Introduction to Financial Engineering

Week 38

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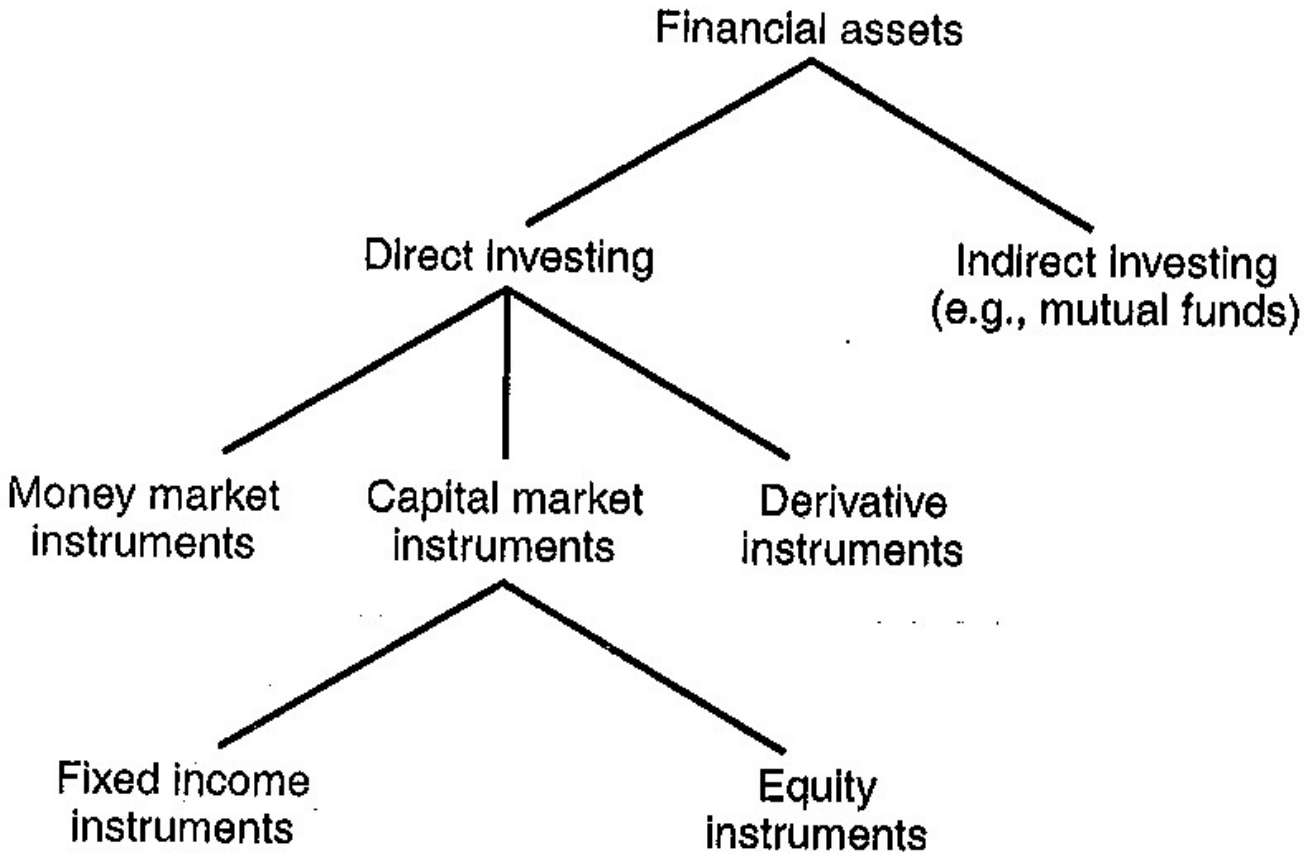
Recall: Financial assets



Stocks

- Owning a stock is owning a part of a company
- Stocks can be publicly traded at stock exchanges or it can be privately held
- When considering stocks, it's interesting to consider
  - returns and average returns
  - standard deviation of returns
  - covariance/correlation of returns
  - remember to annualise!

