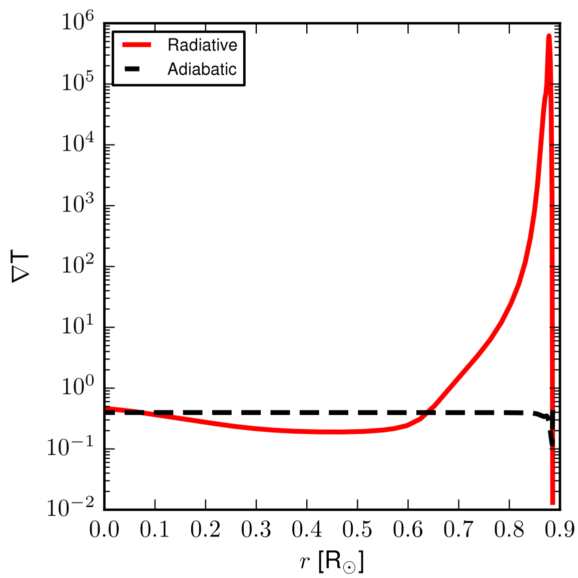
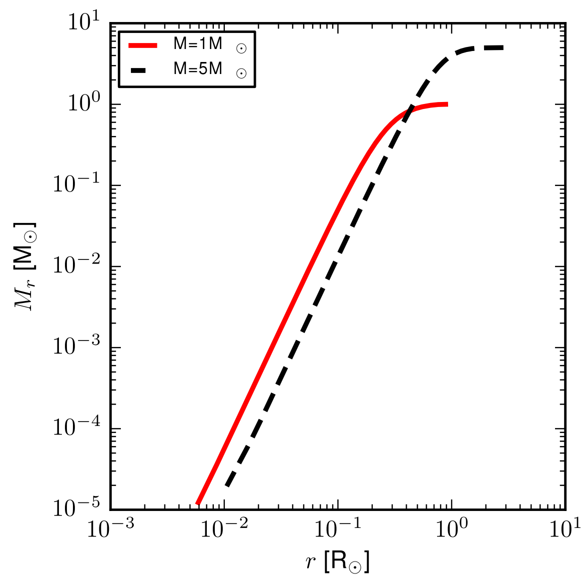
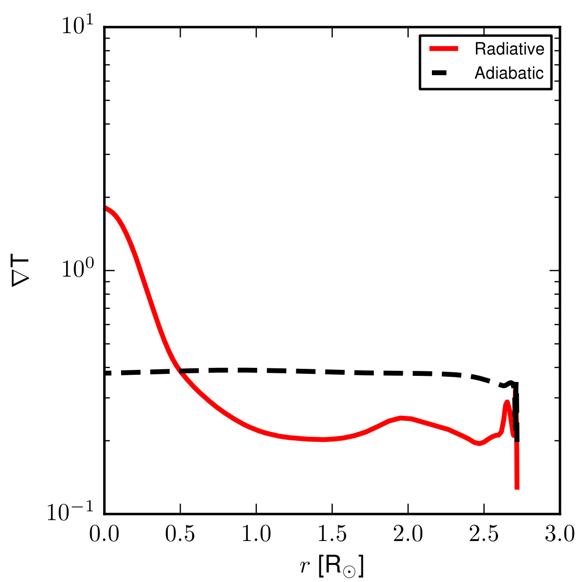
Homework3

1a

A parcel of gas will be stable against convection if the absolute value of the radiative temperature gradient is less than the adiabatic temperature gradient.







