I. APPENDIX C

Let's start from Eq. (9) from the paper,

$$J_L = \frac{2e}{h}\int dE\ \mathcal{T}^{ET}(E)[f_L(E) - f_R(E)] + \frac{2e}{h}\int dE\ \mathcal{T}^{DAR}(E)[f_L(E) - \tilde{f}_L(E)] \\ + \frac{2e}{h}\int dE\ \mathcal{T}^{CAR}(E)[f_L(E) - \tilde{f}_R(E)] + \frac{2e}{h}\int dE\ \mathcal{T}^{QP}(E)[f_L(E) - f_S(E)] \quad (1)$$

where we have used the definitions of Fermi-Dirac distributions functions: $f_\alpha = \{\exp[(E - \mu_\alpha)/k_B T_\alpha] + 1\}^{-1}$ and $\tilde{f}_\alpha = 1 - f_\alpha(-E) = \{\exp[(E + \mu_\alpha)/k_B T_\alpha] + 1\}^{-1}$ for electrons and holes, respectively.

A. Derivation of thermoelectric properties

Let's assume a general Fermi function for a given lead α characterized by a local chemical potential μ_α and temperature T_α such that the corresponding equilibrium quantities are μ and T. In this case the differences between these quantities are expressed as

$$\delta\mu_\alpha = \mu_\alpha - \mu$$
$$\delta T_\alpha = T_\alpha - T$$

where θ_α and V_α are temperature and voltage biases applied to the lead. The Fermi function is then given by:

$$f_\alpha = \left\{1 + \exp\left[\frac{E - \mu_\alpha}{k_B T_\alpha}\right]\right\}^{-1},$$

such that

$$f_0 = \left\{1 + \exp\left[\frac{E - \mu}{k_B T}\right]\right\}^{-1}.$$

Before proceeding to the explicit expansion of the Fermi function, let's consider first some derivatives to be used in further calculations. For simplicity, we define:

$$x = \frac{E - \mu_\alpha}{k_B T_\alpha},$$

$$\frac{\partial f_\alpha}{\partial \mu_\alpha} = \frac{\partial f_\alpha}{\partial x}\frac{\partial x}{\partial \mu_\alpha} = -\frac{1}{k_B T_\alpha}\frac{\partial f_\alpha}{\partial x},$$

$$\frac{\partial f_\alpha}{\partial T_\alpha} = \frac{\partial f_\alpha}{\partial x}\frac{\partial x}{\partial T_\alpha} = -\frac{(E-\mu_\alpha)}{k_B T_\alpha^2}\frac{\partial f_\alpha}{\partial x},$$

and

$$\frac{\partial f_\alpha}{\partial E} = \frac{\partial f_\alpha}{\partial x}\frac{\partial x}{\partial E} = \frac{1}{k_B T_\alpha}\frac{\partial f_\alpha}{\partial x}$$

The first two derivatives may expressed in terms of the last one as follows:

$$\frac{\partial f_\alpha}{\partial \mu_\alpha} = -\frac{\partial f_\alpha}{\partial E} \tag{2}$$

and

$$\frac{\partial f_\alpha}{\partial T_\alpha} = -\frac{(E-\mu_\alpha)}{T_\alpha}\frac{\partial f_\alpha}{\partial E}. \tag{3}$$

A similar calculation may be done for the hole Fermi function, $\tilde{f}_\alpha$, i.e.,

$$\frac{\partial \tilde{f}_\alpha}{\partial \mu_\alpha} = +\frac{\partial \tilde{f}_\alpha}{\partial E} \tag{4}$$

and

$$\frac{\partial \tilde{f}_\alpha}{\partial T_\alpha} = -\frac{(E+\mu_\alpha)}{T_\alpha}\frac{\partial \tilde{f}_\alpha}{\partial E}. \tag{5}$$

B. Expanding the Fermi function

We consider the expansion up to the first terms:

$$f_\alpha = f_0 + \left.\frac{\partial f_\alpha}{\partial \mu_\alpha}\right]_{\mu_\alpha=\mu}(\mu_\alpha - \mu) + \left.\frac{\partial f_\alpha}{\partial T_\alpha}\right]_{T_\alpha=T}(T_\alpha - T)$$

which can be written as follows,

$$f_\alpha = f_0 - \frac{\partial f_0}{\partial E}(\mu_\alpha - \mu) - \frac{(E-\mu)}{T}\frac{\partial f_0}{\partial E}(T_\alpha - T)$$

$$f_\alpha = f_0 - \frac{\partial f_0}{\partial E}\delta\mu_\alpha - \frac{(E-\mu)}{T}\frac{\partial f_0}{\partial E}\delta T_\alpha.$$

A similar expansion may be done for the hole Fermi function:

$$\tilde{f}_\alpha = f_0 + \frac{\partial f_0}{\partial E}\delta\mu_\alpha - \frac{(E+\mu)}{T}\frac{\partial f_0}{\partial E}\delta T_\alpha.$$

C. Expanding the current

By using these expansions on the fermi functinons, Eq. (1) may be expressed as follows:

$$
\begin{aligned}
J_L = \frac{2e}{h}\int dE\, \mathcal{T}^{ET}(E)\left[f_0 - \frac{\partial f_0}{\partial E}\delta\mu_L - \frac{(E-\mu)}{T}\frac{\partial f_0}{\partial E}\delta T_L - \left(f_0 - \frac{\partial f_0}{\partial E}\delta\mu_R - \frac{(E-\mu)}{T}\frac{\partial f_0}{\partial E}\delta T_R\right)\right] \\
+\frac{2e}{h}\int dE\, \mathcal{T}^{DAR}(E)\left[f_0 - \frac{\partial f_0}{\partial E}\delta\mu_L - \frac{(E-\mu)}{T}\frac{\partial f_0}{\partial E}\delta T_L - \left(f_0 + \frac{\partial f_0}{\partial E}\delta\mu_L - \frac{(E+\mu)}{T}\frac{\partial f_0}{\partial E}\delta T_L\right)\right] \\
+\frac{2e}{h}\int dE\, \mathcal{T}^{CAR}(E)\left[f_0 - \frac{\partial f_0}{\partial E}\delta\mu_L - \frac{(E-\mu)}{T}\frac{\partial f_0}{\partial E}\delta T_L - \left(f_0 + \frac{\partial f_0}{\partial E}\delta\mu_R - \frac{(E+\mu)}{T}\frac{\partial f_0}{\partial E}\delta T_R\right)\right] \\
+\frac{2e}{h}\int dE\, \mathcal{T}^{QP}(E)\left[f_0 - \frac{\partial f_0}{\partial E}\delta\mu_L - \frac{(E-\mu)}{T}\frac{\partial f_0}{\partial E}\delta T_L - \left(f_0 - \frac{\partial f_0}{\partial E}\delta\mu_S - \frac{(E-\mu)}{T}\frac{\partial f_0}{\partial E}\delta T_S\right)\right]
\end{aligned} \tag{6}
$$

