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K-Means Problem
(Problem 10.2, Han, Kamber, Pei, 2012)

Question 10.2

K-means

Centers of clusters

$A_1(2,10)$ $B_1(5,8)$ $C_1(1,2)$ $\Rightarrow k=3$

Calculate Euclidian distance of all points to the center points and find minimum distance

a) First round of execution

Points \ centers	$A_1(2,10)$	$B_1(5,8)$	$C_1(1,2)$
$A_1(2,10)$	0 ✓	$\sqrt{(5-2)^2+(8-10)^2}=3.61$	$\sqrt{(2-1)^2+(10-2)^2}=8.06$
$A_2(2,5)$	$\sqrt{(2-2)+(10-5)^2}=5$	$\sqrt{(5-2)^2+(8-5)^2}=4.24$	$\sqrt{(2-1)^2+(5-2)^2}=3.16$ ✓
$A_3(8,4)$	$\sqrt{(8-2)^2+(10-4)^2}=8.49$	$\sqrt{(8-5)^2+(8-4)^2}=5$ ✓	$\sqrt{(8-1)^2+(4-2)^2}=7.28$
$B_1(5,8)$	$\sqrt{(5-2)^2+(10-8)^2}=3.61$	0 ✓	$\sqrt{(5-1)^2+(8-2)^2}=7.21$
$B_2(7,5)$	$\sqrt{(7-2)^2+(10-5)^2}=7.07$	$\sqrt{(7-5)^2+(8-5)^2}=3.61$ ✓	$\sqrt{(7-1)^2+(5-2)^2}=6.71$
$B_3(6,4)$	$\sqrt{(6-2)^2+(10-4)^2}=7.21$	$\sqrt{(6-5)^2+(8-4)^2}=4.12$ ✓	$\sqrt{(6-1)^2+(4-2)^2}=5.39$
$C_1(1,2)$	$\sqrt{(2-1)^2+(10-2)^2}=8.06$	$\sqrt{(5-1)^2+(8-2)^2}=7.21$	0 ✓
$C_2(4,9)$	$\sqrt{(4-2)^2+(10-9)^2}=2.24$	$\sqrt{(5-4)^2+(8-9)^2}=1.41$ ✓	$\sqrt{(4-1)^2+(9-2)^2}=7.62$

Cluster 1	Cluster 2	cluster 3
$A_1(2,10)$	$A_3(8,4)$	$A_2(2,5)$
	$B_1(5,8)$	$C_1(1,2)$
	$B_2(7,5)$	
	$B_3(6,4)$	
	$C_2(4,9)$	

$K_1 = (2,10)$

$K_3 = \left(\frac{2+1}{2}, \frac{5+2}{2}\right)$

$K_3 = (1.5, 3.5)$

$K_2 = \left(\frac{8+5+7+6+4}{5}, \frac{4+8+5+4+9}{5}\right)$

$K_2 = (6, 6)$

$k_1 = (2,10)$

$k_2 = (6,6)$

$k_3 (1.5, 3.5)$

Three cluster centers after the first round of execution.

b) $k_1 = (2,10)$ $k_2 = (6,6)$ $k_3 = (1.5, 3.5)$

points \ centers	$k_1(2,10)$	$k_2(6,6)$	$k_3(1.5-3.5)$
$A_1(2,10)$	0 ✓	$\sqrt{(6-2)^2+(10-6)^2}=$	$\sqrt{(1.5-2)^2+(3.5-10^2}=$
$A_2(2,5)$	$\sqrt{(2-2)^2+(10-5)^2}=5$	$\sqrt{(6-2)^2+(6-5)^2}=$	$\sqrt{(1.5-2)^2+(3.5-5)^2}$ ✓
$A_3(8,4)$	$\sqrt{(2-8)^2+(10-4)^2}$ $=6^2+6^2$	$\sqrt{(6-8)^2+(6-4)^2}=$ $=2^2+2^2$ ✓	$\sqrt{(1.5-8)^2(3.5-4)^2}$
$B_1(5,8)$	$\sqrt{(2-5)^2+(10-8)^2}$	$\sqrt{(6-5)^2(6-8)^2}$ ✓	$\sqrt{(1.5-5)^2+(3.5-8)^2}$
$B_2(7,5)$	$\sqrt{(2-7)^2+(10-5)^2}$	$\sqrt{(6-7)^2+(6-5)^2}$ ✓	$\sqrt{(1.5-7)^2+(3.5-5)^2}$
$B_3(6,4)$	$\sqrt{(6-2)^2+(10-4)^2}$	$\sqrt{(6-6)^2(6-4)^2}$ ✓	$\sqrt{(1.5-6)^2+(3.5-4)^2}$
$C_1(1,2)$	$\sqrt{(2-1)^2+(10-2)^2}$	$\sqrt{(6-1)^2+(6-2)^2}$	$\sqrt{(1.5-1)^2+(3.5-2)^2}$ ✓
$C_2(4,9)$	$\sqrt{(2-4)^2+(10-9)^2}$ ✓	$\sqrt{(6-4)^2+(6-9)^2}$	$\sqrt{(1.5-4)^2+(3.5-9)^2}$

