I. APPENDIX C

Let's start from Eq. (9) from the paper,

$$J_L = \frac{2e}{h}\int dE\ \mathcal{T}^{ET}(E)[f_L(E) - f_R(E)] + \frac{2e}{h}\int dE\ \mathcal{T}^{DAR}(E)[f_L(E) - \tilde{f}_L(E)]$$
$$+ \frac{2e}{h}\int dE\ \mathcal{T}^{CAR}(E)[f_L(E) - \tilde{f}_R(E)] + \frac{2e}{h}\int dE\ \mathcal{T}^{QP}(E)[f_L(E) - f_S(E)] \quad (1)$$

where we have used the definitions of Fermi-Dirac distributions functions: $f_\alpha = \{\exp[(E - \mu_\alpha)/k_B T_\alpha] + 1\}^{-1}$ and $\tilde{f}_\alpha = 1 - f_\alpha(-E) = \{\exp[(E + \mu_\alpha)/k_B T_\alpha] + 1\}^{-1}$ for electrons and holes, respectively.

A. Derivation of thermoelectric properties

Let's assume a general Fermi function for a given lead α characterized by a local chemical potential μ_α and temperature T_α such that the corresponding equilibrium quantities are μ and T. In this case the differences between these quantities are expressed as

$$\delta\mu_\alpha = \mu_\alpha - \mu$$
$$\delta T_\alpha = T_\alpha - T$$

where θ_α and V_α are temperature and voltage biases applied to the lead. The Fermi function is then given by:

$$f_\alpha = \left\{1 + \exp\left[\frac{E - \mu_\alpha}{k_B T_\alpha}\right]\right\}^{-1},$$

such that

$$f_0 = \left\{1 + \exp\left[\frac{E - \mu}{k_B T}\right]\right\}^{-1}.$$

Before proceeding to the explicit expansion of the Fermi function, let's consider first some derivatives to be used in further calculations. For simplicity, we define:

$$x = \frac{E - \mu_\alpha}{k_B T_\alpha},$$

$$\frac{\partial f_\alpha}{\partial \mu_\alpha} = \frac{\partial f_\alpha}{\partial x}\frac{\partial x}{\partial \mu_\alpha} = -\frac{1}{k_B T_\alpha}\frac{\partial f_\alpha}{\partial x},$$

$$\frac{\partial f_\alpha}{\partial T_\alpha} = \frac{\partial f_\alpha}{\partial x}\frac{\partial x}{\partial T_\alpha} = -\frac{(E-\mu_\alpha)}{k_B T_\alpha^2}\frac{\partial f_\alpha}{\partial x},$$

and

$$\frac{\partial f_\alpha}{\partial E} = \frac{\partial f_\alpha}{\partial x}\frac{\partial x}{\partial E} = \frac{1}{k_B T_\alpha}\frac{\partial f_\alpha}{\partial x}$$

The first two derivatives may expressed in terms of the last one as follows:

$$\frac{\partial f_\alpha}{\partial \mu_\alpha} = -\frac{\partial f_\alpha}{\partial E} \tag{2}$$

and

$$\frac{\partial f_\alpha}{\partial T_\alpha} = -\frac{(E-\mu_\alpha)}{T_\alpha}\frac{\partial f_\alpha}{\partial E}. \tag{3}$$

A similar calculation may be done for the hole Fermi function, $\tilde{f}_\alpha$, i.e.,

$$\frac{\partial \tilde{f}_\alpha}{\partial \mu_\alpha} = +\frac{\partial \tilde{f}_\alpha}{\partial E} \tag{4}$$

and

$$\frac{\partial \tilde{f}_\alpha}{\partial T_\alpha} = -\frac{(E+\mu_\alpha)}{T_\alpha}\frac{\partial \tilde{f}_\alpha}{\partial E}. \tag{5}$$

B. Expanding the Fermi function

We consider the expansion up to the first terms:

$$f_\alpha = f_0 + \left.\frac{\partial f_\alpha}{\partial \mu_\alpha}\right]_{\mu_\alpha=\mu}(\mu_\alpha-\mu) + \left.\frac{\partial f_\alpha}{\partial T_\alpha}\right]_{T_\alpha=T}(T_\alpha-T)$$

which can be written as follows,

$$f_\alpha = f_0 - \frac{\partial f_0}{\partial E}(\mu_\alpha-\mu) - \frac{(E-\mu)}{T}\frac{\partial f_0}{\partial E}(T_\alpha-T)$$

$$f_\alpha = f_0 - \frac{\partial f_0}{\partial E}\delta\mu_\alpha - \frac{(E-\mu)}{T}\frac{\partial f_0}{\partial E}\delta T_\alpha.$$

