Astro 735: Cosmology
Lecture 2: The Composition of the Universe; Stars

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1 Composition of the universe

A brief census of the universe. We will discuss these components in more detail later.

1.1 Baryonic matter

This refers to protons, neutrons and electrons (even though electrons aren't baryons). There isn't very much of this. The density of visible matter (i.e. stars) in units of the critical density is

$$\Omega_{\rm vis} = \frac{\rho_{\rm vis}}{\rho_c} \sim 0.002. \tag{1}$$

From analysis of the cosmic microwave background radiation and the theory of nucleosynthesis, we can estimate the *total* density of baryonic matter, including stuff we can't see:

$$\Omega_b = 0.0456 \pm 0.015 \tag{2}$$

(this result is from the WMAP analysis of the CMB, which we'll hear more about later). This is still very small; as we'll see later, there is good evidence that the universe is flat, i.e. $\Omega_{\rm tot} = 1$, which means that less than 5% of the matter and energy in the universe is the stuff we are familiar with.

Note that $\Omega_b \sim 10 \times \Omega_{\rm vis}$, which means that even most of the baryonic matter in the universe is stuff we can't see. Where is it? Most of it is probably in the form of ionized gas in the intergalactic medium, which is difficult to detect, with some additional fraction in the form of dark, compact objects such as stellar remnants (white dwarfs and neutron stars) and brown dwarfs (stars not massive enough to burn hydrogen). There may also be significant baryonic matter in cold molecular clouds.

1.2 Dark matter

The formation of structure (galaxies and clusters of galaxies) requires gravitational collapse of matter that is not coupled to photons, since we know that the distribution of photons and baryons must have been extremely smooth at early times. Analysis of the temperature anisotropies of the CMB also shows that most of the matter in the universe must be in non-baryonic form. We call this dark matter.

Dark matter comes in a variety of theoretical flavors, principally divided between "hot dark matter" and "cold dark matter." Hot and cold refer to speed; hot dark matter was relativistic in the early universe, and cold dark matter non-relativistic. Comparisons of simulations of the gravitational collapse of structure with observations show that the universe has **cold dark matter** (CDM). This is generally believed to be in the form of WIMPs: weakly interacting massive particles. This means that they are affected by gravity but interact very weakly with photons and baryons. We don't yet know what the WIMPs actually are, however; a variety of particle detectors are working on this.

The density of cold dark matter today is

$$\Omega_{\rm CDM} = 0.228 \pm 0.013. \tag{3}$$

This is ~ 5 times larger than the density of baryonic matter, but still much less than the critical density.

For many cosmological calculations, the total non-relativistic matter density is important. This is the sum of baryonic matter, dark matter, and non-relativistic neutrinos ($\Omega_\nu \ll \Omega_{\rm CDM}$):

$$\Omega_{\rm M} = \Omega_{\rm CDM} + \Omega_b + \Omega_\nu = 0.27 \pm 0.01. \tag{4}$$

1.3 Photons

The most abundant particles in the universe, and possibly the most useful for the study of cosmology, are the photons of the cosmic microwave background radiation (CMB). The spectrum of these photons is a nearly perfect blackbody with $T = 2.725$ K. The CMB photon number density is $n_\gamma = 411$ cm^{-3} (recall that the critical matter density corresponds to about 5 protons per m^3, so the density of photons is much higher than that of baryons). The energy density of the CMB is small, however, because of its low temperature.

The thermal spectrum of the CMB photons arises from the conditions in the early universe, when the temperature was too high for protons and electrons to combine into atoms. The photons scattered off the electrons, and the universe was not transparent. The protons and electrons "recombined" when the universe cooled to $T \sim 0.26$ eV; at this point the universe became transparent to photons because of the decrease in the photon-matter cross-section, and the CMB photons we see today have been freely streaming ever since.

The temperature of the CMB varies very slightly with position:

$$\frac{\delta T}{T} \sim 10^{-5}. \tag{5}$$

These temperature variations are due to inhomogeneities in the matter distribution at the time of recombination, and so they contain important information about the initial conditions of structure formation.

