# Monte Carlo Simulation Analysis

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## Fall 2022

In this analysis, we calculate the prices of the European call options and European put options of the assets in our investment portfolio using Monte Carlo Simulations.

The Monte Carlo Simulation is a process of generating a large number of possible values of factors that impact the outcomes of an event. The distributions of the factors are defined, and the distribution of the outcomes is estimated with the simulated factor values. This process can be easily implemented with a few lines of codes or using existing packages.

The pandas-montecarlo package ( https://github.com/ranaroussi/pandas-montecarlo) is a lightweight Python library for running simple Monte Carlo Simulations on Pandas Series data. We may use the pandas-montecarlo package for our analysis. However, we found the following problems of the package after reviewing the codes of the library.

- The package performs **bootstrap simulation without replacement** from historical data instead of generating new independent samples from an estimated distribution. It may be fixed by performing **bootstrap simulation with replacement**. However, this will give us very limited number of discrete outcomes instead of countless outcomes from a continuous distribution.
- The package calculates the aggregated return by summing the daily percent changes. This is an unforgivable mistake. For example, if a stock is up by 10% on one and down by 10% on the next day, the total of change is 1.1 * 0.9 - 1 = -0.01. The calculation of the library gives 0%.
- Caused by the two problems above, the aggregated returns for all simulations have the same designation, which is the sum of all historical daily percent changes. It is apparently wrong to calculate option prices based on this only outcome.

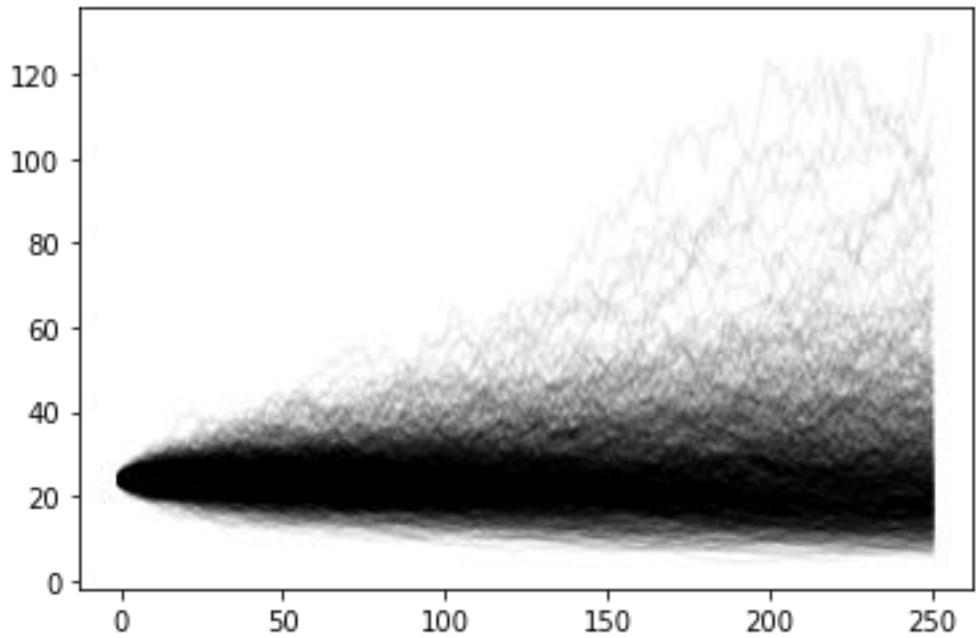
Therefore, we will create our own coding for Monte Carlo simulation in this analysis. One thing to keep in mind when defining the distribution of the asset price for our simulation is that Monte Carlo simulation assumes perfectly efficient markets. The Efficient Market Hypothesis suggests a no arbitrage pricing model. The Black-Scholes Model is one of such models and it is commonly used for option pricing. The Black-Scholes Model assumes that the future log return of an asset followings the following distribution.