A similar expansion may be done for the hole Fermi function:

$$\tilde{f}_\alpha = f_0 + \frac{\partial f_0}{\partial E}\delta\mu_\alpha - \frac{(E+\mu)}{T}\frac{\partial f_0}{\partial E}\delta T_\alpha.$$

C. Expanding the current

By using these expansions on the fermi functinons, Eq. (1) may be expressed as follows:

$$
\begin{aligned}
J_L = &\frac{2e}{h}\int dE\, \mathcal{T}^{ET}(E)\left[f_0 - \frac{\partial f_0}{\partial E}\delta\mu_L - \frac{(E-\mu)}{T}\frac{\partial f_0}{\partial E}\delta T_L - \left(f_0 - \frac{\partial f_0}{\partial E}\delta\mu_R - \frac{(E-\mu)}{T}\frac{\partial f_0}{\partial E}\delta T_R\right)\right]\\
&+\frac{2e}{h}\int dE\, \mathcal{T}^{DAR}(E)\left[f_0 - \frac{\partial f_0}{\partial E}\delta\mu_L - \frac{(E-\mu)}{T}\frac{\partial f_0}{\partial E}\delta T_L - \left(f_0 + \frac{\partial f_0}{\partial E}\delta\mu_L - \frac{(E+\mu)}{T}\frac{\partial f_0}{\partial E}\delta T_L\right)\right]\\
&+\frac{2e}{h}\int dE\, \mathcal{T}^{CAR}(E)\left[f_0 - \frac{\partial f_0}{\partial E}\delta\mu_L - \frac{(E-\mu)}{T}\frac{\partial f_0}{\partial E}\delta T_L - \left(f_0 + \frac{\partial f_0}{\partial E}\delta\mu_R - \frac{(E+\mu)}{T}\frac{\partial f_0}{\partial E}\delta T_R\right)\right]\\
&+\frac{2e}{h}\int dE\, \mathcal{T}^{QP}(E)\left[f_0 - \frac{\partial f_0}{\partial E}\delta\mu_L - \frac{(E-\mu)}{T}\frac{\partial f_0}{\partial E}\delta T_L - \left(f_0 - \frac{\partial f_0}{\partial E}\delta\mu_S - \frac{(E-\mu)}{T}\frac{\partial f_0}{\partial E}\delta T_S\right)\right]
\end{aligned}
\tag{6}
$$

which leads to

$$
\begin{aligned}
J_L = &\frac{2e}{h}\int dE\ \mathcal{T}^{ET}(E)\left[\left(-\frac{\partial f_0}{\partial E}\right)(\delta\mu_L - \delta\mu_R) + \frac{(E-\mu)}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L - \delta T_R)\right]\\
&+\frac{2e}{h}\int dE\ \mathcal{T}^{DAR}(E)\left[2\left(-\frac{\partial f_0}{\partial E}\right)\delta\mu_L - 2\frac{\mu}{T}\left(-\frac{\partial f_0}{\partial E}\right)\delta T_L\right]\\
&+\frac{2e}{h}\int dE\ \mathcal{T}^{CAR}(E)\left[\left(-\frac{\partial f_0}{\partial E}\right)(\delta\mu_L + \delta\mu_R) + \frac{E}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L - \delta T_R) - \frac{\mu}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L + \delta T_R)\right]\\
&+\frac{2e}{h}\int dE\ \mathcal{T}^{QP}(E)\left[\left(-\frac{\partial f_0}{\partial E}\right)(\delta\mu_L - \delta\mu_S) + \frac{(E-\mu)}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L - \delta T_S)\right].
\end{aligned}
\tag{7}
$$

