Assignment 2: Solving LP problems
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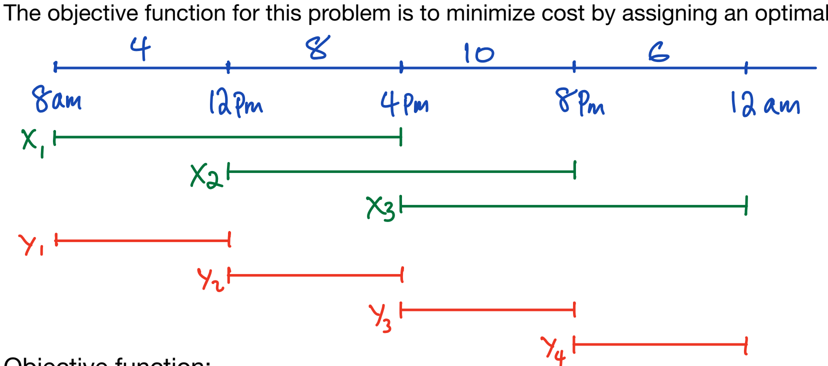
Question 1: Computer Center staffing
The decision variables for this staffing LP problem are full-time(X) and part-time(Y) employees working in the 4 different shifts. Here's the list:

- X1 = full-time employees in morning shift
- X2 = full-time employees in afternoon shift
- X3 = full-time employees in evening shift
- Y1 = part-time employees in morning shift
- Y2 = part-time employees in afternoon shift
- Y2 = part-time employees in evening shift

The objective function for this problem is to minimize cost by assigning an optimal mix of part-time and full-time employees.



Objective function:

Min Z: 112 * (X1 + X2 + X3) + 48 * (Y1 + Y2 +Y3)

$14 x 8 hrs = 112
$12 x 4hrs = 48

S.T.

X1 + Y1 >= 4
X1 + X2 + Y2 >= 8
X2 + X3 + Y3 >= 10
X3 + Y4 >= 6

Minimum number of consultants required on duty constraint

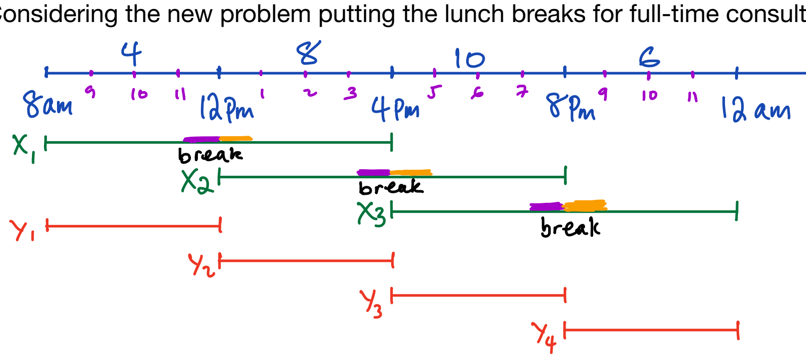
Mix of full-time & part-time consultants constraint (i.e. at least one full-time for every part-time)

X1 >= 2(Y1)
X1 + X2 >= 2(Y2)
X2 + X3 >= 2(Y3)
X3 >= 2(Y4)

Non-negativity constraint

X1 >=0, X2 >=0, X3 >=0, Y1 >=0, Y2 >=0, Y3 >=0

B) Considering the new problem putting the lunch breaks for full-time consultants into factor:



To accommodate full-time employees going on a 1 hr lunch break during their 8 hrs shift, we can add this constraint to still maintain an optimal number of consultants during the full shift:

