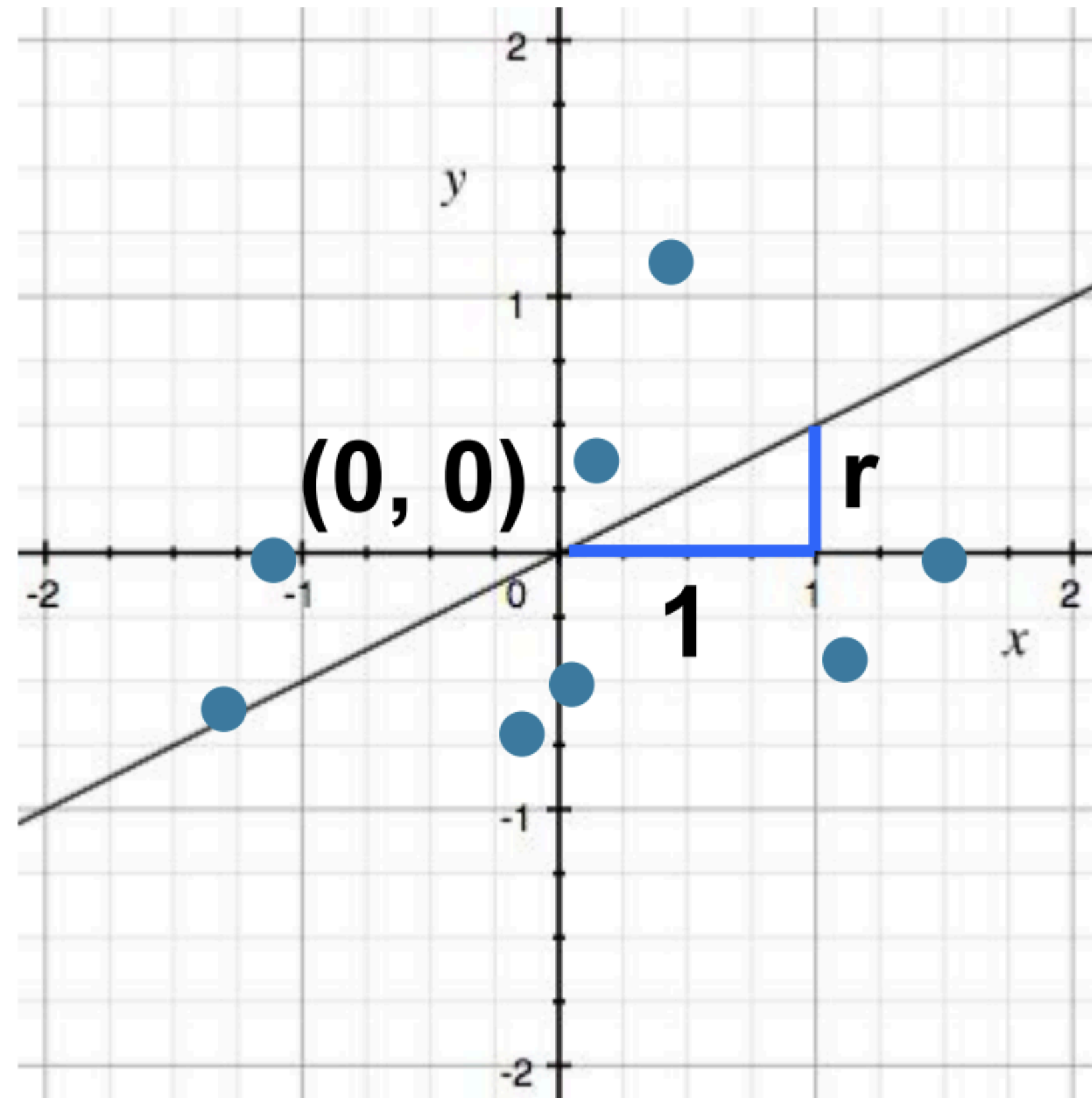
Regression Line: Original Units

$$y_{\text{su}} = r \times x_{\text{su}}$$

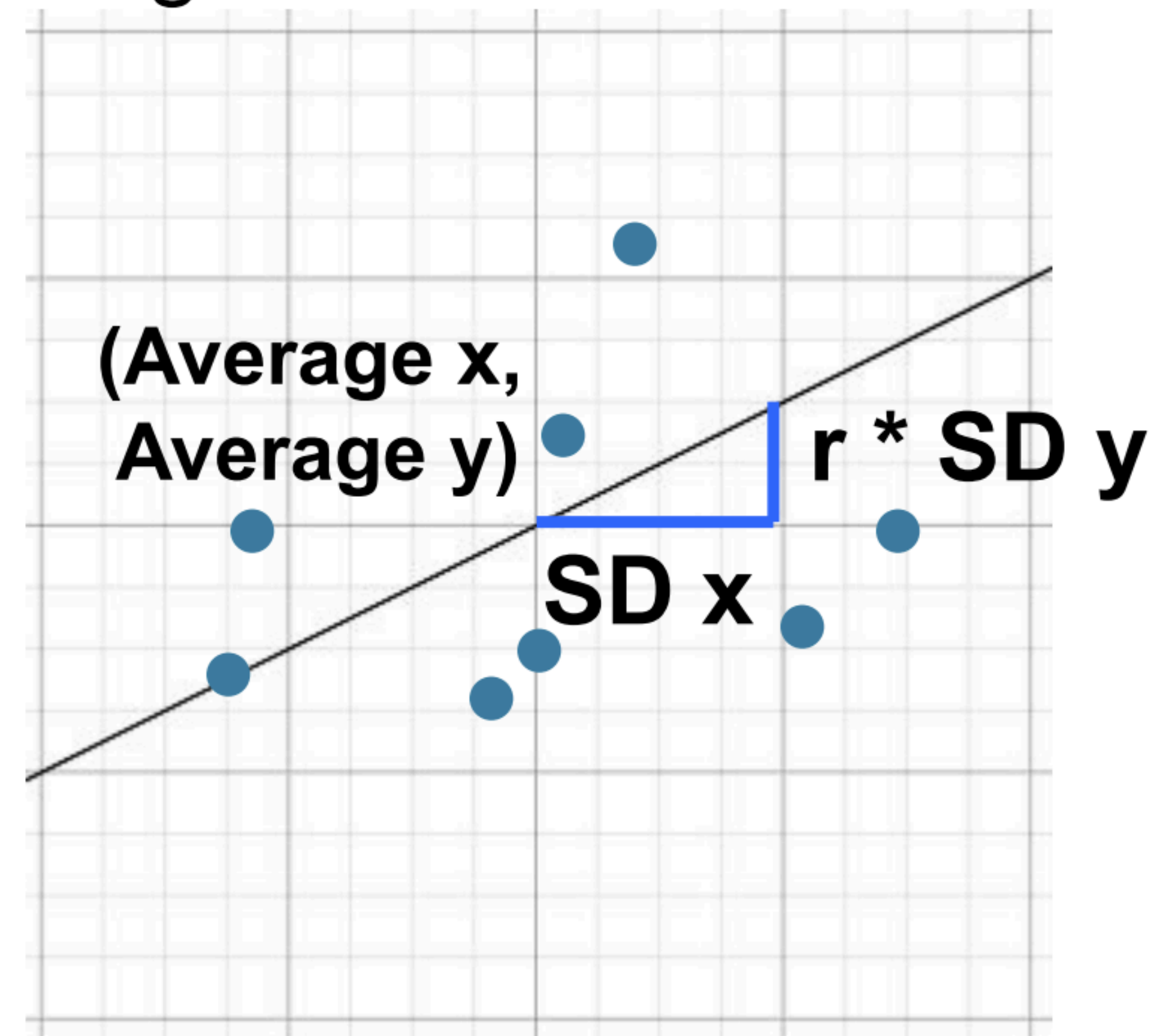
$$\frac{\text{estimate of } y - \text{avg}(y)}{\text{SD of } y} = r \times \frac{x - \text{avg}(x)}{\text{SD of } x}$$

