misc_equations

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```
library(tidyverse)
```

```
## -- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
## v dplyr     1.1.4     v readr     2.1.5
## v forcats   1.0.0     v stringr   1.5.1
## v ggplot2   3.4.4     v tibble    3.2.1
## v lubridate 1.9.3     v tidyr     1.3.0
## v purrr     1.0.2
## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become
```

```
library(kableExtra)
```

```
##
## Attaching package: 'kableExtra'
##
## The following object is masked from 'package:dplyr':
##
##     group_rows
```

```
library(knitr)
```

Kernel machine regression defines the regression relationship using a flexible function $h : \mathbb{R}^M \to \mathbb{R}$, where

$$Y_i = h(\mathbf{x}_i) + \mathbf{z}_i^\top \boldsymbol{\beta}_\mathbf{z} + \varepsilon_i,$$

Here, Y_i is the outcome at a given point, $\mathbf{x}_i = [x_1, \ldots, x_M]^\top$ is a vector of M chemicals, $\mathbf{z}_i$ and $\boldsymbol{\beta}_\mathbf{z}$ are vectors of covariates and their weights, respectively, and $\varepsilon_i \overset{\text{iid}}{\sim} N(0, \sigma^2)$.

$h(\cdot)$ is obtained using the kernel $k : \mathbb{R}^M \times \mathbb{R}^M \to \mathbb{R}$, defined as

$$k(\mathbf{x}, \mathbf{x}') = \exp\left\{ - \frac{||\mathbf{x} - \mathbf{x}'||^2}{\rho} \right\},$$

where x are the predictor values of a second subject, and ρ is a tuning parameter.

$$Y = x_1 + x_2$$

$$Y = x_1 + x_2 + 0.5(x_1 * x_2)$$

$$\widehat{Y} = \widehat{\beta}_0 + \widehat{\beta}_1 x_1 + \widehat{\beta}_2 x_2 + \widehat{\beta}_{12} x_1 x_2$$

```
equations <- data.frame(
  type = c("Multiplicative", "Polynomial",
           "Smaller category", "Larger category"),
  small = c("0.35Hg$*$Ni", "0.13Hg$*($Ni$-1)^2$",
            "0.5Hg$*\\text{race}_{\\text{black}}$",
            "0.5Hg$*\\text{race}_{\\text{hisp.non}}$"),
  large = c("0.7Hg$*$Ni", "0.26Hg$*($Ni$-1)^2$",
            "Hg$*\\text{race}_{\\text{black}}$",
            "Hg$*\\text{race}_{\\text{hisp.non}}$")
)
labels <- c(
  "Chemical by chemical" = 2,
  "Chemical by race" = 2
)

equations |>
  kbl(booktabs = TRUE, escape = FALSE,
      col.names = c("", "Lower", "Higher"),
      align = "lcc",
      caption = "Specification of interaction terms in simulations.") |>
  column_spec(1, width = "10em") |>
  add_header_above(header = c(" " = 1, "Effect size" = 2)) |>
  pack_rows(index = labels)
```

Table 1: Specification of interaction terms in simulations.

	Effect size	
	Lower	Higher
Chemical by chemical		
Multiplicative	$0.35\text{Hg}*\text{Ni}$	$0.7\text{Hg}*\text{Ni}$
Polynomial	$0.13\text{Hg}*(\text{Ni}-1)^2$	$0.26\text{Hg}*(\text{Ni}-1)^2$
Chemical by race		
Smaller category	$0.5\text{Hg}*\text{race}_{\text{black}}$	$\text{Hg}*\text{race}_{\text{black}}$
Larger category	$0.5\text{Hg}*\text{race}_{\text{hisp.non}}$	$\text{Hg}*\text{race}_{\text{hisp.non}}$