X1 + Y1 + X2 + Y2 >= 12
X2 + X3 + Y3 + Y4 >= 16

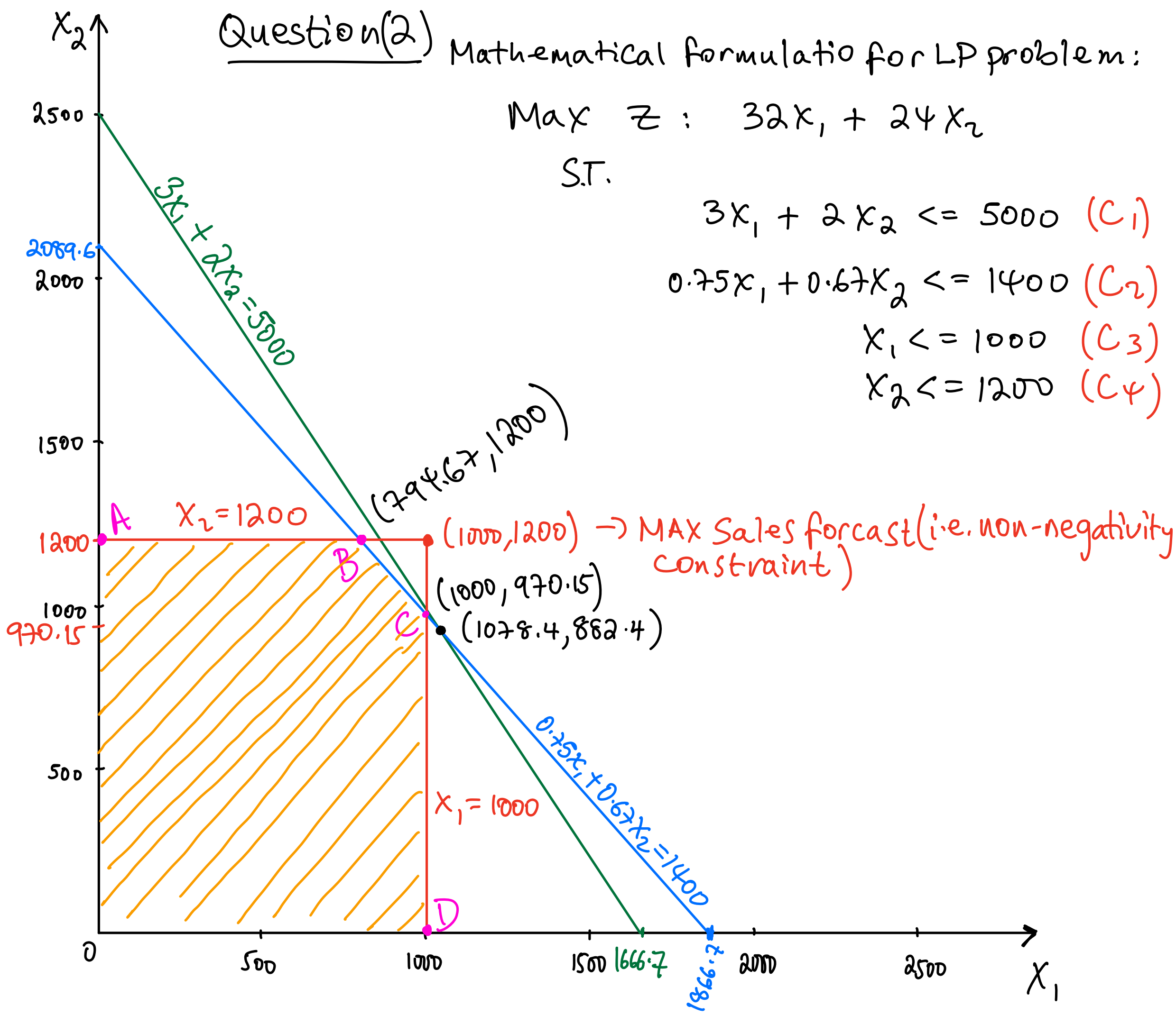
Question(2)

Mathematical formulatio for LP problem:

$$\text{Max } Z: \quad 32x_1 + 24x_2$$

S.T.

$$3x_1 + 2x_2 <= 5000 \quad (C_1)$$
$$0.75x_1 + 0.67x_2 <= 1400 \quad (C_2)$$
$$x_1 <= 1000 \quad (C_3)$$
$$x_2 <= 1200 \quad (C_4)$$



- Plotting the constraints (c_1, c_2, c_3, c_4), I've arrived at the above graph.
- Points B and C are computed by solving the simultaneous equations of the intersecting lines. That is:

$$B \rightarrow \begin{cases} 0.75x_1 + 0.67x_2 = 1400 \\ x_2 = 1200 \end{cases} \qquad C \rightarrow \begin{cases} 0.75x_1 + 0.67x_2 = 1400 \\ x_1 = 1200 \end{cases}$$

- The feasible solution for this problem is within the boundaries of ABCDØ, where:

  A = (0,1200) B = (795,1200) C = (1000,970) D = (1000,0)

