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ACCRETION DISKS IN CATACLYSMIC VARIABLES. I. THE ECLIPSE-RELATED PHASE SHIFTS IN DQ HERCULIS AND UX URSAE MAJORIS

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ABSTRACT

A model is described for a cataclysmic-variable binary system in which a large accretion disk reflects radiation from a progradely rotating source on or near the white dwarf surface, thus producing rapid oscillations in the optical light. Numerical calculations of the reflection process are presented which simulate the behavior of the oscillations as the system goes through eclipse.

The model reproduces the phase shift and amplitude variations of the DQ Herculis oscillations during eclipse very well. Best results are obtained with inclination angles so close to 90° that the outer edge of the disk obscures the entire front half of the disk, including the white dwarf.

The same model, with a *progradely* rotating source, also reproduces the completely different eclipse-related phase shift behavior of UX Ursae Majoris, when it is viewed at a somewhat smaller inclination angle (~75°). Only *one* new element is added: a small fraction (~0.1%) of the oscillating light near the white dwarf is observed directly, without reflection by the disk.

Obviously, the direct light cannot be seen from systems in which the white dwarf is permanently obscured. Thus, a single model, viewed at two different inclination angles, reproduces the observed phase shifts of both UX Ursae Majoris and DQ Herculis.

Subject headings: stars: accretion — stars: eclipsing binaries — stars: individual — stars: U Geminorum

I. INTRODUCTION

DQ Herculis and UX Ursae Majoris resemble each other in many respects. Both are binaries in which a Roche-lobe–filling late-type star transfers matter through the inner Lagrangian point to an accretion disk around a white dwarf. Both systems are optically dominated by the disk. The hot spot, located where the stream of transferred matter feeds into the disk, makes only a small contribution to the light curve in either case. This can be recognized from the presence of a small hump and a slight eclipse asymmetry (an egress shoulder) in Figure 1. The two systems have very similar spectra (Walker 1956) and almost identical orbital periods (4^h39^m for DQ Her and 4^h43^m for UX UMa).

The fact that DQ Her is classified as a "classical nova" and UX UMa as a "nova-like variable" simply reflects the circumstance that the former erupted in 1934 (Nova Herculis), while no eruptions have been reported yet for the latter. Since the interval between subsequent nova eruptions is likely to be much longer than the time we have observed either system, there is no reason to believe that this difference is fundamental.

Like many other cataclysmic variables, DQ Her and UX UMa have shown evidence for *rapid oscillations*, i.e., low magnitude, sinusoidal light variations, which maintain coherence over many cycles. Superficially looked at, these oscillations provide several further similarities between the two systems: their periods are of the same order (respectively, 71 s[1] and 29 s), both are fairly good sinusoids, and both show evidence for a phase shift when the disk is eclipsed by the companion star. However, closer inspection reveals some striking differences. First of all, the oscillation period in DQ

[1] The possibility that the period for DQ Her is 142 s is less likely, but not conclusively ruled out (Patterson 1978).

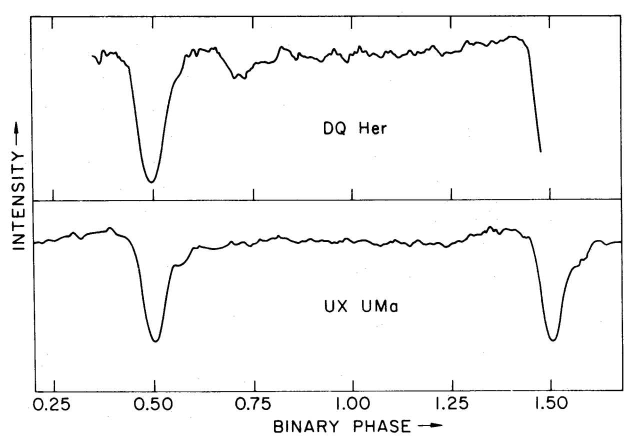

FIG. 1.—Light curves of DQ Her and UX UMa—*top*: DQ Her V light curve of 1975 July 12 (based on data from Nelson and Olson 1976); *bottom*: UX UMa (unfiltered) light curve of 1973 March 7 (from Nather and Robinson 1974). These light curves do not repeat accurately from cycle to cycle.

Her seems to be rock-steady (data can be phased together over a period of many years), while the period in UX UMa has been reported to show small variations of about a second from day to day (Nather and Robinson 1974). Next, oscillations in DQ Her have an amplitude of about 0.04 mag and seem to be always present, while in UX UMa they are sometimes seen and sometimes not, but they never have an amplitude much larger than 0.002 mag. Finally, and perhaps most mysteriously, the phase shifts during eclipse proceed in entirely different ways.

