Monte Carlo Simulation Analysis

CUNY SPS MSDS

Professor Samuel Gralnick

Team Sugar Cane: Euclid Zhang, Jie Zou, Zhenni Xie

Fall 2022

In this analysis, we calculate the prices of the European call options and European put options of the assets in our investment portfolio using Monte Carlo Simulations.

The Monte Carlo Simulation is a process of generating a large number of possible values of factors that impact the outcomes of an event. The distributions of the factors are defined, and the distribution of the outcomes is estimated with the simulated factor values. This process can be easily implemented with a few lines of codes or using existing packages.

The pandas-montecarlo package (https://github.com/ranaroussi/pandas-montecarlo) is a lightweight Python library for running simple Monte Carlo Simulations on Pandas Series data. We may use the pandas-montecarlo package for our analysis. However, we found the following problems of the package after reviewing the codes of the library.

- The package performs **bootstrap simulation without replacement** from historical data instead of generating new independent samples from an estimated distribution. It may be fixed by performing **bootstrap simulation with replacement**. However, this will give us very limited number of discrete outcomes instead of countless outcomes from a continuous distribution.
- The package calculates the aggregated return by summing the daily percent changes. This is an unforgivable mistake. For example, if a stock is up by 10% on one and down by 10% on the next day, the total of change is 1.1 * 0.9 - 1 = -0.01. The calculation of the library gives 0%.
- Caused by the two problems above, the aggregated returns for all simulations have the same designation, which is the sum of all historical daily percent changes. It is apparently wrong to calculate option prices based on this only outcome.

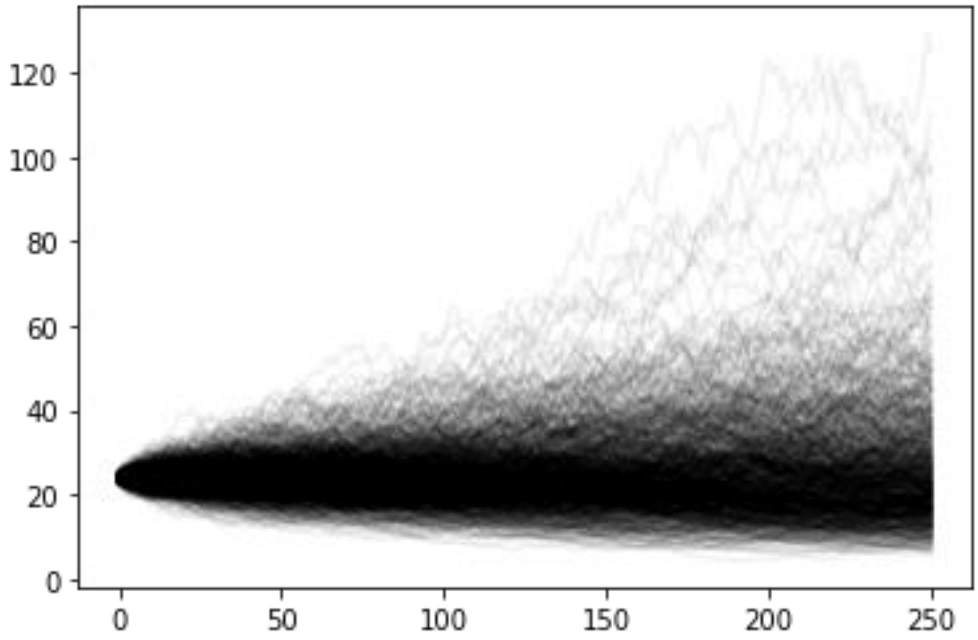
Therefore, we will create our own coding for Monte Carlo simulation in this analysis. One thing to keep in mind when defining the distribution of the asset price for our simulation is that Monte Carlo simulation assumes perfectly efficient markets. The Efficient Market Hypothesis suggests a no arbitrage pricing model. The Black-Scholes Model is one of such models and it is commonly used for option pricing. The Black-Scholes Model assumes that the future log return of an asset followings the following distribution.

$N(r - 0.5\sigma^2, \sigma^2)$ where r is the risk-free rate and σ can be estimated using the standard deviation of the historical log returns. The return is r if σ is 0. It is impossible to get higher return without additional risk (σ). This is consistent with the Efficient Market Hypothesis. We will perform our simulation using this distribution.

