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Bilkent University Electrical and Electronics Engineering

EE202-Circuit Theory

Lab 3:

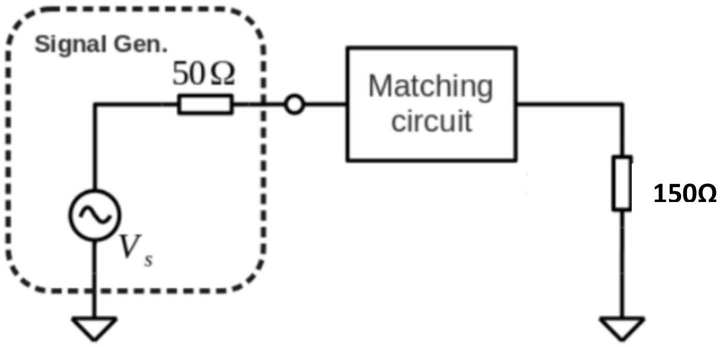
Maximum Power Transfer

Introduction:

This lab aims to design and understand the maximum power transfer theory with two different circuits. The lab expects us to transfer maximum power to 150Ω load from a voltage source with output impedance 50Ω at a frequency between 10 and 15MhZ. I used the L and T section model to transfer maximum power to 150Ω load resistance.

Part 1: Software Implementation

Analysis:



To find the maximum power transfer we write the equation (eq. 1.1) of power according to the circuit and take the derivative with respect to R_L. When we make it equal to zero, we should get the condition when power is maximum. When we substitute the current equation (eq. 1.2) into the power's equation we get our final equation (eq. 1.3) and take the derivative for the real part of the equation with respect to R_L (eq. 1.4). Considering this equation, if we take R_L = R_S we maximize $R_e\{P_L\}$. Also, we choose $X_L = -X_S$. So, maximum power in the load will be eq.1.5.

$$P_L = I^2{}_{rms} \cdot R_L \quad \text{(eq. 1.1)}$$

$$P_L = \frac{I^2}{2} R_L \quad \text{(eq. 1.1)}$$

$$I = \frac{V_S}{Z_L + Z_S} \quad \text{(eq. 1.2)}$$

$$P_L = Re\{\frac{|V_S|^2}{2|Z_L+Z_S|^2} \,.\, Z_L\} = \frac{|V_s|^2}{2(R_L+R_s)^2+(X_L+X_s)^2} \,.\, R_L \quad \text{(eq. 1.3)}$$

$$\frac{|V_s|^2({R_S}^2-{R_L}^2)}{2(R_L+R_s)^4} = 0 \quad \text{(eq. 1.4)}$$

$$P_{Lmax} = \frac{|V_s|^2}{8R_S} \quad \text{(eq. 1.5)}$$

To transform 150Ω to 50Ω we use impedence matching circuits. First I tried L section impedence matching then, T section impdence matching . At the resonance frequency $\omega_0 = 14MHzx2\pi f$ which I choose myself the circuit will act like a parallel tuned circuit. For the L section, the equations are below:

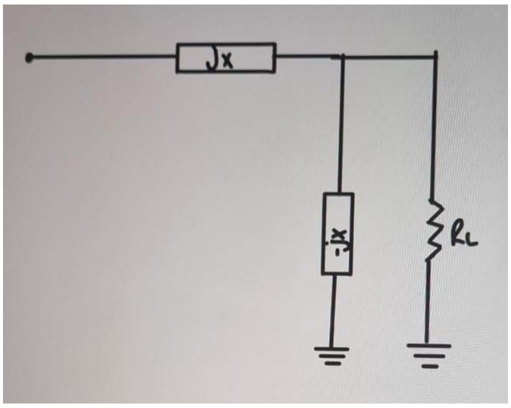


Figure 1: The General Circuit of the L Section Model

$$R_p = (Q^2 + 1)\, R_s \quad \text{(eq.2.1)}$$

$$L_p = L_s\, (1+\frac{1}{Q^2}) \quad \text{(eq.2.2)}$$

$$C = C_p = C_L \quad \text{(eq.2.3)}$$

$$Q = \frac{\omega_o L_S}{R_s} \quad \text{(eq.2.4)}$$

$$\omega_o = \frac{1}{\sqrt{CL_p}} \quad \text{(eq.2.5)}$$

Now, I calculated all of these equations with respect to the values given and I picked. However, in the hardware part, since I couldn't find the proper inductance values available so I needed to change the frequency value I picked (f is now f= 11.2 MHz in the hardware part, so the equations will be recalculated according to the new value, therefore ω will be 70.37 rps) Also, I choose 3Vpp for input.