which leads to

$$
\begin{aligned}
J_L = \frac{2e}{h}\int dE\ \mathcal{T}^{ET}(E)\left[\left(-\frac{\partial f_0}{\partial E}\right)(\delta\mu_L - \delta\mu_R) + \frac{(E-\mu)}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L - \delta T_R)\right] \\
+\frac{2e}{h}\int dE\ \mathcal{T}^{DAR}(E)\left[2\left(-\frac{\partial f_0}{\partial E}\right)\delta\mu_L - 2\frac{\mu}{T}\left(-\frac{\partial f_0}{\partial E}\right)\delta T_L\right] \\
+\frac{2e}{h}\int dE\ \mathcal{T}^{CAR}(E)\left[\left(-\frac{\partial f_0}{\partial E}\right)(\delta\mu_L + \delta\mu_R) + \frac{E}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L - \delta T_R) - \frac{\mu}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L + \delta T_R)\right] \\
+\frac{2e}{h}\int dE\ \mathcal{T}^{QP}(E)\left[\left(-\frac{\partial f_0}{\partial E}\right)(\delta\mu_L - \delta\mu_S) + \frac{(E-\mu)}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L - \delta T_S)\right].
\end{aligned} \tag{7}
$$

We rewrite the term corresponding to the CAR in a different way, in order to make the definitions easier to understand, thus we have:

$$
\begin{aligned}
J_L = \frac{2e}{h}\int dE\ \mathcal{T}^{ET}(E)\left[\left(-\frac{\partial f_0}{\partial E}\right)(\delta\mu_L - \delta\mu_R) + \frac{(E-\mu)}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L - \delta T_R)\right] \\
+\frac{2e}{h}\int dE\ \mathcal{T}^{DAR}(E)\left[2\left(-\frac{\partial f_0}{\partial E}\right)\delta\mu_L - 2\frac{\mu}{T}\left(-\frac{\partial f_0}{\partial E}\right)\delta T_L\right] \\
+\frac{2e}{h}\int dE\ \mathcal{T}^{CAR}(E)\left[\left(-\frac{\partial f_0}{\partial E}\right)(\delta\mu_L + \delta\mu_R) + \frac{(E-\mu)}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L - \delta T_R)\right. \\
\left. +\frac{\mu}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L - \delta T_R) - \frac{\mu}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L + \delta T_R)\right] \\
+\frac{2e}{h}\int dE\ \mathcal{T}^{QP}(E)\left[\left(-\frac{\partial f_0}{\partial E}\right)(\delta\mu_L - \delta\mu_S) + \frac{(E-\mu)}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L - \delta T_S)\right],
\end{aligned}
$$

which may be simplified to the form:

$$\begin{aligned}
J_L = \frac{2e}{h}\int dE\ \mathcal{T}^{ET}(E)\left[\left(-\frac{\partial f_0}{\partial E}\right)(\delta\mu_L-\delta\mu_R)+\frac{(E-\mu)}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L-\delta T_R)\right] \\
+\frac{2e}{h}\int dE\ \mathcal{T}^{DAR}(E)\left[2\left(-\frac{\partial f_0}{\partial E}\right)\delta\mu_L-2\frac{\mu}{T}\left(-\frac{\partial f_0}{\partial E}\right)\delta T_L\right] \\
+\frac{2e}{h}\int dE\ \mathcal{T}^{CAR}(E)\left[\left(-\frac{\partial f_0}{\partial E}\right)(\delta\mu_L+\delta\mu_R)+\frac{(E-\mu)}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L-\delta T_R)\right. \\
\left.-2\frac{\mu}{T}\left(-\frac{\partial f_0}{\partial E}\right)\delta T_R\right] \\
+\frac{2e}{h}\int dE\ \mathcal{T}^{QP}(E)\left[\left(-\frac{\partial f_0}{\partial E}\right)(\delta\mu_L-\delta\mu_S)+\frac{(E-\mu)}{T}\left(-\frac{\partial f_0}{\partial E}\right)(\delta T_L-\delta T_S)\right],
\end{aligned} \tag{8}$$

In order to further simplify the above expression, we use the following definitions:

$$\mathcal{L}^{\kappa}_{\alpha\beta,\mu} = \frac{2e}{h}\int dE\ \mathcal{T}^{\kappa}(E)\left(-\frac{\partial f_0}{\partial E}\right), \tag{9}$$

$$\mathcal{L}^{\kappa}_{\alpha\beta,T} = \frac{2e}{h}\int dE\ \frac{(E-\mu)}{T}\mathcal{T}^{\kappa}(E)\left(-\frac{\partial f_0}{\partial E}\right), \tag{10}$$

with $\alpha,\beta=\{L,R,S\}$ and $\kappa=\{QP,ET,CAR,DAR\}$.

By using Eqs. (9) and (10) into Eq. (8), we obtain:

$$\begin{aligned}
J_L = \mathcal{L}^{ET}_{LR,\mu}(\delta\mu_L-\delta\mu_R)+\mathcal{L}^{ET}_{LR,T}(\delta T_L-\delta T_R)+2\mathcal{L}^{DAR}_{LL,\mu}\left(\delta\mu_L-\frac{\mu}{T}\delta T_L\right)+\mathcal{L}^{CAR}_{LR,\mu}(\delta\mu_L+\delta\mu_R) \\
+\mathcal{L}^{CAR}_{LR,T}(\delta T_L-\delta T_R)-2\frac{\mu}{T}\mathcal{L}^{CAR}_{LR,\mu}\delta T_R+\mathcal{L}^{QP}_{LS,\mu}(\delta\mu_L-\delta\mu_S)+\mathcal{L}^{QP}_{LS,T}(\delta T_L-\delta T_S)
\end{aligned}$$

which leads to

$$\begin{aligned}
J_L = \mathcal{L}^{ET}_{LR,\mu}(\delta\mu_L-\delta\mu_R)+\mathcal{L}^{ET}_{LR,T}(\delta T_L-\delta T_R)+2\mathcal{L}^{DAR}_{LL,\mu}\left(\delta\mu_L-\frac{\mu}{T}\delta T_L\right) \\
+\mathcal{L}^{CAR}_{LR,\mu}\left(\delta\mu_L-\frac{\mu}{T}\delta T_R+\delta\mu_R-\frac{\mu}{T}\delta T_R\right)+\mathcal{L}^{CAR}_{LR,T}(\delta T_L-\delta T_R) \\
+\mathcal{L}^{QP}_{LS,\mu}(\delta\mu_L-\delta\mu_S)+\mathcal{L}^{QP}_{LS,T}(\delta T_L-\delta T_S),
\end{aligned}$$

and by grouping similar terms, we obtain:

$$\begin{aligned}
J_L = \mathcal{L}^{ET}_{LR,\mu}(\delta\mu_L-\delta\mu_R)+(\mathcal{L}^{ET}_{LR,T}+\mathcal{L}^{CAR}_{LR,T})(\delta T_L-\delta T_R)+2\mathcal{L}^{DAR}_{LL,\mu}\left(\delta\mu_L-\frac{\mu}{T}\delta T_L\right) \\
+\mathcal{L}^{CAR}_{LR,\mu}\left(\delta\mu_L-\frac{\mu}{T}\delta T_R+\delta\mu_R-\frac{\mu}{T}\delta T_R\right) \\
+\mathcal{L}^{QP}_{LS,\mu}(\delta\mu_L-\delta\mu_S)+\mathcal{L}^{QP}_{LS,T}(\delta T_L-\delta T_S).
\end{aligned} \tag{11}$$