We rewrite the term corresponding to the CAR in a different way, in order to make the definitions easier to understand, thus we have:

$$
\begin{aligned}
J_L = &\frac{2e}{h}\int dE\ \mathcal{T}^{ET}(E)\left[\left(-\frac{\partial f_0}{\partial E}\right)(\delta\mu_L - \delta\mu_R) + \frac{(E-\mu)}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L - \delta T_R)\right]\\
&+\frac{2e}{h}\int dE\ \mathcal{T}^{DAR}(E)\left[2\left(-\frac{\partial f_0}{\partial E}\right)\delta\mu_L - 2\frac{\mu}{T}\left(-\frac{\partial f_0}{\partial E}\right)\delta T_L\right]\\
&+\frac{2e}{h}\int dE\ \mathcal{T}^{CAR}(E)\left[\left(-\frac{\partial f_0}{\partial E}\right)(\delta\mu_L + \delta\mu_R) + \frac{(E-\mu)}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L - \delta T_R)\right.\\
&\left.+\frac{\mu}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L - \delta T_R) - \frac{\mu}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L + \delta T_R)\right]\\
&+\frac{2e}{h}\int dE\ \mathcal{T}^{QP}(E)\left[\left(-\frac{\partial f_0}{\partial E}\right)(\delta\mu_L - \delta\mu_S) + \frac{(E-\mu)}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L - \delta T_S)\right],
\end{aligned}
$$

which may be simplified to the form:

$$\begin{aligned}
J_L = \frac{2e}{h}\int dE\ \mathcal{T}^{ET}(E)\left[\left(-\frac{\partial f_0}{\partial E}\right)(\delta\mu_L-\delta\mu_R)+\frac{(E-\mu)}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L-\delta T_R)\right] \\
+\frac{2e}{h}\int dE\ \mathcal{T}^{DAR}(E)\left[2\left(-\frac{\partial f_0}{\partial E}\right)\delta\mu_L-2\frac{\mu}{T}\left(-\frac{\partial f_0}{\partial E}\right)\delta T_L\right] \\
+\frac{2e}{h}\int dE\ \mathcal{T}^{CAR}(E)\left[\left(-\frac{\partial f_0}{\partial E}\right)(\delta\mu_L+\delta\mu_R)+\frac{(E-\mu)}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L-\delta T_R)\right. \\
\left.-2\frac{\mu}{T}\left(-\frac{\partial f_0}{\partial E}\right)\delta T_R\right] \\
+\frac{2e}{h}\int dE\ \mathcal{T}^{QP}(E)\left[\left(-\frac{\partial f_0}{\partial E}\right)(\delta\mu_L-\delta\mu_S)+\frac{(E-\mu)}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L-\delta T_S)\right],
\end{aligned} \tag{8}$$

In order to further simplify the above expression, we use the following definitions:

$$\mathcal{L}^{\kappa}_{\alpha\beta,\mu} = \frac{2e}{h}\int dE\ \mathcal{T}^{\kappa}(E)\left(-\frac{\partial f_0}{\partial E}\right), \tag{9}$$

$$\mathcal{L}^{\kappa}_{\alpha\beta,T} = \frac{2e}{h}\int dE\ \frac{(E-\mu)}{T}\mathcal{T}^{\kappa}(E)\left(-\frac{\partial f_0}{\partial E}\right), \tag{10}$$

with $\alpha,\beta=\{L,R,S\}$ and $\kappa=\{QP,ET,CAR,DAR\}$.

By using Eqs. (9) and (10) into Eq. (8), we obtain:

$$\begin{aligned}
J_L = \mathcal{L}^{ET}_{LR,\mu}(\delta\mu_L-\delta\mu_R)+\mathcal{L}^{ET}_{LR,T}(\delta T_L-\delta T_R)+2\mathcal{L}^{DAR}_{LL,\mu}\left(\delta\mu_L-\frac{\mu}{T}\delta T_L\right)+\mathcal{L}^{CAR}_{LR,\mu}(\delta\mu_L+\delta\mu_R) \\
+\mathcal{L}^{CAR}_{LR,T}(\delta T_L-\delta T_R)-2\frac{\mu}{T}\mathcal{L}^{CAR}_{LR,\mu}\delta T_R+\mathcal{L}^{QP}_{LS,\mu}(\delta\mu_L-\delta\mu_S)+\mathcal{L}^{QP}_{LS,T}(\delta T_L-\delta T_S)
\end{aligned}$$