$$50 = (\,Q^2 + 1)\,150$$

$$Q = \sqrt{2}$$

$$L_S = \frac{R_S Q}{\omega_o} = 1.005 \mu H$$

$$C_p = \frac{1}{\omega_o{}^2 L_p (1+\frac{1}{Q^2})} = 133.95\ \rho F$$

In the T section model, again our aim is to create a resonation between L and C so that we ,make the input and load impedences equal in order to get zero impedence through the circuit in the remaining section. I applied 8Vpp (peak to peak voltage) and 14MHz frequncy this time. Fort he T section the equations are below:

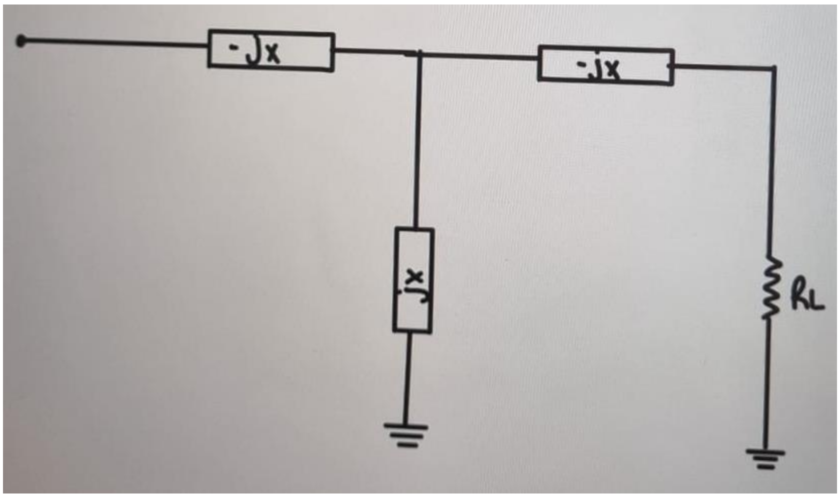


Figure 2: The General Circuit of the T Section Model

$$Z_{in} = jX + (-jX) \,//\, (\,jX + Z_L\,) \quad \text{(eq. 3.1)}$$

$$X = \sqrt{Z_L Z_{in}} \quad \text{(eq. 3.2)}$$

$$L = X\omega^{-1} \quad \text{(eq. 3.3)}$$

$$C = (X\omega)^{-1} \quad \text{(eq. 3.4)}$$

Substituting my values into the equation I get results as below:

$$X = \sqrt{150x50} = 50\sqrt{3}\Omega \approx 86.6\Omega$$

$$L = 984.5\, nH \;\; , C = 131.3 \;\; pF$$

Simulations:

Power transfer compared in load an in the source is slightly different. As seen in Figure 2 for T section the expected value from calculations is 320mW and it is distributed equally in the circuit so the error is 7.597% and as seen in Figure 4 for L section the expected value from calculations is 45mW and it is distributed equally in the circuit so the error is 9% because I found 40.95mW in the simulation.