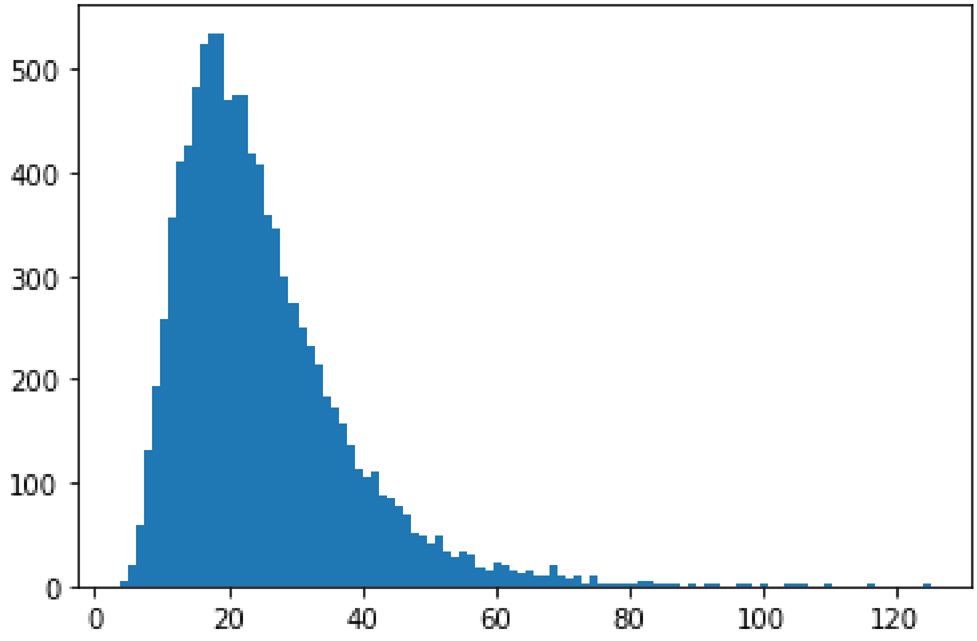
Our investment portfolio includes the following assets, with purchasing prices (close prices) from 9/16/2022. In this analysis, we will not include the U.S. 30 Year Treasury bonds in our calculation.

Ticker	Type	Sector	Units	Amount
US30Y	U.S. 30 Year Treasury	NaN	467317.00	42262981.19
HSON	Common Stock	COMMERCIAL SERVICES	36138.24	1263031.49
GOGO	Common Stock	COMMUNICATIONS	94822.19	1263031.57
JAKK	Common Stock	CONSUMER DURABLES	54044.99	1263031.42
CALM	Common Stock	CONSUMER NON-DURABLES	22216.91	1263031.33
HRB	Common Stock	CONSUMER SERVICES	28111.09	1263031.27
HDSN	Common Stock	DISTRIBUTION SERVICES	159071.98	1263031.52
BELFA	Common Stock	ELECTRONIC TECHNOLOGY	44285.81	1263031.30
ARLP	Common Stock	ENERGY MINERALS	52890.76	1263031.35
CI	Common Stock	HEALTH SERVICES	4350.03	1263031.21
SRTS	Common Stock	HEALTH TECHNOLOGY	85339.97	1263031.56
LNG	Common Stock	INDUSTRIAL SERVICES	7551.75	1263030.19
BSM	Common Stock	MISCELLANEOUS	80447.87	1263031.56
HUDI	Common Stock	NON-ENERGY MINERALS	42612.40	1263031.54
CF	Common Stock	PROCESS INDUSTRIES	12810.95	1263031.56
CSL	Common Stock	PRODUCER MANUFACTURING	4335.84	1263030.19
MUSA	Common Stock	RETAIL TRADE	4561.65	1263029.65
AZPN	Common Stock	TECHNOLOGY SERVICES	5579.50	1263031.42
ASC	Common Stock	TRANSPORTATION	126303.15	1263031.50
ED	Common Stock	UTILITIES	12966.13	1263030.72
YCS	ETF	NaN	25261.96	1599839.93
UUP	ETF	NaN	54416.32	1599839.81
EUO	ETF	NaN	47870.73	1599839.80
EWV	ETF	NaN	79832.33	1599839.89
DIG	ETF	NaN	44390.67	1599839.75
TTT	ETF	NaN	23645.28	1599839.64
ERX	ETF	NaN	29349.47	1599839.61
TMV	ETF	NaN	13430.49	1599839.97
TBT	ETF	NaN	54509.02	1599839.74
TYO	ETF	NaN	127782.74	1599839.90

First, let's try to perform a simulation for 'ARLP' and see how the asset price is moving. Below shows 1000 simulated movements of the asset price. We can see that the prices are centered at the starting price and very unlikely to move up by a lot.



