

MIS 64018: Assignment_4: Transportation / Transshipment problem

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Project Objective

The purpose of this assignment is to formulate and solve a transportation / transshipment problem.

Problem Formulation: Since we are required to find a solution that minimizes the combined cost of production and shipment, this is obviously a minimization problem. In addition, we can see that Plants A and B have a combined production capacity(i.e. supply) of 220. Whereas, the warehouses have a combined demand of 210. This makes for an unbalanced problem which requires introducing a dummy warehouse. The formulation below puts that into consideration.

Plant	W1	W2	W3	W4*	Output
Plant A	22	14	30	M(0)	100
Plant B	16	20	24	M(0)	120
Demand	80	60	70	10	

Objective function:

Min: $Z = 622 x_{11} + 614 x_{12} + 630 x_{13} + 641 x_{21} + 645 x_{22} + 649 x_{23}$ Subject to:

$x_{11} + x_{12} + x_{13} + M_{14} = 100$

$x_{21} + x_{22} + x_{23} + M_{24} = 120$

$x_{11} + x_{21} = 80$

$x_{12} + x_{22} = 60$

$x_{13} + x_{23} = 70$

$M_{14} + M_{24} = 10$

$(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}) \geq 0$

Solving the problem using lpsolve, or any other equivalent library in R.

Loading the lpSolveAPI library

```
library(lpSolveAPI)
```

```
/*Objective function*/
```

```
Min: 622 x11 + 614 x12 + 630 x13 + 641 x21 + 645 x22 + 649 x23;
```

```
/*Constraints*/
```

```

x11 + x12 + x13 + M14 = 100;
x21 + x22 + x23 + M24 = 120;
x11 + x21 = 80;
x12 + x22 = 60;
x13 + x23 = 70;
M14 + M24 = 10;

```

Reading the lp formulation from HS.lp file and assign it to p

```

p <- read.lp("HS.lp")
p

```

```

## Model name:
##          x11  x12  x13  x21  x22  x23  M14  M24
## Minimize 622  614  630  641  645  649    0    0
## R1       1    1    1    0    0    0    1    0 = 100
## R2       0    0    0    1    1    1    0    1 = 120
## R3       1    0    0    1    0    0    0    0 = 80
## R4       0    1    0    0    1    0    0    0 = 60
## R5       0    0    1    0    0    1    0    0 = 70
## R6       0    0    0    0    0    0    1    1 = 10
## Kind      Std  Std  Std  Std  Std  Std  Std  Std
## Type      Real Real Real Real Real Real Real Real
## Upper     Inf  Inf  Inf  Inf  Inf  Inf  Inf  Inf
## Lower      0    0    0    0    0    0    0    0

```

Question 1: Solve this transportation problem using lpsolve library in r.

```

solve(p)

```

```

## [1] 0

```

```

# the objective value for this problem is:
get.objective(p)

```

```

## [1] 132790

```

```

# the values of decision variables for the optimal solution are:
get.variables(p)

```

```

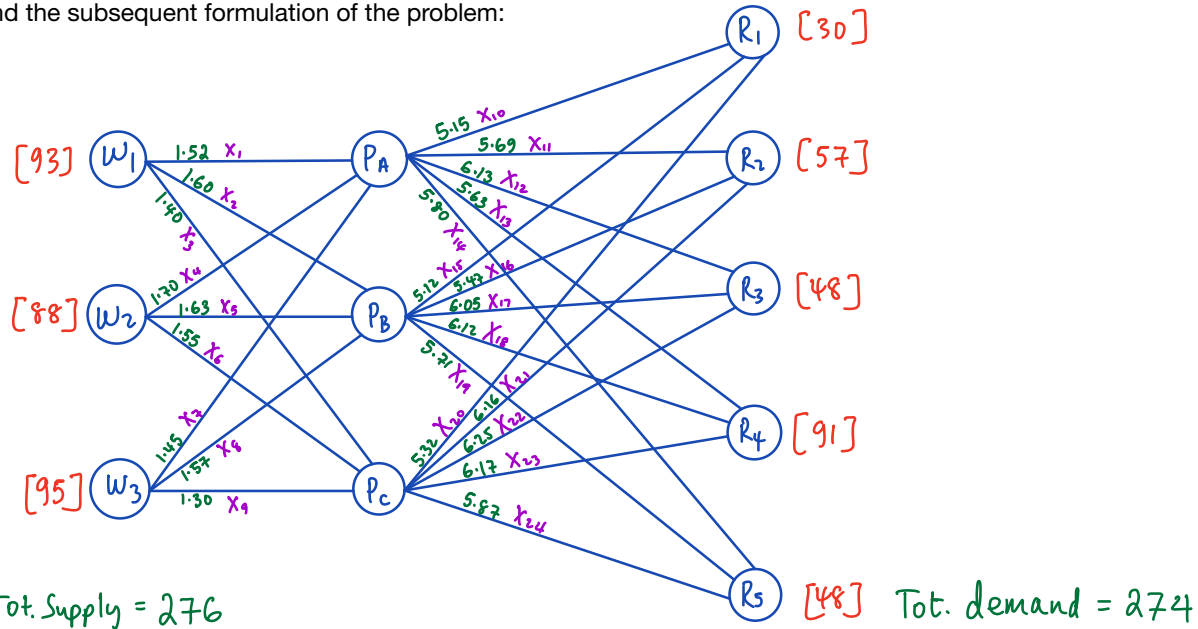
## [1] 0 60 40 80 0 30 0 10

```

The overall total cost is \$132,790. As for shipments from plants to warehouses, Plant A should ship 60 AEDs to Warehouse2 and 40 AEDs to Warehouse3; Plant B should ship 80 AEDs to Warehouse1 and 30 to Warehouse3.

