Black Scholes Analysis

**CUNY SPS MSDS**

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In this analysis, we calculate the prices of the European call options and European put options of the equities in our investment portfolio.

In the calculations, we calculate the returns and volatilities using the assets' prices for 1 calendar year. The risk-free rate is from <https://www.marketwatch.com/investing/bond/tmubmusd01y?countrycode=bx>. We assume that there are 251 trade days in a calendar year, this is used to annualize the returns and standard deviations of the assets. We will set the strike price equal to 80% of the current price for the option calculations. We will calculate the prices of option with expiration period 1 calendar year.

Our investment portfolio includes the following assets, with purchasing prices (close prices) from 9/16/2022. In this analysis, **we do not include the U.S. 30 Year Treasury bonds in our calculation**.

| **Ticker** | **Type** | **Sector** | **Units** | **Amount** |
| --- | --- | --- | --- | --- |
| US30Y | U.S. 30 Year Treasury | NaN | 467317.00 | 42262981.19 |
| HSON | Common Stock | COMMERCIAL SERVICES | 36138.24 | 1263031.49 |
| GOGO | Common Stock | COMMUNICATIONS | 94822.19 | 1263031.57 |
| JAKK | Common Stock | CONSUMER DURABLES | 54044.99 | 1263031.42 |
| CALM | Common Stock | CONSUMER NON-DURABLES | 22216.91 | 1263031.33 |
| HRB | Common Stock | CONSUMER SERVICES | 28111.09 | 1263031.27 |
| HDSN | Common Stock | DISTRIBUTION SERVICES | 159071.98 | 1263031.52 |
| BELFA | Common Stock | ELECTRONIC TECHNOLOGY | 44285.81 | 1263031.30 |
| ARLP | Common Stock | ENERGY MINERALS | 52890.76 | 1263031.35 |
| CI | Common Stock | HEALTH SERVICES | 4350.03 | 1263031.21 |
| SRTS | Common Stock | HEALTH TECHNOLOGY | 85339.97 | 1263031.56 |
| LNG | Common Stock | INDUSTRIAL SERVICES | 7551.75 | 1263030.19 |
| BSM | Common Stock | MISCELLANEOUS | 80447.87 | 1263031.56 |
| HUDI | Common Stock | NON-ENERGY MINERALS | 42612.40 | 1263031.54 |
| CF | Common Stock | PROCESS INDUSTRIES | 12810.95 | 1263031.56 |
| CSL | Common Stock | PRODUCER MANUFACTURING | 4335.84 | 1263030.19 |
| MUSA | Common Stock | RETAIL TRADE | 4561.65 | 1263029.65 |
| AZPN | Common Stock | TECHNOLOGY SERVICES | 5579.50 | 1263031.42 |
| ASC | Common Stock | TRANSPORTATION | 126303.15 | 1263031.50 |
| ED | Common Stock | UTILITIES | 12966.13 | 1263030.72 |
| YCS | ETF | NaN | 25261.96 | 1599839.93 |
| UUP | ETF | NaN | 54416.32 | 1599839.81 |
| EUO | ETF | NaN | 47870.73 | 1599839.80 |
| EWV | ETF | NaN | 79832.33 | 1599839.89 |
| DIG | ETF | NaN | 44390.67 | 1599839.75 |
| TTT | ETF | NaN | 23645.28 | 1599839.64 |
| ERX | ETF | NaN | 29349.47 | 1599839.61 |
| TMV | ETF | NaN | 13430.49 | 1599839.97 |
| TBT | ETF | NaN | 54509.02 | 1599839.74 |
| TYO | ETF | NaN | 127782.74 | 1599839.90 |

In our calculations, we used annualized standard deviation of the daily log returns as the volatility and the log return of the risk-free rate. Additionally, we calculate the ratio of option price to the current asset price. The following is the result of our calculations.