We can also check the distribution of the final prices. The distribution is log-normal which is consistent with the assumption of the Black-Scholes model.



We can then perform simulations for all assets and calculate the option prices based on the results of the simulation.

$P_{call} = mean(\max(final\_price - strike\_price, 0) * e^{-r})$

$P_{put} = mean(\max(strike\_price - \ final\_price, 0) * e^{-r})$

We calculate the option price with strike price equals the starting price of the asset, period equals 1 year (251 trade days), and risk-free rate based on the current yield rate of the 1-year US Treasury Bill. We also calculate the option prices using the Black-Scholes formula for comparison.

	Stock price	Strike price	Risk free rate	Time period	volatility	call_BS	put_BS	call_sim	put_sim
ARLP	23.950001	23.950001	0.046798	1	0.473192	4.945446	3.850463	4.921912	3.914618
ASC	14.030000	14.030000	0.046798	1	0.591182	3.517236	2.875790	3.355578	2.925548
AZPN	233.009995	233.009995	0.046798	1	0.367275	38.766395	28.113277	39.220157	27.553737
BELFA	32.630001	32.630001	0.046798	1	0.486784	6.904918	5.413088	6.875883	5.410121
BSM	19.420000	19.420000	0.046798	1	0.344013	3.059152	2.171278	3.048242	2.143808
CALM	57.480000	57.480000	0.046798	1	0.326349	8.668199	6.040237	8.876806	5.999748
CF	105.379997	105.379997	0.046798	1	0.497390	22.720428	17.902499	23.267359	17.716548
CI	322.130005	322.130005	0.046798	1	0.267453	41.357090	26.629443	41.076799	26.689906
CSL	225.570007	225.570007	0.046798	1	0.350068	36.052693	25.739729	35.761443	25.749973
DIG	47.529999	47.529999	0.046798	1	0.702704	13.870545	11.697493	13.887907	11.752958
ED	90.040001	90.040001	0.046798	1	0.207722	9.519397	5.402806	9.523660	5.554149
ERX	73.110001	73.110001	0.046798	1	0.709946	21.528930	18.186372	21.385508	18.064281
EUO	33.750000	33.750000	0.046798	1	0.191595	3.362852	1.819816	3.421166	1.765561
EWV	20.420000	20.420000	0.046798	1	0.384106	3.527904	2.594310	3.386756	2.683251
GOGO	14.850000	14.850000	0.046798	1	0.489196	3.155936	2.477001	3.126995	2.488528
HDSN	10.540000	10.540000	0.046798	1	0.772048	3.341431	2.859547	3.388370	2.851450
HRB	39.759998	39.759998	0.046798	1	0.387050	6.913667	5.095856	7.047203	5.055023
HSON	36.380001	36.380001	0.046798	1	0.634772	9.708179	8.044901	9.409704	8.006942
HUDI	180.000000	180.000000	0.046798	1	1.934502	121.381473	113.151949	110.099115	113.924059
JAKK	18.730000	18.730000	0.046798	1	0.754775	5.820856	4.964529	5.641459	5.005652
LNG	171.779999	171.779999	0.046798	1	0.391189	30.139869	22.286161	30.376845	22.118005
MUSA	303.619995	303.619995	0.046798	1	0.331189	46.346412	32.465035	47.312407	31.876947
SRTS	6.340000	6.340000	0.046798	1	1.150743	2.841531	2.551669	2.962890	2.533454
TBT	37.060001	37.060001	0.046798	1	0.380679	6.354512	4.660144	6.389366	4.629047
TMV	167.240005	167.240005	0.046798	1	0.573141	40.802649	33.156507	40.610356	33.105586
TTT	95.000000	95.000000	0.046798	1	0.569049	23.032821	18.689461	23.436374	18.555845
TYO	14.610000	14.610000	0.046798	1	0.284387	1.969869	1.301906	1.973230	1.302188
UUP	29.830000	29.830000	0.046798	1	0.084688	1.812882	0.449067	1.798310	0.468144
YCS	67.019997	67.019997	0.046798	1	0.201335	6.923971	3.859846	6.849995	3.868441