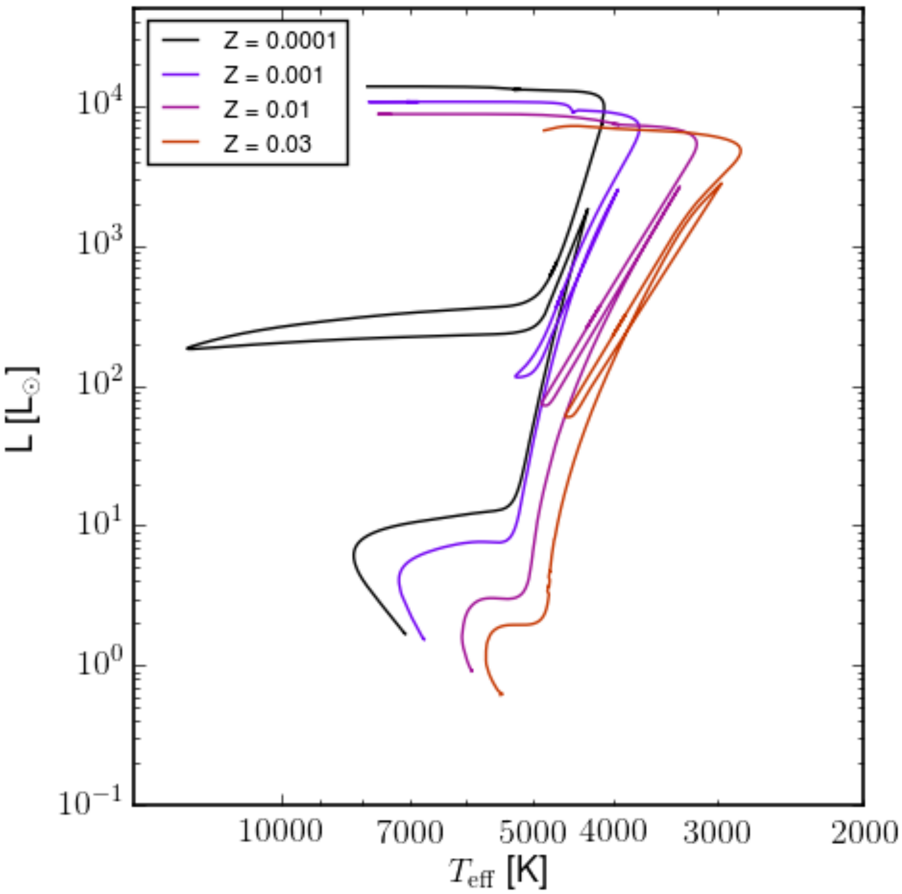
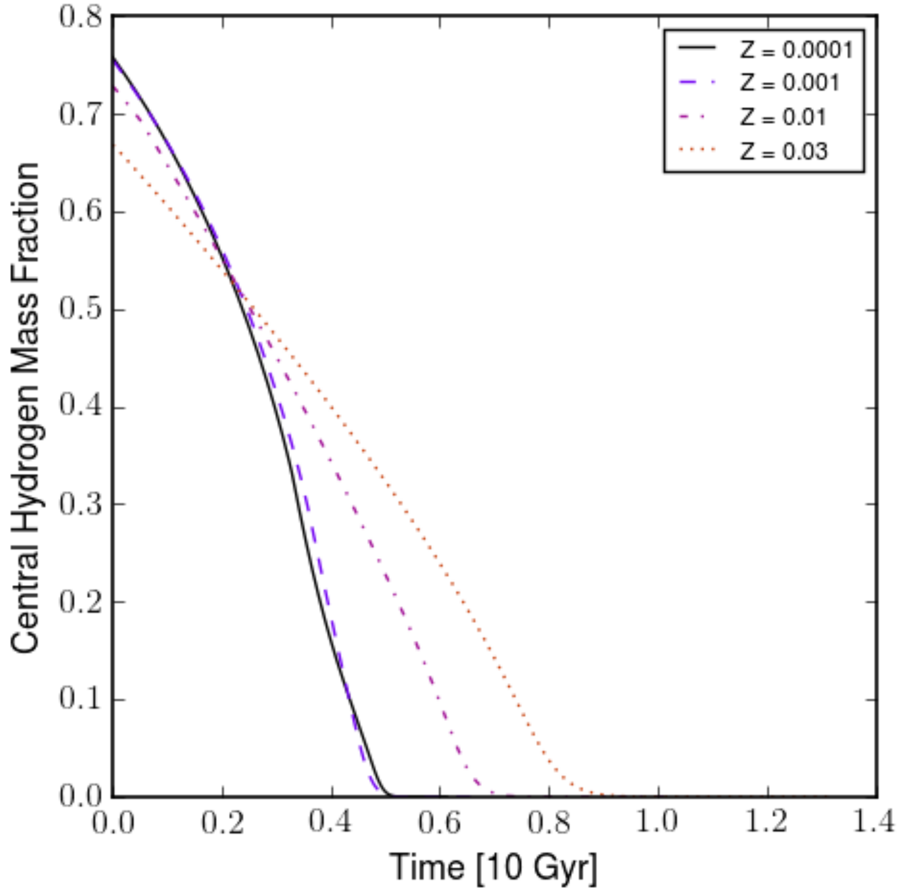
Homework7
Astro 715 Elijah Bernstein-Cooper