The first of these differences, the steadiness in oscillation periods, is likely to be intimately related to the mechanism which ultimately causes the rapid oscillations. Although the nature of this mechanism is still much in debate, all agree that the shortness of the periods argues for a process very near the white dwarf. The extremely high quality factor of the oscillations in DQ Her ($Q = |\dot{P}^{-1}| \approx 10^{12}$) is consistent with solid-body rotation as the underlying mechanism in that system, but the changes in the period of UX UMa (and those of many other cataclysmic variables) argue against the possibility that this mechanism is the clock in *all* the rapidly oscillating cataclysmic variables. A variation on the rotation model which allows a thin surface layer to rotate at a different rate than the rest of the white dwarf could perhaps overcome this difficulty. However, to produce the 0.4% change in period observed for UX UMa within 1 night (Nather and Robinson 1974) the surface layer would have to be uncomfortably thin, while of course it must still be capable of holding some sort of emission spot in place. Other models have been suggested, for instance bright spots at the inner edge of the accretion disk, rotating around the white dwarf (Bath 1973), or—always good if all else fails—nonradial oscillations (Warner and Robinson 1972). The resolution of this issue is not crucial for the present paper because we focus here on processes in the disk, some distance away from the white dwarf surface. For our purposes, the three mentioned proposals are similar; all feature the steady rotation of a radiating spot (be it a magnetic pole, a blob of disk material, or a wave) at a radius comparable to that of the white dwarf. We shall adopt the rotation model for the sake of definiteness when we build a numerical model for the system in § II, but this choice is fairly arbitrary and no claim is made that another choice could not have led to equally well fitting model phase shifts.

The oscillating light we observe optically cannot all come directly from the white dwarf. The fact that the duration of the eclipse-related phase shift roughly equals the total duration of the eclipse, both in DQ Her and in UX UMa, shows that the oscillating part of the visible light comes from a region with the same dimensions as the *entire* disk, not just from its center. If the true origin of the oscillations lies near the white dwarf, as is probable, it follows that a significant fraction of the observed light must be reflected. There is further strong support for this in DQ Her, from a phenomenon called "the wavelength-dependent phase shift" (Chanan, Nelson, and Margon 1978), and, as we hope to demonstrate in § III, also from the details of the eclipse-related phase shift itself. (See also Chanan, Nelson, and Margon 1978, on this latter phase shift.) The reason that unreflected radiation from the white dwarf does not completely dominate the observed oscillating light is probably that much of the direct light is emitted at wavelengths shorter than optical. Several authors (Herbst, Hesser, and Ostriker 1974; Rees 1974) recognized the importance of the reflection of oscillating light by the disk in DQ Her long ago. Chester (1979) has studied the reflection of oscillating light from a disk model, in order to explain binary-phase–related phase shifts seen in DQ Her and UX UMa (and probably in other systems as well). Alpar (1979) considered binary-phase–related shifts in the He II λ4686 line.

In this paper we study the reflection by the disk of a rotating beam of radiation, to see whether this process can explain the behavior of the oscillations during eclipse in DQ Her and UX UMa. We present (§ II) a model of a slightly concave accretion disk, which reflects pulses from a *progradely* rotating light source at its center (progradely, for both systems!). Next, we determine numerically (§ III) how the amplitude and phase of the oscillations behave during the eclipse of the disk, for a set of different inclination angles.

For DQ Her the model reproduces the observed eclipse-related phase-shift behavior very well if an inclination angle close to 90° is chosen. Our results suggest the possibility that the white dwarf in this system is permanently obscured by the outer rim of the disk, a possibility which could also explain why no X-rays are seen from this object. For UX UMa the same model (at a somewhat smaller inclination angle) can be made to reproduce the data surprisingly well if we assume that a small fraction ($\sim$0.1%) of the total radiation from the emission spot lies in the optical band and is observed directly when the white dwarf is not eclipsed. These results indicate that it is probably not necessary to invoke retrograde rotation of any part in the system to explain the fact that the phase shift in UX UMa progresses in a way opposite to that in DQ Her.

The assumption that a small portion of the radiation from the emission spot lies at optical wavelengths does not affect the results obtained for DQ Her, because our best-fitting models have the white dwarf permanently obscured by the disk. Thus, a *single model* can explain the phase shift of DQ Her when $i \approx 85°$–$90°$ and that of UX UMa when $i \approx 70°$–$75°$ (Fig. 2). The hope that a model for the phase shifts of these systems would accommodate with equal ease these two very different phase shifts could hardly be fulfilled more completely.