Regression Line: Original Units

Standard Units

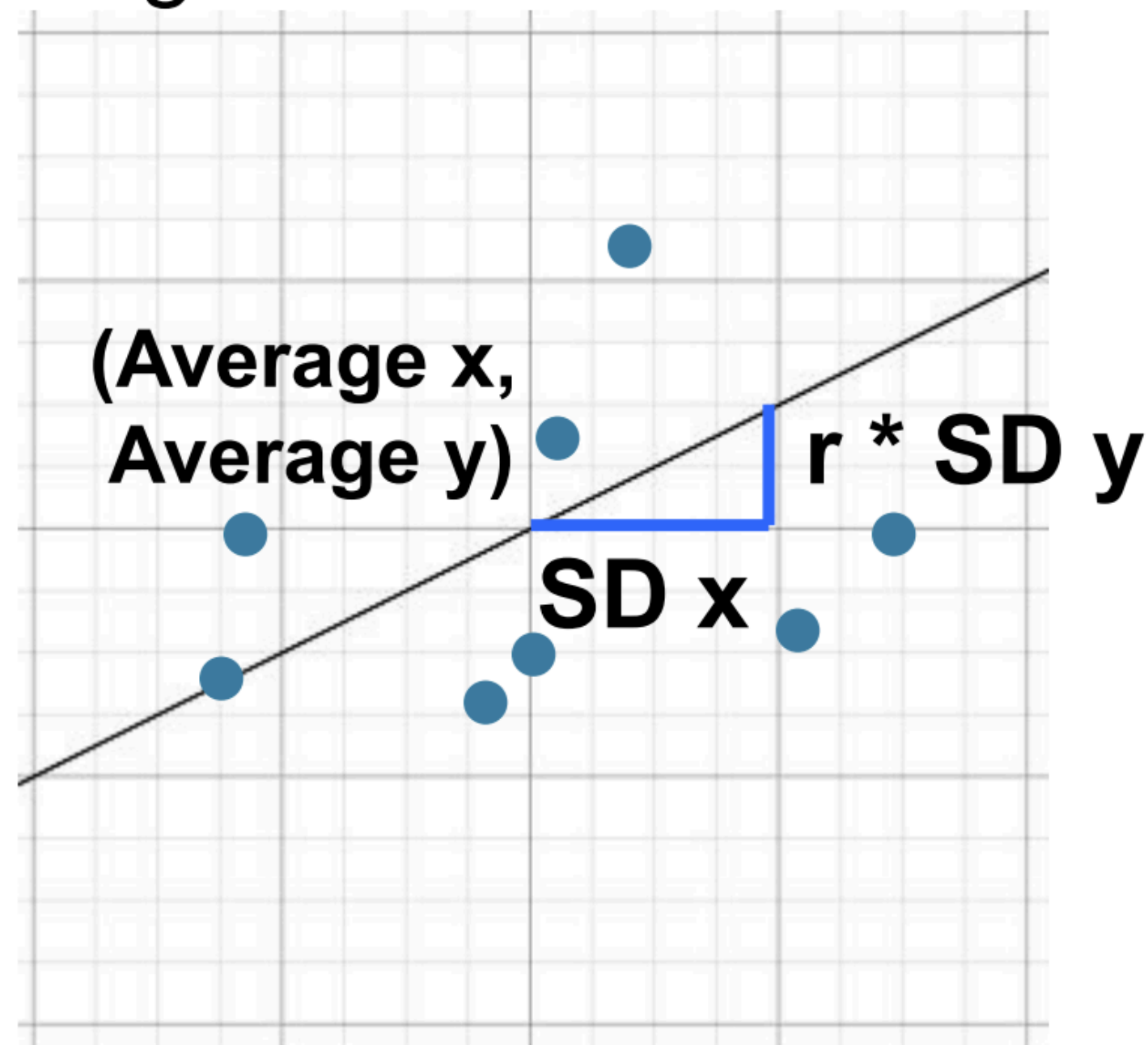


Original Units



Regression Line: Original Units

Original Units



estimate of $y = \text{slope} \times x + \text{intercept}$

$$\text{slope} = r \times \frac{\text{SD of } y}{\text{SD of } x}$$

$$\text{intercept} = \text{avg}(y) - \text{slope} \times \text{avg}(x)$$

Notebook Demo: Regression Line for Height

Next time

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