D. Setting the reference chemical potencial to zero: $\mu = 0$

By using the chemical potencial reference to zero, we obtain a simplified version of Eq. (11). In this way, we have:

$$J_L = \mathcal{L}^{ET}_{LR,\mu}(\delta\mu_L - \delta\mu_R) + \mathcal{L}^{CAR}_{LR,\mu}(\delta\mu_L + \delta\mu_R) + (\mathcal{L}^{ET}_{LR,T} + \mathcal{L}^{CAR}_{LR,T})(\delta T_L - \delta T_R) + 2\mathcal{L}^{DAR}_{LL,\mu}\delta\mu_L \\ + \mathcal{L}^{QP}_{LS,\mu}(\delta\mu_L - \delta\mu_S) + \mathcal{L}^{QP}_{LS,T}(\delta T_L - \delta T_S). \quad (12)$$

where we have definitions,

$$\mathcal{L}^{\kappa}_{\alpha\beta,\mu} = \frac{2e}{h}\int dE\ \mathcal{T}^{\kappa}(E)\left(-\frac{\partial f_0}{\partial E}\right), \quad (13)$$

$$\mathcal{L}^{\kappa}_{\alpha\beta,T} = \frac{2e}{hT}\int dE\ E\mathcal{T}^{\kappa}(E)\left(-\frac{\partial f_0}{\partial E}\right). \quad (14)$$

We also have the current from the right lead which is obtained by performing the exchange $L \leftrightarrow R$ in Eq. (12). In this way, we have:

$$J_R = \mathcal{L}^{ET}_{RL,\mu}(\delta\mu_R - \delta\mu_L) + \mathcal{L}^{CAR}_{RL,\mu}(\delta\mu_R + \delta\mu_L) + (\mathcal{L}^{ET}_{RL,T} + \mathcal{L}^{CAR}_{RL,T})(\delta T_R - \delta T_L) + 2\mathcal{L}^{DAR}_{RR,\mu}\delta\mu_R \\ + \mathcal{L}^{QP}_{RS,\mu}(\delta\mu_R - \delta\mu_S) + \mathcal{L}^{QP}_{RS,T}(\delta T_R - \delta T_S). \quad (15)$$

We can also determine the current flowing into the S lead by using current conservation: $J_S = -J_L - J_R$ which leads to:

$$-J_S = 2\mathcal{L}^{CAR}_{LR,\mu}(\delta\mu_L + \delta\mu_R) + 2\mathcal{L}^{DAR}_{LL,\mu}\delta\mu_L + 2\mathcal{L}^{DAR}_{RR,\mu}\delta\mu_R \\ + \mathcal{L}^{QP}_{LS,\mu}(\delta\mu_L - \delta\mu_S) + \mathcal{L}^{QP}_{LS,T}(\delta T_L - \delta T_S) + \mathcal{L}^{QP}_{RS,\mu}(\delta\mu_R - \delta\mu_S) + \mathcal{L}^{QP}_{RS,T}(\delta T_R - \delta T_S). \quad (16)$$

E. Reproducing the equations of appendix C

1. Eq. (C2)

We consider the scenario in which the L lead will be the voltage probe under isothermal conditions. In addition, we also set the voltage at S equal to zero, $\delta\mu_S = 0$. In this way, by using Eq. (12), we have:

$$0 = \mathcal{L}^{ET}_{LR,\mu}(\delta\mu_L - \delta\mu_R) + \mathcal{L}^{CAR}_{LR,\mu}(\delta\mu_L + \delta\mu_R) + 2\mathcal{L}^{DAR}_{LL,\mu}\delta\mu_L + \mathcal{L}^{QP}_{LS,\mu}\delta\mu_L$$

which leads to

$$0 = (\mathcal{L}^{CAR}_{LR,\mu} - \mathcal{L}^{ET}_{LR,\mu})\delta\mu_R + (\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})\delta\mu_L$$

thus,

$$\delta\mu_R = \frac{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L. \tag{17}$$

In order to derive Eq. (C1), we use the isothermal version of Eq. (15)

$$J_R = J_{RS} = (-\mathcal{L}^{ET}_{RL,\mu} + \mathcal{L}^{CAR}_{RL,\mu})\delta\mu_L + (\mathcal{L}^{ET}_{RL,\mu} + \mathcal{L}^{CAR}_{RL,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})\delta\mu_R$$

and by substituting Eq. (17) we have:

$$\begin{aligned} J_{RS} = &(-\mathcal{L}^{ET}_{RL,\mu} + \mathcal{L}^{CAR}_{RL,\mu})\delta\mu_L \\ &+ (\mathcal{L}^{ET}_{RL,\mu} + \mathcal{L}^{CAR}_{RL,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})\frac{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L \end{aligned}$$

which leads to

$$\begin{aligned} (\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})\frac{J_{RS}}{\delta\mu_L} = &(-\mathcal{L}^{ET}_{RL,\mu} + \mathcal{L}^{CAR}_{RL,\mu})(\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}) \\ &+ (\mathcal{L}^{ET}_{RL,\mu} + \mathcal{L}^{CAR}_{RL,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu}) \end{aligned}$$

and by using the symmetry properties, we can further simplify the equation above

$$\begin{aligned} (\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})\frac{J_{RS}}{\delta\mu_L} = &-(\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})^2 \\ &+ (\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu}) \end{aligned}$$

$$\begin{aligned} (\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})\frac{J_{RS}}{\delta\mu_L} = &-(\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})^2 + (\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu})^2 + (\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu})(2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu}) \\ &+ (2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu}) + (2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})(2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu}) \end{aligned}$$

$$\begin{aligned} (\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})\frac{J_{RS}}{\delta\mu_L} = &4\mathcal{L}^{ET}_{LR,\mu}\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{ET}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{CAR}_{LR,\mu} + \mathcal{L}^{QP}_{LS,\mu}\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{QP}_{LS,\mu}\mathcal{L}^{CAR}_{LR,\mu} \\ &+ 2\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{QP}_{RS,\mu}\mathcal{L}^{ET}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{CAR}_{LR,\mu} + \mathcal{L}^{QP}_{RS,\mu}\mathcal{L}^{CAR}_{LR,\mu} \\ &+ 4\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{DAR}_{RR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{QP}_{RS,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{QP}_{LS,\mu} + \mathcal{L}^{QP}_{RS,\mu}\mathcal{L}^{QP}_{LS,\mu} \end{aligned}$$

and performing the multiplications, we have:

$$
\begin{aligned}
(\mathcal{L}^{ET}_{LR,\mu}-\mathcal{L}^{CAR}_{LR,\mu})\frac{J_{RS}}{\delta\mu_L} = 4\mathcal{L}^{ET}_{LR,\mu}\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{ET}_{LR,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{CAR}_{LR,\mu}+\mathcal{L}^{QP}_{LS,\mu}\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{QP}_{LS,\mu}\mathcal{L}^{CAR}_{LR,\mu} \\
+ 2\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{QP}_{RS,\mu}\mathcal{L}^{ET}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{CAR}_{LR,\mu} + \mathcal{L}^{QP}_{RS,\mu}\mathcal{L}^{CAR}_{LR,\mu} \\
+ 4\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{DAR}_{RR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{QP}_{RS,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{QP}_{LS,\mu} + \mathcal{L}^{QP}_{RS,\mu}\mathcal{L}^{QP}_{LS,\mu}.
\end{aligned}
$$

