

$$\text{Ito: } \int_0^t f(x_s) dW_s = \lim_{N \rightarrow \infty} \sum_{i=0}^{N-1} f(x_{t_i}) \Delta W_i$$

$$\text{Stratonovich: } \int_0^t f(x_s) \circ dW_s = \lim_{N \rightarrow \infty} \sum_{i=0}^{N-1} f\left(\frac{x_{t_i} + x_{t_{i+1}}}{2}\right) \Delta W_i$$

for small time step)

$$f\left(\frac{x_{t_i} + x_{t_{i+1}}}{2}\right) \approx f(x_{t_i}) + \frac{1}{2} f'(x_{t_i})(x_{t_{i+1}} - x_{t_i})$$

plugging it in,

$$\int_0^t f(x_s) \circ dW_s = \int_0^t f(x_s) dW_s + \frac{1}{2} \sum_{i=0}^{N-1} f'(x_{t_i})(x_{t_{i+1}} - x_{t_i}) \Delta W_i$$

$$= \int_0^t f(x_s) dW_s + \frac{1}{2} \sum_{i=0}^{N-1} f'(x_{t_i}) \Delta t$$

$$= \int_0^t f(x_s) dW_s + \frac{1}{2} \int_0^t \frac{\partial f(x_s)}{\partial x_s} dt_s$$