1.4 Neutrinos

The universe is believed to be filled with neutrinos in addition to photons. Neutrinos have been suggested as a non-baryonic dark matter candidate, but because they are hot dark matter, i.e. relativistic at early times, they do not match observations. The neutrino temperature is slightly less than the photon temperature, and the expected neutrino density today is $n_\nu = 112$ cm^{-3} per species (recall that neutrinos come in three "flavors").

The strictest limit on the contribution of neutrinos to the mass density of the universe comes from the lack of any observed effect on the spectrum of fluctuations of the CMB. This gives

$$\Omega_\nu < 0.015. \tag{6}$$

We do believe that neutrinos have mass, however, from searches for neutrino oscillations (the transformation of neutrinos from one flavor to another). Limits from these studies combine with limits from the density inhomogeneities to give

$$0.0009\, h_{70}^{-2} < \Omega_\nu < 0.015. \tag{7}$$

1.5 Dark energy

Apparently the most significant component of the universe, and also the most surprising, is dark energy. Evidence for dark energy comes from studies of supernovae which indicate that the expansion of the universe is accelerating. Dark energy is also known as vacuum energy or the cosmological constant Λ, and its current contribution to the universe is

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} = \frac{\Lambda}{3H_0^2} = 0.726 \pm 0.015 \tag{8}$$

(we are using natural units here, with a factor of $c^2 = 1$ in the numerator). The corresponding density is

$$\rho_\Lambda = 3.75\ h_{70}^2 \times 10^9 \text{ eV m}^{-3} \tag{9}$$

Vacuum energy isn't associated with particles, so it isn't diluted as the universe expands; the energy density is constant, meaning that the contribution of vacuum energy to the total energy of the universe increases as the universe expands and the matter density decreases.

We do not currently have a theory which successfully explains the value of the cosmological constant. We can associate the energy density with a mass scale M:

$$\rho_\Lambda \sim \frac{M^4}{(\hbar c)^3}. \tag{10}$$

A natural suggestion is that the fundamental mass scale is the Planck mass, $m_{\rm pl} = (\hbar c^5/G)^{1/2} \sim 10^{19}$ GeV. This is very large, however, and gives $\rho_\Lambda \sim 3 \times 10^{132}$ eV m^{-3}, which differs from the observed value by 123 orders of magnitude (!). The actual mass scale implied by observations is $M \sim 10^{-3}$ eV, which doesn't correspond to any fundamental scale we know about.

2 Astrophysical background: Stars

An understanding of observational cosmology requires some astrophysical background about the visible objects in the universe and how they work. This could be (and often is) several courses on its own, but we will cover it very briefly.

Nearly all of the astronomical objects we can see with the naked eye are nearby stars. Cosmology is generally concerned with galactic and (especially) super-galactic scales, but an understanding of observational cosmology requires a basic understanding of stars.

A star begins life as a cloud of gas with approximately the primordial composition of the universe: 75% hydrogen and 25% helium. Later generations of stars contain $\sim$ 1–2% heavier elements (called "metals" by astronomers). The cloud gravitationally contracts, compensating for the negative gravitational binding energy by increasing the temperature. The gravitational contraction is halted when the core of the star becomes hot enough to fuse hydrogen into helium, and the star can exist in a steady state for as long as its nuclear fuel lasts. The internal structure of the star can then be described by four differential equations known as the **equations of stellar structure**.

2.1 The equations of stellar structure

A star in equilibrium balances the force of gravity with a pressure gradient, so the first equation of stellar structure is the **equation of hydrostatic equilibrium**:

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}. \tag{11}$$

This assumes that the star is spherical and non-rotating, neither expanding nor contracting, and that gravity and pressure gradients provide the only forces. Even with known boundary conditions, this by itself isn't enough to determine the structure of the star (the pressure and density as a function of radius), but it's easy to use it for a quick estimate of the pressure at the core of the Sun. A rough approximation to the equation of hydrostatic equilibrium is

$$\frac{\Delta P}{\Delta r} \approx -\frac{G\langle M\rangle\langle\rho\rangle}{\langle r^2\rangle}, \tag{12}$$