Here we can group some terms as follows:

$$
\begin{aligned}
(\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})\frac{J_{RS}}{\delta\mu_L} = \mathcal{L}^{ET}_{LR,\mu}(4\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{LS,\mu} + \mathcal{L}^{QP}_{RS,\mu}) \\
+ \mathcal{L}^{CAR}_{LR,\mu}(2\mathcal{L}^{DAR}_{LL,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{LS,\mu} + \mathcal{L}^{QP}_{RS,\mu}) \\
+ 4\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{DAR}_{RR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{QP}_{RS,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{QP}_{LS,\mu} + \mathcal{L}^{QP}_{RS,\mu}\mathcal{L}^{QP}_{LS,\mu}.
\end{aligned}
$$

or,

$$
\begin{aligned}
(\mathcal{L}^{ET}_{LR,\mu}-\mathcal{L}^{CAR}_{LR,\mu})\frac{J_{RS}}{\delta\mu_L} = \mathcal{L}^{ET}_{LR,\mu}(4\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}+2\mathcal{L}^{DAR}_{RR,\mu})+\mathcal{L}^{CAR}_{LR,\mu}(2\mathcal{L}^{DAR}_{LL,\mu}+2\mathcal{L}^{DAR}_{RR,\mu})+4\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{DAR}_{RR,\mu} \\
+ 2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{QP}_{RS,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{QP}_{LS,\mu} + (\mathcal{L}^{QP}_{LS,\mu} + \mathcal{L}^{QP}_{RS,\mu})(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu}) + \mathcal{L}^{QP}_{RS,\mu}\mathcal{L}^{QP}_{LS,\mu}. \quad (18)
\end{aligned}
$$

In order to comply with the definitions appearing on the paper, we define:

$$
\begin{aligned}
D = \mathcal{L}^{ET}_{LR,\mu}(2\mathcal{L}^{CAR}_{LR,\mu} + \mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{DAR}_{RR,\mu}) + \mathcal{L}^{CAR}_{LR,\mu}(\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{DAR}_{RR,\mu}) + 2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{DAR}_{RR,\mu} \\
+ \mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{QP}_{RS,\mu} + \mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{QP}_{LS,\mu} + \frac{1}{2}(\mathcal{L}^{QP}_{LS,\mu} + \mathcal{L}^{QP}_{RS,\mu})(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu}) + \frac{1}{2}\mathcal{L}^{QP}_{RS,\mu}\mathcal{L}^{QP}_{LS,\mu}. \quad (19)
\end{aligned}
$$

with this definition, we can rewrite Eq. (18) as follows:

$$
(\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})\frac{J_{RS}}{\delta\mu_L} = 2D \quad (20)
$$

which leads to

$$
\frac{\delta\mu_L}{eJ_{RS}} = \frac{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}{2eD}. \quad (21)
$$

Now, by considering $\delta\mu_S = 0$, we can write:

$$
\delta\mu_L = \delta\mu_L - \delta\mu_S = \Delta\mu_{LS}.
$$

Next, we can write:

$$
R_{RS,LS} = \frac{\Delta\mu_{LS}}{eJ_{RS}} = \frac{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}{2eD}. \quad (22)
$$

2. Eq. (C1)

We start from Eq. (17)

$$\delta\mu_R = \frac{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L$$

which allows us to calculate:

$$\Delta\mu_{RL} = \delta\mu_R - \delta\mu_L = \frac{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L - \delta\mu_L$$

which leads to

$$\Delta\mu_{RL} = \frac{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L - \frac{(\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L$$

then we have:

$$\Delta\mu_{RL} = \frac{(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L. \tag{23}$$

In order to derive Eq. (C1), we use the isothermal version of Eq. (15)

$$eJ_{RS} = \frac{2eD}{(\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})}\delta\mu_L$$

which leads to,

$$\frac{\Delta\mu_{RL}}{eJ_{RS}} = \frac{(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L \frac{(\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})}{2eD\delta\mu_L}$$

$$R_{RS,RL} = \frac{\Delta\mu_{RL}}{eJ_{RS}} = \frac{2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu}}{2eD}. \tag{24}$$

3. Eq. (C3)

Next, we need to determine the voltage difference $\Delta\mu_{RS}$,

$$\Delta\mu_{RS} = \delta\mu_R - \delta\mu_S = \delta\mu_R,$$

since $\delta\mu_S$ is zero. In this case, by using (17)

$$\delta\mu_R = \frac{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L$$

we have:

$$\Delta\mu_{RS} = \frac{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L,$$

Next, we use the isothermal version of Eq. (15)

$$eJ_{RS} = \frac{2eD}{(\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})}\delta\mu_L$$

which allows us to write:

$$R_{RS,RS} = \frac{\Delta\mu_{RS}}{eJ_{RS}} = \frac{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L \frac{(\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})}{2eD\delta\mu_L}$$

$$R_{RS,RS} = \frac{\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu}}{2eD}. \tag{25}$$

4. Eq. (C4)

In order to derive the last three equations of appendix B, we consider the superconducting electrode to be floating ($J_S = 0$) and denote $J_R = -J_L = J_{RL}$.

We start from Eq. (16),

$$\begin{aligned}-J_S = {} & 2\mathcal{L}^{CAR}_{LR,\mu}(\delta\mu_L + \delta\mu_R) + 2\mathcal{L}^{DAR}_{LL,\mu}\delta\mu_L + 2\mathcal{L}^{DAR}_{RR,\mu}\delta\mu_R \\ & + \mathcal{L}^{QP}_{LS,\mu}(\delta\mu_L - \delta\mu_S) + \mathcal{L}^{QP}_{LS,T}(\delta T_L - \delta T_S) + \mathcal{L}^{QP}_{RS,\mu}(\delta\mu_R - \delta\mu_S) + \mathcal{L}^{QP}_{RS,T}(\delta T_R - \delta T_S).\end{aligned}$$

and by setting $J_S = 0$ we obtain:

$$\begin{aligned}& 2\mathcal{L}^{CAR}_{LR,\mu}(\delta\mu_L + \delta\mu_R) + 2\mathcal{L}^{DAR}_{LL,\mu}\delta\mu_L + 2\mathcal{L}^{DAR}_{RR,\mu}\delta\mu_R \\ & + \mathcal{L}^{QP}_{LS,\mu}(\delta\mu_L - \delta\mu_S) + \mathcal{L}^{QP}_{LS,T}(\delta T_L - \delta T_S) + \mathcal{L}^{QP}_{RS,\mu}(\delta\mu_R - \delta\mu_S) + \mathcal{L}^{QP}_{RS,T}(\delta T_R - \delta T_S) = 0,\end{aligned}$$

and by setting $\delta\mu_S = 0$ and isothermal conditions, we have:

$$2\mathcal{L}^{CAR}_{LR,\mu}(\delta\mu_L + \delta\mu_R) + 2\mathcal{L}^{DAR}_{LL,\mu}\delta\mu_L + 2\mathcal{L}^{DAR}_{RR,\mu}\delta\mu_R + \mathcal{L}^{QP}_{LS,\mu}\delta\mu_L + \mathcal{L}^{QP}_{RS,\mu}\delta\mu_R = 0$$

and by grouping similar terms we obtain:

$$(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})\delta\mu_L + (2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})\delta\mu_R = 0$$

which leads to

$$\delta\mu_R = -\frac{(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu}}\delta\mu_L.$$

Next, we consider the current J_R under isothermal conditions:

$$J_R = J_{RL} = -J_L = \mathcal{L}^{ET}_{RL,\mu}(\delta\mu_R - \delta\mu_L) + \mathcal{L}^{CAR}_{RL,\mu}(\delta\mu_R + \delta\mu_L) + 2\mathcal{L}^{DAR}_{RR,\mu}\delta\mu_R + \mathcal{L}^{QP}_{RS,\mu}\delta\mu_R$$

which can be written as follows:

$$J_{RL} = (\mathcal{L}^{ET}_{RL,\mu} + \mathcal{L}^{CAR}_{RL,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})\delta\mu_R + (\mathcal{L}^{CAR}_{RL,\mu} - \mathcal{L}^{ET}_{RL,\mu})\delta\mu_L,$$

and by substituting $\delta\mu_R$ we obtain:

$$J_{RL} = (\mathcal{L}^{ET}_{RL,\mu} + \mathcal{L}^{CAR}_{RL,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})\left[-\frac{(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu}}\delta\mu_L\right] + (\mathcal{L}^{CAR}_{RL,\mu} - \mathcal{L}^{ET}_{RL,\mu})\delta\mu_L,$$

and multiplying both sides by $2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu}$ we obtain,

$$\begin{aligned}\frac{J_{RL}}{\delta\mu_L}(2\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu}) &= -(\mathcal{L}^{ET}_{RL,\mu}+\mathcal{L}^{CAR}_{RL,\mu}+2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu})(2\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}+\mathcal{L}^{QP}_{LS,\mu})\\ &\quad + (\mathcal{L}^{CAR}_{RL,\mu} - \mathcal{L}^{ET}_{RL,\mu})(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu}),\end{aligned}$$

and performing the products and by using the symmetry $\mathcal{L}^{CAR}_{RL,\mu} = \mathcal{L}^{CAR}_{LR,\mu}$, we have:

$$\begin{aligned}\frac{J_{RL}}{\delta\mu_L}(2\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu}) &= -(\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu})(2\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}+\mathcal{L}^{QP}_{LS,\mu})\\ &\quad + (\mathcal{L}^{CAR}_{LR,\mu} - \mathcal{L}^{ET}_{LR,\mu})(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu}),\end{aligned}$$

or

$$\begin{aligned}\frac{J_{RL}}{\delta\mu_L}(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu}) &= -(2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})\\ &\quad - (\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})\\ &\quad - (\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu})(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})\end{aligned}$$

and performing some manipulations,

$$\begin{aligned}&\frac{J_{RL}}{\delta\mu_L}(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})\\ &\quad = -\mathcal{L}^{ET}_{LR,\mu}(4\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu}) - \mathcal{L}^{CAR}_{LR,\mu}(2\mathcal{L}^{DAR}_{RR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu}) - 4\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{DAR}_{RR,\mu}\\ &\qquad - 2\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{QP}_{LS,\mu} - 2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{QP}_{RS,\mu} - (\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu})(\mathcal{L}^{QP}_{LS,\mu} + \mathcal{L}^{QP}_{RS,\mu}) - \mathcal{L}^{QP}_{LS,\mu}\mathcal{L}^{QP}_{RS,\mu}.\end{aligned}$$

Let's divide by $-1/2$ both side of equation above, which allows us to write:

$$
\begin{aligned}
&-\frac{J_{RL}}{2\delta\mu_L}(2\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu}) \\
&\qquad=\mathcal{L}^{ET}_{LR,\mu}(2\mathcal{L}^{CAR}_{LR,\mu}+\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{DAR}_{LL,\mu})+\mathcal{L}^{CAR}_{LR,\mu}(\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{DAR}_{LL,\mu})+2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{DAR}_{RR,\mu} \\
&\frac{1}{2}\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{QP}_{LS,\mu}+\frac{1}{2}\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{QP}_{RS,\mu}+\frac{1}{2}(\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu})(\mathcal{L}^{QP}_{LS,\mu}+\mathcal{L}^{QP}_{RS,\mu})+\frac{1}{2}\mathcal{L}^{QP}_{LS,\mu}\mathcal{L}^{QP}_{RS,\mu}.
\end{aligned}
\tag{26}
$$

Once again we define the denominator D as,

$$
\begin{aligned}
D&=\mathcal{L}^{ET}_{LR,\mu}(2\mathcal{L}^{CAR}_{LR,\mu}+\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{DAR}_{LL,\mu})+\mathcal{L}^{CAR}_{LR,\mu}(\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{DAR}_{LL,\mu})+2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{DAR}_{RR,\mu} \\
&\frac{1}{2}\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{QP}_{LS,\mu}+\frac{1}{2}\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{QP}_{RS,\mu}+\frac{1}{2}(\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu})(\mathcal{L}^{QP}_{LS,\mu}+\mathcal{L}^{QP}_{RS,\mu})+\frac{1}{2}\mathcal{L}^{QP}_{LS,\mu}\mathcal{L}^{QP}_{RS,\mu},
\end{aligned}
\tag{27}
$$

which allows us to write Eq. (26) as follows:

$$
-\frac{J_{RL}}{\delta\mu_L}=\frac{2D}{2\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu}}. \tag{28}
$$

By using Eq. (28), we can write $R_{RL,LS}$, we obtain:

$$
R_{RL,LS}=\frac{\Delta\mu_{LS}}{eJ_{RL}}=-\frac{\mathcal{L}^{CAR}_{LR,\mu}+\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu}/2}{eD}, \tag{29}
$$

where we have used $\delta\mu_L=\delta\mu_L-\delta\mu_S=\Delta\mu_{LS}$.

