Astro 735: Cosmology
Lecture 1: Fundamental Observations and Overview of the Universe

1 Basics: units and scales

How do we measure things that are very large?

Astronomical Distances: A common distance is the AU (astronomical unit), the distance from the Earth to the Sun, 1.5×10^{11} m. This is not so useful beyond the solar system, so we use the parsec (pc), the distance at which 1 AU subtends an angle of 1 arcsec: 1 pc = 3.1×10^{16} m = 3.26 light years.

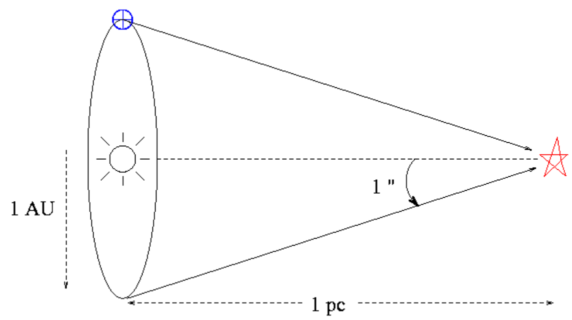

Figure 1: A parsec is the distance at which 1 AU subtends an angle of 1 arcsec. 1 arcsec = 1/3600 deg = 4.8×10^{-6} radians, so $d/1$ AU $\simeq 4.8 \times 10^{-6}$ and $d =$ 206,265 AU = 1 pc.

We are 1.3 pc from Proxima Centauri (the nearest star) and 8000 pc = 8 kpc from the center of our Galaxy. For intergalactic distances, we use the megaparsec (Mpc): 1 Mpc = 3.1×10^{22} m. We are 0.7 Mpc from M31 (Andromeda galaxy) and 15 Mpc from the Virgo Cluster (the nearest big cluster of galaxies) or gigaparsec (Gpc): the most distant galaxies we can see are $\sim$ 10 Gpc away. It is important to have a rough idea of these numbers, to have a sense of the scale of the universe and to know whether or not your calculations are reasonable.

Mass: The standard unit of mass is the solar mass, $M_\odot$: 1 $M_\odot$= 2.0×10^{30} kg. Mass of Milky Way $\approx 10^{12}$ $M_\odot$. The Sun also provides the standard unit of power or luminosity: 1 $L_\odot$= 3.8×10^{26} W. Total luminosity of Milky Way, $L_{\rm gal} = 3.6 \times 10^{10}$ $L_\odot$. (Approximately how many stars are in the galaxy?)

Time: 1 year = 3.2×10^7 s. In a cosmological context we use Gyr, 1 Gyr = 10^9 yr = 3.2×10^{16} s. The Universe is 13.7 Gyr old.

Natural units: Many cosmologists and cosmology texts use natural or geometric units, in which $c = \hbar = k = 1$ (and sometimes $G = 1$ as well). Observational astronomers generally don't use this system, but it will come up occasionally.

2 Fundamental Observations

2.1 Olbers' Paradox: Why is the sky dark at night?

Perhaps the simplest astronomical observation we can make has surprisingly important consequences. This is the question "Why is the sky dark at night?" This is known as Olbers' Paradox, named after Heinrich Olbers, who wrote a paper on the subject in 1826, but it was first proposed by Thomas Digges in 1576.

Suppose the universe is infinite and static, as Isaac Newton believed; a universe that isn't infinite will collapse inward due to its own self-gravity. Now let's compute how bright we expect the night sky to

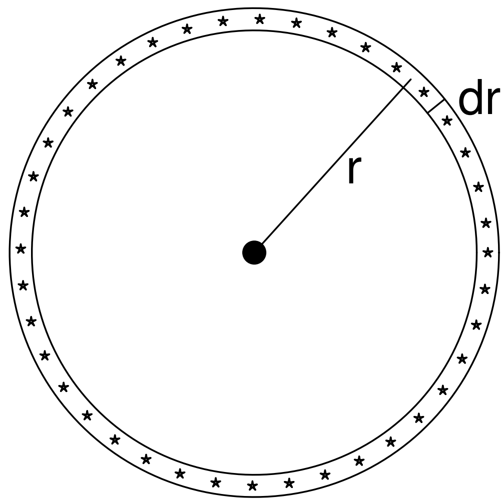

Figure 2: A thin spherical shell of stars, centered on the Earth.

