Foundations of Data Science & Analytics
Association Rule Mining

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Introduction to Data Mining, 2nd Edition
by
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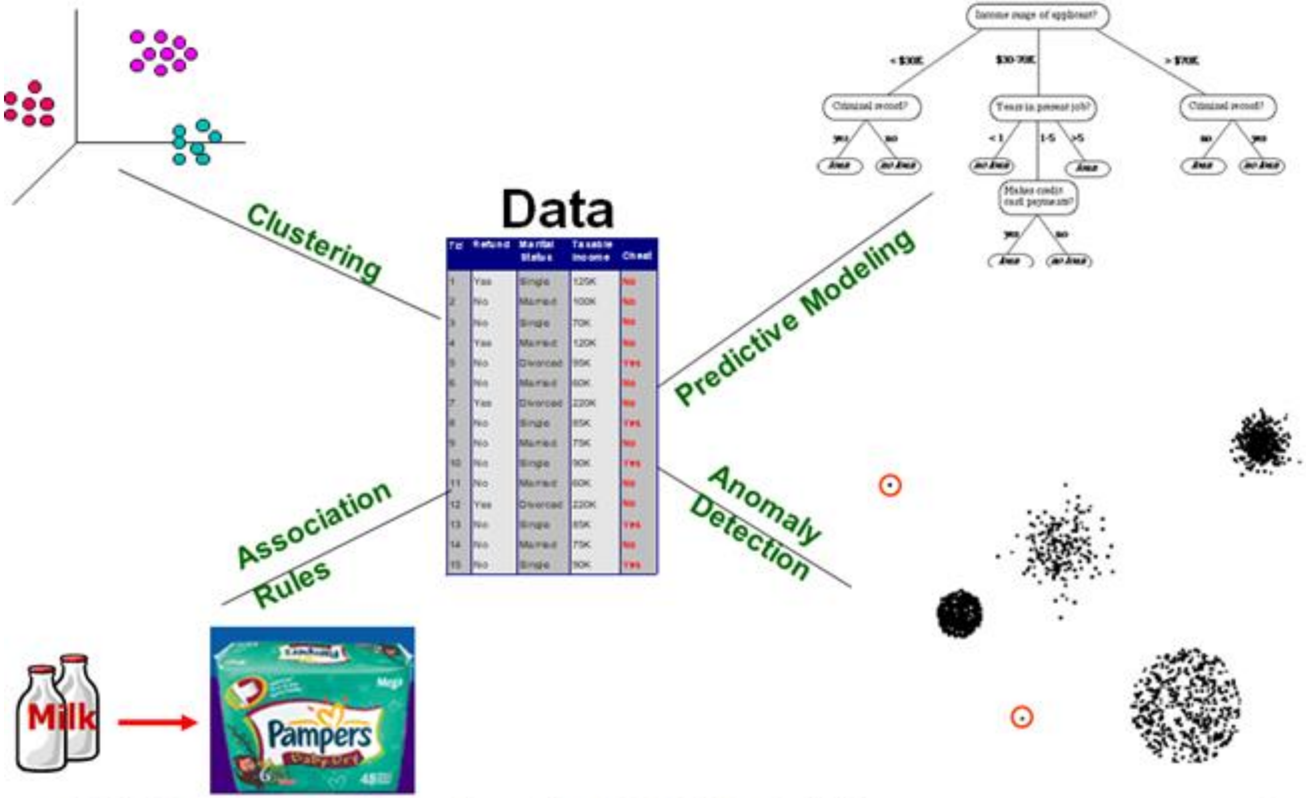
Tasks



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Association Rule Mining

Market-Basket transactions

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction.