$N(r - 0.5\sigma^2, \sigma^2)$ where $r$ is the risk-free rate and $\sigma$ can be estimated using the standard deviation of the historical log returns. The return is $r$ if $\sigma$ is 0. It is impossible to get higher return without additional risk ($\sigma$). This is consistent with the Efficient Market Hypothesis. We will perform our simulation using this distribution.

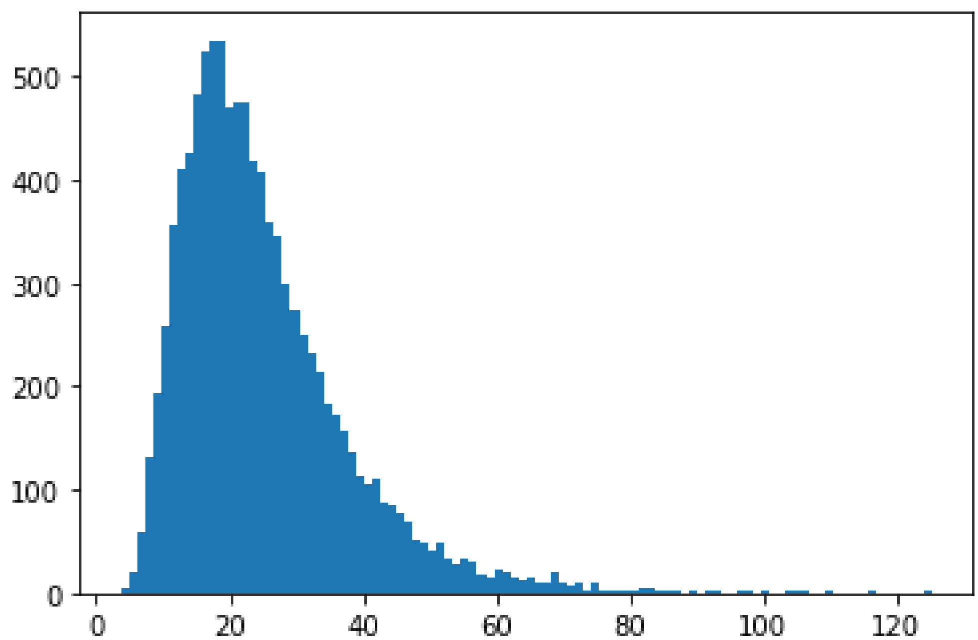
Our investment portfolio includes the following assets, with purchasing prices (close prices) from 9/16/2022. In this analysis, we will not include the U.S. 30 Year Treasury bonds in our calculation.

