Introduction to Financial Engineering

Week 37

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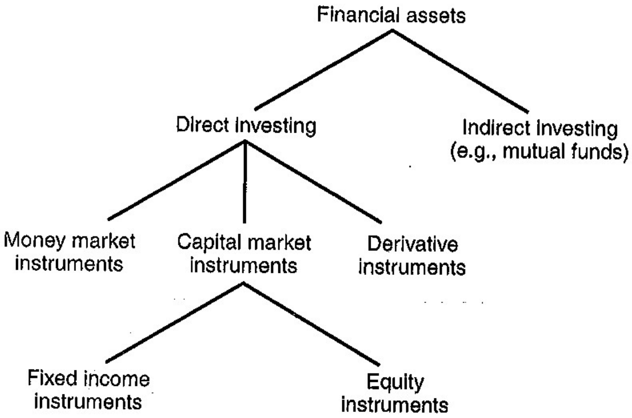
Financial assets



Stocks

- Owning a stock is owning a part of a company
- Stocks can be publicly traded at stock exchanges...
- ...or it can be privately held
- Do you know any *privately held* companies?
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- Do you know any *listed* companies?
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Bonds

- Owning a bond is lending to a company
- . . . or a country or a group of home owners
- Bond holders get their money before stock owners
- Again, (corporate) bonds can be publicly or privately traded
- Why do you think that most corporate bonds do not trade in an exchange?
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What affects the prices of stocks?

Spend five minutes researching and/or discussing with the student next to you `https://e.ggtimer.com/5%20minutes`

Shorting of stocks

- Shorting a stock means you sell a stock you don't own
- This is relevant if you think the market will go down
- You need to borrow (or in fact rent) a stock from someone else to sell in and then repurchase it and hand it back
- Nice if prices decrease, not so nice if prices increase

Is shorting good or bad?

Spend three minutes discussing with the student next to you
https://e.ggtimer.com/3%20minutes

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The rate of return

- The rate of return (or just the **return**) over a time-period is defined as the difference in stock price at the two points in time measured as a fraction of initial value:

$$K_S = \frac{S(1) - S(0)}{S(0)}$$

- Often in finance, log returns are used instead:

$$k_S = \log \frac{S(1)}{S(0)}$$

Note: $\log$ means the natural logarithm, sometimes denoted as $\ln$ in some computer programs or calculators

Returns vs log returns

- The relationship between K_S and k_S can be expressed as

$$1 + K_S = \frac{S(1)}{S(0)} = e^{\log \frac{S(1)}{S(0)}} = e^{k_S}$$

- Log returns are usually used is to replace the real returns, i.e., $K_S = e^{k_S} - 1 \approx k_S$
- This approximation is okay, when returns are small:

$$e^{k_S} - 1 = -1 + e^0 + e^0(k_S - 0) + \tfrac{1}{2}e^0(k_S - 0)^2 + \dots$$

- Note: Last week, the Taylor expansion was around the $\log$ function. The results are the same!
- Are log returns lower or higher than returns?

Geometric average 1/2

- When using returns, the geometric average should be used. When looking at N periods, the average rate of return per period is the solution $\bar{K}_S$ to the following equation

$$(1+\bar{K}_S)^N = (1+K_S(1))(1+K_S(2))\cdots(1+K_S(N))$$

  which can be expressed as:
- The average rate of return per time period is:

$$\bar{K}_S = \left(\prod_{i=1}^{N}(1+K_S(i))\right)^{(1/N)} - 1$$

Geometric average 2/2

- Actually, only the first and last price is needed to compute an average:

$$\bar{K}_S = \left(\prod_{i=1}^{N} \frac{S(i)}{S(i-1)} \right)^{(1/N)} - 1$$
$$= \left(\frac{S(N)}{S(0)} \right)^{(1/N)} - 1$$

- But often the we are interested in the variation in individual returns, so this is not necessarily an advantage

Aritmetric average

- When using log returns, the arithmetic average should be used. When looking at N periods, the average rate of return per period is the simple average of log returns.
- Again, only the first and last price is needed to compute an average:

$$\bar{k}_S = \tfrac{1}{N} \sum_{i=1}^{N} \log\left(\frac{S(i)}{S(i-1)}\right)$$
$$= \tfrac{1}{N} \log\left(\frac{S(N)}{S(0)}\right)$$

Annualising average returns 1/2

- For comparison, the convention is to report average returns per year
- This DOES NOT mean that we use just one year of data, but that we report the average return if this average was obtained for each time period for an entire year.
- Example: If we have used weekly data and obtained an average of K_S^w, the corresponding annual average is the solution to

$$(1 + \bar{K}_S^a) = (1 + K_S^w)^{52}$$

$$\bar{K}_S^a = (1 + K_S^w)^{52} - 1$$

- In similar way, we convert an average daily rate to an annual average rate by $\bar{K}_S^a = (1 + K_S^d)^{252} - 1$

Annualising average returns 2/2

- For log returns, annualising is a bit simpler:

$$\bar{k}_S^a = 52 \times \bar{k}_S^w$$
$$\bar{k}_S^a = 252 \times \bar{k}_S^d$$

Standard deviation of returns

- It is useful to get an idea of how much returns vary
- For this we use the normal definition of standard deviation and use the build-in functions in the data tool we are using
- It can be seen that the standard deviation of annual returns can be extracted from the standard deviation of daily or weekly returns
- At least if we assume that returns are independent, have constant standard deviation and we are using log returns (see argument on blackboard)
- In conclusion, the standard deviation of daily returns are converted into an annual number by

$$\sigma_k^a = \sqrt{252}\sigma_k^d$$

- The annualised standard deviation of returns is often also denoted volatility

Summary

- Don't mix the two - simple returns plus geometric average or log returns plus arithmetic average
- For standard deviations, we use the basic way of computing standard deviation no matter the choice of returns
- Log returns are always smaller

What else could we think of doing with data

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Next week we will talk about

- finding cash flow of bonds
- analysis of bonds