Bonds

- Owning a bond is lending to the issuer of the bond
- Bond holders get their money before stock owners
- Government bonds are generally viewed as safe assets, where the bond owner gets coupons and principal back
- Corporate bonds are generally not completely safe assets, as the company can go bankrupt
- The risk of a bond is reflected in the bond price

Time value of money

Future value

What is the future value of an amount of money invested or borrowed today?

Present value

What is the present value of an amount of money to be paid or received at a given future date?

Simple interest rate

- Suppose you deposit P in a bank account, so $V(0) = P$
- The principal earns interest at rate r, so $V(t) = P(1 + tr)$, where t is measured in years
- This is the *simple interest*, where the interest payments are not earning any interest – think of the interests being paid out in cash or deposited in another account
- Interest rates refers to a period of one year and looking at the return for the bank account over one year, you get exactly the interest rate r as rate of return:

$$K(0,1) = \frac{V(1) - V(0)}{V(0)} = r$$

Some examples

- \$9,000 paid into a bank account for two months will produce \$9,020 after two months. What was the interest r?
- How much needs to be paid in to an account attracting a simple interest rate of 9% if \$1,000 is needed after three months?
- The first question is related to future value of present money and the last question to the present value of future money
- We can rewrite the relation from the previous slide to

$$V(0) = \frac{V(t)}{1+rt}$$

- $V(0)$ is referred to as the *present value* of $V(t)$ and $1/(1+rt)$ is called the *discount factor*

Periodic compounding

- Suppose you deposit P in a bank account, so $V(0) = P$
- But now, interest is earned m times per year
- We still get an interest rate r, but this is divided into m parts, so m times per year an interest payment of $\frac{r}{m}$ is made on the value of the bank account
- After t (a multiple of $1/m$), the bank account has grown to

$$V(t) = \left(1 + \frac{r}{m}\right)^{tm} V(0)$$

- Try to set $m = 1/12$ and $t = 1$, if you find this too abstract

Continuous compounding

- What happens if interest is added with very small intervals?
- Let's see what happens when $m \to \infty$

$$\begin{aligned} V(t) &= \left(1 + \frac{r}{m}\right)^{tm} V(0) \\ &= \left(\left(1 + \frac{r}{m}\right)^{m/r}\right)^{rt} V(0) \\ &\to e^{rt} V(0) \text{ for } m \to \infty \end{aligned}$$

- When m is big enough, e turns up! It's convenient to work with and therefore often seen in finance (but not so much in this course)

