Homework 1

Geophysical Fluids

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1ai)

For most cases I saw some turbulent flow. The turbulence was especially prominent when the smoke bomb was near an obstruction.

1aii)

The Reynold's number is the speed u, times the scale length, L, divided by the viscosity *v*:

Re = $\frac{u\,L}{v}$

For smoke near a building's corner, we estimate the scale length L = 1 m. The speed of the smoke is about u = 0.5 m/s. The viscosity of air is $1.4 \times 10^{-5}\ \frac{m^{2}}{s}$. Thus

Re = 35,000

This is consistent with an extremely turbulent medium, which we see.

1aiii)

We see evidence of blocked flow. Attached eddies are present in the flow when the smoke source is at the bottom of a building corner.

1b)

The diffusion of turbulence can be described by

$$\frac{\partial n}{\partial t} = K \frac{\partial^2 n}{\partial x^2}$$

To estimate the turblent diffusion coefficient K, we simplify the above equation to

$$\frac{1}{t} \sim \frac{K}{x^2}$$
$$K \sim \frac{x^2}{t}$$

We estimate $\frac{x^2}{t} = \frac{(1/2\,m\ \ )^2}{s}$. So $K = 1/4\ \frac{m\ \ ^2}{s}$.

2a)

Given a speed of u = $\frac{u_0\,y}{b}$ for a Couette flow, we can calculate the following quantities:

Shear strain: $\frac{\partial u}{\partial y} = \frac{u_0}{b}$

Linear strain: $\frac{\partial u}{\partial x} = 0$

Vorticity: $\boldsymbol{w} = \nabla \times \boldsymbol{u} = \left(0,\ 0,\ 0 - \frac{\partial u_x}{\partial y}\right) = -\frac{u_0}{b}\,\boldsymbol{z}$

2b)

Given the streamfunction definition of

$u = -\frac{\partial \psi}{\partial y},\ \ v = \frac{\partial \psi}{\partial x}$

we can express the streamfunction in terms of u_0 and b by integrating

$\partial\,\psi = -u\,\partial\,y$

$\partial\,\psi = -\frac{u_0\,y}{b}\,\partial\,y$

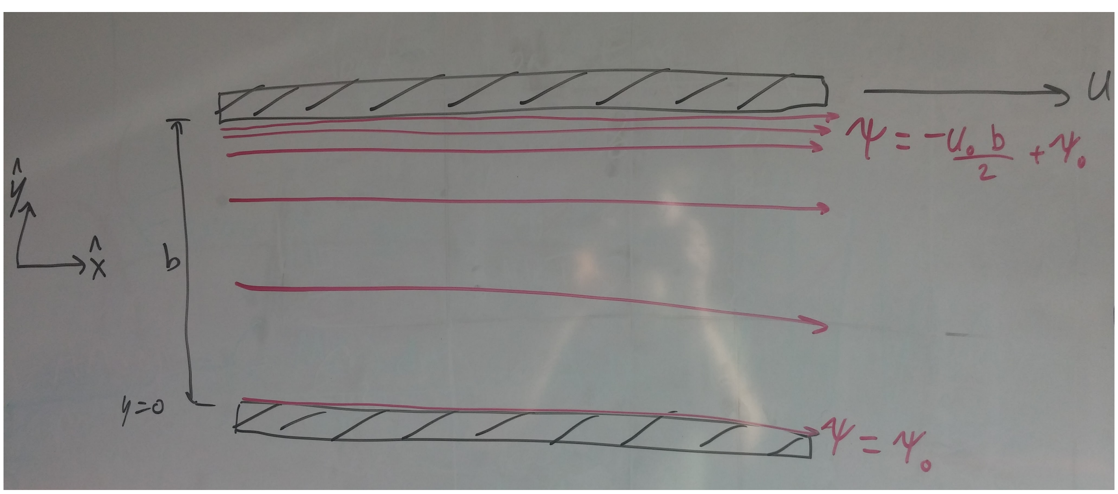
$\psi(y) = -\frac{u_0\,y^2}{2\,b} + \psi_0$

and

$\partial\,\psi = v\,\partial\,x$

$\psi(x) = \psi_0$

2c)



3)

Since Karman Vortex streets are present, we can be sure that the flow is in the high Reynold's number regime. This means that the force on the string is proportional to the velocity of the flow squared:

$F = c_D u^2$

where c_D is the drag coefficient. Fitting for the relationship between F and u^2 we find $c_D = 3\, m^2$. If we assume that the drag coefficient is inversely proportional to the Reynold's number, then an increase in viscosity will lead to a lower Re and thus a lower c_D because

$c_D \propto \mathrm{Re}^{-1} \propto \nu$

thus the drag coefficient with the new fluid will be

$c_{D,\mathrm{new}} = 5\, c_D = 15\, m^2$

So we predict the following forces for given velocities with the new fluid:
U = 0.4 m/s, F = 2.15 N
U = 1.0 m/s, F =15.0 N
U = 5.5 m/s, F = 450 N

4a)

We assume the following about the properties of the cells in the Earth's mantle:
speed: u = 10^{-9} m/s
viscosity: ν = 10^17 m^2/s
coefficient of expansion $\alpha = 10^{-5}\, K^{-1}$
size scale: d = 10^6m
acceleration due to gravity: g = 9.8 m/s^2

We would like to know what the temperature difference between the bottom and top of the mantle will lead to convection. We first assume the Rayleigh number must be greater than 1,700 for convection. However we need to determine the thermal diffusivity, κ. If we use thte temperature diffusion equation:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial z} \sim \kappa \frac{\partial^2 T}{\partial z^2}$$

Then over long periods of time:

$$\frac{uT'}{z} \sim \frac{\kappa T'}{z^2}$$
$$u \sim \frac{\kappa}{d}, \ \kappa \sim u\, d$$

We can now use the Rayleigh number to determine if convection will occur:

$\text{Ra} = \frac{g\alpha T' d^3}{\nu\kappa} > 1,700$
$T' > \frac{\text{Ra}\,\nu\,\kappa}{g\alpha\,d^3} = \frac{\text{Ra}\,\nu\,u\,d}{g\alpha\,d^3} = \frac{\text{Ra}\,\nu\,u}{g\alpha\,d^2}$
thus

$T' > 1,700\,K$

for convection.

4b)

With the change in thermal diffusivity, the thickness of the boundary layer will be about:

$\text{d} = \frac{\kappa}{u} = 10^2\,m$

leading to a temperature difference needed for convection of

$T' = 10^{11}\text{K}$

which means that convection in Earth’s mantle would never happen.