5. Eq. (C6)

Next, we determine $R_{RL,RS}$ which may be obtained by using the expression,

$$
\delta\mu_R=\Delta\mu_{RS}=-\frac{(2\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}+\mathcal{L}^{QP}_{LS,\mu})}{2\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu}}\delta\mu_L.
$$

and we notice that $\Delta\mu_{RS}=\delta\mu_R-\delta\mu_S$, since $\delta\mu_S=0$.

In this way, by dividing Eq. (28) by $\Delta\mu_{RS}$ we obtain:

$$
R_{RL,RS}=\frac{\Delta\mu_{RS}}{eJ_{RL}}=-\frac{(2\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}+\mathcal{L}^{QP}_{LS,\mu})}{2\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu}}\delta\mu_L\left[-\frac{2\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu}}{2eD\delta\mu_L}\right]
$$

$$
R_{RL,RS}=\frac{\Delta\mu_{RS}}{eJ_{RL}}=\frac{(2\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}+\mathcal{L}^{QP}_{LS,\mu})}{2eD}, \tag{30}
$$

which is Eq. (C5) of the paper.

6. Eq. (C7)

The last equation of appendix C is obtained first through the calculation of $\Delta\mu_{LR}$, in this way, we have:

$$\Delta\mu_{LR} = \delta\mu_L - \delta\mu_R$$

$$\Delta\mu_{LR} = \delta\mu_L - \left[-\frac{(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu}} \delta\mu_L \right]$$

$$\Delta\mu_{LR} = \delta\mu_L - \left[-\frac{(2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu}} \delta\mu_L \right]$$

$$\frac{\Delta\mu_{LR}}{\delta\mu_L} = 1 + \frac{2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu}}{2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu}}$$

$$\Delta\mu_{LR} = \frac{(4\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{LS,\mu} + \mathcal{L}^{QP}_{RS,\mu})}{2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu}} \delta\mu_L$$

and by noticing that $\Delta\mu_{RL} = -\Delta\mu_{LR}$, we have:

$$\Delta\mu_{RL} = -\frac{(4\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{LS,\mu} + \mathcal{L}^{QP}_{RS,\mu})}{2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu}} \delta\mu_L$$

Next, we divide Eq. (28) by this expression, thus we have:

$$\frac{\Delta\mu_{RL}}{eJ_{RL}} = -\frac{(4\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{LS,\mu} + \mathcal{L}^{QP}_{RS,\mu})}{2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu}} \delta\mu_L \left[-\frac{2\mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu}}{2eD\delta\mu_L} \right],$$

then we obtain eq. (C7):

$$R_{RL,RL} = \frac{\Delta\mu_{RL}}{eJ_{RL}} = \frac{\mathcal{L}^{DAR}_{LL,\mu} + 2\mathcal{L}^{CAR}_{LR,\mu} + \mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{LS,\mu}/2 + \mathcal{L}^{QP}_{RS,\mu}/2}{eD}. \tag{31}$$

II. APPENDIX D

Next, we move forward to determine the equations appearing in Appendix D in the paper, the currents for nonisothermal conditions. We still consider the more general case in which the quaisparticles is included.

A. Equations (D1), (D2) and (D3)

For equations (D1), (D2) and (D3) the electrode L is taken as the probe. By using Eq. (12),

$$J_L = \mathcal{L}^{ET}_{LR,\mu}(\delta\mu_L - \delta\mu_R) + \mathcal{L}^{CAR}_{LR,\mu}(\delta\mu_L + \delta\mu_R) + (\mathcal{L}^{ET}_{LR,T} + \mathcal{L}^{CAR}_{LR,T})(\delta T_L - \delta T_R) + 2\mathcal{L}^{DAR}_{LL,\mu}\delta\mu_L + \mathcal{L}^{QP}_{LS,\mu}(\delta\mu_L - \delta\mu_S) + \mathcal{L}^{QP}_{LS,T}(\delta T_L - \delta T_S).$$

By setting $J_L = 0$, $\Delta T_{LS} = \delta T_L - \delta T_S$, $\Delta T_{LR} = \delta T_L - \delta T_R$, $\delta\mu_S = 0$, we obtain:

$$0 = \mathcal{L}^{ET}_{LR,\mu}(\delta\mu_L - \delta\mu_R) + \mathcal{L}^{CAR}_{LR,\mu}(\delta\mu_L + \delta\mu_R) + (\mathcal{L}^{ET}_{LR,T} + \mathcal{L}^{CAR}_{LR,T})\Delta T_{LR} + 2\mathcal{L}^{DAR}_{LL,\mu}\delta\mu_L + \mathcal{L}^{QP}_{LS,\mu}\delta\mu_L + \mathcal{L}^{QP}_{LS,T}\Delta T_{LS},$$

and by grouping the similar terms we have:

$$0 = (\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})\delta\mu_L + (\mathcal{L}^{CAR}_{LR,\mu} - \mathcal{L}^{ET}_{LR,\mu})\delta\mu_R + (\mathcal{L}^{ET}_{LR,T} + \mathcal{L}^{CAR}_{LR,T})\Delta T_{LR} + \mathcal{L}^{QP}_{LS,T}\Delta T_{LS},$$

which allows us express $\delta\mu_R$ in terms of $\delta\mu_L$ and the temperature differences. Thus, we have:

$$\delta\mu_R = \frac{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})\delta\mu_L + (\mathcal{L}^{ET}_{LR,T} + \mathcal{L}^{CAR}_{LR,T})\Delta T_{LR} + \mathcal{L}^{QP}_{LS,T}\Delta T_{LS}}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}},$$

and it is convenient to split the chemical and thermal differences, so we have:

$$\delta\mu_R = \frac{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L + \frac{(\mathcal{L}^{ET}_{LR,T} + \mathcal{L}^{CAR}_{LR,T})\Delta T_{LR} + \mathcal{L}^{QP}_{LS,T}\Delta T_{LS}}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}} \quad (32)$$

where we have exchanged the order of the terms in the denominator to get ride of the minus signal.

1. Eq. (D1)

The current J_{RS} is given by Eq. (15),

$$J_{RS} = \mathcal{L}^{ET}_{RL,\mu}(\delta\mu_R - \delta\mu_L) + \mathcal{L}^{CAR}_{RL,\mu}(\delta\mu_R + \delta\mu_L) + (\mathcal{L}^{ET}_{RL,T} + \mathcal{L}^{CAR}_{RL,T})(\delta T_R - \delta T_L) + 2\mathcal{L}^{DAR}_{RR,\mu}\delta\mu_R + \mathcal{L}^{QP}_{RS,\mu}(\delta\mu_R - \delta\mu_S) + \mathcal{L}^{QP}_{RS,T}(\delta T_R - \delta T_S).$$

which may be written as follows:

$$J_{RS} = (\mathcal{L}^{ET}_{RL,\mu} + \mathcal{L}^{CAR}_{RL,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})\delta\mu_R + (\mathcal{L}^{CAR}_{RL,\mu} - \mathcal{L}^{ET}_{RL,\mu})\delta\mu_L + (\mathcal{L}^{ET}_{RL,T} + \mathcal{L}^{CAR}_{RL,T})\Delta T_{RL} + \mathcal{L}^{QP}_{RS,T}\Delta T_{RS}.$$