II. THE MODEL

a) General Characteristics

Consider an axisymmetric disk, lying in the binary plane, with inner radius r_i, outer radius r_0, and half

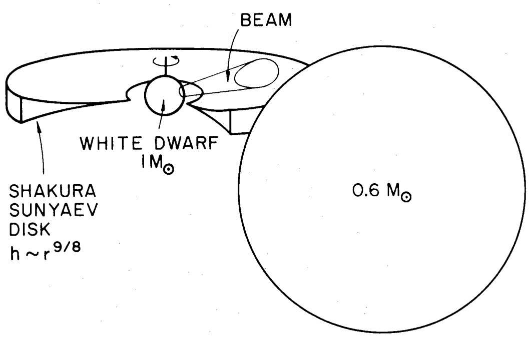

FIG. 2.—Rough explanation of the phase-shift phenomenon occurring during eclipse for DQ Her and UX UMa. Six stages during eclipse are shown with the direction of the illuminating beam from the emission spot on the white dwarf drawn in at the moment the oscillating light goes through a maximum. Note how the difference in the inclination angle of the system is essential to the explanation.

thickness $h(r) = h_0(r/r_0)^\epsilon$. Assume this disk is thin ($h_0 \ll r_0$) and concave ($\epsilon > 1$). Let it be illuminated by an emission spot on the equator of the white dwarf which rotates progradely with frequency ω_1 around an axis perpendicular to the orbital plane. Assume that the dimensions of the emission spot are small compared to the radius R_1 of the white dwarf.

The distance d between the spot and the point P on the disk surface with cylindrical coordinates r, ψ, h then satisfies

$$d^2(r, \psi, t) = r^2 + R_1{}^2 - 2rR_1 \cos(\psi - \omega_1 t) \quad (1)$$

to first order in h/r, where $\psi = 0$ is taken in the direction of the observer. The angle α_1 between the normal to the stellar surface at the emission spot and the light ray to the point $P(r, \psi, h)$ follows from

$$d \cos \alpha_1 = r \cos(\psi - \omega_1 t) - R_1 \quad (2)$$

and the angle α_2 between this light ray and the normal to the disk surface at P from

$$d \cos \alpha_2 = [r - R_1 \cos(\psi - \omega_1 t)] \frac{dh}{dr} - h \,. \quad (3)$$

When expressions (2) or (3) become negative, the corresponding angle α_1 or α_2 becomes larger than 90°, implying that the emission spot is no longer in view from that point of the disk surface. For α_2 this may happen when the disk is not sufficiently concave [i.e., when ϵ is not larger than $r_i/(r_i - R_1)$] to bring the disk surface completely out of the shadow of the inner edge. The part of the disk which is obscured from the spot by the white dwarf has $\alpha_1 < 0$.

If every point of the emission spot radiates isotropically, the total flux received per unit disk surface area at point P is

$$\mathscr{I} = \frac{I_0}{d^2} \cos \alpha_1 \cos \alpha_2 \, H(\cos \alpha_1) H(\cos \alpha_2) \,, \quad (4)$$

where I_0 is the flux per steradian emitted by the spot in the normal direction, and $H(x)$ equals unity for $x \geq 0$ and zero for $x < 0$. The two step functions in equation (4) take the possible obscuration of the emission spot into account. The disk reprocesses the incoming radiation $\mathscr{I}$, degrades the ultraviolet part to visible wavelengths, and emits per unit surface area a flux of $\mathscr{I}/2\pi$ per steradian.

The total flux from the emission spot that reaches an observer after having been reprocessed by the disk is then

$$F_{\rm refl} = \int_{ri}^{ro} \int_0^{2} \frac{\mathscr{I}}{2\pi} \cos \theta \, H_0 H_1 H_2 r dr d\psi \,. \quad (5)$$

The meaning of the different variables is as follows:

1. The angle between the observer's direction and the normal to the disk surface is θ. This angle is found from

$$\cos \theta = \cos i \cos \sigma - \cos \psi \sin i \sin \sigma \,, \quad (6)$$

where

i = the inclination angle of the binary with respect to the observer ($i = 90°$ implies that the system is seen edge-on),

σ = the slope of the disk surface ($\tan \sigma = dh/dr$) . (7)

Since $h \ll r$, it follows that $\cos \sigma \approx 1$, so that

$$\cos \theta \approx \cos i - \frac{dh}{dr} \cos \psi \sin i \,, \quad (8)$$

and the disk surface area $(r/\cos \sigma) dr d\psi$ may be approximated by $r dr d\psi$, as we have done in equation (5).

2. The step functions,

$$H_0 = H[y_0 - y_p] \,, \quad (9a)$$

$$H_1 = H\left[s_1{}^2 + \frac{\cos \psi}{|\cos \psi|} R_1{}^2\right] , \quad (9b)$$

$$H_2 = H[s_2{}^2 - R_2{}^2] \,, \quad (9c)$$

eliminate from the integral in equation (5) those surface points P which are obscured from the observer by, respectively, the outer disk rim, the white dwarf, and the companion star (near eclipse). The coordinates of P, projected onto a plane perpendicular to the line of sight, are

$$x_p = r \sin \psi \,, \quad (10)$$

$$y_p = r \cos \psi \cos i - h \sin i \,, \quad (11)$$

and the projected vertical coordinate of the point on the outer disk rim with projected horizontal coordinate x_p is

$$y_0 = \sqrt{r_0{}^2 - r^2 \sin^2 \psi} \cos i - h_0 \sin i \,. \quad (12)$$

The projected distances of P to the center of the white dwarf (radius R_1) and the companion star (radius R_2), respectively, satisfy

$$s_1{}^2 = x_p{}^2 + y_p{}^2 \,, \quad (13)$$

$$s_2{}^2 = (y_p - a \cos \phi \cos i)^2 + (x_p - a \sin \phi)^2 \,, \quad (14)$$

where a is the binary separation, and $\phi = \omega_2 t$ the orbital phase of the binary (defined such that a white dwarf eclipse occurs at $\phi = 0$).