The last two values for the decision variables are shipments for the dummy Warehouse4.

Q2. Oil distribution transshipment problem: In this specific problem we can see that the supply is higher than the total demand, which indicates an unbalanced problem. Below the network diagram related to the problem and the subsequent formulation of the problem:



$$\text{Min: } Z = 1.52 \cdot X_1 + 1.60 \cdot X_2 + 1.40 \cdot X_3 + 1.7 \cdot X_4 + 1.63 \cdot X_5 + 1.55 \cdot X_6 + 1.45 \cdot X_7 + 1.57 \cdot X_8 + 1.30 \cdot X_9 + 5.15 \cdot X_{10} + 5.69 \cdot X_{11} + 6.13 \cdot X_{12} + 5.63 \cdot X_{13} + 5.80 \cdot X_{14} + 5.12 \cdot X_{15} + 5.47 \cdot X_{16} + 6.05 \cdot X_{17} + 6.12 \cdot X_{18} + 5.71 \cdot X_{19} + 5.32 \cdot X_{20} + 6.16 \cdot X_{21} + 6.25 \cdot X_{22} + 6.17 \cdot X_{23} + 5.87 \cdot X_{24}$$

Subject to:

$$\begin{aligned} X_1 + X_2 + X_3 &\leq 93 \\ X_4 + X_5 + X_6 &\leq 88 \\ X_7 + X_8 + X_9 &\leq 95 \\ X_{10} + X_{15} + X_{20} &\geq 30 \\ X_{11} + X_{16} + X_{21} &\geq 57 \\ X_{12} + X_{17} + X_{22} &\geq 48 \\ X_{13} + X_{18} + X_{23} &\geq 91 \\ X_{14} + X_{19} + X_{24} &\geq 48 \\ X_1 + X_4 + X_7 - X_{10} - X_{11} - X_{12} - X_{13} - X_{14} &= 0 \\ X_2 + X_5 + X_8 - X_{15} - X_{16} - X_{17} - X_{18} - X_{19} &= 0 \\ X_3 + X_6 + X_9 - X_{20} - X_{21} - X_{22} - X_{23} - X_{24} &= 0 \\ X_1, X_2, X_3, \dots, X_{27} &\geq 0 \end{aligned}$$

Since there is an excess supply than demand (i.e. $276 > 274$), we can re-write the constraints by introducing the concept of dummy refinery R_6 in order to solve the problem. The constraints are as follows:

$$\begin{aligned} X_1 + X_2 + X_3 &= 93 \\ X_4 + X_5 + X_6 &= 88 \\ X_7 + X_8 + X_9 &= 95 \\ X_{10} + X_{15} + X_{20} &= 30 \\ X_{11} + X_{16} + X_{21} &= 57 \\ X_{12} + X_{17} + X_{22} &= 48 \\ X_{13} + X_{18} + X_{23} &= 91 \\ X_{14} + X_{19} + X_{24} &= 48 \\ M_{16} + M_{26} + M_{36} &= 2 \\ X_1 + X_4 + X_7 - X_{10} - X_{11} - X_{12} - X_{13} - X_{14} - M_{16} &= 0 \\ X_2 + X_5 + X_8 - X_{15} - X_{16} - X_{17} - X_{18} - X_{19} - M_{26} &= 0 \\ X_3 + X_6 + X_9 - X_{20} - X_{21} - X_{22} - X_{23} - X_{24} - M_{36} &= 0 \\ X_1, X_2, X_3, \dots, X_{27} &\geq 0 \end{aligned}$$

In the next page, I have solved this transshipment problem in R, using the lpSolveAPI library.

Part 2: Oil Distribution transshipment problem.

```
/*Objective Function*/
Min: 1.52 X1 + 1.60 X2 + 1.40 X3 + 1.7 X4 + 1.63 X5 + 1.55 X6 + 1.45 X7 + 1.57 X8 + 1.30 X9 +
      5.15 X10 + 5.69 X11 + 6.13 X12 + 5.63 X13 + 5.80 X14 + 5.12 X15 + 5.47 X16 + 6.05 X17 + 6.12 X18 +
      5.71 X19 + 5.32 X20 + 6.16 X21 + 6.25 X22 + 6.17 X23 + 5.87 X24;
/*Constraints*/
X1 + X2 + X3 = 93;
X4 + X5 + X6 = 88;
X7 + X8 + X9 = 95;
X10 + X15 + X20 = 30;
X11 + X16 + X21 = 57;
X12 + X17 + X22 = 48;
X13 + X18 + X23 = 91;
X14 + X19 + X24 = 48;
M16 + M26 + M36 = 2;
X1 + X4 + X7 - X10 - X11 - X12 - X13 - X14 - M16 = 0;
X2 + X5 + X8 - X15 - X16 - X17 - X18 - X19 - M26 = 0;
X3 + X6 + X9 - X20 - X21 - X22 - X23 - X24 - M36 = 0;
```

Reading the lp formulation from TOD.lp file and assign it to x

```
x <- read.lp("TOD2.lp")
x
```

```
## Model name:
## a linear program with 27 decision variables and 12 constraints
```

Question 1: Solve this transportation problem using lpsolve library in r.

```
solve(x)
```

```
## [1] 0
```

```
# the objective value for this problem is:
get.objective(x)
```

```
## [1] 1966.68
```

```
# the values of decision variables for the optimal solution are:
get.variables(x)
```

```
## [1] 93 0 0 0 88 0 28 0 67 30 0 0 91 0 0 57 31 0 0 0 0 17 0 48 0
## [26] 0 2
```

The minimum cost is 1966.68. Wells 1 and 2 are operating at capacity. Note that the last three values for the decision variables are for the dummy refineries.