Next, we substitute the expression for $\delta\mu_R$ which leads to

$$J_{RS} = (\mathcal{L}^{ET}_{RL,\mu} + \mathcal{L}^{CAR}_{RL,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})\left[\frac{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L + \frac{(\mathcal{L}^{ET}_{LR,T} + \mathcal{L}^{CAR}_{LR,T})\Delta T_{LR} + \mathcal{L}^{QP}_{LS,T}\Delta T_{LS}}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\right] + (\mathcal{L}^{CAR}_{RL,\mu} - \mathcal{L}^{ET}_{RL,\mu})\delta\mu_L + (\mathcal{L}^{ET}_{RL,T} + \mathcal{L}^{CAR}_{RL,T})\Delta T_{RL} + \mathcal{L}^{QP}_{RS,T}\Delta T_{RS},$$

which may be written as follows:

$$J_{RS} = \frac{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L + (\mathcal{L}^{CAR}_{RL,\mu} - \mathcal{L}^{ET}_{RL,\mu})\delta\mu_L + (\mathcal{L}^{ET}_{RL,\mu} + \mathcal{L}^{CAR}_{RL,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})\left[\frac{(\mathcal{L}^{ET}_{LR,T} + \mathcal{L}^{CAR}_{LR,T})\Delta T_{LR} + \mathcal{L}^{QP}_{LS,T}\Delta T_{LS}}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\right] + (\mathcal{L}^{ET}_{RL,T} + \mathcal{L}^{CAR}_{RL,T})\Delta T_{RL} + \mathcal{L}^{QP}_{RS,T}\Delta T_{RS},$$

and putting all the terms of common denominator,

$$J_{RS} = \left[\frac{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu}) - (\mathcal{L}^{CAR}_{RL,\mu} - \mathcal{L}^{ET}_{RL,\mu})^2}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\right]\delta\mu_L + (\mathcal{L}^{ET}_{RL,\mu} + \mathcal{L}^{CAR}_{RL,\mu} + 2\mathcal{L}^{DAR}_{RR,\mu} + \mathcal{L}^{QP}_{RS,\mu})\left[\frac{(\mathcal{L}^{ET}_{LR,T} + \mathcal{L}^{CAR}_{LR,T})\Delta T_{LR} + \mathcal{L}^{QP}_{LS,T}\Delta T_{LS}}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\right] + (\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu})\left[\frac{(\mathcal{L}^{ET}_{RL,T} + \mathcal{L}^{CAR}_{RL,T})\Delta T_{RL} + \mathcal{L}^{QP}_{RS,T}\Delta T_{RS}}{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}\right]. \quad (33)$$

The numerator appearing in Eq. (33) may be expressed as follows:

$$
\begin{aligned}
&(\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu})(\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}+\mathcal{L}^{QP}_{LS,\mu})-(\mathcal{L}^{CAR}_{RL,\mu}-\mathcal{L}^{ET}_{RL,\mu})^2\\
&=(\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu})^2-(\mathcal{L}^{CAR}_{RL,\mu}-\mathcal{L}^{ET}_{RL,\mu})^2\\
&+(\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu})(2\mathcal{L}^{DAR}_{LL,\mu}+\mathcal{L}^{QP}_{LS,\mu})+(\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu})(2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu})\\
&+(2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu})(2\mathcal{L}^{DAR}_{LL,\mu}+\mathcal{L}^{QP}_{LS,\mu})
\end{aligned}
$$

which leads to

$$
\begin{aligned}
&(\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu})(\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}+\mathcal{L}^{QP}_{LS,\mu})-(\mathcal{L}^{CAR}_{RL,\mu}-\mathcal{L}^{ET}_{RL,\mu})^2\\
&=4\mathcal{L}^{ET}_{LR,\mu}\mathcal{L}^{CAR}_{LR,\mu}+(\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu})(2\mathcal{L}^{DAR}_{LL,\mu}+2\mathcal{L}^{DAR}_{RR,\mu})\\
&+(\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu})(\mathcal{L}^{QP}_{LS,\mu}+\mathcal{L}^{QP}_{RS,\mu})+(2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu})(2\mathcal{L}^{DAR}_{LL,\mu}+\mathcal{L}^{QP}_{LS,\mu})
\end{aligned}
$$

and with some algebra we write,

$$
\begin{aligned}
&(\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu})(\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}+\mathcal{L}^{QP}_{LS,\mu})-(\mathcal{L}^{CAR}_{RL,\mu}-\mathcal{L}^{ET}_{RL,\mu})^2\\
&=\mathcal{L}^{ET}_{LR,\mu}(2\mathcal{L}^{DAR}_{LL,\mu}+4\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{RR,\mu})+\mathcal{L}^{CAR}_{LR,\mu}(2\mathcal{L}^{DAR}_{LL,\mu}+2\mathcal{L}^{DAR}_{RR,\mu})+4\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{DAR}_{RR,\mu}\\
&+(\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu})(\mathcal{L}^{QP}_{LS,\mu}+\mathcal{L}^{QP}_{RS,\mu})+2\mathcal{L}^{DAR}_{RR,\mu}\mathcal{L}^{QP}_{LS,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}\mathcal{L}^{QP}_{RS,\mu}+\mathcal{L}^{QP}_{LS,\mu}\mathcal{L}^{QP}_{RS,\mu}
\end{aligned}
$$

and by comparing this expression, we see the right hand side is equal to Eq. (27) multiplied by 2. In this way, we write:

$$(\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu})(\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}+\mathcal{L}^{QP}_{LS,\mu})-(\mathcal{L}^{CAR}_{RL,\mu}-\mathcal{L}^{ET}_{RL,\mu})^2=2D.$$

By substituting the expression above into Eq. (33)

$$
\begin{aligned}
J_{RS}=&\left[\frac{2D}{\mathcal{L}^{ET}_{LR,\mu}-\mathcal{L}^{CAR}_{LR,\mu}}\right]\delta\mu_L\\
&+(\mathcal{L}^{ET}_{RL,\mu}+\mathcal{L}^{CAR}_{RL,\mu}+2\mathcal{L}^{DAR}_{RR,\mu}+\mathcal{L}^{QP}_{RS,\mu})\left[\frac{(\mathcal{L}^{ET}_{LR,T}+\mathcal{L}^{CAR}_{LR,T})\Delta T_{LR}+\mathcal{L}^{QP}_{LS,T}\Delta T_{LS}}{\mathcal{L}^{ET}_{LR,\mu}-\mathcal{L}^{CAR}_{LR,\mu}}\right]\\
&+(\mathcal{L}^{ET}_{LR,\mu}-\mathcal{L}^{CAR}_{LR,\mu})\left[\frac{(\mathcal{L}^{ET}_{RL,T}+\mathcal{L}^{CAR}_{RL,T})\Delta T_{RL}+\mathcal{L}^{QP}_{RS,T}\Delta T_{RS}}{\mathcal{L}^{ET}_{LR,\mu}-\mathcal{L}^{CAR}_{LR,\mu}}\right],
\end{aligned}\tag{34}
$$