{Diaper} → {Beer},
{Milk, Bread} → {Diaper},
{Beer, Bread} → {Milk, Diaper}

co-occurrence, not **causality**

Frequent Itemset

Market-Basket transactions

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- **Itemset**
  - A collection of one or more items
    - {Milk, Bread, Diaper}
  - k-itemset
    - An itemset that contains k items
- **Support count**
  - Frequency of occurrence of an itemset
    - σ({Milk, Bread, Diaper}) = 2
- **Support**
  - Fraction of transactions that contain an itemset
    - s({Milk, Bread, Diaper}) = 2/5
- **Frequent Itemset**
  - An itemset whose support is greater than or equal to a *minsup* threshold

Association Rules

Market-Basket transactions

TID	*Items*
1	Bread, Milk
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- **Association Rule**
  - An implication expression of the form $X \rightarrow Y$, where X and Y are itemsets
  - {Milk, Diaper} → {Beer}
- Rule Evaluation Metrics
  - Support (s)
    - Fraction of transactions that contain both X and Y
  - Confidence (c)
    - Measures how often items in Y appear in transactions that contain X

Association Rules

Market-Basket transactions

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Example:

$$\{\text{Milk}, \text{Diaper}\} \Rightarrow \{\text{Beer}\}$$

$$s = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{|\text{T}|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

Association Rule Mining

Market-Basket transactions

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Given a set of transactions T, the goal of association rule mining is to find all rules having

- support ≥ *minsup* threshold
- confidence ≥ *minconf* threshold

How?

Market-Basket transactions

TID	*Items*
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Two-step approach:

- **Frequent Itemset Generation**
  - Generate all itemsets whose support >= *minsup*
- **Rule Generation**
  - Generate high confidence (>= *minconf*) rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset

How?

Market-Basket transactions

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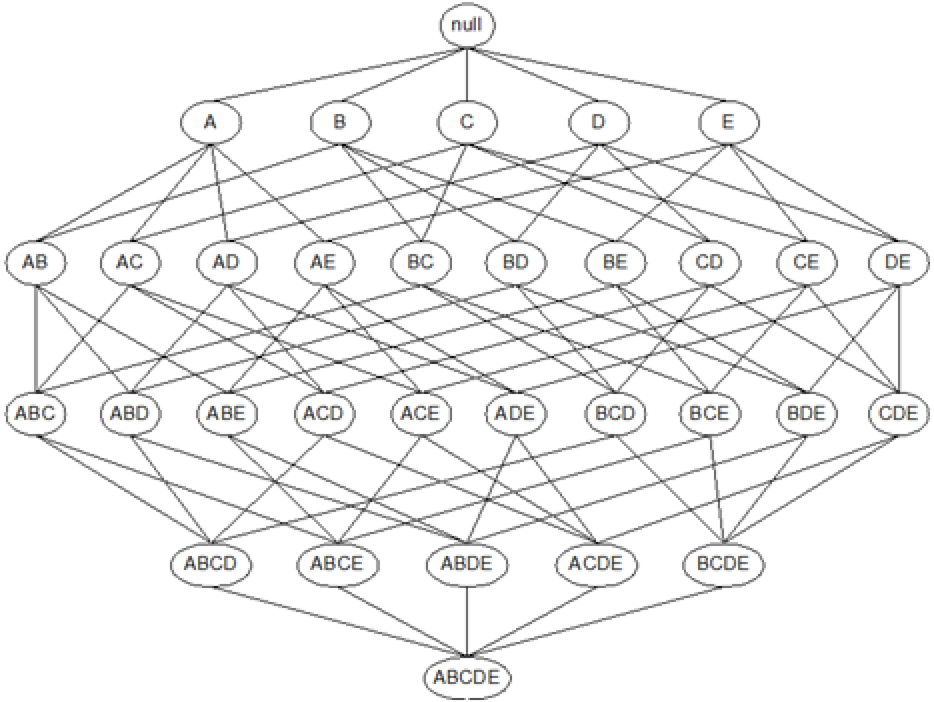
Frequent Itemset Generation



Given d items, there are 2^d possible candidate itemsets

Frequent Itemset Generation

Apriori principle:

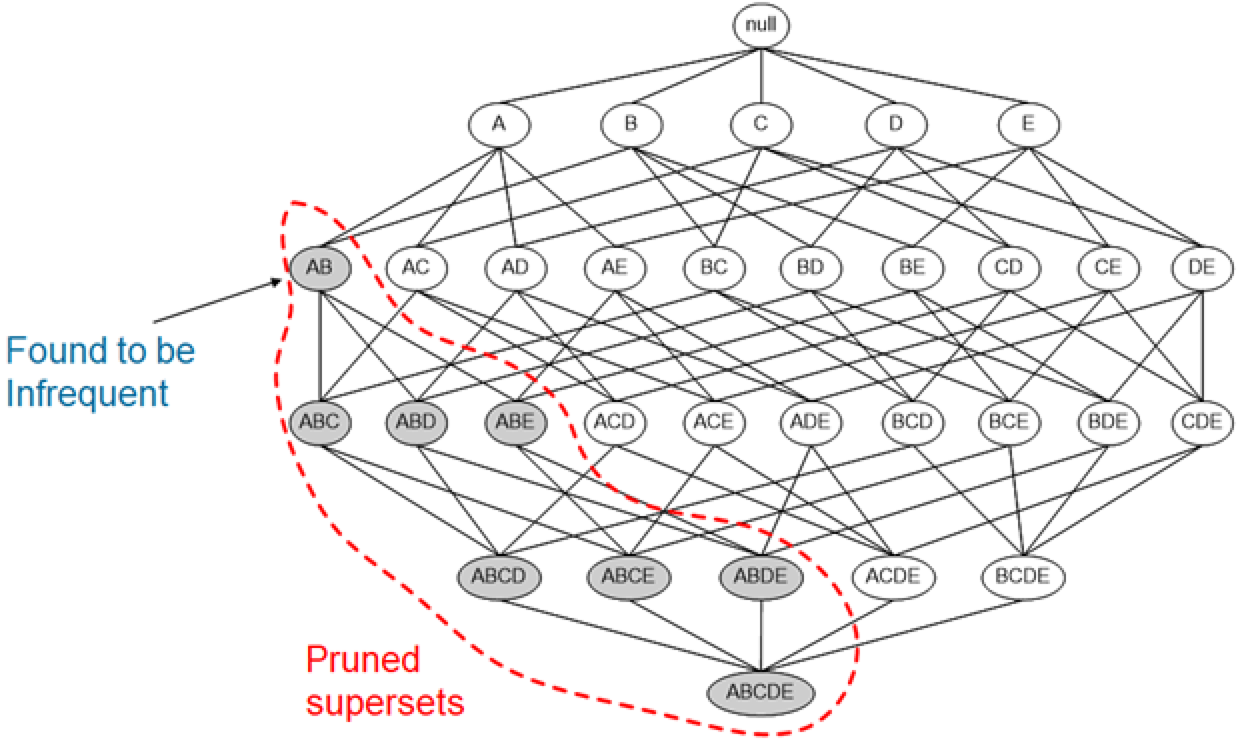
- If an itemset is frequent, then all of its subsets must also be frequent

Apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

- Support of an itemset never exceeds the support of its subsets
- This is known as the ***anti-monotone*** property of support

Apriori Principle



Apriori Algorithm

- F_k: frequent k-itemsets
- L_k: candidate k-itemsets

- Algorithm
  - Let k=1
  - Generate F_1 = {frequent 1-itemsets}
  - Repeat until F_k is empty
    - **Candidate Generation**: Generate L_{k+1} from F_k
    - **Candidate Pruning**: Prune candidate itemsets in L_{k+1} containing subsets of length k that are infrequent
    - **Support Counting**: Count the support of each candidate in L_{k+1} by scanning the DB
    - **Candidate Elimination**: Eliminate candidates in L_{k+1} that are infrequent, leaving only those that are frequent => F_{k+1}

Generate 1-Itemset

TID	*Items*
1	Bread, Milk
2	Beer, Bread, Diaper, Eggs
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Bread, Coke, Diaper, Milk

Minimum Support = 3

Count and Elimination

TID	Items
1	Bread, Milk
2	Beer, Bread, Diaper, Eggs
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Bread, Coke, Diaper, Milk

Items (1-itemsets)