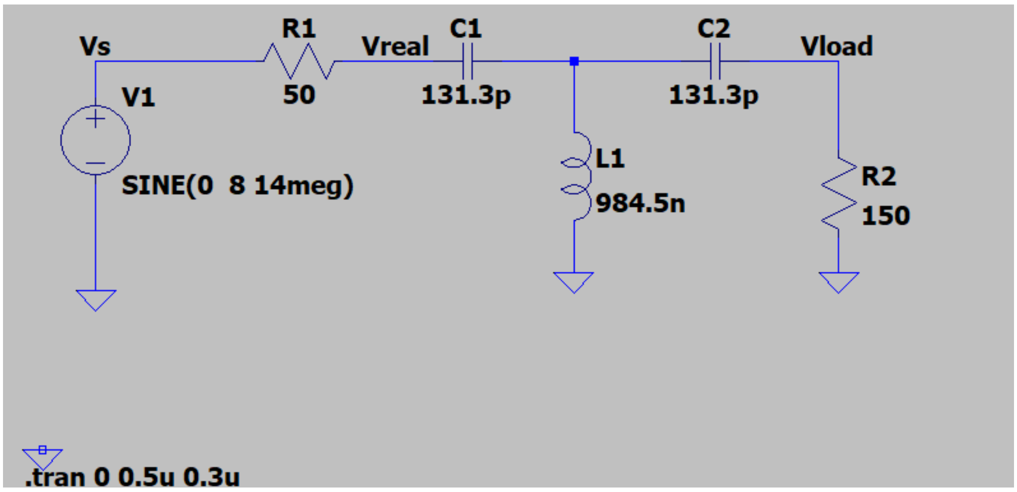


Figure 3: The Circuit of the T Section Model

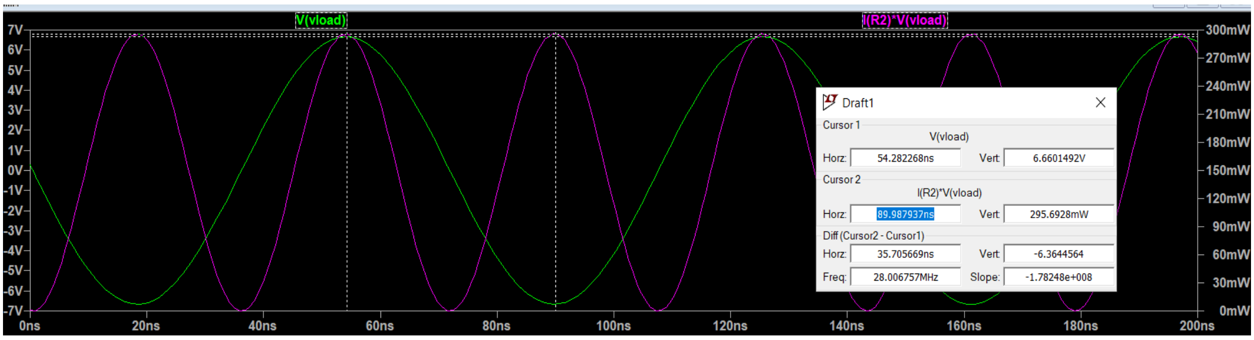


Figure 4: Simulation of T section model , voltage and power transfered to load can be seen by cursors

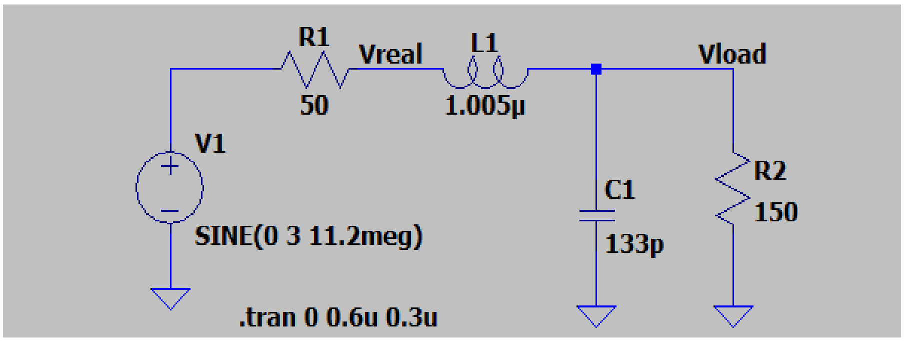


Figure 5: The circuit of the L section model

(Note: as seen frequency has been changed because of the problems in hardware part)

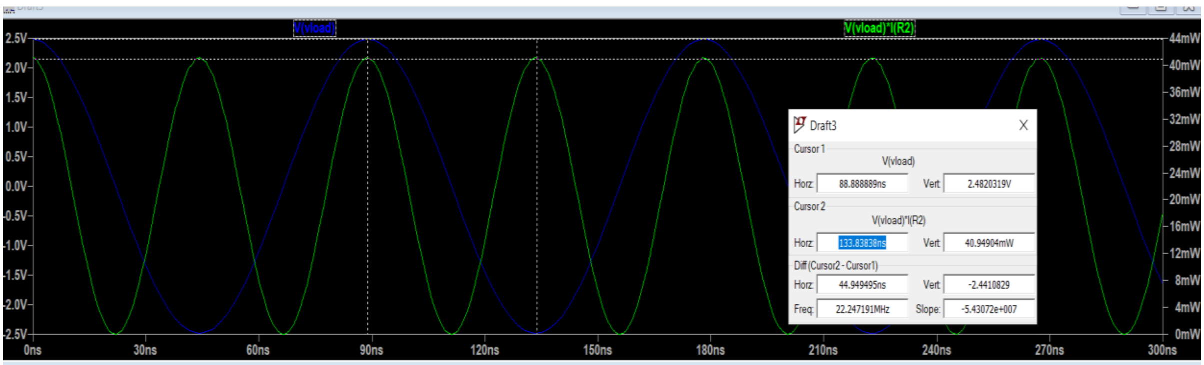


Figure 6: Simulation of L section model , voltage and power transfered to load can be seen by cursors