| Ticker | Type | Sector | Units | Amount |
| --- | --- | --- | --- | --- |
| US30Y | U.S. 30 Year Treasury | NaN | 467317.00 | 42262981.19 |
| HSON | Common Stock | COMMERCIAL SERVICES | 36138.24 | 1263031.49 |
| GOGO | Common Stock | COMMUNICATIONS | 94822.19 | 1263031.57 |
| JAKK | Common Stock | CONSUMER DURABLES | 54044.99 | 1263031.42 |
| CALM | Common Stock | CONSUMER NON-DURABLES | 22216.91 | 1263031.33 |
| HRB | Common Stock | CONSUMER SERVICES | 28111.09 | 1263031.27 |
| HDSN | Common Stock | DISTRIBUTION SERVICES | 159071.98 | 1263031.52 |
| BELFA | Common Stock | ELECTRONIC TECHNOLOGY | 44285.81 | 1263031.30 |
| ARLP | Common Stock | ENERGY MINERALS | 52890.76 | 1263031.35 |
| CI | Common Stock | HEALTH SERVICES | 4350.03 | 1263031.21 |
| SRTS | Common Stock | HEALTH TECHNOLOGY | 85339.97 | 1263031.56 |
| LNG | Common Stock | INDUSTRIAL SERVICES | 7551.75 | 1263030.19 |
| BSM | Common Stock | MISCELLANEOUS | 80447.87 | 1263031.56 |
| HUDI | Common Stock | NON-ENERGY MINERALS | 42612.40 | 1263031.54 |
| CF | Common Stock | PROCESS INDUSTRIES | 12810.95 | 1263031.56 |
| CSL | Common Stock | PRODUCER MANUFACTURING | 4335.84 | 1263030.19 |
| MUSA | Common Stock | RETAIL TRADE | 4561.65 | 1263029.65 |
| AZPN | Common Stock | TECHNOLOGY SERVICES | 5579.50 | 1263031.42 |
| ASC | Common Stock | TRANSPORTATION | 126303.15 | 1263031.50 |
| ED | Common Stock | UTILITIES | 12966.13 | 1263030.72 |
| YCS | ETF | NaN | 25261.96 | 1599839.93 |
| UUP | ETF | NaN | 54416.32 | 1599839.81 |
| EUO | ETF | NaN | 47870.73 | 1599839.80 |
| EWV | ETF | NaN | 79832.33 | 1599839.89 |
| DIG | ETF | NaN | 44390.67 | 1599839.75 |
| TTT | ETF | NaN | 23645.28 | 1599839.64 |
| ERX | ETF | NaN | 29349.47 | 1599839.61 |
| TMV | ETF | NaN | 13430.49 | 1599839.97 |
| TBT | ETF | NaN | 54509.02 | 1599839.74 |
| TYO | ETF | NaN | 127782.74 | 1599839.90 |

First, let's try to perform a simulation for 'ARLP' and see how the asset price is moving. Below shows 1000 simulated movements of the asset price. We can see that the prices are centered at the starting price and very unlikely to move up by a lot.




We can also check the distribution of the final prices. The distribution is log-normal which is consistent with the assumption of the Black-Scholes model.




We can then perform simulations for all assets and calculate the option prices based on the results of the simulation.

$$P_{call} = mean(\max(final\_price - strike\_price, 0) * e^{-r})$$

$$P_{put} = mean(\max(strike\_price - \ final\_price, 0) * e^{-r})$$

We calculate the option price with strike price equals the starting price of the asset, period equals 1 year (251 trade days), and risk-free rate based on the current yield rate of the 1-year US Treasury Bill. We also calculate the option prices using the Black-Scholes formula for comparison.