which leads to

$$\begin{aligned}
J_L = \mathcal{L}^{ET}_{LR,\mu}(\delta\mu_L-\delta\mu_R)+\mathcal{L}^{ET}_{LR,T}(\delta T_L-\delta T_R)+2\mathcal{L}^{DAR}_{LL,\mu}\left(\delta\mu_L-\frac{\mu}{T}\delta T_L\right) \\
+\mathcal{L}^{CAR}_{LR,\mu}\left(\delta\mu_L-\frac{\mu}{T}\delta T_R+\delta\mu_R-\frac{\mu}{T}\delta T_R\right)+\mathcal{L}^{CAR}_{LR,T}(\delta T_L-\delta T_R) \\
+\mathcal{L}^{QP}_{LS,\mu}(\delta\mu_L-\delta\mu_S)+\mathcal{L}^{QP}_{LS,T}(\delta T_L-\delta T_S),
\end{aligned}$$

and by grouping similar terms, we obtain:

$$\begin{aligned}
J_L = \mathcal{L}^{ET}_{LR,\mu}(\delta\mu_L-\delta\mu_R)+(\mathcal{L}^{ET}_{LR,T}+\mathcal{L}^{CAR}_{LR,T})(\delta T_L-\delta T_R)+2\mathcal{L}^{DAR}_{LL,\mu}\left(\delta\mu_L-\frac{\mu}{T}\delta T_L\right) \\
+\mathcal{L}^{CAR}_{LR,\mu}\left(\delta\mu_L-\frac{\mu}{T}\delta T_R+\delta\mu_R-\frac{\mu}{T}\delta T_R\right) \\
+\mathcal{L}^{QP}_{LS,\mu}(\delta\mu_L-\delta\mu_S)+\mathcal{L}^{QP}_{LS,T}(\delta T_L-\delta T_S).
\end{aligned} \tag{11}$$

D. Setting the reference chemical potencial to zero: $\mu = 0$

By using the chemical potencial reference to zero, we obtain a simplified version of Eq. (11). In this way, we have:

$$J_L = \mathcal{L}^{ET}_{LR,\mu}(\delta\mu_L - \delta\mu_R) + \mathcal{L}^{CAR}_{LR,\mu}(\delta\mu_L + \delta\mu_R) + (\mathcal{L}^{ET}_{LR,T} + \mathcal{L}^{CAR}_{LR,T})(\delta T_L - \delta T_R) + 2\mathcal{L}^{DAR}_{LL,\mu}\delta\mu_L \\ + \mathcal{L}^{QP}_{LS,\mu}(\delta\mu_L - \delta\mu_S) + \mathcal{L}^{QP}_{LS,T}(\delta T_L - \delta T_S). \quad (12)$$

where we have definitions,

$$\mathcal{L}^{\kappa}_{\alpha\beta,\mu} = \frac{2e}{h}\int dE\ \mathcal{T}^{\kappa}(E)\left(-\frac{\partial f_0}{\partial E}\right), \quad (13)$$

$$\mathcal{L}^{\kappa}_{\alpha\beta,T} = \frac{2e}{hT}\int dE\ E\mathcal{T}^{\kappa}(E)\left(-\frac{\partial f_0}{\partial E}\right). \quad (14)$$

We also have the current from the right lead which is obtained by performing the exchange $L \leftrightarrow R$ in Eq. (12). In this way, we have:

$$J_R = \mathcal{L}^{ET}_{RL,\mu}(\delta\mu_R - \delta\mu_L) + \mathcal{L}^{CAR}_{RL,\mu}(\delta\mu_R + \delta\mu_L) + (\mathcal{L}^{ET}_{RL,T} + \mathcal{L}^{CAR}_{RL,T})(\delta T_R - \delta T_L) + 2\mathcal{L}^{DAR}_{RR,\mu}\delta\mu_R \\ + \mathcal{L}^{QP}_{RS,\mu}(\delta\mu_R - \delta\mu_S) + \mathcal{L}^{QP}_{RS,T}(\delta T_R - \delta T_S). \quad (15)$$

We can also determine the current flowing into the S lead by using current conservation: $J_S = -J_L - J_R$ which leads to:

$$-J_S = 2\mathcal{L}^{CAR}_{LR,\mu}(\delta\mu_L + \delta\mu_R) + 2\mathcal{L}^{DAR}_{LL,\mu}\delta\mu_L + 2\mathcal{L}^{DAR}_{RR,\mu}\delta\mu_R \\ + \mathcal{L}^{QP}_{LS,\mu}(\delta\mu_L - \delta\mu_S) + \mathcal{L}^{QP}_{LS,T}(\delta T_L - \delta T_S) + \mathcal{L}^{QP}_{RS,\mu}(\delta\mu_R - \delta\mu_S) + \mathcal{L}^{QP}_{RS,T}(\delta T_R - \delta T_S). \quad (16)$$