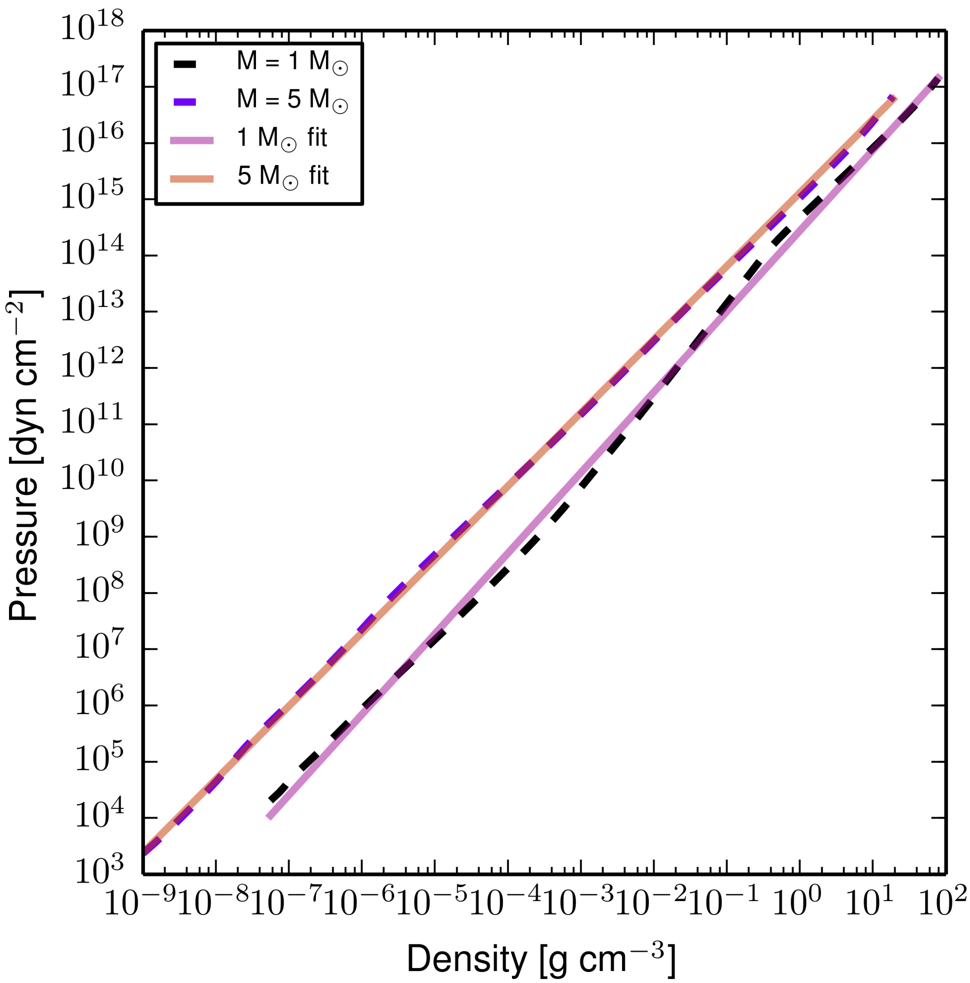
Radiative and adiabatic temperature gradients for a one solar mass star, top left, and a five solar mass star, bottom left. The solar mass star will be stable against convection up to about 0.6 $R_{\odot}$, and will have convection at larger radii to the surface of the star. The 5 solar mass star will have convection in its core, out to about 0.5 $R_{\odot}$. Another way to state the criterion for stability against convection is that the density gradient must be sufficiently large. In the top right we plot mass interior to a radius r. We see that the convection zone found using temperature gradients is consistent with the low density gradient shown for the one solar mass star above 0.6 $R_{\odot}$. However this is not consistent with the 5 solar mass star.

1b

We test the hypothesis that the stars in our sample are polytropic determining the relationship between pressure as a function of density given by