| | Stock price | Strike price | Risk free rate | Time period | volatility | call_BS | put_BS | call_sim | put_sim |
|---|---|---|---|---|---|---|---|---|---|
| **ARLP** | 23.950001 | 23.950001 | 0.046798 | 1 | 0.473192 | 4.945446 | 3.850463 | 4.921912 | 3.914618 |
| **ASC** | 14.030000 | 14.030000 | 0.046798 | 1 | 0.591182 | 3.517236 | 2.875790 | 3.355578 | 2.925548 |
| **AZPN** | 233.009995 | 233.009995 | 0.046798 | 1 | 0.367275 | 38.766395 | 28.113277 | 39.220157 | 27.553737 |
| **BELFA** | 32.630001 | 32.630001 | 0.046798 | 1 | 0.486784 | 6.904918 | 5.413088 | 6.875883 | 5.410121 |
| **BSM** | 19.420000 | 19.420000 | 0.046798 | 1 | 0.344013 | 3.059152 | 2.171278 | 3.048242 | 2.143808 |
| **CALM** | 57.480000 | 57.480000 | 0.046798 | 1 | 0.326349 | 8.668199 | 6.040237 | 8.876806 | 5.999748 |
| **CF** | 105.379997 | 105.379997 | 0.046798 | 1 | 0.497390 | 22.720428 | 17.902499 | 23.267359 | 17.716548 |
| **CI** | 322.130005 | 322.130005 | 0.046798 | 1 | 0.267453 | 41.357090 | 26.629443 | 41.076799 | 26.689906 |
| **CSL** | 225.570007 | 225.570007 | 0.046798 | 1 | 0.350068 | 36.052693 | 25.739729 | 35.761443 | 25.749973 |
| **DIG** | 47.529999 | 47.529999 | 0.046798 | 1 | 0.702704 | 13.870545 | 11.697493 | 13.887907 | 11.752958 |
| **ED** | 90.040001 | 90.040001 | 0.046798 | 1 | 0.207722 | 9.519397 | 5.402806 | 9.523660 | 5.554149 |
| **ERX** | 73.110001 | 73.110001 | 0.046798 | 1 | 0.709946 | 21.528930 | 18.186372 | 21.385508 | 18.064281 |
| **EUO** | 33.750000 | 33.750000 | 0.046798 | 1 | 0.191595 | 3.362852 | 1.819816 | 3.421166 | 1.765561 |
| **EWV** | 20.420000 | 20.420000 | 0.046798 | 1 | 0.384106 | 3.527904 | 2.594310 | 3.386756 | 2.683251 |
| **GOGO** | 14.850000 | 14.850000 | 0.046798 | 1 | 0.489196 | 3.155936 | 2.477001 | 3.126995 | 2.488528 |
| **HDSN** | 10.540000 | 10.540000 | 0.046798 | 1 | 0.772048 | 3.341431 | 2.859547 | 3.388370 | 2.851450 |
| **HRB** | 39.759998 | 39.759998 | 0.046798 | 1 | 0.387050 | 6.913667 | 5.095856 | 7.047203 | 5.055023 |
| **HSON** | 36.380001 | 36.380001 | 0.046798 | 1 | 0.634772 | 9.708179 | 8.044901 | 9.409704 | 8.006942 |
| **HUDI** | 180.000000 | 180.000000 | 0.046798 | 1 | 1.934502 | 121.381473 | 113.151949 | 110.099115 | 113.924059 |
| **JAKK** | 18.730000 | 18.730000 | 0.046798 | 1 | 0.754775 | 5.820856 | 4.964529 | 5.641459 | 5.005652 |
| **LNG** | 171.779999 | 171.779999 | 0.046798 | 1 | 0.391189 | 30.139869 | 22.286161 | 30.376845 | 22.118005 |
| **MUSA** | 303.619995 | 303.619995 | 0.046798 | 1 | 0.331189 | 46.346412 | 32.465035 | 47.312407 | 31.876947 |
| **SRTS** | 6.340000 | 6.340000 | 0.046798 | 1 | 1.150743 | 2.841531 | 2.551669 | 2.962890 | 2.533454 |
| **TBT** | 37.060001 | 37.060001 | 0.046798 | 1 | 0.380679 | 6.354512 | 4.660144 | 6.389366 | 4.629047 |
| **TMV** | 167.240005 | 167.240005 | 0.046798 | 1 | 0.573141 | 40.802649 | 33.156507 | 40.610356 | 33.105586 |
| **TTT** | 95.000000 | 95.000000 | 0.046798 | 1 | 0.569049 | 23.032821 | 18.689461 | 23.436374 | 18.555845 |
| **TYO** | 14.610000 | 14.610000 | 0.046798 | 1 | 0.284387 | 1.969869 | 1.301906 | 1.973230 | 1.302188 |
| **UUP** | 29.830000 | 29.830000 | 0.046798 | 1 | 0.084688 | 1.812882 | 0.449067 | 1.798310 | 0.468144 |
| **YCS** | 67.019997 | 67.019997 | 0.046798 | 1 | 0.201335 | 6.923971 | 3.859846 | 6.849995 | 3.868441 |

The option prices calculated using two different methods are very close. This proves that the Black-Scholes formula gives a fair price of the options. One advantage of Monte Carlo simulation over the Black-Scholes model is that Monte Carlo simulation can handle any kind of distributions of the returns while the B-S model is limited to the normal distribution. The Monte Carlo simulations can also provide a clear view of the price movements while most of the other methods focus only on the results.