E. Reproducing the equations of appendix C

1. *Eq. (C2)*

We consider the scenario in which the L lead will be the voltage probe under isothermal conditions. In addition, we also set the voltage at S equal to zero, $\delta\mu_S = 0$. In this way, by using Eq. (12), we have:

$$0 = \mathcal{L}^{ET}_{LR,\mu}(\delta\mu_L - \delta\mu_R) + \mathcal{L}^{CAR}_{LR,\mu}(\delta\mu_L + \delta\mu_R) + 2\mathcal{L}^{DAR}_{LL,\mu}\delta\mu_L + \mathcal{L}^{QP}_{LS,\mu}\delta\mu_L$$

which leads to

$$0 = (\mathcal{L}^{CAR}_{LR,\mu} - \mathcal{L}^{ET}_{LR,\mu})\delta\mu_R + (\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})\delta\mu_L$$

thus,

$$\delta\mu_R = \frac{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L. \tag{17}$$

In order to derive Eq. (C1), we use the isothermal version of Eq. (15)

$$J_R = J_{RS} = (-\mathcal{L}^{ET}_{RL,\mu} + \mathcal{L}^{CAR}_{RL,\mu})\delta\mu_L + (\mathcal{L}^{ET}_{RL,\mu} + \mathcal{L}^{CAR}_{RL,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})\delta\mu_R$$

and by substituting Eq. (17) we have:

$$\begin{aligned} J_{RS} = {} & (-\mathcal{L}^{ET}_{RL,\mu} + \mathcal{L}^{CAR}_{RL,\mu})\delta\mu_L \\ & + (\mathcal{L}^{ET}_{RL,\mu} + \mathcal{L}^{CAR}_{RL,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})\frac{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L \end{aligned}$$

which leads to

$$\begin{aligned} (\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})\frac{J_{RS}}{\delta\mu_L} = {} & (-\mathcal{L}^{ET}_{RL,\mu} + \mathcal{L}^{CAR}_{RL,\mu})(\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}) \\ & + (\mathcal{L}^{ET}_{RL,\mu} + \mathcal{L}^{CAR}_{RL,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu}) \end{aligned}$$

and by using the symmetry properties, we can further simplify the equation above

$$\begin{aligned} (\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})\frac{J_{RS}}{\delta\mu_L} = {} & -(\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})^2 \\ & + (\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu}) \end{aligned}$$

$$\begin{aligned} (\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})\frac{J_{RS}}{\delta\mu_L} = {} & -(\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})^2 + (\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu})^2 + (\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu})(2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu}) \\ & + (2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu}) + (2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})(2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu}) \end{aligned}$$

$$\begin{aligned} (\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})\frac{J_{RS}}{\delta\mu_L} = {} & 4\mathcal{L}^{ET}_{LR,\mu}\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{ET}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{CAR}_{LR,\mu} + \mathcal{L}^{QP}_{LS,\mu}\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{QP}_{LS,\mu}\mathcal{L}^{CAR}_{LR,\mu} \\ & + 2\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{QP}_{RS,\mu}\mathcal{L}^{ET}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{CAR}_{LR,\mu} + \mathcal{L}^{QP}_{RS,\mu}\mathcal{L}^{CAR}_{LR,\mu} \\ & + 4\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{DAR}_{RR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{QP}_{RS,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{QP}_{LS,\mu} + \mathcal{L}^{QP}_{RS,\mu}\mathcal{L}^{QP}_{LS,\mu} \end{aligned}$$

and performing the multiplications, we have:

$$(\mathcal{L}^{ET}_{LR,\mu}-\mathcal{L}^{CAR}_{LR,\mu})\frac{J_{RS}}{\delta\mu_L} = 4\mathcal{L}^{ET}_{LR,\mu}\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{ET}_{LR,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{CAR}_{LR,\mu}+\mathcal{L}^{QP}_{LS,\mu}\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{QP}_{LS,\mu}\mathcal{L}^{CAR}_{LR,\mu}$$
$$+ 2\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{QP}_{RS,\mu}\mathcal{L}^{ET}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{CAR}_{LR,\mu} + \mathcal{L}^{QP}_{RS,\mu}\mathcal{L}^{CAR}_{LR,\mu}$$
$$+ 4\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{DAR}_{RR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{QP}_{RS,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{QP}_{LS,\mu} + \mathcal{L}^{QP}_{RS,\mu}\mathcal{L}^{QP}_{LS,\mu}.$$

