Homework 4
Astro 500 Elijah Bernstein-Cooper

1

The magnification factor is given by

$w_{\theta'} = \frac{f_2}{f_1} w$

where $w_{\theta'}$ is the reimaged spatial width, and w is the physical slit width. f_2 is the detector focal length and f_1 is the collimator focal length. The magnifcation factor is then

$\frac{f_2}{f_1} = \frac{f/1.33}{f/3.32} = 1.33/3.32 = 0.4$

2

$w = 300\ \mu\text{m}$
$w_{\theta'} = f_2/f_1\, w$
$w_{\theta'} = 0.4 \times 300\ \mu\text{m}$
$w_{\theta'} = 120.2\ \mu\text{m} = 8\ \text{pix}$

3

$\frac{f_1}{D_1} = 3.32$

$D_1 = 115\ \text{mm}$

$f_1 = 3.32\, D_1 = 381.8\ \text{mm}$

and in Littrow $D_1 = D_2$

$f_2 = 1.33\, D_2 = 152.5\ mm$

4

$\alpha = \beta = 100$ deg for Littrow. $\lambda_c = 653$ nm. Given $m = 1$
the grating equation gives

$\lambda = \sigma[\sin\alpha + \sin\beta]$

thus the separation σ is

$$\frac{[\sin\alpha + \sin\beta]}{\lambda} = \frac{1}{\sigma}$$
$$\frac{1}{\sigma} = 2\sin\left(100\,\text{deg}\,\frac{\pi}{180}\right)\left[653\times 10^{-3}\,\text{mm}\right]^{-1}$$
$$\frac{1}{\sigma} = 3016\,\text{mm}^{-1}$$

5

$$\gamma = \frac{d\beta}{d\lambda}$$
$$\gamma\ =\ [\sigma\cos\beta]^{-1}\left[\text{mm}^{-1}\right]$$
$$\gamma = 0.0019\,\text{mm}^{-1}$$

6

linear dispersion in Å/mm is given by

$$\frac{d\lambda}{dx} = \frac{\sigma\cos\beta}{m f_2}\lambda_{\text{pix}}$$

$$\frac{d\lambda}{dx} = \left[\frac{1}{3016}\,\text{mm}\ \times 10^7\,\frac{\text{Å}}{\text{mm}}\right]\cos\left(100\,\text{deg}\,\frac{\pi}{180}\right)[152.5\,\text{mm}]^{-1}\left[15\ \times 10^{-3}\,\frac{\text{mm}}{\text{pix}}\right]$$
$$\frac{d\lambda}{dx} = 3.77\,\frac{\text{Å}}{\text{mm}} = 0.057\,\frac{\text{Å}}{\text{pix}} = 3.77\times 10^{-7}$$

7

The anamorphic magnification is 1 for Littrow spectrographs.

8

The spectral resolution, R, of the system is given by

$$R = \frac{\lambda}{d\lambda}$$

which in Littrow is

$$R\ =\ (f_1/w)\,2\tan\alpha$$

where w is the physical slit width and f_1 is the collimater focal length.

$$R\ = \left(\frac{381.8\,\text{mm}\times 10^3\,\frac{\mu\text{m}}{\text{mm}}}{300\,\mu\text{m}}\right)2\tan\left(100\,\text{deg}\,\frac{\pi}{180}\right)$$
$$R =\ 14,\ 435$$

9a

A resolution element is the reimaged slit width. We have 8 pixels per reimaged slit width w_θ'.

$$d\lambda = \frac{w_{\theta}'}{dl/d\lambda} = \frac{8\text{ pix}}{\left(0.057\ \frac{\text{Å}}{\text{mm}}\right)^{-1}} = 0.0456\text{ nm}$$

$$R = \frac{\lambda}{d\lambda} = \frac{653\text{ nm}}{0.0456\text{ nm}} = 14,\ 320$$

9b

The resolution element determines the maximum spectral resolution an instrument can have. With more pixels across the resolution element we could have a higher spectral resolution.

9c

To correctly Nyquist sample the slit, we need two pixels. We can bin by four pixels since our reimaged slit width is 8 pixels.