EEE342 Feedback Control Systems-Preliminary Work 2
Bode Plot of DC motor's Transfer Function

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1. Introduction

This report focuses on analyzing the first order transfer function of a DC motor. The objective is to explore the correlation between the Fast Fourier Transform (FFT) results of the input signal and the transfer function itself. By comparing the frequency-domain magnitude and phase responses derived from the FFT with those obtained from the analytical transfer function, we aim to validate the consistency and accuracy of both approaches.

2. Laboratory Content

Part 1

In the first part of the preliminary we are given a transfer function of our DC motor (Eq.2) in a form of first order transfer function Eq.1 [1][2]. We are asked to plot the Bode of this function without using the "bode" method of matlab but rather using "logspace" method we generate logarithmically separated points.

$$G(s) = \frac{K}{\tau s + 1} \quad (Eq.1)$$

$$G_p(s) = \frac{20}{0.5s + 1} \quad (Eq.2)$$

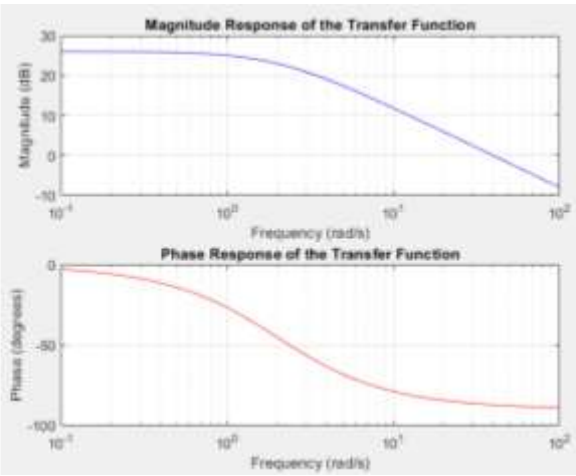

Fig. 1: Magnitude and Phase Response Plots of Eq.2

Part 2

Now we will use this transfer function and apply a sinusoidal input (Eq.3) and try to derive the input output phase and magnitude difference plots by using "fft" method of matlab [3]. The reason why we use FFT is that, fft efficiently converts finite time-domain signals into frequency components using the Discrete Fourier Transform (DFT) but in a faster way (O(N log N) complexity).

Bode plot is drawn with 10 sample points (0.1 rad-10 rad). When we combine 10-point length FFT of input and output (Eq.4) and we take the difference of phase and magnitude we derive the 10-point discrete transfer function which we see that this alligns with our transfer function at the desired points given us before.

$$x(t) = A\cos(\omega t)\ (Eq.3)$$

$$y(t) = A|G(j\omega)|\cos\big(\omega t + \angle G(j\omega)\big)\ (Eq.4)$$

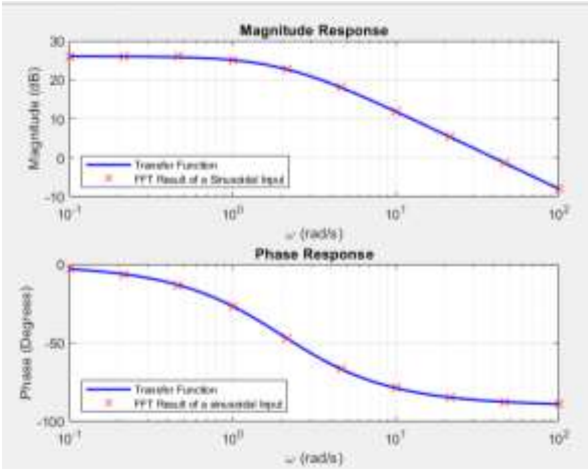

Fig. 2: Magnitude and Phase Response Plots Compared to 10-point FFT of Sinusoidal Input

Matlab Code

```
w1 = logspace(-1,2,100); % Frequency range for
Transfer Function
G = zeros(1,100);

% Compute Transfer Function Response
for k = 1:100
    s = 1i * w_x(k);  % Define s in the Laplace do-
main
    G(k) = 20 / (0.5*s + 1);  % Transfer function
end

% Define test frequencies for sinusoidal response
w = logspace(-1,2,10); % 10 frequencies for sinusoi-
dal input
K = zeros(1,10); % Magnitude ratios
P = zeros(1,10); % Phase differences

% Loop through each test frequency
for i = 1:10
    t = (1:10000); % Time vector
```

```
    x = cos(w(i) * t / 100); % Input sinusoidal

    % Compute system output using magnitude and
phase
    H = 20 / (sqrt(-1) * w(i) * 0.5 + 1); % Frequ-
ency response
    y = abs(H) * cos(w(i) * t / 100 + angle(H)); %
Output sinusoidal

    % Compute FFTs
    x_fft = fft(x);
    y_fft = fft(y);

    % Find dominant frequency component
    [K_x, index_x] = max(abs(x_fft));
    P_x = angle(x_fft(index_x));

    [K_y, index_y] = max(abs(y_fft));
    P_y = angle(y_fft(index_y));


    K(i) = K_y / K_x;  %magnitude difference of in-
put and output
    P(i) = P_y - P_x;   %magnitude difference of in-
put and output
end

% Plot Magnitude Response
subplot(2,1,1)
semilogx(w1, 20*log10(abs(G)), 'b', 'LineWidth',
1.5);
hold on
semilogx(w, 20*log10(K), 'xr', 'MarkerSize',8,'Mar-
kerFaceColor', 'r');  % for discrete fft points
grid on
xlabel('\omega (rad/s)')
ylabel('Magnitude (dB)')
title('Magnitude Response')
legend('Transfer Function', 'FFT Result of a Sinuso-
idal Input', 'FontSize', 7, 'Location', 'southwest')

% Plot Phase Response
subplot(2,1,2)
semilogx(w1, angle(G)*180/pi, 'b', 'LineWidth',
1.5);
hold on
semilogx(w, P*180/pi, 'xr', 'MarkerSize',8, 'Marker-
FaceColor', 'r');
grid on
xlabel('\omega (rad/s)')
ylabel('Phase (Degrees)')
title('Phase Response')
legend('Transfer Function', 'FFT Result of a sinuso-
idal Input', 'FontSize', 7, 'Location', 'southwest')
```

3. Conclusion

In the first part of the assignment, the magnitude and phase responses of the transfer function were directly computed and plotted. In the second part, we derived the magnitude and phase responses by analyzing the input-output relationship using 10 data points. After generating the time-domain input-output signals, the FFT (Fast Fourier Transform) was applied to obtain the corresponding frequency-domain responses. The results obtained from both the analytical and FFT-based approaches were consistent with each other, validating the accuracy of the analysis.

REFERENCES

1. R. H. Bishop and R. C. Dorf, Modern Control Systems: International Edition, 10th ed. Upper Saddle River, NJ: Pearson, 2005.
2. Arif Bülent Özgüler, Spring 2025 Lecture Notes
3. https://www.mathworks.com/help/matlab/ref/fft.html