Matching Circuit Type	Calculated Available Power at Source/Calculated Max Power Delivered to Load with Matching Circuit (mW)	Measured Max Power Delivered to Load in Simulation (mW)	Error (%)
T section model	320	295.69	7.597
L section model	45	40.95	9

Table 1 : Power Comparison in Simulation in Both T and L section models

Part 2: Hardware Implementation

In the first step of the hardware lab, a 47Ω resistor is directly connected to the source and the input voltage. V_{pp} is 4.80V .Therefore, the powerr at the source is,

$$P = (\frac{4.8}{2})^2 \times \frac{1}{47} = 122.55mW$$

Figure 7: When 47Ω directly connected to the signal generator

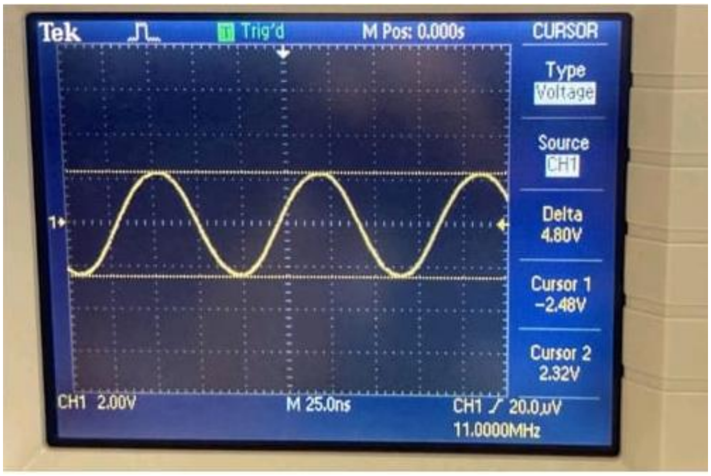


Figure 8 : When 47Ω directly connected to the signal generator values shown in the oscilloscope screen

In the hardware part, I implemented first L section circuit, in this part the reason why I could't find the inductors with the corresponding value I picked in the software part, I changed my frequency value to get a value very close to 1µH. Therefore, chose 11.2MHz. Also L = 1µH and C = 120 *pF are* picked and V = 2.56 V measured (Figure 10). Moreover, the power found:

$$Pmax = \frac{|2.56|^2}{8\times 50} \times 2 = 32.77mW$$

Secondly, in T section model. I again used L = 1µH and C = 120 pF for both capacitors. This time 14MHz applied. V = 6.24 V was measured.