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Minimum Support = 3

Generate 2-Itemset and prune

Items (1-itemsets)

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Itemset
{Bread,Milk}
{Bread, Beer }
{Bread,Diaper}
{Beer, Milk}
{Diaper, Milk}
{Beer,Diaper}

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

Count and Elimination

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Items (1-itemsets)

Itemset	Count
{Bread,Milk}	3
{Beer, Bread}	2
{Bread,Diaper}	3
{Beer,Milk}	2
{Diaper,Milk}	3
{Beer,Diaper}	3

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

Generate 3-Itemset and prune

Items (1-itemsets)

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Itemset	Count
{Bread,Milk}	3
{Bread,Beer}	2
{Bread,Diaper}	3
{Milk,Beer}	2
{Milk,Diaper}	3
{Beer,Diaper}	3

Minimum Support = 3

Triplets (3-itemsets)

Itemset
{ Beer, Diaper, Milk}
{ Beer,Bread,Diaper}
{Bread, Diaper, Milk}
{ Beer, Bread, Milk}

If every subset is considered,

$${}^6C_1 + {}^6C_2 + {}^6C_3$$

$$6 + 15 + 20 = 41$$

With support-based pruning,

$$6 + 6 + 4 = 16$$

Count and Elimination

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Items (1-itemsets)

Itemset	Count
{Bread,Milk}	3
{Bread,Beer}	2
{Bread,Diaper}	3
{Milk,Beer}	2
{Milk,Diaper}	3
{Beer,Diaper}	3

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

Triplets (3-itemsets)

Itemset	Count
{ Beer, Diaper, Milk}	
{ Beer,Bread, Diaper}	
{Bread, Diaper, Milk}	2
{Beer, Bread, Milk}	

If every subset is considered,

$^6C_1 + {}^6C_2 + {}^6C_3$

$6 + 15 + 20 = 41$

With support-based pruning,

$6 + 6 + 4 = 16$

$6 + 6 + 1 = 13$

More Efficient Candidate Generation

Items (1-itemsets)

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Itemset	Count
{Bread,Milk}	3
{Bread,Beer}	2
{Bread,Diaper}	3
{Milk,Beer}	2
{Milk,Diaper}	3
{Beer,Diaper}	3

Triplets (3-itemsets)

Itemset	Count
{Bread, Diaper, Milk}	2

Minimum Support = 3

If every subset is considered,

$^6C_1 + {}^6C_2 + {}^6C_3$

$6 + 15 + 20 = 41$

With support-based pruning,

$6 + 6 + 1 = 13$

How?

Market-Basket transactions

TID	Items
1	Bread, Milk
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Two-step approach:

- **Frequent Itemset Generation**
  - Generate all itemsets whose support >= *minsup*
- **Rule Generation**
  - Generate high confidence (>= *minconf*) rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset

Rule Generation

If {A,B,C,D} is a frequent itemset, candidate rules:

$ABC \rightarrow D$,	$ABD \rightarrow C$,	$ACD \rightarrow B$,	$BCD \rightarrow A$,
$A \rightarrow BCD$,	$B \rightarrow ACD$,	$C \rightarrow ABD$,	$D \rightarrow ABC$
$AB \rightarrow CD$,	$AC \rightarrow BD$,	$AD \rightarrow BC$,	$BC \rightarrow AD$,
$BD \rightarrow AC$,	$CD \rightarrow AB$,		

If $|L| = k$, then there are $2^k - 2$ candidate association rules (ignoring $L \rightarrow \phi$ and $\phi \rightarrow L$)

