Homework 6

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1a

The distribution of stars in our model cluster can be determined by integrating the

$$N = A \int_{0.5}^{40} \phi(m)\, \mathrm{dm}$$

where ϕ(m) is the IMF, i.e., the number of stars born with mass m. We assume a salpeter IMF

$\phi(\mathrm{m}) = \mathrm{A} \;\; m^{-(\Gamma - 1)}\, \mathrm{dm} \qquad \Gamma = 1.3$

And we solve for A

$$A = N \Big/ \int_{0.5}^{40} \phi(m)\, \mathrm{dm}$$
$$A = N \Big/ \int_{0.5}^{40} m^{-(1.3-1)}\, \mathrm{dm}$$
$$A = 5.3 \times 10^3$$

Mass $[M_\odot]$	N_m
0.5	26085
0.6	15347
0.8	9029
1.0	5312
1.3	3125
1.6	1839
2.0	1082
2.5	636
3.2	374
4.0	220
5.0	130
6.3	76
8.0	45
10.0	26
12.6	16
15.9	9
20.0	5
25.2	3
31.8	2
40.0	1

calculations for A below:

```
Nstars = 10^4;
τMS[m_] := m^-2.5 * 10^10;
ϕ[m_] := m^-(1.3+1);
A = Nstars / Integrate[ϕ[m], {m, 0.5, 40}];
StringForm["A = ``", A]
```

A = 5297.425330249122`

1b

The half-mass luminosity is 24.80 $M_\odot$ given $L_{bol} = 7 \times 10^6\, L_\odot$.

1c

To determine the color of cluster, we need to weight each color from a given mass bin by the number of stars and the luminosity of the stars in the bin.

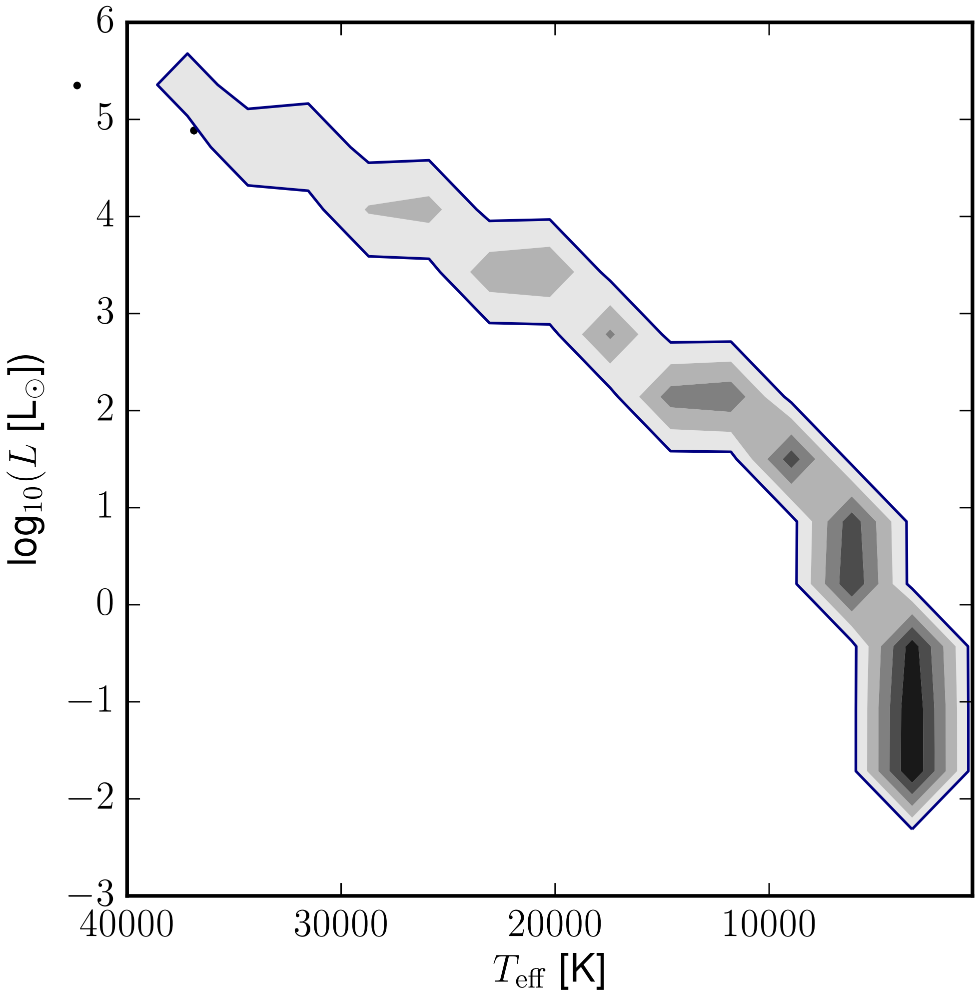
$$\frac{F_2}{F_1} = 2.512^{\Delta m}$$

where Δm is the difference in magnitudes, $B - V$. Given the number of stars per mass, $N(M)$ and the luminosity as a function of mass $L(M)$, and the color as a function of mass $[B - V](M)$ we can calculate the color of the cluster by

$$[B - V](M) = \frac{F_2}{F_1} \frac{L(M)}{L_{TOT}} \frac{N(M)}{N_{TOT}}$$
$$[B - V](M) = 2.512^{[B-V](M)} \frac{L(M)}{L_{TOT}} \frac{N(M)}{N_{TOT}}$$
$$[B - V]_{cluster} = \int_{0.5}^{40} 2.512^{[B-V](M)} \frac{L(M)}{L_{TOT}} \frac{N(M)}{N_{TOT}} dM$$

$$[B - V]_{cluster} = 6 \times 10^{-4} \text{ mag}$$

1d



Above shows the log of the luminosity vs. the effective temperature for EZweb models. The contours represent the number of stars in the bin, spaced logarithmically from 0.