$$Pmax = \frac{|6.24|^2}{8\times 50} \times 2 = 238mW$$

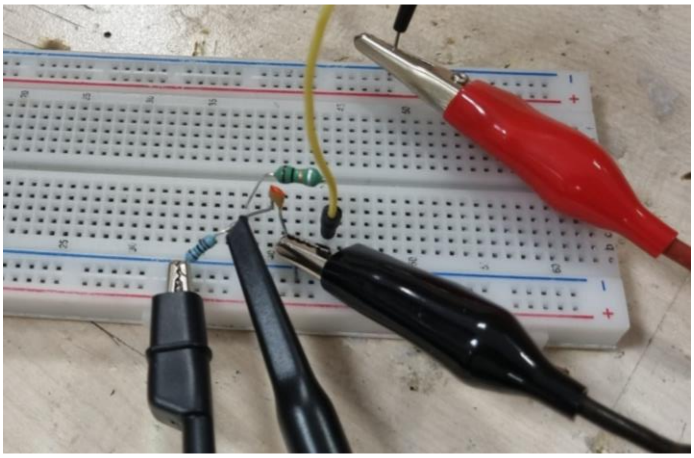


Figure 9 : L section model implemented

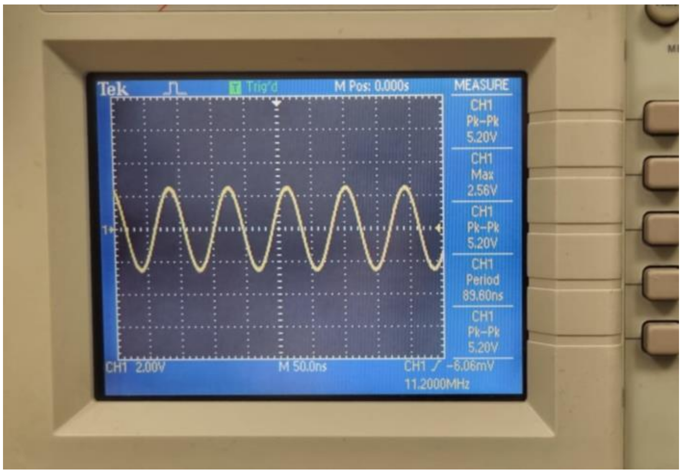


Figure 10 : L section model Vmax value measured from oscilloscope

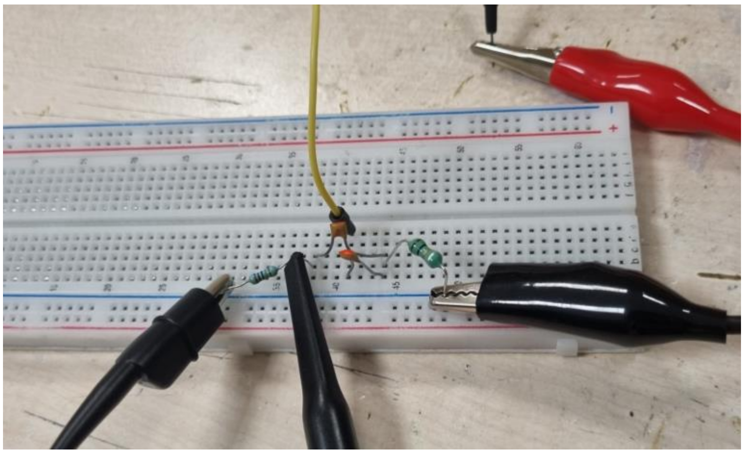


Figure 11 : T section model implemented

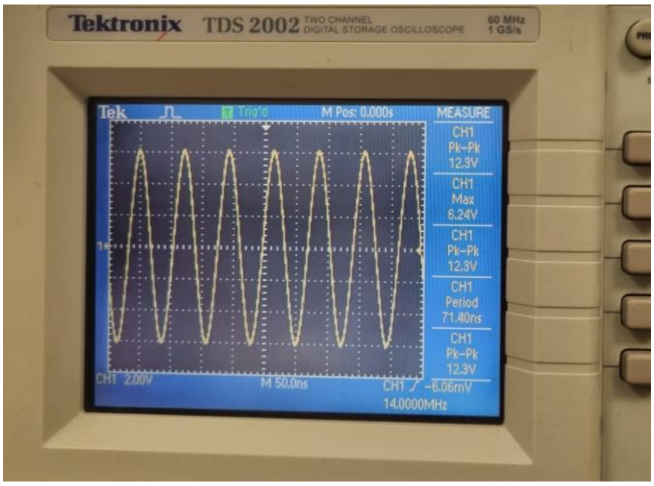


Figure 12 : T section model Vmax value measured from oscilloscope

Matching Circuit Type	Max Power Transfered to the Load in the Simulation (mW)	Max Power Transfered to the Load in the Hardware (mW)	Error (%)	Power Transfer Ratio (%)
T section model	295.69	238	19.51	80.49
L section model	40.95	32.77	19.98	80.02

Table 2 : Comparison of Simulation and Hardware Results

Conclusion:

I needed to make some changes in software part at the very end of my report in order to lower hardware error. I higher the software power transfer error so that I can get a error lower than 20% in the hardware part. My errors in hardware is lower than 20%. These errors in hardware part occured because I needed to use 131.3pF in the T section and 133pF in L section. However I used 120pF which is the closest standard value in the lab. On the other hand, I observed very big changes when I used jumpers instead of connecting my crocodiles directly to the points in the implemented circuit. Also, the reason why I got so big errors I changed the oscilloscope and finally I observed different values this time. As a result, beside the error T section model preferable because I got lower error on this and these errors could stem from so many variables beacuse we are dealing with real life errors.

REFERENCES

Equations from:

Book-Electric-Circuits-9th-ed-J.-Nilsson-S.-Riedel-Prentice-Hall-2011.pdf