Kalman Filter Analysis

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In this analysis, we use the Kalman Filter to estimate the values of the assets in our investment portfolio.

We make the following assumptions in our analysis:

- The asset value follows a Brownian motion. That is, the daily change of the value is normally distributed with expected value 0.
- People's estimation of the true asset value is normally distributed with expected value approximately the same as the true asset value.
- The true asset value doesn't change through the same day. So the trading prices on the same day are the estimations of the same asset value by different traders.

Based on the assumptions, we have the following Predictor Equation and Measurement Equation:

$x_{t+1} = x_t + \epsilon_{x,t+1}$ where $\epsilon_{x,t+1}$ is normally distributed with variance Q.

$z_{t+1} = z_t + \epsilon_{z,t+1}$ where $\epsilon_{z,t+1}$ is normally distributed with variance R.

We define the terms in the Kalman Filter as following:

- State (**x**): The true asset value. This is not the trading price of the asset. In real time transactions, people are estimating the value of the same asset differently and hence the trading price may differ from the true asset value.
- Measurement (**z**): We will use the daily VWAP (volume weighted average price) as the measurement or observation instead of the daily closing price. The daily closing price is just the trading price of the last transaction of the day and so it doesn't reflect the true state of the asset.
- Process Noise Uncertainty (**Q**): The random effect on the change of the state (asset value). Assume that the asset value follows a Brownian motion. The change of the asset value is normally distributed with expected value 0 with a variance Q.
- Measurement Uncertainty (**R**): The variance of the measurement error. In this case, the error is the amount of mispricing of the true asset value, which is *trading price - true asset value*. We assume this error is normally distributed with expected value 0 and variance R.

Since we use the VWAP as the measurement of the asset value, we can estimate Q using the volume weighted variance of (*typical price – VWAP)*, where the typical price = (1/3)*(high + low + close) as it is calculated in the VWAP calculation. For R, we can estimate it using the variance of the percent change of the daily VWAP.

Usually, Q and R are constant in the Kalman Filter calculation, but we would like the values of Q and R to be dynamic, which change based on the predicted value in each iteration. It is reasonable that a person misprices an asset by 10 when the price is 200. However, it is unreasonable that a person misprices an asset by 10 when the price is 20. Therefore, we construct our own Kalman Filter calculation and update Q and R in each iteration.

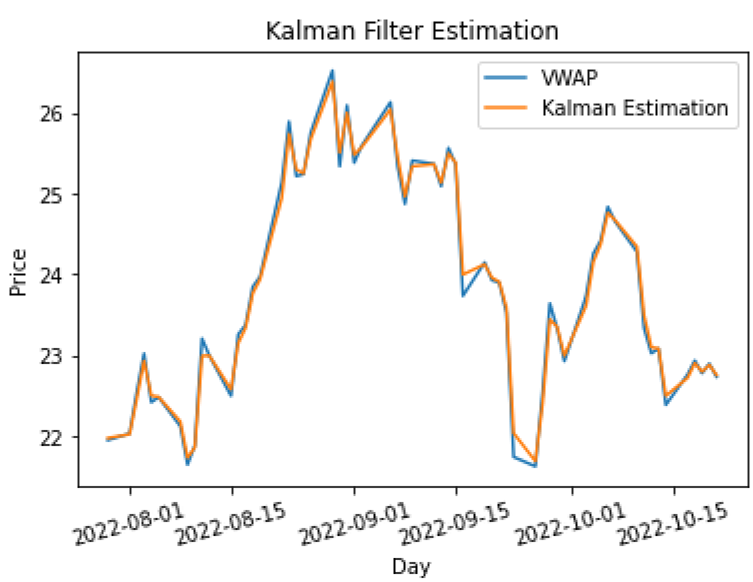
In this analysis, we demonstrate the Kalman Filter estimation for 'ARLP', one of our assets in our investment portfolio. The first and last few iterations of our calculation is showed below.

	VWAP	x_predict	P_predict	innovation	R	Q	P	K	KF_Value
2022-07-29	21.950682	23.828603	6.38588	-1.877921	0.088585	0.39084	0.087373	0.986318	21.976376
2022-08-01	22.032092	21.976376	0.419814	0.055715	0.075349	0.33244	0.063883	0.84783	22.023613
2022-08-02	22.549370	22.023613	0.397754	0.525757	0.075673	0.333871	0.063578	0.840159	22.465333
2022-08-03	23.023072	22.465333	0.410975	0.557739	0.078739	0.347398	0.066079	0.839214	22.933395
2022-08-04	22.420136	22.933395	0.428104	-0.513259	0.082054	0.362025	0.068857	0.839159	22.502689

	VWAP	x_predict	P_predict	innovation	R	Q	P	K	KF_Value
2022-10-17	22.763507	22.499246	0.418301	0.264261	0.078977	0.348447	0.066434	0.841181	22.721537
2022-10-18	22.937638	22.721537	0.421801	0.2161	0.080545	0.355367	0.067631	0.839662	22.902988
2022-10-19	22.779246	22.902988	0.428696	-0.123742	0.081837	0.361065	0.068719	0.839703	22.799082
2022-10-20	22.898098	22.799082	0.426515	0.099016	0.081096	0.357796	0.06814	0.84024	22.882279
2022-10-21	22.734939	22.882279	0.428553	-0.14734	0.081689	0.360413	0.068611	0.839901	22.758528

We can see that the Kalman gain quickly converge to around 0.84. The high Kalman gain indicates that innovation makes the most contribution to the estimated value and the predicted value from the last period only plays a small role. This is consistent with the fact that the asset value is difficult to be predicted.

We can confirm from the following graph that the Kalman Filter estimations are mostly following the actual VWAP values.



To verify that our calculation is reasonable, we can check the percentage of the innovations that are within 1.96 standard deviation of the prediction. The result is 0.9167. The number is close to 90%, which is consistent with our normal distribution assumption. Our calculation is reasonable.

Since we assume that the asset value following a Brownian motion, we predict the asset value of the following four days to be the same as the last Kalman Filter estimation. Let's compare the actual VWAP of the four days and our prediction.

	Actual VWAP	Prediction	Prediction Std	Error	Number of Std
2022-10-24	22.651619	22.758528	0.652024	-0.106909	-0.163965
2022-10-25	23.311725	22.758528	0.884116	0.553197	0.625706
2022-10-26	23.588834	22.758528	1.066858	0.830306	0.778272
2022-10-27	24.052763	22.758528	1.222583	1.294235	1.058607

We calculated the predicted variance increasingly by adding the Process Noise Uncertainty variance for each day. The result shows that the actual VWAP is within 1.1 standard deviation of our prediction. Our Kalman Filter calculation gives plausible predictions and can be used to manage our portfolio volatility / risk.