|  | **Stock price** | **Strike price** | **Risk free**  **rate** | **Time**  **period** | **volatility** | **Call price** | **Put price** | **Strike to price** | **Call to price** | **Put to price** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **ARLP** | 22.76 | 18.208 | 0.044945 | 1 | 0.479219 | 7.044185 | 1.691950 | 0.8 | 0.309498 | 0.074339 |
| **ASC** | 11.69 | 9.352 | 0.044945 | 1 | 0.591874 | 4.008553 | 1.259536 | 0.8 | 0.342904 | 0.107745 |
| **AZPN** | 253.64 | 202.912 | 0.044945 | 1 | 0.363252 | 70.333709 | 10.687801 | 0.8 | 0.277297 | 0.042138 |
| **BELFA** | 30.21 | 24.168 | 0.044945 | 1 | 0.482695 | 9.380423 | 2.276248 | 0.8 | 0.310507 | 0.075348 |
| **BSM** | 17.63 | 14.104 | 0.044945 | 1 | 0.343947 | 4.801065 | 0.655200 | 0.8 | 0.272324 | 0.037164 |
| **CALM** | 58.69 | 46.952 | 0.044945 | 1 | 0.321967 | 15.661778 | 1.860256 | 0.8 | 0.266856 | 0.031696 |
| **CF** | 107.373 | 85.896 | 0.044945 | 1 | 0.494922 | 33.721637 | 8.472539 | 0.8 | 0.314069 | 0.078910 |
| **CI** | 301.34 | 241.072 | 0.044945 | 1 | 0.267241 | 76.662444 | 5.799419 | 0.8 | 0.254405 | 0.019245 |
| **CSL** | 273.88 | 219.104 | 0.044945 | 1 | 0.305211 | 71.989296 | 7.583754 | 0.8 | 0.262850 | 0.027690 |
| **DIG** | 42.94 | 34.352 | 0.044945 | 1 | 0.704232 | 16.189409 | 6.091652 | 0.8 | 0.377024 | 0.141864 |
| **ED** | 84.87 | 67.896 | 0.044945 | 1 | 0.207839 | 20.672917 | 0.714912 | 0.8 | 0.243583 | 0.008424 |
| **ERX** | 66.37 | 53.096 | 0.044945 | 1 | 0.711636 | 25.172985 | 9.565434 | 0.8 | 0.379283 | 0.144123 |
| **EUO** | 34.41 | 27.528 | 0.044945 | 1 | 0.183209 | 8.266907 | 0.175061 | 0.8 | 0.240247 | 0.005088 |
| **EWV** | 21.81 | 17.448 | 0.044945 | 1 | 0.378152 | 6.133650 | 1.004817 | 0.8 | 0.281231 | 0.046071 |
| **GOGO** | 13.57 | 10.856 | 0.044945 | 1 | 0.503786 | 4.297139 | 1.106022 | 0.8 | 0.316665 | 0.081505 |
| **HDSN** | 8.01 | 6.408 | 0.044945 | 1 | 0.763522 | 3.164850 | 1.281220 | 0.8 | 0.395112 | 0.159953 |
| **HRB** | 41.02 | 32.816 | 0.044945 | 1 | 0.390116 | 11.667851 | 2.021600 | 0.8 | 0.284443 | 0.049283 |
| **HSON** | 35.55 | 28.440 | 0.044945 | 1 | 0.659718 | 12.921052 | 4.561124 | 0.8 | 0.363461 | 0.128302 |
| **HUDI** | 58.92 | 47.136 | 0.044945 | 1 | 1.377616 | 33.817026 | 19.961417 | 0.8 | 0.573948 | 0.338788 |
| **JAKK** | 18.78 | 15.024 | 0.044945 | 1 | 0.755675 | 7.375260 | 2.958961 | 0.8 | 0.392719 | 0.157559 |
| **LNG** | 175.79 | 140.632 | 0.044945 | 1 | 0.388281 | 49.915142 | 8.576418 | 0.8 | 0.283948 | 0.048788 |
| **MUSA** | 283.18 | 226.544 | 0.044945 | 1 | 0.324245 | 75.725746 | 9.133222 | 0.8 | 0.267412 | 0.032252 |
| **SRTS** | 13.16 | 10.528 | 0.044945 | 1 | 0.871809 | 5.632925 | 2.538223 | 0.8 | 0.428034 | 0.192874 |
| **TBT** | 38.26 | 30.608 | 0.044945 | 1 | 0.375972 | 10.737701 | 1.740491 | 0.8 | 0.280651 | 0.045491 |
| **TMV** | 175.67 | 140.536 | 0.044945 | 1 | 0.565985 | 58.871140 | 17.560635 | 0.8 | 0.335123 | 0.099964 |
| **TTT** | 99.96 | 79.968 | 0.044945 | 1 | 0.562354 | 33.390239 | 9.883675 | 0.8 | 0.334036 | 0.098876 |
| **TYO** | 14.77 | 11.816 | 0.044945 | 1 | 0.281966 | 3.804227 | 0.330918 | 0.8 | 0.257564 | 0.022405 |
| **UUP** | 30.11 | 24.088 | 0.044945 | 1 | 0.080390 | 7.080895 | 0.000236 | 0.8 | 0.235168 | 0.000008 |
| **YCS** | 67.41 | 53.928 | 0.044945 | 1 | 0.195375 | 16.299521 | 0.447404 | 0.8 | 0.241797 | 0.006637 |

We observe that for a call option with strike price 20% lower than the current asset price, the option price is more than 20% of the current asset price. This is reasonable since the call option is in the money and the option price must be higher than the asset price - strike price as of now.

The price of the put options are lower than the call options since the call options are out of the money.

The option price of a highly volatile asset is extremely expansive. For example, the price of the call option for HUDI is more than 50% of the current asset price and the price of the put option is more than 30% of the current asset price. This indicates the asset is a risky one. However, with great risk comes great reward. Since the asset only take a very small portion of our portfolio, we can afford the risk and aim for a return boost to our portfolio.