Finally, assume that a small fraction f_0 of the light from the emission spot is radiated at optical wavelengths and, thus, contributes an amount

$$F_{\text{direct}} = I_0 f_0 \cos \alpha_3 \, H(\cos \alpha_3) H(y_3) H(s_3{}^2 - R_2{}^2) \tag{15}$$

to the total optical light seen by the observer. The angle α_3 between the normal to the white dwarf's surface at the emission spot and the line to the observer follows from

$$\cos \alpha_3 = \cos i \cos (\omega_1 t) , \tag{16}$$

and the step function $H(\cos \alpha_3)$ prevents the flux F_{direct} from becoming negative when the emission spot is on the back side of the white dwarf and invisible to the observer. Since

$$y_3 = r_0 \cos i - h_0 \sin i , \tag{17}$$

and $s_3 = s_2$ (for $x_p = y_p = 0$), the two last step functions in equation (15) set the flux F_{direct} to zero when the white dwarf is obscured by the disk rim, or by the companion (during eclipse).

The total amount of light originating from the emission spot and optically detected by an observer at different orbital phases of the binary system is

$$F = F_{\text{refl}} + F_{\text{direct}} . \tag{18}$$

The computation of this total amount of light will be performed numerically in § III. At orbital phases *not* near eclipse it is also possible to evaluate the optical fluxes analytically for a simple choice of disk parameters. We shall turn to this computation first, because it gives some insight into the question how the observed oscillations attain their sinusoidal shape.

b) *Analytical Computation of Oscillations in a Special Case*

We can analytically calculate the total observed optical flux from the emission spot under the following assumptions:

1. The radius of the inner disk edge is zero. This implies that the radius of the white dwarf is zero.
2. No part of the disk surface is ever obscured by either the companion star or the outer disk edge.
3. The emission spot radiates no optical light directly, i.e., $f_0 = 0$.

With these assumptions the observed flux equals F_{refl}, and the integral (5) simplifies to

$$F = \frac{I_0}{2\pi} \int_0^{r_0} \int_0^{2\pi} \cos \theta \frac{d}{dr}\left(\frac{h}{r}\right) \cos (\psi - \omega_1 t) \times H[\cos (\psi - \omega_1 t)] dr d\psi , \tag{19}$$

as follows from equations (1)–(4). Now use equation (8) to write

$$F = \frac{I_0}{2\pi} (F_1 + F_2) , \tag{20}$$

where

$$F_1 = \cos i \int_0^{h_0/r_0} \int_0^{2\pi} \cos (\psi - \omega_1 t) \times H[\cos (\psi - \omega_1 t)] d\left(\frac{h}{r}\right) d\psi , \tag{21}$$

$$F_2 = -\sin i \int_0^{h_0/r_0} \int_0^{2\pi} \left(\frac{dh}{dr}\right) \cos \psi \cos (\psi - \omega_1 t) \times H[\cos (\psi - \omega_1 t)] d\left(\frac{h}{r}\right) d\psi . \tag{22}$$

A transformation $\psi \to \psi + \omega_1 t$ in equations (21) and (22) shows that F_1 is not a function of t, so that it only contributes to the steady luminosity. We obtain

$$F_1 = 2 \cos i \left(\frac{h_0}{r_0}\right) , \tag{23}$$

$$F_2 = -\frac{\pi \epsilon}{4} \sin i \left(\frac{h_0}{r_0}\right)^2 \cos \omega_1 t , \tag{24}$$

indicating that *the time-variable part of the observed flux from the emission spot is roughly a factor* $(h_0/r_0) \tan i$ *smaller than the steady flux*, and that *the variable part is a pure sinusoid.* Because of the negative sign in equation (24), F_2 reaches a maximum when the spot illuminates the back side of the disk ($\omega_1 t = \pi$), a situation which appears to be realized in DQ Her (see § III). The above calculation, which results in a purely sinusoidal oscillation, assumes an axisymmetric disk and the absence of obscuration effects from disk rim or companion star. Any deviation from these assumptions may introduce higher harmonics in the oscillations.

c) *Parameter Choice for the Numerical Computations*

Using masses of $M_1 = 1 \, M_\odot$ and $M_2 = 0.6 \, M_\odot$ for, respectively, the white dwarf and the companion star, and a binary period of $4^{\text{h}}40^{\text{m}}$, we find from Kepler's third law that there is a separation a of 1.5×10^{11} cm between the two stars. The radius of the white dwarf's Roche lobe is then approximately 4.9×10^{10} cm (Paczyński 1971), and we shall adopt an outer disk radius $r_0 = 4.2 \times 10^{10}$ cm ($\sim 86\%$ that of the lobe), roughly in accordance with disk radii estimated by Papaloizou and Pringle (1977) for disks whose excess angular momentum is removed by tidal torques from the companion. The radius of the white dwarf is taken to be 6×10^8 cm, and assuming that the companion star fills its Roche lobe, we set $R_2 = 3.9 \times 10^{10}$ cm (see Fig. 3).