where ΔP and Δr are the differences in pressure and radius between the Sun's photosphere (the radius at which the density is low enough for photons to escape, after a random walk from the center) and center, and $\langle M\rangle$, $\langle\rho\rangle$ and $\langle r\rangle$ are typical values of mass, density and radius in the Sun's interior. We'll use the average density of the Sun, $\langle\rho\rangle \approx \rho_\odot \approx 1400$ kg m^{-3} and assume $\langle M\rangle \approx M_\odot/2 \approx 1.0\times10^{30}$ kg and $\langle r\rangle \approx R_\odot/2 \approx 3.5\times10^8$ m. Also assuming that the pressure is zero at $r = R_\odot$, we find a central pressure

$$P_c \approx \frac{2GM_\odot}{\rho_\odot}R_\odot \approx \frac{8\pi}{3}G\rho_\odot^2 R_\odot^2 \approx 5\times10^9 \text{ atm.} \tag{13}$$

This simple estimate shows that the central pressure in the Sun is high.

The second equation of stellar structure is the **equation of continuity**,

$$\frac{dM}{dr} = 4\pi r^2\rho(r), \tag{14}$$

which just tells us that the total mass of the star is the sum of all the infinitesimally thin spherical shells of which it is made.

We will also need to know the **equation of state**, which gives the relationship between pressure, density and temperature. For most stars, the appropriate equation of state is the ideal gas law:

$$P(r) = \frac{\rho(r)kT(r)}{\mu m_p}, \tag{15}$$

where μ is the mean molecular mass, which depends on the mix of elements and degree of ionization. This means that μ is a function of r, since the chemical composition and ionization state change with radius. Using our previous estimate of the central pressure and assuming $\mu \approx 0.6$ (easily calculated assuming full ionization, but we won't do it here), we can estimate the temperature at the center of the Sun:

$$T_c \approx P_c\frac{\mu_\odot m_p}{\rho_\odot k} \approx \frac{2GM_\odot\mu_\odot m_p}{R_\odot k} \approx 3\times10^7 \text{ K.} \tag{16}$$

More sophisticated models give a central temperature $T_c = 1.47\times10^7$ K, so we aren't all that far off.

Note that

$$T_c \propto \frac{M\mu}{R} \tag{17}$$

for any sphere of ideal gas in hydrostatic equilibrium.

The third equation of stellar structure is the **equation of energy transport**, which links the luminosity of the star to its temperature gradient in much the same way as the equation of hydrostatic equilibrium links the mass to the pressure gradient. Thermal energy transport in stars occurs via either **radiation** or **convection**. Radiation is the transport of energy by photons, and so usually happens in transparent media, and convection is the transport of energy by bulk motion of hot fluids. Convection dominates in relatively opaque liquids and gases. There are two different equations of energy transport, which we won't derive here, for stars dominated by convection and radiation:

$$\frac{dT}{dr} = -\frac{3\kappa(r)\rho(r)L(r)}{64\pi\sigma_{\rm SB}r^2T(r)^3} \quad \text{[radiative]} \tag{18}$$

where κ is the opacity and $\sigma_{\rm SB}$ is the Stefan-Boltzmann constant, and

$$\frac{dT}{dr} = \left(1-\frac{1}{\gamma}\right)\frac{T(r)}{P(r)}\frac{dP}{dr} \quad \text{[convective]} \tag{19}$$

where γ is the adiabatic index (PV^γ = constant for a blob of gas undergoing an adiabatic process, and $\gamma = 5/3$ for simple atomic gases and fully ionized gases). The appropriate equation of energy transport is the one which gives the smaller temperature gradient. In the Sun, energy transport is primarily radiative out to $r = 0.7$ R$_\odot$, and primarily convective for the outer 30% of the radius.

The last equation of stellar structure is the **equation of energy generation**:

$$\frac{dL}{dr} = 4\pi r^2 \rho(r)\epsilon(r), \tag{20}$$

where $\epsilon(r)$ is the rate of energy production (in power per unit mass). This makes no assumption about the process by which energy is generated; that process is the nuclear fusion of hydrogen into helium (and the fusion of heavier elements during later stages of the lives of massive stars) by two different processes we won't go into here (the PP chain and the CNO cycle).