Here we can group some terms as follows:

$$(\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})\frac{J_{RS}}{\delta\mu_L} = \mathcal{L}^{ET}_{LR,\mu}(4\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{LS,\mu} + \mathcal{L}^{QP}_{RS,\mu})$$
$$+ \mathcal{L}^{CAR}_{LR,\mu}(2\mathcal{L}^{DAR}_{LL,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{LS,\mu} + \mathcal{L}^{QP}_{RS,\mu})$$
$$+ 4\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{DAR}_{RR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{QP}_{RS,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{QP}_{LS,\mu} + \mathcal{L}^{QP}_{RS,\mu}\mathcal{L}^{QP}_{LS,\mu}.$$

or,

$$(\mathcal{L}^{ET}_{LR,\mu}-\mathcal{L}^{CAR}_{LR,\mu})\frac{J_{RS}}{\delta\mu_L} = \mathcal{L}^{ET}_{LR,\mu}(4\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}+2\mathcal{L}^{DAR}_{RR,\mu})+\mathcal{L}^{CAR}_{LR,\mu}(2\mathcal{L}^{DAR}_{LL,\mu}+2\mathcal{L}^{DAR}_{RR,\mu})+4\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{DAR}_{RR,\mu}$$
$$+ 2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{QP}_{RS,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{QP}_{LS,\mu} + (\mathcal{L}^{QP}_{LS,\mu} + \mathcal{L}^{QP}_{RS,\mu})(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu}) + \mathcal{L}^{QP}_{RS,\mu}\mathcal{L}^{QP}_{LS,\mu}. \quad (18)$$

In order to comply with the definitions appearing on the paper, we define:

$$D = \mathcal{L}^{ET}_{LR,\mu}(2\mathcal{L}^{CAR}_{LR,\mu} + \mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{DAR}_{RR,\mu}) + \mathcal{L}^{CAR}_{LR,\mu}(\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{DAR}_{RR,\mu}) + 2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{DAR}_{RR,\mu}$$
$$+ \mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{QP}_{RS,\mu} + \mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{QP}_{LS,\mu} + \frac{1}{2}(\mathcal{L}^{QP}_{LS,\mu} + \mathcal{L}^{QP}_{RS,\mu})(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu}) + \frac{1}{2}\mathcal{L}^{QP}_{RS,\mu}\mathcal{L}^{QP}_{LS,\mu}. \quad (19)$$

with this definition, we can rewrite Eq. (18) as follows:

$$(\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})\frac{J_{RS}}{\delta\mu_L} = 2D \quad (20)$$

which leads to

$$\frac{\delta\mu_L}{eJ_{RS}} = \frac{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}{2eD}. \quad (21)$$

Now, by considering $\delta\mu_S = 0$, we can write:

$$\delta\mu_L = \delta\mu_L - \delta\mu_S = \Delta\mu_{LS}.$$

Next, we can write:

$$R_{RS,LS} = \frac{\Delta\mu_{LS}}{eJ_{RS}} = \frac{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}{2eD}. \quad (22)$$

2. *Eq. (C1)*

We start from Eq. (17)

$$\delta\mu_R = \frac{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L$$

which allows us to calculate:

$$\Delta\mu_{RL} = \delta\mu_R - \delta\mu_L = \frac{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L - \delta\mu_L$$

which leads to

$$\Delta\mu_{RL} = \frac{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L - \frac{(\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L$$

then we have:

$$\Delta\mu_{RL} = \frac{(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L. \tag{23}$$

In order to derive Eq. (C1), we use the isothermal version of Eq. (15)

$$eJ_{RS} = \frac{2eD}{(\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})}\delta\mu_L$$

which leads to,

$$\frac{\Delta\mu_{RL}}{eJ_{RS}} = \frac{(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L \frac{(\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})}{2eD\delta\mu_L}$$

$$R_{RS,RL} = \frac{\Delta\mu_{RL}}{eJ_{RS}} = \frac{2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu}}{2eD}. \tag{24}$$