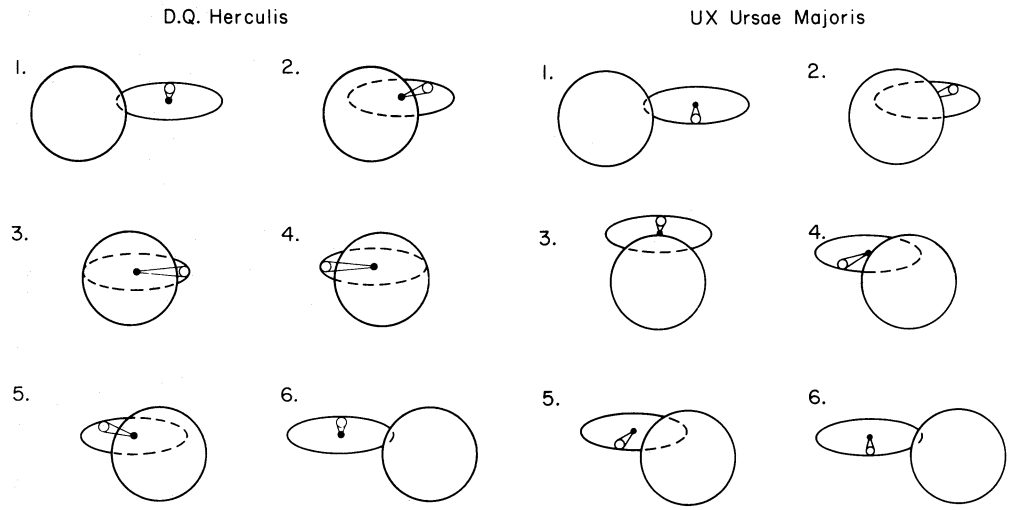

FIG. 3.—Sketch of the model for DQ Her and UX UMa used in the numerical computations in this paper. The binary system consists of a companion star (radius R_2) and a white dwarf (radius R_1) with a large accretion disk (radius R_0). For clarity, white dwarf radius and disk thickness have been drawn larger than is realistic.

Next we must choose for the half thickness h of the disk. We do this by assuming the "standard" accretion disk model (Shakura and Sunyaev 1973) which, in its formulation by Novikov and Thorne (1973), gives a half thickness in the "outer disk region" of

$$h = 6 \times 10^8 \text{ cm } \alpha^{-1/10} \dot{M}_*^{3/20}\left(\frac{r}{r_0}\right)^{9/8} \tag{25}$$

for the above adopted values of M_1 and r_0. The thickness is only weakly dependent on the values of the viscosity parameter α, and on $\dot{M}_*$, the mass accretion rate in units of 10^{17} g s^{-1}. Arbitrarily choosing $\alpha \dot{M}_*^{-3/2} = 10^{-3}$, we obtain

$$h = (1 \times 10^9 \text{ cm})\left(\frac{r}{r_0}\right)^{9/8}, \tag{26}$$

which gives the values $h_0 = 1 \times 10^9$ cm and $\epsilon = 9/8$.

Finally, we adopt $r_i = 2.1 \times 10^9$ cm for the inner boundary of the disk, unless where specifically stated otherwise. Reflection from the surface at the inner edge is neglected. One may physically justify the fact that we do not take $r_i = R_1$, by hypothesizing a magnetosphere which prevents the disk from extending all the way to the white dwarf surface. However, we have taken the above stated value for r_i mainly for computational convenience (avoiding contact between the hot spot and the disk).

III. NUMERICAL RESULTS AND DISCUSSION

a) Models without Direct Optical Light from the Emission Spot

Using the disk model discussed in § II, we numerically determine the total amount of light (steady plus pulsed) seen from the oscillating source at any moment in time. The steady part is not equal to the observed steady luminosity because there are other steady light sources in the system. We Fourier analyze the oscillating part and determine the amplitude and phase of the two lowest harmonics at any orbital phase of the binary system.

Figure 4 shows the phase of the fundamental as the model system goes through an eclipse, for six different inclination angles i between 70° and 90°. As i increases, the eclipse widens somewhat, and the maximum phase shift increases until it reaches $\pm\pi/2$ for near edge-on situations. The shape of the phase shift depends remarkably strongly on i, being like a sinusoid for $i \approx 70°$, and like a sawtooth for $i \approx 90°$.