We can show that gravitational potential energy U is not sufficient to power the Sun for very long. This is the Kelvin-Helmholtz timescale:

$$t_{\rm KH} \equiv \frac{U}{L}, \tag{21}$$

which for the Sun is about 50 Myr. However, the fusion timescale $t_{\rm fus} = E_{\rm fus}/L$, where $E_{\rm fus}$ is the energy generated by fusing about 10% of the Sun's mass from hydrogen to helium (the temperature is high enough to fuse hydrogen in only about 10% of the Sun), is about 10 Gyr, the expected lifetime of the Sun.

The equations of stellar structure are often written in Lagrangian form, in which the independent variable is the mass M instead of the radius r. These equations can be solved to find the interior structure of a star. We need to know the boundary conditions at the photosphere, and we need to know how the mean molecular mass $\mu(\rho, T)$, the opacity $\kappa(\rho, T)$, and the energy generation rate $\epsilon(\rho, T)$ depend on density and temperature within the star. The energy generation rate in particular is extremely sensitive to temperature, with $\epsilon \propto T^4$ for the PP chain and $\epsilon \propto T^{20}$ for the CNO cycle. Modeling of stellar structure is computationally intensive, but fairly easily done with computers these days.

2.2 Mass, radius and luminosity relations

Observation and modeling of stars show that they have well-defined relationships between mass, radius and luminosity. The mass-radius relation is well-fit by two power laws:

$$\frac{R}{R_\odot} = 1.06\left(\frac{M}{M_\odot}\right)^{0.945} \quad (M < 1.66 M_\odot) \tag{22}$$

$$\frac{R}{R_\odot} = 1.33\left(\frac{M}{M_\odot}\right)^{0.55} \quad (M > 1.66 M_\odot) \tag{23}$$

Stars that follow this relation are "main sequence" stars; the main sequence is the majority of the lifetime of a star, during which it fuses hydrogen into helium.

There is also a reasonably well-defined relationship between mass and luminosity:

$$\frac{L}{L_\odot} = 0.35\left(\frac{M}{M_\odot}\right)^{2.62} \quad (M < 0.7 M_\odot) \tag{24}$$

$$\frac{L}{L_\odot} = 1.02\left(\frac{M}{M_\odot}\right)^{3.92} \quad (M > 0.7 M_\odot) \tag{25}$$

Note that the dependence of luminosity on mass is very strong, especially for high mass stars. This has very important consequences for stellar evolution. Because the amount of fuel a star has depends on its mass and the rate at which it uses its fuel is proportional to its luminosity, the lifetime of a star before it exhausts its fuel is $\tau \propto M/L$. We can then use the mass-luminosity relation to determine the how the lifetimes of stars depend on mass:

$$\tau \propto M/L \propto M^{-1.62} \quad (M < 0.7 M_\odot) \tag{26}$$
$$\tau \propto M/L \propto M^{-2.92} \quad (M > 0.7 M_\odot). \tag{27}$$

These are inverse relationships: massive stars are hotter and able to burn through their fuel more quickly, so their lifetimes are shorter.

2.3 Stellar classification and evolution

Stars are classified according to the strength of absorption lines in their spectra, which depends on surface temperature and thus on mass. The classification sequence, from most massive and hottest to least massive and coldest, is OBAFGKM. The Sun is a G star. An O star has a mass of about 60 $\mathrm{M}_\odot$ and a lifetime of about 1 Myr, and an M star has $M \sim 0.2$ $\mathrm{M}_\odot$ and lifetime $\sim 10^{12}$ yrs. See Table 2.1 of the text for more information about the properties of different types of stars.

Distant galaxies are dominated by light from the brightest (OB) stars, and only the most massive stars produce photons energetic enough to ionize hydrogen. For reasons that we will discuss in more detail later, the cosmologically important first generation of stars to form, which would have been composed of only hydrogen and helium and are known as Population III stars, are expected to have been much more massive, perhaps ~ 300 $\mathrm{M}_\odot$ (these have never been observed).

Stars can be very usefully plotted on a diagram of temperature vs. luminosity; this is called the Hertzprung-Russell (H-R) diagram.