Cluster 1

$A_1(2,10)$

$C_2(4,9)$

↓

$k_1 = \left(\frac{2+4}{2}, \frac{10+9}{2}\right)$

$k_1 = (3, 9.5)$

Cluster 2

$A_3(8,4)$

$B_1(5,8)$

$B_2(7,5)$

$B_3(6,4)$

↓

$k_2 = \left(\frac{8+5+7+6}{4}, \frac{4+8+5+4}{4}\right)$

$k_2 = (6.5, 5.25)$

cluster 3

$A_2(2,5)$

$C_1(1,2)$

$k_3\left(\frac{2+1}{2}, \frac{5+2}{2}\right)$

$k_3 = (1.5, 3.5)$

⇒

$k_1 = (3, 9.5)$ $k_2 = (6.5, 5.25)$ $k_3 = (1.5, 3.5)$

points \ centers	$k_1(3, 9.5)$	$k_2(6.5, 5.25)$	$k_3(1.5, 3.5)$
$A_1(2, 10)$	$\sqrt{(3-2)^2+(9.5-10)^2}$	$\sqrt{(6.5-2)^2+(5.25-10)^2}$	$\sqrt{(1.5-2)^2+(3.5-10)^2}$
$A_2(2, 5)$	$\sqrt{(3-2)^2+(9.5-5)^2}$	$\sqrt{(6.5-2)^2+(5.25-5)^2}$	$\sqrt{(1.5-2)^2+(3.5-5)^2}$
$A_3(8, 4)$	$\sqrt{(3-8)^2+(9.5-4)^2}$	$\sqrt{(6.5-8)^2+(5.25-4)^2}$	$\sqrt{(1.5-8)^2+(3.5-4)^2}$
$B_1(5, 8)$	$\sqrt{(3-5)^2+(9.5-8)^2}$	$\sqrt{(6.5-5)^2+(5.25-8)^2}$	$\sqrt{(1.5-5)^2+(3.5-8)^2}$
$B_2(7, 5)$	$\sqrt{(3-7)^2+(9.5-5)^2}$	$\sqrt{(6.5-7)^2+(5.25-5)^2}$	$\sqrt{(1.5-7)^2+(3.5-5)^2}$
$B_3(6, 4)$	$\sqrt{(3-6)^2+(9.5-4)^2}$	$\sqrt{(6.5-6)^2+(5.25-4)^2}$	$\sqrt{(1.5-6)^2+(3.5-4)^2}$
$C_1(1, 2)$	$\sqrt{(3-1)^2+(9.5-2)^2}$	$\sqrt{(6.5-1)^2+(5.25-2)^2}$	$\sqrt{(1.5-1)^2+(3.5-2)^2}$
$C_2(4, 9)$	$\sqrt{(3-4)^2+(9.5-9)^2}$	$\sqrt{(6.5-4)^2+(5.25-9)^2}$	$\sqrt{(1.5-4)^2+(3.5-9)^2}$

Cluster 1	Cluster 2	Cluster 3
$A_1(2, 10)$	$A_3(8, 4)$	$A_2(2, 5)$
$B_1(5, 8)$	$B_2(7, 5)$	$C_1(1, 2)$
$C_2(4, 9)$	$B_3(6, 4)$	

$k_3 = \left(\frac{2+1}{2}, \frac{5+2}{2}\right)$

$k_3 = (1.5, 3.5)$

$k_1 = \left(\frac{2+5+4}{3}, \frac{10+8+9}{3}\right)$

$k_1 = (3.67, 9)$

$k_2 = \left(\frac{8+7+6}{3}, \frac{4+5+4}{3}\right)$

$k_2 = (7, 4.33)$

$\Rightarrow$

$K_1 = (3.67, 9)$ $K_2 = (7, 4.33)$ $K_3 = (1.5, 3.5)$

points \ centers	$K_1(3.67, 9)$	$K_2(7, 4.33)$	$K_3(1.5, 3.5)$
$A_1(2, 10)$	$\sqrt{(3.67-2)^2+(9-10)^2}$ ✓	$\sqrt{(7-2)^2+(4.33-10)^2}$	$\sqrt{(1.5-2)^2+(3.5-10)^2}$
$A_2(2, 5)$	$\sqrt{(3.67-2)^2+(9-5)^2}$	$\sqrt{(7-2)^2+(4.33-5)^2}$	$\sqrt{(1.5-2)^2+(3.5-5)^2}$ ✓
$A_3(8, 4)$	$\sqrt{(3.67-8)^2+(9-4)^2}$	$\sqrt{(7-8)^2+(4.33-4)^2}$ ✓	$\sqrt{(1.5-8)^2+(3.5-4)^2}$
$B_1(5, 8)$	$\sqrt{(3.67-5)^2+(9-8)^2}$ ✓	$\sqrt{(7-5)^2+(4.33-8)^2}$	$\sqrt{(1.5-5)^2+(3.5-8)^2}$
$B_2(7, 5)$	$\sqrt{(3.67-7)^2+(9-5)^2}$	$\sqrt{(7-7)^2+(4.33-5)^2}$ ✓	$\sqrt{(1.5-7)^2+(3.5-5)^2}$
$B_3(6, 4)$	$\sqrt{(3.67-6)^2+(9-4)^2}$	$\sqrt{(7-6)^2+(4.33-4)^2}$ ✓	$\sqrt{(1.5-6)^2+(3.5-4)^2}$
$C_1(1, 2)$	$\sqrt{(3.67-1)^2+(9-2)^2}$	$\sqrt{(7-1)^2+(4.33-2)^2}$	$\sqrt{(1.5-1)^2+(3.5-2)^2}$ ✓
$C_2(4, 9)$	$\sqrt{(3.67-4)^2+(9-9)^2}$ ✓	$\sqrt{(7-4)^2+(4.33-9)^2}$	$\sqrt{(1.5-4)^2+(3.5-9)^2}$

Cluster 1	Cluster 2	Cluster 3
$A_1(2, 10)$	$A_3(8, 4)$	$A_2(2, 5)$
$B_1(5, 8)$	$B_2(7, 5)$	$C_1(1, 2)$
$C_2(4, 9)$	$B_3(6, 4)$	

These are final clusters. Since the clusters do not change this is the final round.