In comparing these model curves to the observed DQ Her shifts (points with error bars in Fig. 4), one must keep two things in mind. First, the model disk is axisymmetric and does not take account of the hot spot, so that it must necessarily display phase shift behavior which is symmetrical about the center of eclipse. This may explain some differences with the observed shifts, which are somewhat asymmetric. Second, if we estimate the center of the observed phase-shift curve as well as we can (the asymmetry obstructs the attainment of great accuracy here), it lies at approximately $\phi = 0.98$ (using orbital phases defined in Patterson, Robinson, and Nather 1978), i.e., clearly *before* the minimum in the eclipse curve of the total light (defined as $\phi = 0$). Hutchings, Cowley, and Crampton (1979) present spectroscopic evidence suggesting that stellar conjunction also occurs before mid-eclipse, at $\phi \approx 0.96$. The differences in mid-eclipse, stellar conjunction, and phase-shift center are likely to all be aspects of disk asymmetry.

Figure 4 shows that the DQ Her model fits the data well for an inclination angle very close to 90°. The best value is $i = 89°$; however, this way of determining i

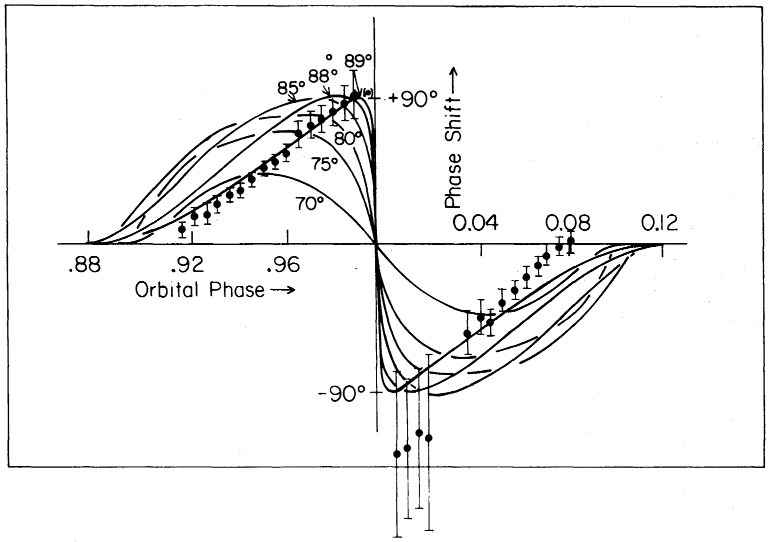

FIG. 4.—Eclipse-related phase shift of DQ Her (*filled circles with error bars*) compared to curves from a model without direct light for different inclination angles. Phase shifts in the rapid oscillations are plotted such that positive values indicate that the pulses come early. Agreement with data is best for models with inclination angles close to 90°. Orbital phase $\phi = 0$ in this figure corresponds to $\phi = 0.98$ in Patterson, Robinson, and Nather (1978).

must be viewed with some caution. When the inclination angle is 89°, the entire front sector of the disk is obscured by the outer disk edge, whose thickness is roughly 1°.5 in the model. If it is this obscuration which makes the fit so good, it is possible that a disk model with a thick outer rim would achieve an equally good fit at a somewhat lower value of i, perhaps as low as 85°. Our simplistic disk model does not contain such a rim, but it is not unreasonable to assume that the real disk might, as a result of, e.g., the mass-transfer process.

In Figure 5 the amplitude of the fundamental is plotted as a function of orbital phase, again for six different inclination angles. Only orbital phases near eclipse are shown. The same comments about asymmetry of the observed curve which we made for Figure 4 apply here. However, in this figure we have no freedom to adjust the eclipse center of the observed points, once we have made a choice in Figure 4. Again, we see that a reasonable fit to the data occurs for angles close to 90°, the best value being again about 89°.

Interestingly, the model behavior in Figure 5 at inclination angles below 80° is strikingly different from that at angles close to 90°. The increase in oscillation amplitude at the smaller angles during eclipse is caused by the fact that the companion star obscures more of the front disk sector than of the back sector, both of which reflect light to the observer outside of eclipse. Since the oscillation amplitude in our model is the difference between reflection from the two disk sectors, it must increase during eclipse if the *less* reflecting sector (in this case the front sector) is more obscured than the *more* reflecting one. The slight wings in the 85° and 88° curves may thus have diagnostic potential.