be in this infinite universe, where every line of sight ends at a star. Let n be the average number density of stars in the universe, and let L be the average stellar luminosity. The flux received here at Earth from a star of luminosity L at a distance r is given by an inverse square law:

$$f(r) = \frac{L}{4\pi r^2} \tag{1}$$

Now consider a thin spherical shell of stars, with radius r and thickness dr, centered on the Earth. The intensity of radiation from the shell of stars (that is, the power per unit area per steradian of the sky) will be

$$dJ(r) = \frac{L}{4\pi r^2} \cdot n \cdot r^2 dr = \frac{nL}{4\pi} dr. \tag{2}$$

The total intensity of starlight from a shell thus depends only on its thickness, not on its distance from us. We can compute the total intensity of starlight from all the stars in the universe by integrating over shells of all radii:

$$J = \int_{r=0}^{\infty} dJ(r) = \frac{nL}{4\pi} \int_{r=0}^{\infty} dr = \infty. \tag{3}$$

We have shown that the night sky is infinitely bright. Where did we go wrong? Possible issues include:

- Stars have finite size, so we don't actually have an unobstructed line of sight to all stars. But still, each line of sight ends at a star, so the sky should have the surface brightness of a typical star.
- Interstellar matter that absorbs starlight? No, because the matter would be heated by starlight until it has the same temperature as the surface of a star, and then it would emit as much light as it absorbs and glow as brightly as the stars.
- We assumed that number density and mean luminosity of stars are constant throughout the universe; distant stars might be less numerous or less luminous than nearby stars.

- We assumed that universe is infinitely large. If the universe has size $r_{\max}$, then the total intensity of starlight we see in the night sky will be $J \sim nLr_{\max}/(4\pi)$. Note that this result will also be found if the universe is infinite in space, but has no stars beyond a distance $r_{\max}$.
- We assumed that the universe is infinitely old. When we see stars farther away, we're also seeing stars farther back in time. If universe has finite age t_0, intensity of starlight will be at most $J \sim nLct_0/(4\pi)$. This also applies if stars have only existed for time t_0.
- We assumed that flux of light from a distant source is given by the inverse square law; i.e. we have assumed that the universe obeys laws of Euclidean geometry and that the source of light is stationary with respect to the observer. However, the universe may not be Euclidean, and if the universe is expanding or contracting then the light will be red or blueshifted to lower or higher energies.
- The primary resolution to the paradox is that the universe has a finite age, and the light from stars beyond some distance—called the horizon distance—hasn't had time to reach us yet. Surprisingly, the first person to suggest this was Edgar Allen Poe in 1848:

  "Were the succession of stars endless, then the background of the sky would present us an uniform density ...since there could be absolutely no point, in all that background, at which would not exist a star. The only mode, therefore, in which, under such a state of affairs, we could comprehend the voids which our telescopes find in innumerable directions, would be by supposing the distance of the invisible background so immense that no ray from it has yet been able to reach us at all."

3 The cosmological principle

Another basic observation, which is true to our ability to test it, is known as the cosmological principle:

- The universe is homogenous. There are no preferred locations: the universe looks the same anywhere.
- The universe is isotropic. There is no preferred direction: the universe looks the same in all directions.

Note that homogeneity doesn't imply isotropy, and vice versa. This is only true on large scales; obviously it isn't true on the scale of a person or a planet or even a galaxy or cluster of galaxies. Galaxies are grouped into clusters and voids, a fact that is cosmologically important, but on scales of $\gtrsim 100$ Mpc, the universe is homogenous and isotropic; this is roughly the scale of superclusters of galaxies and the voids between them. Of course we have no way of knowing whether or not this is true on scales larger than our horizon, and some cosmological theories propose that the universe is very inhomogeneous on super-horizon scales.