1a

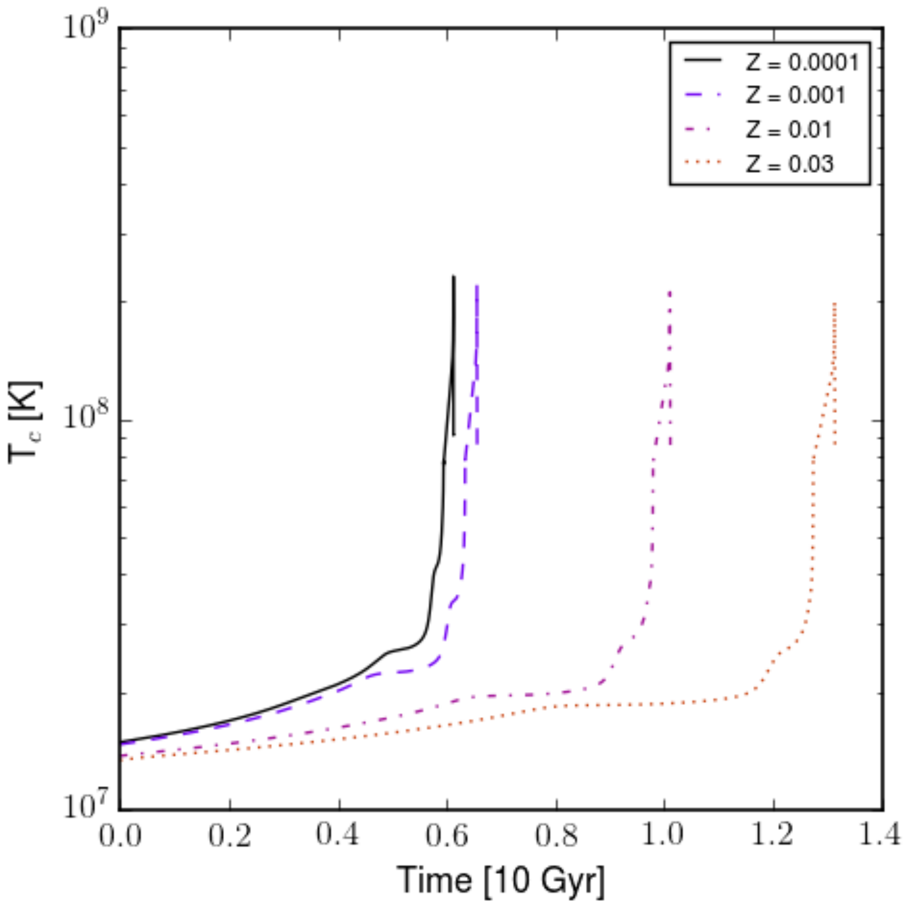


We can determine the length of the main sequence lifetime, $\tau_{\rm MS}$, for each stellar model based on the central hydrogen mass fraction. Stars on the MS will fuse H in their cores, so a sharp drop in H mass fraction in the core represents the turn off from the main sequence. Below is shows the central H mass fraction as a function of time.