Eq. (34) can be written in a more compact form by using the set of equations of appendix C. In fact, we have:

$$
\begin{aligned}
\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}+\mathcal{L}^{QP}_{LS,\mu}&=2eDR_{RS,RS}\\
\mathcal{L}^{ET}_{LR,\mu}-\mathcal{L}^{CAR}_{LR,\mu}&=2eDR_{RS,LS}
\end{aligned}
$$

which allows us to write,

$$\frac{\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}+\mathcal{L}^{QP}_{LS,\mu}}{\mathcal{L}^{ET}_{LR,\mu}-\mathcal{L}^{CAR}_{LR,\mu}}=\frac{2eDR_{RS,RS}}{2eDR_{RS,LS}}=\frac{R_{RS,RS}}{R_{RS,LS}}.$$

and the first term of Eq. (34) is expressed in terms of $R_{RS,LS}$ as well. In fact, we write Eq. (34) as follows:

$$J_{RS}=\frac{\delta\mu_L}{eR_{RS,LS}}+\frac{R_{RS,RS}}{R_{RS,LS}}[(\mathcal{L}^{ET}_{LR,T}+\mathcal{L}^{CAR}_{LR,T})\Delta T_{LR}+\mathcal{L}^{QP}_{LS,T}\Delta T_{LS}]$$
$$+(\mathcal{L}^{ET}_{RL,T}+\mathcal{L}^{CAR}_{RL,T})\Delta T_{RL}+\mathcal{L}^{QP}_{RS,T}\Delta T_{RS},$$

or

$$J_{RS}=\frac{\delta\mu_L}{eR_{RS,LS}}+\left[\frac{R_{RS,RS}}{R_{RS,LS}}-1\right](\mathcal{L}^{ET}_{LR,T}+\mathcal{L}^{CAR}_{LR,T})\Delta T_{LR}+\frac{R_{RS,RS}}{R_{RS,LS}}(\mathcal{L}^{QP}_{RS,T}\Delta T_{RS}+\mathcal{L}^{QP}_{LS,T}\Delta T_{LS}). \tag{35}$$

Next, we notice that

$$R_{RS,RS}-R_{RS,LS}=\frac{\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}+\mathcal{L}^{QP}_{LS,\mu}}{2eD}-\frac{(\mathcal{L}^{ET}_{LR,\mu}-\mathcal{L}^{CAR}_{LR,\mu})}{2eD}$$

which leads to

$$R_{RS,RS}-R_{RS,LS}=\frac{2\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}+\mathcal{L}^{QP}_{LS,\mu}}{2eD}=\frac{\mathcal{L}^{CAR}_{LR,\mu}+\mathcal{L}^{DAR}_{LL,\mu}+\mathcal{L}^{QP}_{LS,\mu}/2}{eD}=R_{RL,RS}.$$

Substituting this expression back into Eq. (35)

$$J_{RS}=\frac{\Delta\mu_{LS}}{eR_{RS,LS}}+\frac{R_{RL,RS}}{R_{RS,LS}}(\mathcal{L}^{ET}_{LR,T}+\mathcal{L}^{CAR}_{LR,T})\Delta T_{LR}+\frac{R_{RS,RS}}{R_{RS,LS}}(\mathcal{L}^{QP}_{RS,T}\Delta T_{RS}+\mathcal{L}^{QP}_{LS,T}\Delta T_{LS}), \tag{36}$$

where we have used $\delta\mu_S=0$ such that $\Delta\mu_{LS}=\delta\mu_L-\delta\mu_S=\delta\mu_L$.

2. Eq. (D2)

Next, we proceed to demonstrate Eq. (D2). In this case, we start from Eq. (32)

$$\delta\mu_R = \frac{(\mathcal{L}^{ET}_{LR,\mu}+\mathcal{L}^{CAR}_{LR,\mu}+2\mathcal{L}^{DAR}_{LL,\mu}+\mathcal{L}^{QP}_{LS,\mu})}{\mathcal{L}^{ET}_{LR,\mu}-\mathcal{L}^{CAR}_{LR,\mu}}\delta\mu_L + \frac{(\mathcal{L}^{ET}_{LR,T}+\mathcal{L}^{CAR}_{LR,T})\Delta T_{LR}+\mathcal{L}^{QP}_{LS,T}\Delta T_{LS}}{\mathcal{L}^{ET}_{LR,\mu}-\mathcal{L}^{CAR}_{LR,\mu}}$$

and express $\delta\mu_L$ in terms of $\delta\mu_R$. Thus, we have:

$$\delta\mu_L = \frac{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}\delta\mu_R - \frac{(\mathcal{L}^{ET}_{LR,T} + \mathcal{L}^{CAR}_{LR,T})\Delta T_{LR} + \mathcal{L}^{QP}_{LS,T}\Delta T_{LS}}{\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu}},$$

and since $\delta\mu_S = 0$, we can rewrite both $\delta\mu_L$ and $\delta\mu_R$ as $\Delta\mu_L$ and $\Delta\mu_R$, respectively, which leads to:

$$\Delta\mu_{LS} = \frac{\mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu}}{(\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu})}\Delta\mu_{RS} - \frac{(\mathcal{L}^{ET}_{LR,T} + \mathcal{L}^{CAR}_{LR,T})\Delta T_{LR} + \mathcal{L}^{QP}_{LS,T}\Delta T_{LS}}{\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu}}. \tag{37}$$

Here it is convenient to rewrite Eq. (37) in terms of previous definitions:

$$\begin{aligned}\mathcal{L}^{ET}_{LR,\mu} + \mathcal{L}^{CAR}_{LR,\mu} + 2\mathcal{L}^{DAR}_{LL,\mu} + \mathcal{L}^{QP}_{LS,\mu} &= 2eDR_{RS,RS},\\ \mathcal{L}^{ET}_{LR,\mu} - \mathcal{L}^{CAR}_{LR,\mu} &= 2eDR_{RS,LS,}\end{aligned}$$

which leads to

$$\Delta\mu_{LS} = \frac{R_{RS,LS}}{R_{RS,RS}}\Delta\mu_{RS} - \frac{(\mathcal{L}^{ET}_{LR,T} + \mathcal{L}^{CAR}_{LR,T})\Delta T_{LR} + \mathcal{L}^{QP}_{LS,T}\Delta T_{LS}}{2eDR_{RS,RS}}. \tag{38}$$

Substituting Eq. (37) into Eq. (36) we obtain,

$$J_{RS} \;=\; \frac{\Delta\mu_{LS}}{eR_{RS,LS}} + \frac{R_{RL,RS}}{R_{RS,LS}}(\mathcal{L}^{ET}_{LR,T} + \mathcal{L}^{CAR}_{LR,T})\Delta T_{LR} + \frac{R_{RS,RS}}{R_{RS,LS}}(\mathcal{L}^{QP}_{RS,T}\Delta T_{RS} + \mathcal{L}^{QP}_{LS,T}\Delta T_{LS}),$$