This is also called the Copernican principle, after Copernicus, who determined that the Earth is not the center of the universe. There is no center.

4 Galaxies are redshifted

Consider light at a particular wavelength observed from a distant galaxy: $\lambda_{\rm obs}$ is the observed wavelength of some absorption or emission feature in the galaxy's spectrum. $\lambda_{\rm em}$ is the wavelength at which that feature is emitted in the galaxy. In general, $\lambda_{\rm obs} \neq \lambda_{\rm em}$; the galaxy has a **redshift** z given by

$$z \equiv \frac{\lambda_{\rm obs} - \lambda_{\rm em}}{\lambda_{\rm em}}. \tag{4}$$

Most galaxies have redshifts.

Modern cosmology started in 1929, with Edwin Hubble's discovery that galaxy redshifts are directly proportional to their distance R (redshifts are easy to measure, but distances are hard). This is now known as Hubble's law:

$$z = \frac{H_0}{c} R, \tag{5}$$

where H_0 is a constant now called the **Hubble constant**.

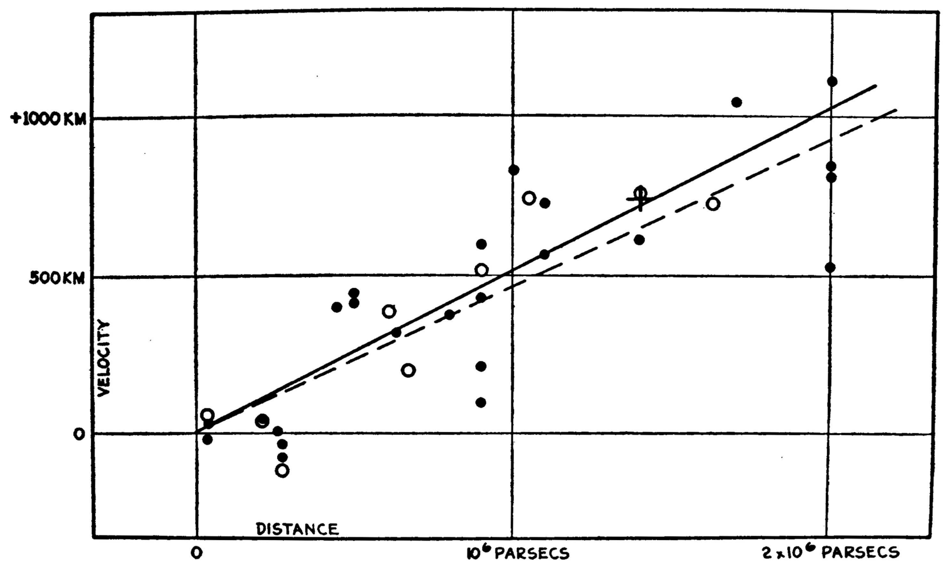

Figure 3: Edwin Hubble's original plot of the relation between redshift (vertical axis) and distance (horizontal axis). Note that the vertical axis actually plots cz rather than z, and that the units are mistakenly written as km rather than km s^{-1} (Hubble 1929, Proc. Nat. Acad. Sci., 15, 168).

The explanation for the redshifts of galaxies is of course that the universe is expanding. If we interpret redshifts as Doppler shifts (not strictly true, but we'll talk about that later), $z = v/c$ and Hubble's law takes the form

$$v = H_0 R, \tag{6}$$

where v is the radial velocity of the galaxy. Therefore we can get the Hubble constant from dividing velocity by distance, and it has the units km s^{-1} Mpc^{-1} (note that the actual units are inverse time). Hubble's original estimate was $H_0 = 500$ km s^{-1} Mpc^{-1}, but he was severely underestimating the distances to galaxies. The Hubble constant has been measured with precision only in the last $\sim$ 10 years; the best current value is $H_0 = 70.4^{+1.3}_{-1.4}$ km s^{-1} Mpc^{-1} (2010, from WMAP seven-year results, with priors from other estimates). We will discuss the methods by which this is measured later.