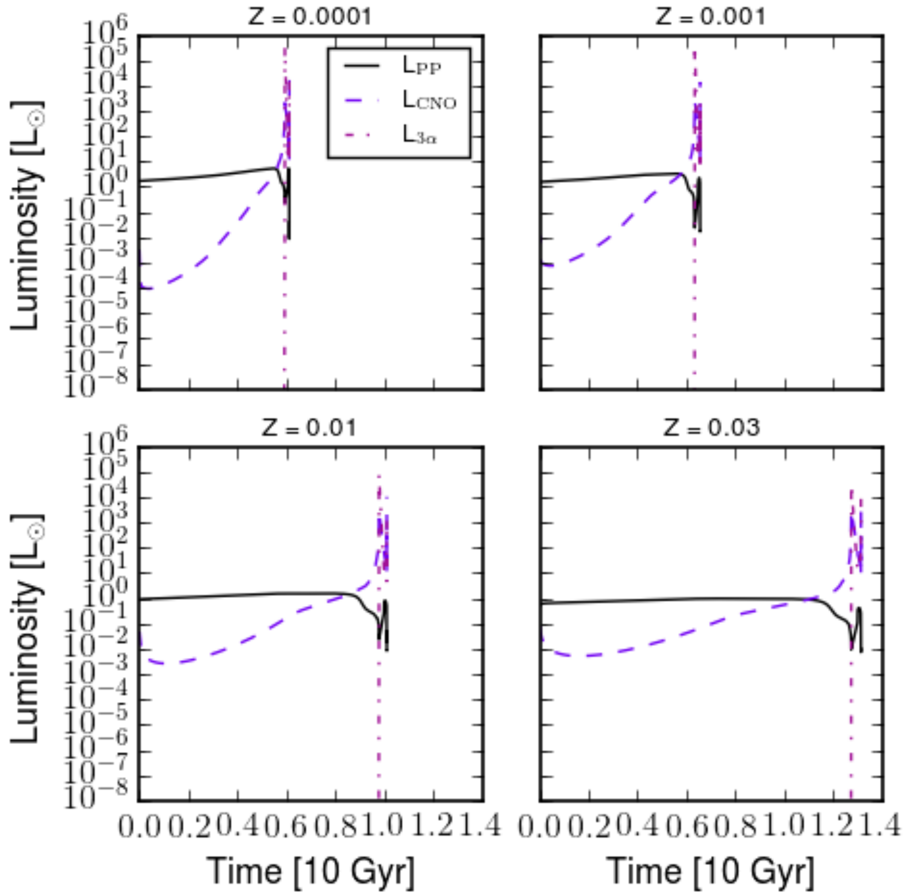


We can see the lifetimes are
Z = 0.0001 $\longrightarrow$ τ_{MS} ~ 5 Gyr
Z = 0.001 $\longrightarrow$ τ_{MS} ~ 5 Gyr
Z = 0.01 $\longrightarrow$ τ_{MS} ~ 7 Gyr
Z = 0.03 $\longrightarrow$ τ_{MS} ~ 9 Gyr

We find that the stars with lower metal contents will have shorter lifetimes. Metals will have larger opacities than non-metals. A lower metallicity star will thus have a lower internal gas pressure, leading to a more compact core, thus a higher core temperature. A higher core temperature will raise the number of core fusion reactions, thus higher luminosity. See the core temperatures as a function of time below. The lower metallicity star will have a higher effective temperature due to the higher luminosity. Finally, because the lower-metallicity stars are fusing at higher rates, their lifetimes will be shorter.



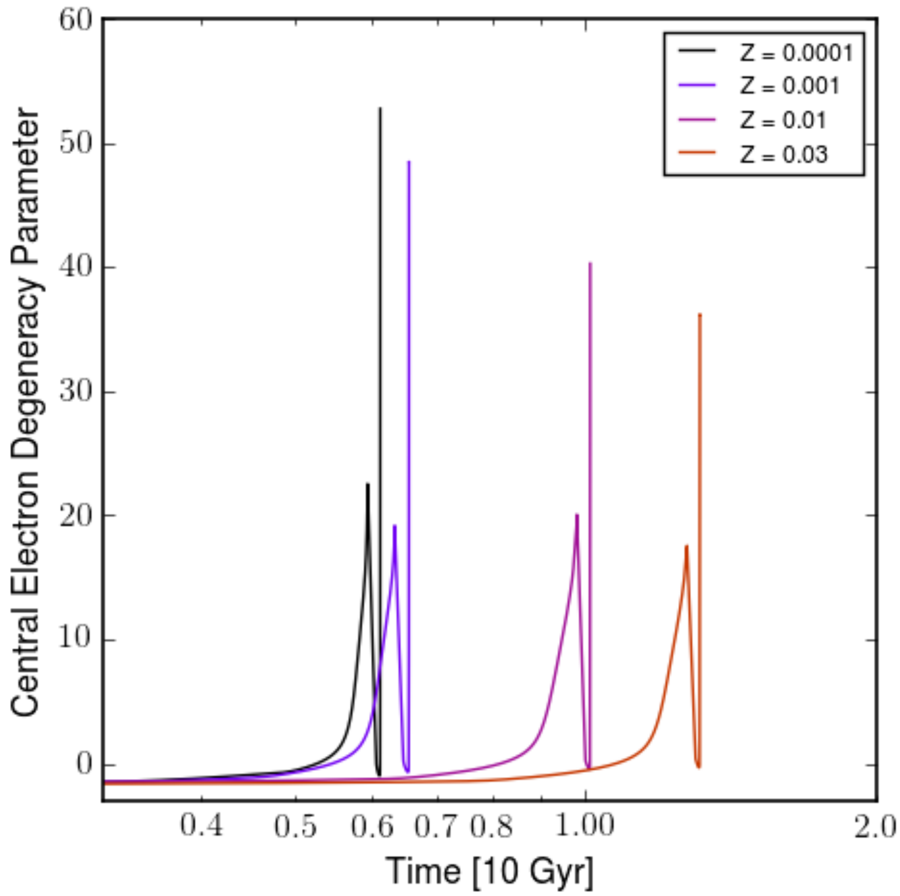
Three nuclear processes contribute substantially to the luminosities of our stellar models throughout their lifetimes. These are the PP-chain luminosity L_{PP}, the CNO cycle luminosity L_{CNO}, the triple-α process luminosity $L_{3\,\alpha}$, and metal burning luminosity L_Z. Below is a plot of the luminosities of each nuclear process over time for each stellar model. We can see that L_{CNO} contributes several orders of magnitude more to the total luminosity for the most metal-rich star than the most metal poor star for the majority of both stars life-
times.