  Ø = (0,0)

<u>Note</u>: I have rounded off values, since a decimal place doesn't make sense (i.e. ther's no such thing as 794.67 backpacks)

- The optimal solution is obtained by the corner point method. As we will substitute values of A,B,C,D,Ø into the Objective function :

Corner Point (x_1, x_2)	value of $Z: 32x_1 + 24x_2$
Ø (0,0)	0
A (0,1200)	28,800
B (794.67, 1200)	54,229.44
C (1000, 970.15)	55,283.6
D (1000, 0)	32,000

As we can see from the table above, point C gives us the optimum solution with 1000 units of X_1 (the Collegiate) and 970 units of X_2 (the Mini).

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Question 3:

a. Define the decision variables:

The decision variables are what products (L, M, S) to be produced, at which plant (P1, P2, P3). LP1: Large at Plant1 LP2: Large at Plant2 LP3: Large at Plant3 MP1: Medium at Plant1 MP2: Medium at Plant2 MP3: Medium at Plant3 SP1: Small at Plant1 SP2: Small at Plant2 SP3: Small at Plant3

b. The objective funtion for this problem is to maximize profit by producing optimum amounts of each product in the respective plants.

```
MAX z:

420xLP1 + 420xLP2 + 420xLP3 + 360xMP1 + 360xMP2 + 360xMP3 + 300xSP1 + 300xSP2 + 300xSP3

S.T.

**capacity to produce constraint:**
LP1 + MP1 + SP1 <= 750
LP2 + MP2 + SP2 <= 900
LP3 + MP3 + SP3 <= 450

**available storage constraint:**
20 x LP1 + 15 x MP1 + 12 x SP1 <= 750
20 x LP2 + 15 x MP2 + 12 x SP2 <= 900
20 x LP3 + 15 x MP3 + 12 x SP3 <= 450

**sales constraint:**
LP1 + LP2 + LP3 <= 900
MP1 + MP2 + MP3 <= 1200
SP1 + SP2 + SP3 <= 750

Percentage of excess capacity constraint:
i.e. (Products/Plant)x100% should be the same across all plants

100 x ((LP1 + MP1 + SP1)/750) = ((LP2 + MP2 + SP2)/900) x 100
re-arranging the equation:

    (LP1 + MP1 + SP1)/750 = (LP2 + MP2 + SP2)/900

    (1/750) x (LP1 + MP1 + SP1) = (1/900) x (LP2 + MP2 + SP2)
```

```
0.0013 x LP1 + 0.0013 x MP1 + 0.0013 xSP1 - 0.0011 x LP1 - 0.0011 x MP1 - 0.0011 x SP1 = 0

doing the same as above between (P1,P3) and (P2,P3):
```

Percentage of excess capacity constraint: 0.0013 x LP1 + 0.0013 x MP1 + 0.0013 xSP1 - 0.0022 x LP1 - 0.0022 x MP1 - 0.0022 x SP1 = 0 0.0011 x LP1 + 0.0011 x MP1 + 0.0011 xSP1 - 0.0022 x LP1 - 0.0022 x MP1 - 0.0022 x SP1 = 0 0.0013 x LP1 + 0.0013 x MP1 + 0.0013 xSP1 - 0.0011 x LP1 - 0.0011 x MP1 - 0.0011 x SP1 = 0

Solving the problem using lpsolve, or any other equivalent library in R.

Loading the lpSolveAPI library

```
library(lpSolveAPI)
```

Reading the lp formulation from assignment2.lp file and assign it to WeigeltProd

```
WeigeltProd <- read.lp("assignment2.lp")
WeigeltProd
```

```
## Model name:
##   a linear program with 9 decision variables and 12 constraints
```

Solve the lp model

```
solve(WeigeltProd)    # if output is 0, it means there's a successful solution
```

```
## [1] 0
```

```
get.objective(WeigeltProd)        # This is the objective value
```

```
## [1] 33750
```

```
get.variables(WeigeltProd)        # get values of decision variables
```

```
## [1]  0.0  0.0  0.0  0.0  0.0  0.0  0.0 75.0 37.5
```

```
get.constraints(WeigeltProd)      # get constraint RHS values
```

```
##  [1]   0.0  75.0  37.5   0.0 900.0 450.0   0.0   0.0 112.5   0.0   0.0   0.0
```