Hubble's law in this form only applies to galaxies near enough that $v \ll c$. For galaxies that are far enough away, the distance R may change significantly during the time it takes for photons from the galaxy to reach us, and we need to be more careful about what we mean by "distance." We will return to this later.

The Hubble constant is a measurement of the current expansion rate of the universe, and should be distinguished from the **Hubble parameter** $H(t)$, which gives the expansion rate as a function of time. In other words, the Hubble "constant" is not (necessarily) constant with time.

The inverse of the Hubble constant, H_0^{-1}, defines the timescale for significant changes in the scale of the universe. If galaxies are moving apart from each other, they must have been together at some point in the past. Consider two galaxies separated by a distance R and moving at a constant velocity v with respect to each other. The time elapsed since the galaxies were in contact is

$$t_0 = \frac{R}{v} = \frac{R}{H_0 R} = H_0^{-1}, \tag{7}$$

independent of R. The time $t_H = H_0^{-1}$ is called the **Hubble time**, and is an approximate timescale for the age of the universe (it is only equal to the age of the universe if galaxy velocities were the same at all times in the past). $H_0^{-1} = 13.8$ Gyr.

Since H_0 appears in nearly every cosmological formula, it is useful to define

$$h = \frac{H_0}{100 \text{ km s}^{-1} \text{ Mpc}^{-1}} \tag{8}$$

so that $H_0 = 100\ h$ km s^{-1} Mpc^{-1} and $h = 0.70$. More recently, since the value of H_0 has become better known, it's popular to define

$$h_{70} = \frac{H_0}{70 \text{ km s}^{-1} \text{ Mpc}^{-1}} \tag{9}$$

so that $H_0 = 70\ h_{70}$ km s^{-1} Mpc^{-1} and $h_{70} = 1$; then the numerical factors in the formulas reflect our current best measurements of H_0.

What are the implications of Hubble's law?

1. The universe is dynamic. The universe was denser (and hotter) in the past. We extrapolate backwards to a point of infinite density called the "Big Bang," but because we don't understand the laws of physics under such conditions this extrapolation is suspect. The Hubble time gives the order of magnitude since the time when the known laws of physics began to apply. Also note that in one Hubble time light can travel a distance $d_H = ct_H$, which implies that there is a horizon distance d_H from beyond which light hasn't reached us. This is the resolution to Olbers' paradox; more on this later.
2. Universal expansion. The linearity of Hubble's law means that every point is receding from every other point with the same velocity; an observer in another galaxy will also see universal expansion with the same H_0. This is another statement of the cosmological principle: there is no privileged position at the center of the universe from which the expansion radiates. We can see how this works by considering a triangle defined by the positions of three galaxies (Figure 4). Uniform expansion means that the triangle must maintain its shape as the galaxies move away from each other. This requires an expansion law of the form

$$\begin{aligned} R_{12}(t) &= a(t)\ R_{12}(t_0) & (10)\\ R_{23}(t) &= a(t)\ R_{23}(t_0) & (11)\\ R_{13}(t) &= a(t)\ R_{13}(t_0). & (12) \end{aligned}$$

In these equations $a(t)$ is the *scale factor*, equal to one at the present time $t = t_0$ and independent of location or direction. At any time t, an observer in galaxy 1 will see the other two galaxies receding with a speed

$$\begin{aligned} v_{12}(t) &= \frac{dR_{12}}{dt} = \dot{a}\, R_{12}\,(t_0) = \frac{\dot{a}}{a} R_{12}(t) & (13)\\ v_{13}(t) &= \frac{dR_{13}}{dt} = \dot{a}\, R_{13}\,(t_0) = \frac{\dot{a}}{a} R_{13}(t). & (14) \end{aligned}$$

The same is true for an observer in the other two galaxies. We have recovered the Hubble law $v = HR$, where $H(t) = \dot{a}/a$. This is an important definition of the Hubble parameter, to which we will return.