We have not corrected the observed data points in Figure 5 for the orbital-phase–related smooth, but large, change in the amplitude of the oscillation (see, e.g., Patterson, Robinson, and Nather 1978). This variation is probably caused by the presence of a hot spot, an asymmetrical outer disk rim (Chester 1979), and/or an ellipticity of the disk, none of which are

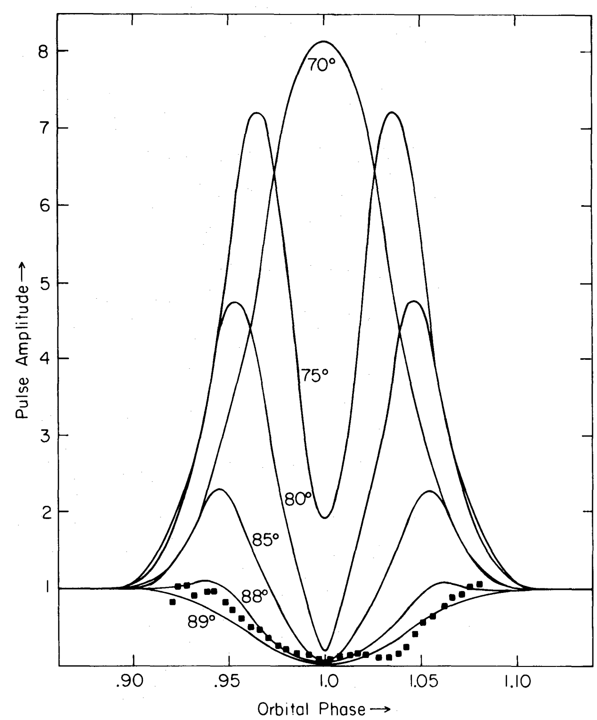

FIG. 5.—Amplitude of the fundamental as a function of binary phase at different inclination angles. Data for DQ Her (*filled squares*) are indicated for comparison. As in Fig. 4, agreement with observations is best for inclination angles close to 90°.

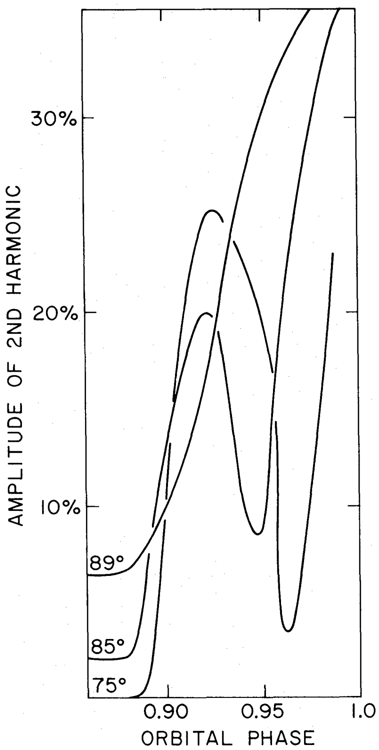

FIG. 6.—Sinusoidality of the oscillations as a function of binary phase, at 3 different inclination angles. Plotted is the ratio of amplitudes of the second harmonic and the fundamental in percentages. Outside of eclipse (phase <0.90) the oscillations are good sinusoids, while during eclipse the amplitude of the second harmonic generally grows.

included in our model. Disks with a rim at the outer edge might again fit the observations at inclination angles somewhat smaller than 89°.

Figure 6 gives an indication of the quality of the sinusoid that the model produces, by showing the amplitude ratio of the second harmonic (first overtone) and the fundamental. Outside of eclipse, the second harmonic is always more than an order of magnitude down in amplitude, for the values of i considered here. During eclipse, it increases, but it only becomes comparable in amplitude to the fundamental at orbital phases close to the eclipse center, assuming values of i near 90°. Throughout the rest of the eclipse the oscillation remains a fairly good sinusoid, in agreement with observations. Data by Kiplinger and Nather (1975) for DQ Her suggest that the second harmonic is probably present at the $\sim 4\%$ level outside eclipse, compatible with Figure 6 for an inclination angle between 85° and 90°.

So far we have only compared our model curves with observations of DQ Her. Our reason for leaving UX UMa out of the picture is of course that its phase shift characteristics are very different from those shown by the model curves in Figures 4–6. Things will change, however, when we allow a small fraction of the light from the emission spot to be seen directly by the optical observer.

b) *Models with Some Direct Optical Light from the Emission Spot*

By allowing some optical radiation from the spot on the white dwarf to be observed directly, i.e., without reflection by the disk, we create a situation in which an optical observer may receive more light when the spot is pointed *at* him than when it is pointed to the back side of the disk. With such a model one may hope to reproduce the eclipse-related phase-shift behavior of UX UMa, as we shall now demonstrate. Since the eclipse-related phase-shift extends over the entire width of the eclipse and shows no abrupt jumps, the *direct* contribution to the oscillating light, which is cut off sharply at eclipse because of the small size of the white dwarf, cannot be much larger than the contribution from the reflected part. We experimented with different values for the parameter f_0, the fraction of light emitted by the spot in the optical band, to see what would fit the data. Good results were obtained with values $f_0 \approx 0.05$–0.2%.

