Exercises – Week 40&41

Introduction to Financial Engineering

Note: You may choose to work in R or Matlab. Sometimes solutions will be available in one language, sometimes in both.

1. (returns, portfolios, diversification) Collect 10 years of weekly data for McDonalds, Coca Cola and Microsoft for the period January 1, 1991 to January 1, 2001.

   (a) Calculate the weekly returns for each of the stocks

   (b) Calculate the annualised mean and covariance matrix for the returns of the stocks

   (c) Consider all portfolios weights $(0, 0, 1), (0, 0.1, 0.9), (0.1, 0.1, 0.8) \ldots, (0.9, 0.1, 0), (1, 0, 0)$. For each of the portfolios calculate the annualised mean and annualised standard deviation. Plot mean against standard deviation for all these portfolios in one single plot.

   (d) Which of the portfolios has the maximal mean? (Could you have answered this question without doing c)?)

   (e) Which of the portfolios has the lowest standard deviation? (Could you have answered this question without doing c)?)

   (f) Which of the portfolios has the highest ratio of mean to standard deviation? Why is this portfolio interesting?

2. (portfolios, diversification, efficient frontier) Assume that

   - The risk free rate is 2%
   - Asset 1 has an annual expected return of 10% and the standard deviation is 10%

- Asset 1 has an annual expected return of 20% and the standard deviation is 20%

(a) Find the Efficient Frontier (combinations of $(\mu; \sigma)$) when short selling is allowed and with no riskless lending/borrowing for each of the following correlation coefficient between the returns of assets 1 and 2

- $\rho = 0$
- $\rho = 0.5$
- $\rho = -0.5$

Use different colours for different values of the correlation coefficient. Comment on your findings. What happens to the efficient frontier when $\rho \to \pm 1$? (Hint: Check textbook p. 67)

(b) Now include the risk-free asset and compute the Capital Market Line (CML) for each of the three different correlation coefficients. Use different colours for different values of the correlation coefficient. Comment on your findings.

(c) Bonus question: What happens if the risk-free borrowing rate is 4% and the risk-free lending rate is 2%?

3. (portfolios, diversification, efficient frontier) Use the data from Exercise 1. Assume the risk free rate is 2% (or otherwise estimate it based on the USD 3m LIBOR rate from the British Banking Association webpage).

(a) Calculate the efficient frontier for a combination of McDonalds and Coca Cola without a risk-free asset and the CML with the risk-free asset included.

(b) Now add Microsoft to the portfolio and redo the previous question. Add the new efficient frontier and the CML to the same graph. Comment on your findings.

4. (portfolios, diversification, efficient frontier) Use the data and code from Exercise 3 and modify to include:

   (a) No bank account (risk-less lending) and no short selling allowed. Plot this in the same graph as with short selling allowed and comment on your findings

   (b) Risk-less lending is allowed and no short selling. Plot this in the same graph as above.