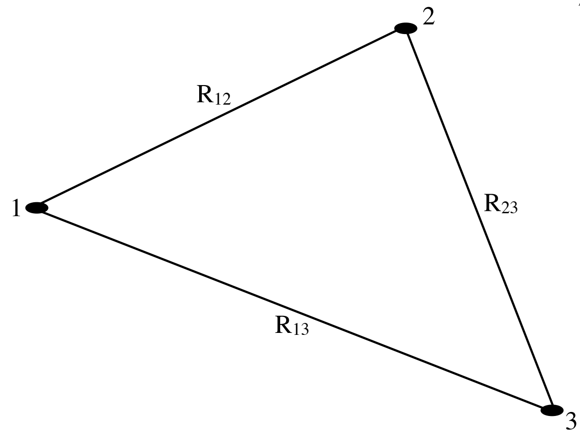

Figure 4: The triangle defined by three galaxies in a uniformly expanding universe.

3. Critical density. With the Hubble constant H_0 and the gravitational constant G, we can form a quantity with the units of density,

$$\rho_c = \frac{3H_0^2}{8\pi G} = 0.91\ h_{70}^2 \times 10^{-26}\ \mathrm{kg\ m^{-3}} \tag{15}$$
$$= 1.34\ h_{70}^2 \times 10^{11}\ \mathrm{M_\odot\ Mpc^{-3}} \tag{16}$$
$$= 0.51\ h_{70}^2 \times 10^{10}\ \mathrm{eV\ m^{-3}} \tag{17}$$

(we'll see later where the numerical factors come from). This is called the "critical density," and the second and third lines show that it corresponds to about one galaxy per Mpc^3 or about 5 protons per m^3, which is much greater than the observed densities of galaxies or protons. The critical density is the density which, in the absence of a cosmological constant, will be sufficient to cause the universe to collapse rather than expand forever. We generally normalize cosmological densities to the critical density: $\Omega = \rho/\rho_c$. A universe with $\Omega = 1$ is known as "flat." The presence of a cosmological constant complicates the relationship between density, geometry and the fate of the universe, but ρ_c is the natural unit of cosmological density.

Let's now return to Olbers' paradox with the implications of Hubble's law in mind. If the universe is of finite age, $t_0 \sim H_0^{-1}$, then the night sky can be dark, even if the universe is infinitely large, because light from distant galaxies has not yet had time to reach us. Galaxy surveys tell us that the luminosity density of galaxies in the local universe is

$$nL \approx 2 \times 10^8\ \mathrm{L_\odot\ Mpc^{-3}} \tag{18}$$

This luminosity density is equivalent to a single 40 watt light bulb within a sphere 1 AU in radius. If the horizon distance is $d_H \sim c/H_0$, then the total flux of light we receive from all the stars from all the galaxies within the horizon will be

$$F_{\rm gal} = 4\pi J_{\rm gal} \approx nL \int_0^{r_H} dr \sim nL\frac{c}{H_0} \sim 9 \times 10^{11}\ \mathrm{L_\odot\ Mpc^{-2}} \sim 2 \times 10^{-11}\ \mathrm{L_\odot\ AU^{-2}}. \tag{19}$$

By the cosmological principle, this is the total flux of starlight you would expect at any randomly located spot in the universe. Comparing this to the flux we receive from the Sun,

$$F_{\rm sun} = \frac{1\ \mathrm{L_\odot}}{4\pi \mathrm{AU}^2} \approx 0.08\ \mathrm{L_\odot\ AU^{-2}}, \tag{20}$$

we find that $F_{\rm gal}/F_{\rm sun} \sim 3 \times 10^{-10}$. Thus, the total flux of starlight at a randomly selected location in the universe is less than a billionth the flux of light we receive from the Sun here on Earth. For the entire universe to be as well-lit as the Earth, it would have to be over a billion times older than it is, and you'd have to keep the stars shining during all that time.