Astron 715- Hw 2

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1)

The grey atmosphere assumes a plane parallel atmosphere in LTE. Using the result from the radiative transfer equation for the intensity at the surface where the optical depth, $\tau_s = 1$,

$I_\nu(0,\ \mu) = S_\nu(\tau_s = \tau_z / \mu = 1)$

where τ_z is the plane parallel atmosphere optical depth and $\mu = \cos[\theta]$.

$\tau_s = \kappa \rho s$

where κ is the opacity which is constant, ρ is the mass density, and s is the path length. The path length will remain constant with μ. For larger μ, lines of sight towards the edges of the star, an optical depth of 1 will proble larger radii, points where the star is cooler. The outer regions of the star will be darker, because the surface is cooler. Limb darkening would not occur if the temperature of a star remained constant with radius.

2a)

At above temperatures above 10^4 $K$the ionization fraction of hydrogen is high and is nearly constant of temperature. O stars have temperatures > 10^4 K. The fraction of ionized H will be similar in each star. Therefore the equivalent width of the Balmer absorption line will be similar between an O5 and an O9 star, and will be much weaker for stars with temperatures < 10^4 K.

2b)

Stark broadening will contribute to the line Balmer line width. The width due to pressure broading, $\Delta\lambda$, is given by

$$\Delta\lambda = \frac{\lambda^2}{c} \frac{n\sigma}{\pi} \sqrt{\frac{2\,kT}{m}}$$

where c is the speed of light, n is the number density of atoms, σ is the collisional cross section, T is the temperature, and m is the particle mass. We can see that for the B0V and B0Ia stars, which have constant T, $\Delta\lambda \propto$ n. The surface gravity of the two starsmust then differ by about the ratio of their line widths because the surface gravity, g, is given by

$P(z) = g \int \rho(z)\, dz$

where P(z) is the pressure, which will remain mostly constant in the atmosphere, and so will the density, thus $g = \frac{P}{\rho} \propto \frac{1}{n}$.

3)

When the temperature of the star is > 10^4 $K$the vast majority of H atoms are ionized, thus the opacity of the star is mainly due to free-free collisions. As the radius increases, the temperature decreases in an LBV. At 10^4 K the ionization fractions of H quickly drops, and the opacity rises due to bound-free absorption. The increase in radiation pressure will drive the luminosity to be greater than the Eddington luminosity. This is an unstable process, whereby the expanding gas will become cooler, and thus more opaque, which will cause the gas to expand further.

4)

The ratio of SFRs derived using the Kroupa and Salpeter IMF is given by

$$\frac{\phi_{\text{salpeter}}}{\phi_{\text{kroupa}}} = \frac{\int M * M^{\Gamma_{\text{salpeter}}-1}\, dM}{\int M * M^{\Gamma_{\text{kroupa}}-1}\, dM} = 1.48$$

In[18]:=
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salpeter = Integrate[M * M^-2.3, {M, 0.1, ∞}];
kroupa = Integrate[Piecewise[{{M * M^-1, M < 0.5}, {M * M^-2.3, M ≥ 0.5}}], {M, 0.1, ∞}];
salpeter / kroupa
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Out[20]= 1.47672