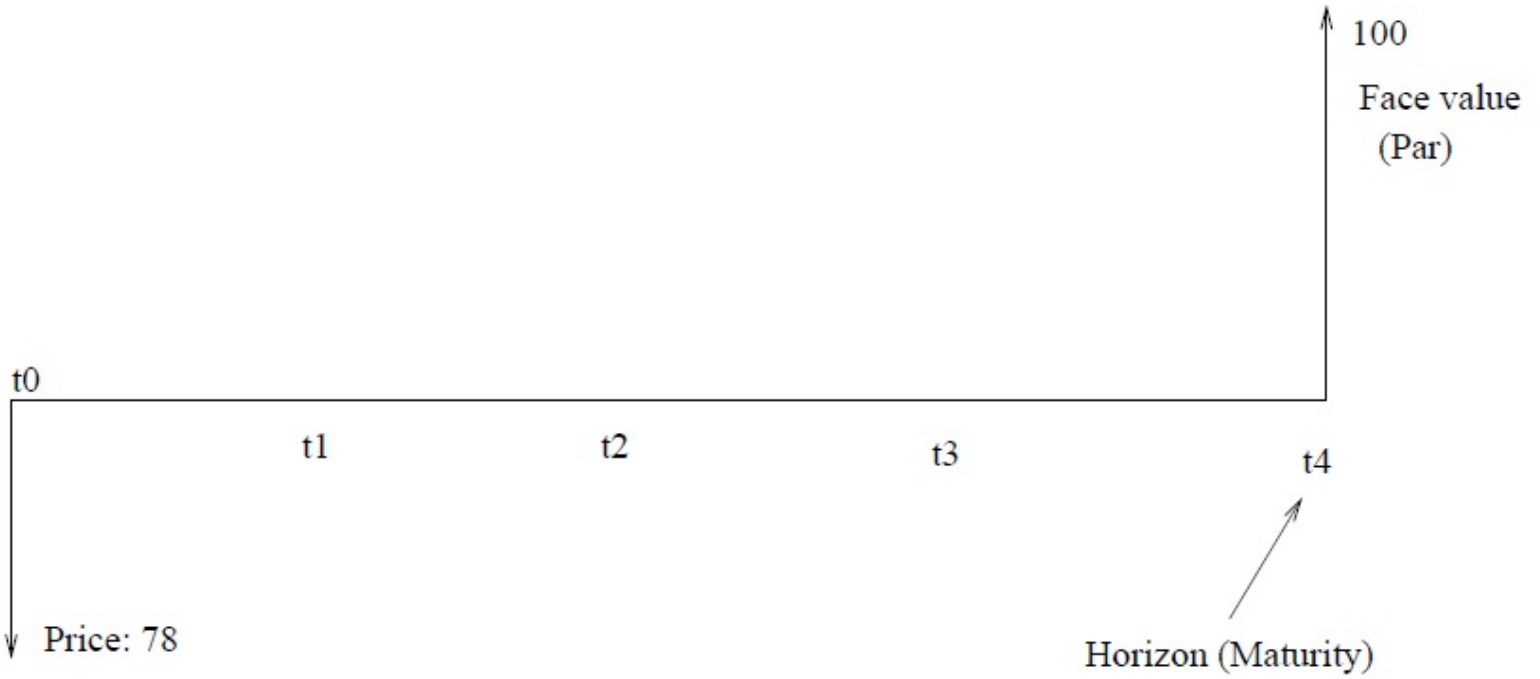
Types of bonds

The basic bonds are

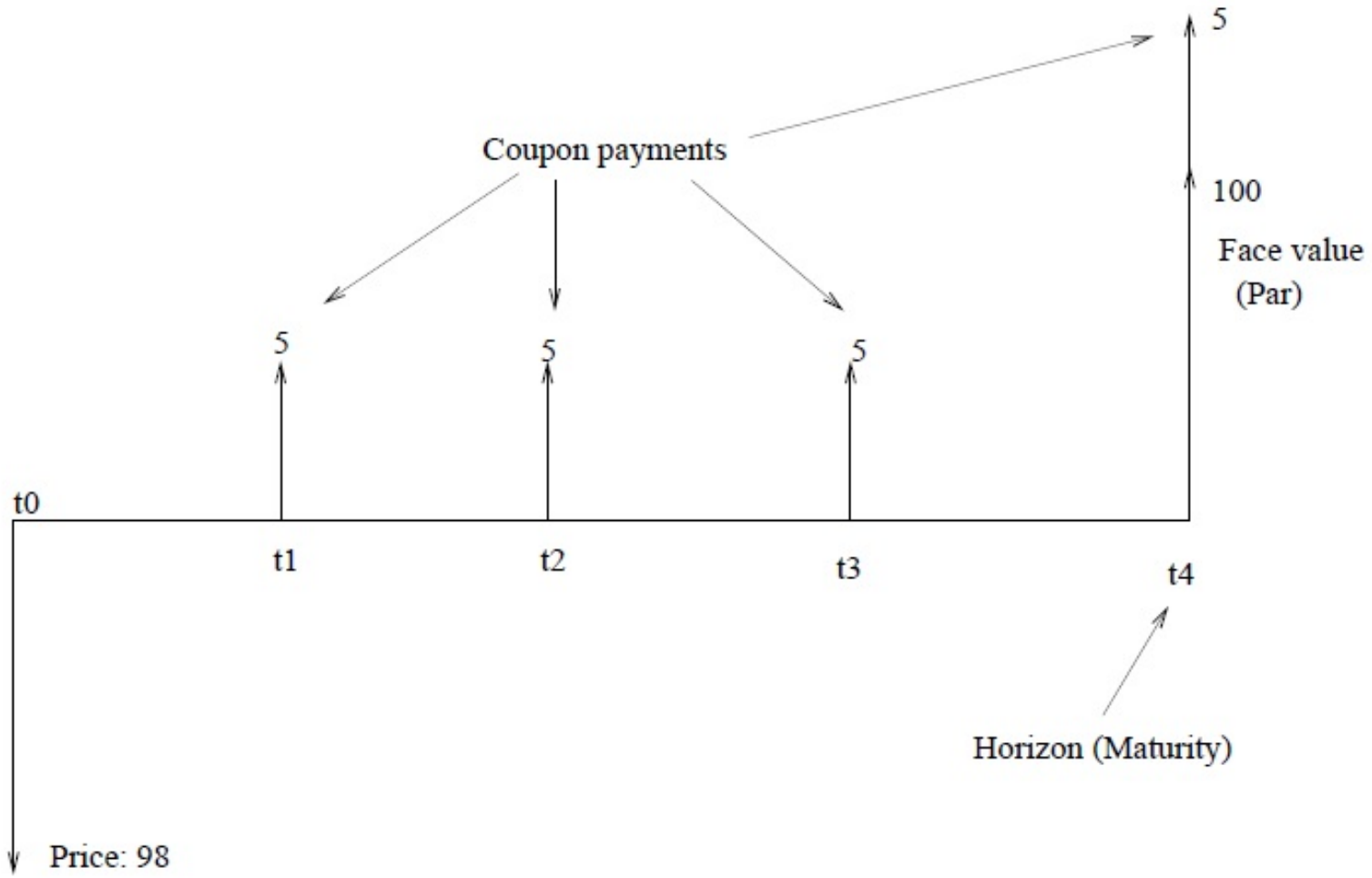
- Zero Coupon Bond
- Bond with coupons
- Annuity
- (Other types, e.g., linear loans)