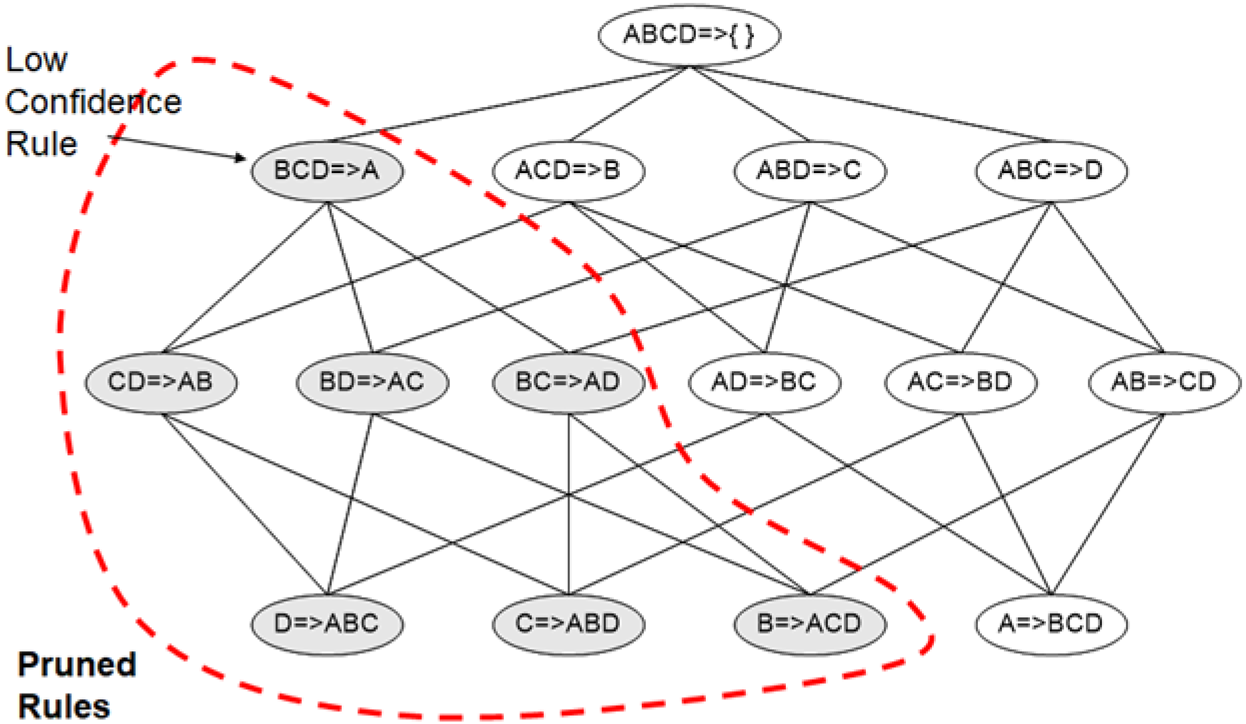
Rule Generation

- Confidence of rules generated from the same itemset has an anti-monotone property
  - E.g., Suppose {A,B,C,D} is a frequent 4-itemset:

$$
\begin{aligned}
& c(ABC \rightarrow D) = \sigma(ABCD) / \sigma(ABC) \\
\geq\ & c(AB \rightarrow CD) = \sigma(ABCD) / \sigma(AB) \\
\geq\ & c(A \rightarrow BCD) = \sigma(ABCD) / \sigma(A)
\end{aligned}
$$

  - Confidence is *anti-monotone* w.r.t. number of items on the RHS of the rule

Rule Generation



Rule Generation

Items (1-itemsets)

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Itemset	Count
{Bread,Milk}	3
{Bread,Beer}	2
{Bread,Diaper}	3
{Milk,Beer}	2
{Milk,Diaper}	3
{Beer,Diaper}	3

Triplets (3-itemsets)

Itemset	Count
{Bread, Diaper, Milk}	2

Minconf = 0.8

c(Bread → Milk) = 3 / 4
c(Milk → Bread) = 3 / 4
c(Bread → Diaper) = 3 / 4
c(Diaper → Bread) = 3 / 4
c(Milk → Diaper) = 3 / 4
c(Diaper → Milk) = 3 / 4
c(Beer → Diaper) = 3 / 3
c(Diaper → Beer) = 3 / 4

Apply Rules

Rules: {Beer → Diaper}

- When a customer buys **Beer**, suggest **Diaper** also.
- Put **Beer** and **Diaper** close.