Sudden Stops: contractionary real exchange rate depreciations

For more details:
http://www.korinek.com/suddenstops

The model

A model of small open economy featuring a continuum of infinitely lived identical agent facing a constant relative risk aversion utility function. There is no production, we don't make a distinction between households and firms.

$$E_0 = \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\} \quad \text{i.e.} \quad u(c_t) = \frac{(c_t)^{1-\sigma}}{1-\sigma} \tag{1}$$

Agents maximize the expected future discounted value of consumption where β is the subjective discount factor. There are two types of goods available in the economy: traded ones, and non-traded ones. The aggregator for consumption will be of the CES type.

$$c_t = \left[\alpha \left(c_t^T \right)^{-\mu} + (1-\alpha) \left(c_t^N \right)^{-\mu} \right]^{-1/\mu}$$

where $\mu > -1$, $\alpha \in (0,1)$ and $1/(1+\mu)$ is the elasticity of substitution between tradable and nontradable goods. Each period household receives an endowment $\left(y^{T,i}, y^{N,i} \right)$ and decides how much to consume and chooses the level of bond holdings according given budget constraint:

$$\frac{b_{t+1}^i}{R} + c_t^T + p_t^N c_t^N = b_t + y_t^T + p_t^N y_t^N \tag{2}$$

The price of tradable goods are determined in the world market and taken as numeraire such that price of nontradables p^N is the relative price of non-tradable goods in terms of tradable goods. The menu of assets available for saving purposes is restricted to a one period, non-state-contingent bond denominated in tradables which hold the price R with $R = (1+r)$ where r is the interest rate determined exogenously.

Access to international financial markets is imperfect. We assume that creditors restrict loans such that the value of debt does not exceed a κ fraction of households' total income.

$$\frac{b_{t+1}}{R} \geq -\kappa \left(p_t^N y_t^N + y^T \right) \tag{3}$$

Equilibrium

The household chooses the stochastic process $\left\{ c_t^T, c_t^N, b_{t+1} \right\}_{t \geq 0}$ to maximize the expected present discounted value of utility 1 subject to 2 and 3 taking b_0 and p_t^N as given. The first order conditions are:

$$\lambda_t = u_T \left(c_t^T, c_t^N \right) \tag{4}$$

$$p_t^N = \frac{u_N \left(c_t^T, c_t^N \right)}{u_T \left(c_t^T, c_t^N \right)} = \frac{(1-\alpha)}{\alpha} \left(\frac{c_t^T}{c_t^N} \right)^{\mu+1} \tag{5}$$

$$\lambda_t = \beta R \lambda_{t+1} + \eta_t \tag{6}$$

$$\frac{b_{t+1}}{R} \geq -\kappa \left(p_t^N y_t^N + y^T \right) \quad \text{with equality if} \quad \eta_t > 0 \tag{7}$$

Since household are identical and non-traded goods should be consumed in the domestic market , market clearing conditions are:

$$c_t^N = y_t^N \tag{8}$$

$$c_t^T = b_t + y_t^T - \frac{b_{t+1}}{R} \tag{9}$$

Rewriting the Euler equation

$$u_T\left(c_t^T, c_t^N\right) = \beta R u_T\left(c_{t+1}^T, c_{t+1}^N\right)$$

$$\alpha\left(c_t^T\right)^{-\mu-1}\left[\alpha\left(c_t^T\right)^{-\mu}+(1-\alpha)\left(c_t^N\right)^{-\mu}\right]^{-\frac{1-\sigma}{\mu}-1} = \beta R\alpha\left(c_{t+1}^T\right)^{-\mu-1}\left[\alpha\left(c_{t+1}^T\right)^{-\mu}+(1-\alpha)\left(c_{t+1}^N\right)^{-\mu}\right]^{-\frac{1-\sigma}{\mu}-1}$$

$$1-\beta R\frac{\alpha\left(c_{t+1}^T\right)^{-\mu-1}\left[\alpha\left(c_{t+1}^T\right)^{-\mu}+(1-\alpha)\left(c_{t+1}^N\right)^{-\mu}\right]^{-\frac{1-\sigma}{\mu}-1}}{\alpha\left(c_t^T\right)^{-\mu-1}\left[\alpha\left(c_t^T\right)^{-\mu}+(1-\alpha)\left(c_t^N\right)^{-\mu}\right]^{-\frac{1-\sigma}{\mu}-1}} = 0$$

$$1-\beta R\left(\frac{c_{t+1}^T}{c_t^T}\right)^{-\mu-1}\left(\frac{\alpha\left(c_{t+1}^T\right)^{-\mu}+(1-\alpha)\left(c_{t+1}^N\right)^{-\mu}}{\alpha\left(c_t^T\right)^{-\mu}+(1-\alpha)\left(c_t^N\right)^{-\mu}}\right)^{-\frac{1}{\mu}(1-\sigma+\mu)} = 0$$