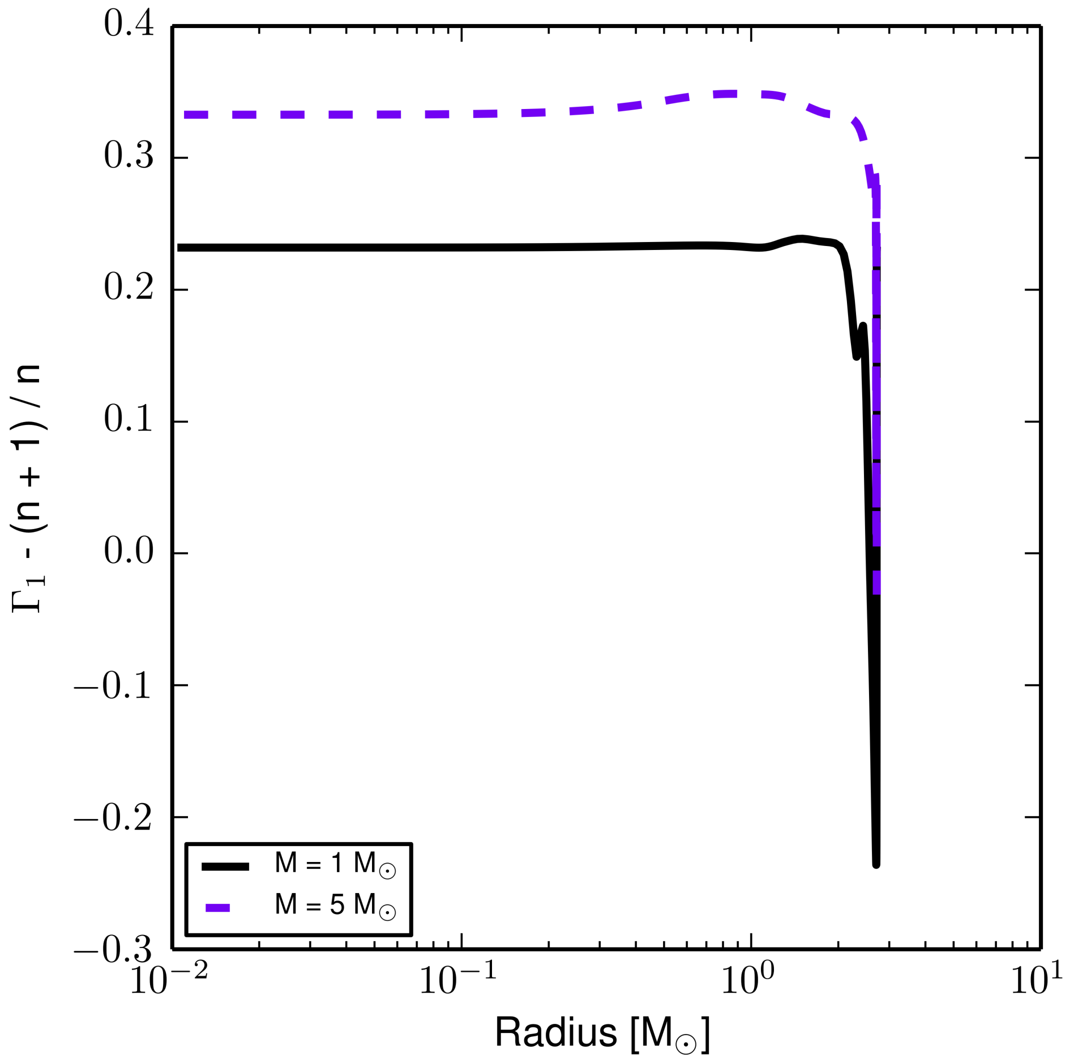
$P = K\rho^{(n+1)/2}$

we find that for the solar mass star n = 2.3, K = 2.7 x 10^{14} cm dyn / g, and for the five the solar mass star n = 1.3, K = 1.3 x 10^{15} cm dyn / g. These stars are likely polytropic, given the vary tight correlation between pressure and density and the model.



1c

Adiabatic behavior will show that the pressure is proportional to the density to the power of the adiabatic index γ. We calculate the difference of the adiabatic index from the model and the power of the density calculated in the previous problem, (n+1)/n. We plot this difference as a function of radius. We find that both stars are better described as polytropic than adiabatic.



2

We begin by writing the equation of motion for a parcel of gas in a plane parallel atmosphere

$$\rho_b \frac{d^2 \Delta r}{dt^2} = -g(\rho_b - \rho_e) \qquad (1)$$

where ρ_b is the blob density, Δr is the distance the blob travels in the z direction, and ρ_e is the environment density. We are assuming pressure balance between the blob and the surroundings, and that the blob undergoes adiabatic motion.

We can also relate

$$\rho_b - \rho_e = \Delta r\left(\frac{d\rho}{dr} - \frac{\rho}{\Gamma_1 P}\frac{dP}{dr}\right) \qquad (2)$$

where P is the pressure, Γ_1 is the first adiabatic constant, and ρ is the overall density of the region. Substituting (2) in to (1) we find

$$\rho_b \frac{d^2 \Delta r}{dt^2} = -\Delta r\, g\left(\frac{d\rho}{dr} - \frac{\rho}{\Gamma_1 P}\frac{dP}{dr}\right)$$

we set

$$N^2 = \frac{g}{\rho_b}\left(\frac{\rho}{\Gamma_1 P}\frac{dP}{dr} - \frac{d\rho}{dr}\right)$$

giving

$$\frac{d^2 \Delta r}{dt^2} = -\Delta r\, N^2$$

This is a differential EQ with a well known solution of

$$\Delta r = C\, e^{N^2 t}$$

where C is a constant and N^2 is the Bruint – Vaisala frequency.