3. *Eq. (C3)*

Next, we need to determine the voltage difference $\Delta\mu_{RS}$,

$$\Delta\mu_{RS} = \delta\mu_R - \delta\mu_S = \delta\mu_R,$$

since $\delta\mu_S$ is zero. In this case, by using (17)

$$\delta\mu_R = \frac{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L$$

we have:

$$\Delta\mu_{RS} = \frac{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L,$$

Next, we use the isothermal version of Eq. (15)

$$eJ_{RS} = \frac{2eD}{(\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})}\delta\mu_L$$

which allows us to write:

$$R_{RS,RS} = \frac{\Delta\mu_{RS}}{eJ_{RS}} = \frac{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L \frac{(\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})}{2eD\delta\mu_L}$$

$$R_{RS,RS} = \frac{\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu}}{2eD}. \tag{25}$$

In order to derive the last three equations of appendix B, we consider the superconducting electrode to be floating ($J_S = 0$) and denote $J_R = -J_L = J_{RL}$.

We start from Eq. (16),

$$\begin{aligned} -J_S = {} & 2\mathcal{L}^{CAR}_{LR,\mu}(\delta\mu_L + \delta\mu_R) + 2\mathcal{L}^{DAR}_{LL,\mu}\delta\mu_L + 2\mathcal{L}^{DAR}_{RR,\mu}\delta\mu_R \\ & + \mathcal{L}^{QP}_{LS,\mu}(\delta\mu_L - \delta\mu_S) + \mathcal{L}^{QP}_{LS,T}(\delta T_L - \delta T_S) + \mathcal{L}^{QP}_{RS,\mu}(\delta\mu_R - \delta\mu_S) + \mathcal{L}^{QP}_{RS,T}(\delta T_R - \delta T_S). \end{aligned}$$

and by setting $J_S = 0$ we obtain:

$$\begin{aligned} & 2\mathcal{L}^{CAR}_{LR,\mu}(\delta\mu_L + \delta\mu_R) + 2\mathcal{L}^{DAR}_{LL,\mu}\delta\mu_L + 2\mathcal{L}^{DAR}_{RR,\mu}\delta\mu_R \\ & + \mathcal{L}^{QP}_{LS,\mu}(\delta\mu_L - \delta\mu_S) + \mathcal{L}^{QP}_{LS,T}(\delta T_L - \delta T_S) + \mathcal{L}^{QP}_{RS,\mu}(\delta\mu_R - \delta\mu_S) + \mathcal{L}^{QP}_{RS,T}(\delta T_R - \delta T_S) = 0, \end{aligned}$$

and by setting $\delta\mu_S = 0$ and isothermal conditions, we have:

$$2\mathcal{L}^{CAR}_{LR,\mu}(\delta\mu_L + \delta\mu_R) + 2\mathcal{L}^{DAR}_{LL,\mu}\delta\mu_L + 2\mathcal{L}^{DAR}_{RR,\mu}\delta\mu_R + \mathcal{L}^{QP}_{LS,\mu}\delta\mu_L + \mathcal{L}^{QP}_{RS,\mu}\delta\mu_R = 0$$

and by grouping similar terms we obtain:

$$(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})\delta\mu_L + (2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})\delta\mu_R = 0$$

which leads to

$$\delta\mu_R = -\frac{(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu}}\delta\mu_L.$$

Next, we consider the current J_R under isothermal conditions:

$$J_R = J_{RL} = -J_L = \mathcal{L}^{ET}_{RL,\mu}(\delta\mu_R - \delta\mu_L) + \mathcal{L}^{CAR}_{RL,\mu}(\delta\mu_R + \delta\mu_L) + 2\mathcal{L}^{DAR}_{RR,\mu}\delta\mu_R + \mathcal{L}^{QP}_{RS,\mu}\delta\mu_R$$

which can be written as follows:

$$J_{RL} = (\mathcal{L}^{ET}_{RL,\mu} + \mathcal{L}^{CAR}_{RL,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})\delta\mu_R + (\mathcal{L}^{CAR}_{RL,\mu} - \mathcal{L}^{ET}_{RL,\mu})\delta\mu_L,$$

and by substituting $\delta\mu_R$ we obtain:

$$J_{RL} = (\mathcal{L}^{ET}_{RL,\mu} + \mathcal{L}^{CAR}_{RL,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})\left[-\frac{(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu}}\delta\mu_L\right] + (\mathcal{L}^{CAR}_{RL,\mu} - \mathcal{L}^{ET}_{RL,\mu})\delta\mu_L,$$