Finally the Euler equation becomes

$$1-\beta R\left(\frac{c_{t+1}^T}{c_t^T}\right)^{-\mu-1}\left(\frac{c_{t+1}}{c_t}\right)^{1-\sigma+\mu} - \eta_t = 0$$

$$\text{where} \quad \frac{b_{t+1}}{R}+\kappa\left(p_t^N y_t^N + y^T\right) \geq 0 \quad \text{with equality if} \quad \eta_t > 0$$

Functional Forms

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$$

$$c_t = \left[\alpha \left(c_t^T\right)^{-\mu} + (1-\alpha)\left(c_t^N\right)^{-\mu}\right]^{-\frac{1}{\mu}}$$

$$u(c_T, c_N) = \frac{\left[\alpha \left(c_t^T\right)^{-\mu} + (1-\alpha)\left(c_t^N\right)^{-\mu}\right]^{-\frac{1-\sigma}{\mu}}}{1-\sigma}$$

$$u_T(c_T, c_N) = \alpha \left(c_t^T\right)^{-\mu-1} \left[\alpha \left(c_t^T\right)^{-\mu} + (1-\alpha)\left(c_t^N\right)^{-\mu}\right]^{-\frac{1-\sigma}{\mu}-1}$$

$$u_N(c_T, c_N) = (1-\alpha) \left(c_t^N\right)^{-\mu-1} \left[\alpha \left(c_t^T\right)^{-\mu} + (1-\alpha)\left(c_t^N\right)^{-\mu}\right]^{-\frac{1-\sigma}{\mu}-1}$$

$$p_t^N = \frac{u_N(c_T, c_N)}{u_T(c_T, c_N)} = \frac{(1-\alpha)}{\alpha}\left(\frac{c_t^T}{c_t^N}\right)^{\mu+1}$$

Calibration

$\beta = 0.96$
$R = 1.03$
$\sigma = 2$
$\alpha = 1/3$
$\mu = 0.8$
$\kappa = 1/3$
$\Delta y = 0.03$
$\pi = 0.05$

Note: I think there is a problem here with the value of μ. I think the authors made a typo about the calibration table because they are calibrating the elasticity of substitution $\frac{1}{1+\mu}$ not μ directly.

In the paper, authors say
"As in Mendoza (2005), we assume an elasticity of substitution $\frac{1}{1+\mu}$ *= 0.8 "*
In that case the true calibration should be $\mu = 1/4$

Rewriting the Euler eq as a fct of the relative price?

Euler equation is:

$$1-\beta R\left(\frac{c_{t+1}^T}{c_t^T}\right)^{-\mu-1}\left(\frac{c_{t+1}}{c_t}\right)^{1-\sigma+\mu}-\eta_t=0$$

$$\text{where}\quad \frac{b_{t+1}}{R}+\kappa\left(p_t^N y_t^N+y^T\right)\geq 0\quad\text{with equality if}\quad \eta_t>0$$

Using the market clearing condition for non-tradable goods ($c_t^N=y_t^N$) we can rewrite the euler equation as

$$1-\beta R\left(\frac{c_{t+1}^T}{c_t^T}\frac{y_{t+1}^N}{c_{t+1}^N}\frac{c_t^N}{y_t^N}\right)^{-\mu-1}\left(\frac{c_{t+1}}{c_t}\right)^{1-\sigma+\mu}-\eta_t=0$$

$$1-\beta R\left(\frac{c_t^N}{c_t^T}\frac{c_{t+1}^T}{c_{t+1}^N}\frac{y_{t+1}^N}{y_t^N}\right)^{-\mu-1}\left(\frac{c_{t+1}}{c_t}\right)^{1-\sigma+\mu}-\eta_t=0$$

$$1-\beta R\left(\frac{\frac{c_t^T}{c_t^N}}{\frac{c_{t+1}^T}{c_{t+1}^N}}\right)^{\mu+1}\left(\frac{y_{t+1}^N}{y_t^N}\right)^{-\mu-1}\left(\frac{c_{t+1}}{c_t}\right)^{1-\sigma+\mu}-\eta_t=0$$

Using the information about price

$$\frac{p_t^N}{p_{t+1}^N}=\frac{\frac{(1-\alpha)}{\alpha}\left(\frac{c_t^T}{c_t^N}\right)^{\mu+1}}{\frac{(1-\alpha)}{\alpha}\left(\frac{c_{t+1}^T}{c_{t+1}^N}\right)^{\mu+1}}=\left(\frac{\frac{c_t^T}{c_t^N}}{\frac{c_{t+1}^T}{c_{t+1}^N}}\right)^{\mu+1}$$

We can rewrite the Euler equation

$$1-\beta R\frac{p_t^N}{p_{t+1}^N}\left(\frac{y_{t+1}^N}{y_t^N}\right)^{-\mu-1}\left(\frac{c_{t+1}}{c_t}\right)^{1-\sigma+\mu}-\eta_t=0$$

Useful?