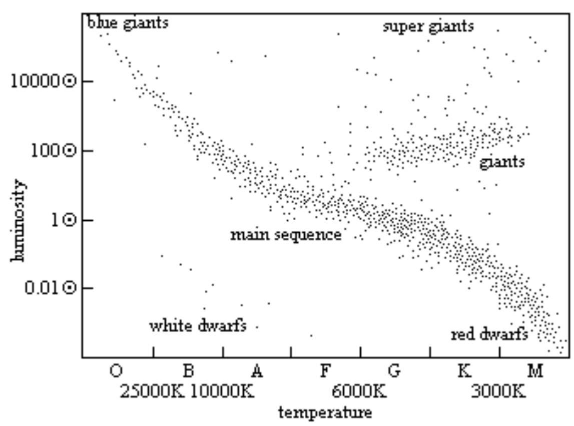

Figure 1: The Hertzprung-Russell diagram of stellar temperature vs luminosity.

Hydrogen burning stars live on the main sequence, with more massive stars on the upper left, and as they run out of hydrogen in the core they move off the main sequence into the giant or supergiant phases.

The spectrum of a star is generally well-approximated by a blackbody, with higher peak temperature for more massive stars. This accounts for the fact that massive stars are blue while low mass stars are red. The H-R diagram is sometimes plotted with color rather than temperature on the x axis.

2.4 The initial mass function

In cosmology we are concerned with stars in galaxies, and particularly with trying to estimate the total mass of a galaxy (including dark matter) from the light that we can see. The light is dominated by massive stars, but the bulk of the stellar mass comes from the large numbers of low mass stars which are too faint to see. This means that we need to know the distribution of stars as a function of mass. This is called the **initial mass function** (IMF; "initial" because it refers to the mass distribution of a newly formed population of stars, before any of the more massive stars have died).

We write this in differential form: the number of stars with masses between M and $M + dM = N_0\, \xi(M)\, dM$, where N_0 depends on the size of the burst of star formation. Like many things in astronomy, this is usually written as a power law:

$$\xi(M) = \frac{dN}{dM} = CM^{-(1+x)} \tag{28}$$

where x may take different values for different mass ranges and C is a normalization constant. The classic slope is $x = 1.35$; this is called the Salpeter IMF and dates from 1955. More recently it has been recognized that the Salpeter IMF appears to overpredict the numbers of low mass stars in local galaxies, so more recent IMFs usually have a broken power law with a shallower slope at the low mass end resulting in fewer low mass stars. The Kroupa IMF (Figure 2) is one common example of this.

There is much discussion and speculation about whether or not the IMF is universal, i.e. the same in all galaxies and throughout most of the history of the universe. Evidence, so far, is limited and somewhat contradictory. It is important to be aware that this is a major systematic uncertainty affecting most studies of the stellar content of galaxies or of total masses inferred from the stellar content.

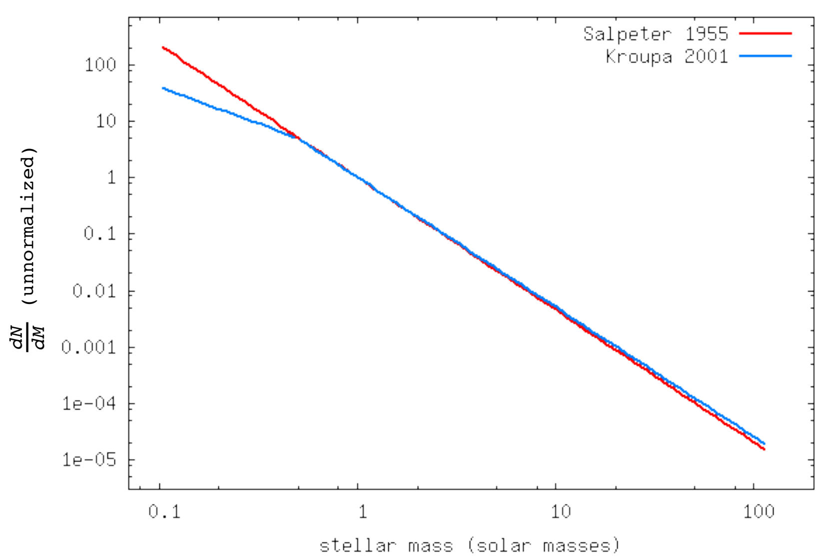

Figure 2: The Salpeter and Kroupa initial mass functions (unnormalized; y axis units are arbitrary) .