and multiplying both sides by $2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu}$ we obtain,

$$\begin{aligned}\frac{J_{RL}}{\delta\mu_L}(2\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu}) = &-(\mathcal{L}^{ET}_{RL,\mu}+\mathcal{L}^{CAR}_{RL,\mu}+2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu})(2\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}+\mathcal{L}^{QP}_{LS,\mu})\\ &+ (\mathcal{L}^{CAR}_{RL,\mu} - \mathcal{L}^{ET}_{RL,\mu})(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu}),\end{aligned}$$

and performing the products and by using the symmetry $\mathcal{L}^{CAR}_{RL,\mu} = \mathcal{L}^{CAR}_{LR,\mu}$, we have:

$$\begin{aligned}\frac{J_{RL}}{\delta\mu_L}(2\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu}) = &-(\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu})(2\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}+\mathcal{L}^{QP}_{LS,\mu})\\ &+ (\mathcal{L}^{CAR}_{LR,\mu} - \mathcal{L}^{ET}_{LR,\mu})(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu}),\end{aligned}$$

or

$$\begin{aligned}\frac{J_{RL}}{\delta\mu_L}(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu}) = &-(2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})\\ &- (\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})\\ &- (\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu})(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})\end{aligned}$$

and performing some manipulations,

$$\begin{aligned}&\frac{J_{RL}}{\delta\mu_L}(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})\\ &\quad= -\mathcal{L}^{ET}_{LR,\mu}(4\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu}) - \mathcal{L}^{CAR}_{LR,\mu}(2\mathcal{L}^{DAR}_{RR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu}) - 4\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{DAR}_{RR,\mu}\\ &\qquad - 2\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{QP}_{LS,\mu} - 2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{QP}_{RS,\mu} - (\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu})(\mathcal{L}^{QP}_{LS,\mu} + \mathcal{L}^{QP}_{RS,\mu}) - \mathcal{L}^{QP}_{LS,\mu}\mathcal{L}^{QP}_{RS,\mu}.\end{aligned}$$

Let's divide by $-1/2$ both side of equation above, which allows us to write:

$$
\begin{aligned}
&-\frac{J_{RL}}{2\delta\mu_L}(2\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu})\\
&\qquad=\mathcal{L}^{ET}_{LR,\mu}(2\mathcal{L}^{CAR}_{LR,\mu}+\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{DAR}_{LL,\mu})+\mathcal{L}^{CAR}_{LR,\mu}(\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{DAR}_{LL,\mu})+2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{DAR}_{RR,\mu}\\
&\frac{1}{2}\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{QP}_{LS,\mu}+\frac{1}{2}\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{QP}_{RS,\mu}+\frac{1}{2}(\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu})(\mathcal{L}^{QP}_{LS,\mu}+\mathcal{L}^{QP}_{RS,\mu})+\frac{1}{2}\mathcal{L}^{QP}_{LS,\mu}\mathcal{L}^{QP}_{RS,\mu}.
\end{aligned}
\tag{26}
$$

Once again we define the denominator D as,

$$
D=\mathcal{L}^{ET}_{LR,\mu}(2\mathcal{L}^{CAR}_{LR,\mu}+\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{DAR}_{LL,\mu})+\mathcal{L}^{CAR}_{LR,\mu}(\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{DAR}_{LL,\mu})+2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{DAR}_{RR,\mu} \tag{27}
$$

$$
\frac{1}{2}\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{QP}_{LS,\mu}+\frac{1}{2}\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{QP}_{RS,\mu}+\frac{1}{2}(\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu})(\mathcal{L}^{QP}_{LS,\mu}+\mathcal{L}^{QP}_{RS,\mu})+\frac{1}{2}\mathcal{L}^{QP}_{LS,\mu}\mathcal{L}^{QP}_{RS,\mu}, \tag{28}
$$

which allows us to write Eq. (26) as follows:

$$
-\frac{J_{RL}}{\delta\mu_L}=\frac{2D}{2\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu}}. \tag{29}
$$

By using Eq. (29), we can write $R_{RL,LS}$, we obtain:

$$
R_{RL,LS}=\frac{\Delta\mu_{LS}}{eJ_{RL}}=-\frac{\mathcal{L}^{CAR}_{LR,\mu}+\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu}/2}{eD}. \tag{30}
$$