The CNO cycle is the most obviously dependent on metallicity. The more metals present in the star at conception, the more fuel the CNO cycle will have.

1b

The central degeneracy parameter, $\psi = \log_{10}\left[\frac{\epsilon_F}{kT}\right]$ where ϵ_F is the fermi energy, remains below 0, i.e. not degenerate, during the main sequence lifetime of each star. Degeneracy as a function of time does not vary much with metallicity. Because the PP-chain is the dominant fusion process in each of these models during their main sequence, the inert He core will build in mass (and degeneracy) proportional to the amount of PP-chain reactions. The CNO reaction rates are similar for each star once they leave the main sequence (see above figure), thus the central electron degeneracy will follow a similar trend for each metallicity.



2a

Sweigart, A.~V., Greggio, L., & Renzini, 1990, apj, 364, 527 describe the evolutionary stage of stars with masses between 1.4 to 3.4 $M_\odot$ along the RGB phase. They define the RGB phase transition as the evolutionary stage at which a hydrogen-shell-burning star spends near the Hyashi line until He core ignition. The age at which a stellar population undergoes the RGB phases, ~0.6 Gyr and the time interval of the transition is ~0.2 Gyr. Both times are is independent of composition. Finally, Figure 2 shows that the maximum luminosity of stars in the RGB phase is constant with mass.

2b

The dispersion in maximum luminosity of stars in the RGB phase is about 0.2 dex as shown in Figure 2 of Sweigart et al. (1990). Salaris et al. (2002) reviewed previous models to obtain an absolute I-band magnitude of $M_I = -4.04 \pm 0.12$ mag. Today with HST we can view this magnitude out to 10s of Mpc.

2c

The red clump is essentially the horizontal branch for metal-rich stars ($Z \sim \frac{1}{10} Z_\odot$) with main sequence masses of > 1 $M_\odot$. Red clump stars will occupy this stage on order of 100 Myr, thus a significant number of stars may be present in the clump. In this stage of evolution stars undergo He core and H shell burning. Red clump stars have similar mass cores because their progenitors were RGB stars, which all have similar mass He cores when He ignites. Because red clump stars have similar internal structures, they will have similar luminosities and in turn can be used as standard candles.

Assuming a constant luminosity of the red clump, we would like to see how much the stars deviate from this assumption. We do this by looking at the dispersion in the derived distance. The uncertainty of the distance modulus is about 0.3 mag (Girardi & Salaris 2001). The distance modulus gives

$m - M = \mu = 5\log_{10}\left(\frac{d}{10\text{ pc}}\right), \quad d = 10^{\frac{(m-M)}{5}+1}$

where the uncertainty in d, δd, is given by

δd= 0.461 d $\delta\mu$

thus the dispersion in the distance is given by the difference of the relative errors

$\sigma_d = \frac{\delta \mathrm{d}}{d} = 0.461 \times 0.3 = 0.14$

There is a dispersion of about 14% in derived distance using the red clump.