Astro 735
Problem Set 2
Due in class, **Monday October 13, 2014**

## 1 The cosmological constant

Einstein originally introduced the cosmological constant $\Lambda$ to stabilize his model of a pressureless matter universe against expansion or contraction.

a) Find an expression for $\Lambda$ in terms of the density $\rho_m$ of a static model of a pressureless matter universe with a cosmological constant.

b) Find an expression for the curvature $k$ for this static model. Is this model universe closed, open or flat?

c) Explain why Einstein's static model is in an unstable equilibrium, so any departure from equilibrium (expansion or contraction) will tend to increase.

## 2 Evolution of a $\Lambda$-dominated universe

The cosmological constant becomes dominant as the scale factor $a$ becomes increasingly large in the $\Lambda$ era.

a) Show that the Hubble parameter is a constant in a flat universe deep in the $\Lambda$ era.

b) Suppose that, starting today ($t = t_0$, when $a = 1$), only the cosmological constant contributes to the Friedmann equation. Solve the Friedmann equation and show that for $\Lambda > 0$, the scale factor will increase exponentially. Note that in this situation the universe is not flat, so do not assume $k = 0$.

c) Use WMAP values to evaluate the characteristic time $\tau$ for the exponential expansion, where exponential expansion has the form $f(t) = f_0 e^{t/\tau}$ with $\tau$ the characteristic time for the system under consideration.

## 3 High redshift galaxies

Astronomers recently announced the discovery of a galaxy likely to be at redshift $z = 10$ (Bouwens et al. 2011, *Nature*, 469, 504). This claim is controversial, partly because it would imply that galaxies formed only a short time after the Big Bang.

Assuming an Einstein-de Sitter ($\Omega_0 = \Omega_m = 1$) universe, calculate:

a) the fraction of the current age of the universe at which this galaxy was observed;

b) the time interval available from the Big Bang to $z = 10$ to form this galaxy.

c) Repeat parts a) and b) for a flat $\Omega_m = 0.3, \Omega_\Lambda = 0.7$ cosmology.

## 4 Absorption line systems

a) Using the Friedmann equation for a universe containing pressureless matter and a cosmological constant

$$\left[\left(\frac{1}{a}\frac{da}{dt}\right)^2 - \frac{8\pi}{3}G\rho - \frac{1}{3}\Lambda c^2\right]a^2 = -kc^2 \tag{1}$$

show that for a population of absorbing gas clouds with constant number density per comoving volume and with constant physical cross-section, the probability that the line of sight to a distant source intersects such a cloud per unit redshift at redshift $z$ is proportional to:

$$\frac{(1+z)^2}{(\Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0})^{1/2}} \tag{2}$$

where $\Omega_{m,0} + \Omega_{\Lambda,0} = 1$.

b) Surveys have shown that, on average, the spectrum of a quasar at redshift $z_{\mathrm{em}} = 2$ shows five sets of absorption lines from elements heavier than helium (i.e. five *metal line systems*) at redshifts $z_{\mathrm{abs}} < z_{\mathrm{em}}$. If such systems are produced by intervening galaxies, randomly distributed along the line of sight to the quasar, estimate the gaseous size of galaxies required to satisfy the absorption line statistics in a $\Omega_{\Lambda,0} = 0.7$, $\Omega_{m,0} = 0.3$ cosmology. (Hint: Use the average density of galaxies implied by the galaxy luminosity function). Comment on the implications of your answer.

## 5 Galaxy counts and quasar counts

a) From deep galaxy counts, it is estimated that there are about 40 billion galaxies within the observable universe (not including probable multitudes of faint dwarf galaxies). Assuming that the mean age of these galaxies is 10 Gyr, that each galaxy goes through one quasar phase in which it accretes large amounts of matter onto a central supermassive black hole, and that the quasar phase has a mean duration of $10^8$ years, estimate the projected surface density of quasars on the sky (number per square degree).

b) If the volume number density of quasars at $z \sim 0$ is about 100 Gpc$^{-3}$, and the normalization of the galaxy luminosity function is $\phi_* \approx 0.01$ Mpc$^{-3}$, what is the probability of a galaxy containing a quasar at $z \sim 0$? There were about a thousand times more quasars per unit comoving volume at $z \sim 2$, the peak of the quasar era. Compare these numbers with the average probability evaluated in a).

c) Using the same assumptions about the galaxy luminosity function, and assuming that the look-back time to the onset of galaxy formation is $\sim$ 10 Gyr, estimate the number of $L \sim L_*$ (i.e. non-dwarf) galaxies in the entire observable universe. State your assumptions. Compare your estimate with the total observed number of galaxies over the entire sky. Discuss the differences, if any.

## 6 Direct measurement of the expanding universe

a) From the relation between redshift and scale factor

$$1 + z(t_0, t_e) \equiv \frac{a(t_0)}{a(t_e)} \tag{3}$$

where $t_0$ is the time of observation and $t_e$ is the time when a photon was emitted (or absorbed), show that the measured redshift of a distant astronomical source varies with $t_0$ as:

$$\dot{z} = \frac{dz}{dt_0} = H(t_0)\,(1+z) - H(t_e) \tag{4}$$

where $H(t)$ is the Hubble parameter at time $t$.