The bond has a cash flow depending on type, price, coupons, maturity.

The Zero Coupon Bond is the most simple



A bond with coupon has payments before maturity



An annuity pays the same every time

Insert picture from blackboard yourself

Quoted prices vs present value of cash flows

- There two types of bond prices
  - Clean price: The price the exchange quotes
  - Dirty price: The price you actually have to pay
  - Clean price = Dirty price - accrued interest
  - The accrued interest is calculated by the simple interest rule
- The dirty price is the present value of all future payments
- Remember: When setting up the cash flow and calculating yields (see later), the dirty price needs to be calculated from the clean price and the coupons
- Clean prices are more "clean". When they change, it's because of changes in the market
- The dirty price can change just because of time passing

Finding yields from ZCB

- The price – or present value of a zero coupon bond – determines the cash flow of the bond
- From the cash flow, it is possible to extract the *yield to maturity* implied by that bond
- The yield at time t for a bond maturing at time T solves the equation

$$B(t,T) = \frac{1}{(1+y(t))^{(T-t)}}$$

Some comments on the textbook

- In the textbook, this is given for continuous time. For now, we use the discrete time version
- In the textbook, they assume that the yield varies over time (calculated two different dates)
- In the textbook, they assume that the yield is constant for bonds with different maturities (but do note that it's not realistic)

Finding yields from bonds with coupons

- The principle for extracting yields from coupon-bearing bonds is the same
- The question is "which rate should I use for discounting the cash flows to obtain the quoted price of the bond?"
- Denote the payment at time t_i from the bond by C_{t_i}
- The yield to maturity solves the equation:

$$P(t,T) = \sum_{i=t_1}^{t_N} \frac{C_i}{(1+y(t))^{i-t}}$$

For todays exercises

- Extract prices from NASDAQ
- Convert the clean prices into dirty prices
- Set up the cash flow (dates, coupons, principals) as a vector or matrix
- Calculate the yield for each bond using the formulas in the slides
- If you have already worked with the exercises and have used the continuous time version of the formula, that's perfectly fine. Come talk to me about the solutions then.

Next week we will talk about

- brush up on bonds (cash flows, definitions etc.)
- much more on the analysis of bonds (rates/yields, duration and convexity)
- Khan Academy has loads of videos explaining bonds, compounding etc if you want to supplement the book
- readings for next week will be published Thursday or Friday