Homework 5

Astro 500

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1

A Littrow grating with a camera-collimater angle, $\Phi = \alpha + \beta = 100$ deg means $\alpha = \beta = 50$ deg. In this case the light is on blaze. For the case of a plane-parallel grating without tilted fringes ($\phi = 0$) the angle of diffraction β is given by equation 29 from Burgh et al. (2007)

$$\sin[\beta] = n \sin\left[\arcsin\left[\frac{\sin[\alpha]}{n}\right]\right] - 2\,\phi\right] \qquad (1)$$

We now have two equations and two unknowns. Using $n = 1.4$, we relate β to α with $\beta = \Phi - \alpha$ and plug in to (1)

$$\sin[\Phi - \alpha] = \sin[\arcsin[\sin[\alpha]]] - 2\,\phi]$$

and solve for $\alpha = 42.2$ deg, and $\beta = 57.8$ deg.

2

The blaze wavelength is given by

$$\lambda = \frac{2\,\sigma\,n}{m} \sin[\alpha - \phi] \cos[\phi]$$

where $\sigma = 331$ nm, the grating spacing. $\lambda_b = 558$ nm.

3

$$\sigma = \frac{\lambda\, m}{2\, n \sin[\alpha - \phi] \cos[\phi]}$$
$$\sigma = 387 \text{ nm}$$

4

The linear dispersion in Å/mm is given by

$$\frac{d\lambda}{dx} = \frac{\sigma \cos\beta}{m f_2} \lambda_{pix}$$

$$\frac{d\lambda}{dx} = \left[387 \text{ mm} \times 10^7 \frac{\text{Å}}{\text{mm}}\right] \cos\left(57.8 \text{ deg} \frac{\pi}{180}\right) [152.5 \text{ mm}]^{-1} \left[15 \times 10^{-3} \frac{\text{mm}}{\text{pix}}\right]$$

$$\frac{d\lambda}{dx} = 0.2\ \frac{\text{Å}}{\text{pix}} = 1.3 \times 10^{-6}$$

5

The anamorphic magnification is given by

$r = \cos[\alpha]/\cos[\beta]$
$r = 1.55$

6

The unblazed case is where $\alpha = \beta = 100$ deg.

$$N_R = \frac{2\sin[\delta]}{r}\ \frac{f_{coll}}{w}$$

where $\delta = \phi = 5$ deg is the blaze angle, r is the anamorphic factor, f_{coll} is the collimator focal length, and w is the slit width.

$f_{coll} = 381.8$ mm

$$N_{R,\text{blazed}} = \frac{2\sin[\delta]}{r}\ \frac{f_{coll}}{w} = \frac{2\sin\left[\frac{5\pi}{180}\right]}{\cos[42.2\,\pi/180]/\cos[57.8\,\pi/180]}\ \frac{381\text{ mm}}{300\ \mu\text{m}}\ \frac{10^{-3}\text{ mm}}{\mu\text{m}} = 159\text{ elements}$$

$$\frac{N_{R,\text{unblazed}}}{N_{R,\text{blazed}}} = \frac{1}{r_{\text{unblazed}}} \Big/ \frac{1}{r_{\text{blazed}}} = \frac{r_{\text{blazed}}}{r_{\text{unblazed}}} = 1.39$$

7

Given $f_{coll} = 381.8$ mm and $w = 300\ \mu$m

$$R = \frac{\lambda}{\Delta\lambda} = \frac{f_{coll}}{w}\ \frac{\sin\left[\frac{\beta\pi}{180}\right]+\sin\left[\frac{\alpha\pi}{180}\right]}{\cos\left[\frac{\alpha\pi}{180}\right]} = 2600$$

8

$$R = \lambda\left(\frac{w_{\theta'}}{dl/d\lambda}\right)^{-1}$$
$$w_{\theta'} = \lambda\left(\frac{R}{dl/d\lambda}\right)^{-1}$$
$$w_{\theta'} = 653\text{ nm} \Big/ \left(\frac{2600}{1.3 \times 10^{-6}}\right) = 653\text{ nm} \Big/ \left(2600\ \ 1.3 \times 10^{-6}\right)\ \frac{1\text{ pix}}{15\ \mu\text{m}}\ \frac{10^{-3}\ \mu\text{m}}{\text{nm}}$$
$w_{\theta'} = 13$ pix

There are 13 pixels per resolution element.

```
β = 50;
α = 50;
ϕ = 5;
n = 1.4;
Sin[β] == n * Sin[ArcSin[Sin[α] / n]];
FindMinimum[Abs[Sin[β * π / 180] - n * Sin[ArcSin[Sin[α * π / 180] / n]]], n];
sol = FindMinimum[
  Abs[Sin[(100 - α1) * π / 180] - 1.4 * Sin[ArcSin[Sin[α1 * π / 180] / 1.4]] - 2 * ϕ * π / 180], α1]
σ = 331;
n = 1.4;
α = 42.2 * π / 180;
ϕ = 5 * π / 180.;
m = 1;
λ = 2 σ n / m Sin[α - ϕ] Cos[ϕ]
λ = 653;
σ = λ m / (2 n Sin[α - ϕ] Cos[ϕ])
```

558.21

387.207

$$\frac{d\lambda}{dx} = \left[387\ \text{mm} * 10^7\ \frac{\text{Å}}{\text{mm}}\right] \cos\left(57.8\ \text{deg}\ \frac{\pi}{180}\right) [152.5\ \text{mm}]^{-1} \left[15 * 10^{-3}\ \frac{\text{mm}}{\text{pix}}\right]$$

```
lineardisp = 387 * 10^1 * Cos[57.8 π / 180] / 152.5 * 15 * 10^-3
```

0.202842

```
ScientificForm[lineardisp]
```

2.02842×10^{-1}

```
r = Cos[α * π / 180] / Cos[β * π / 180]
```

$$\frac{\text{Cos}[42.2 * \pi / 180] / \text{Cos}[57.8 * \pi / 180]}{\text{Cos}[50 * \pi / 180] / \text{Cos}[50 * \pi / 180]}$$

1.3902

In[6]:=
$$\alpha = 42.2\,;$$
$$\beta = 57.8\,;$$
$$f_{\text{coll}} = 381 * 10^{-3};$$
$$w = 300 * 10^{-6};\ \frac{2\,\text{Sin}\left[\frac{5\,\pi}{180}\right]}{\text{Cos}[\alpha\,\pi/180]/\text{Cos}[\beta\,\pi/180]}\,\frac{f_{\text{coll}}}{w}$$
$$R = \frac{f_{\text{coll}}}{w}\,\frac{\text{Sin}\left[\frac{\beta\,\pi}{180}\right] + \text{Sin}\left[\frac{\alpha\,\pi}{180}\right]}{\text{Cos}\left[\frac{\alpha\,\pi}{180}\right]}$$

Out[9]= 159.24

Out[10]= 2602.24

In[14]:=
$$653\ \text{nm}\ \Big/ \left(2600\ \ 1.3 \times 10^{-6}\right)\,\frac{1\ \text{pix}}{15\ \mu\text{m}}\,\frac{10^{-3}\ \mu\text{m}}{\text{nm}}$$

Out[14]= 12.8797 pix