Figure 7 shows the phase shift of the oscillation during eclipse for $f_0 = 0.1\%$ and inclination angles between 65° and 90°. All curves with $i \leq 85°$ show the oscillation losing exactly one cycle during eclipse, similarly to the observed shift in UX UMa, which is indicated by open circles in the figure. Our explanation

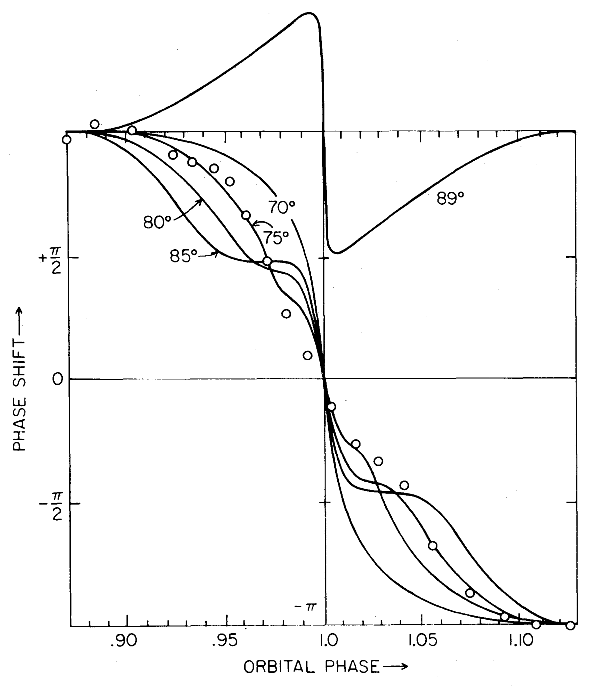

FIG. 7.—Eclipse-related phase shift of UX UMa (*open circles*) compared to curves from a model with $f_0 = 0.1\%$, described in § II, for different inclination angles. The curve for 89° is very different from the curves at other inclination angles, because the outer disk edge is in this case obscuring the entire front half of the disk so that reflection of oscillating light from the back side must always dominate over reflection from the front side of the disk, and no phase shift by 2π is possible. Agreement with data is best for models with an inclination angle near 75°.

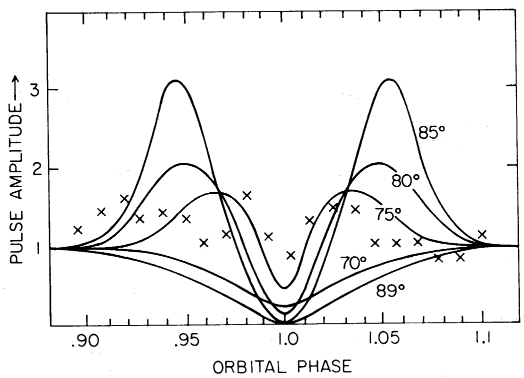

FIG. 8.—Amplitude of the fundamental as a function of binary phase for models with $f_0 = 0.1\%$, at different inclination angles. Data for UX UMa (*crosses*) are indicated for comparison. As in Fig. 5, agreement with the data is best for inclination angles near 75°.

is as follows: Outside of eclipse, maxima of the oscillation sinusoid occur when the illuminating beam hits the front sector of the disk. At eclipse center the companion obscures this front sector, but leaves the back sector partially uncovered if i is not too large (e.g., 70°). Thus, maxima of the oscillation at eclipse center then occur when the beam is pointed at the back sector of the disk, which implies a phase shift of π. In the course of the eclipse these maxima occur when the beam is directed at a disk sector which travels around precisely one cycle in the prograde direction: a total phase shift of 2π (see Fig. 3).

As Figure 7 shows, for $i = 70°$ the shift is gradual, and for $i = 85°$ it becomes steep near eclipse center, as a result of the more complete obscuration of the disk. However, for inclination angles so close to 90° that the entire front sector of the disk is obscured by the disk rim itself, reflection from the back sector always dominates, and no 2π phase shift is possible. This is shown in the curve labeled 89°, which is identical to the similarly labeled one in Figure 4. The model curves seem to fit the UX UMa data reasonably well for

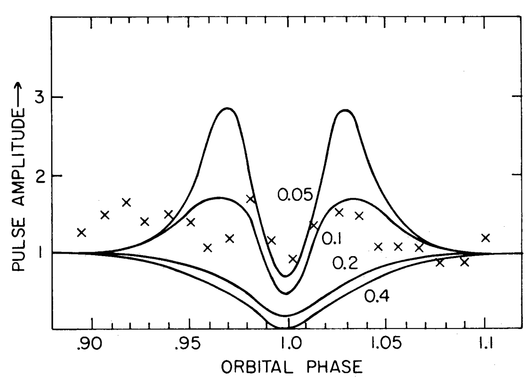

FIG. 9.—Amplitude of the fundamental as a function of binary phase for models with different values of f_0 (indicated with every curve), all at an inclination angle $i = 75°$. Crosses mark observational results of Nather and Robinson (1974).

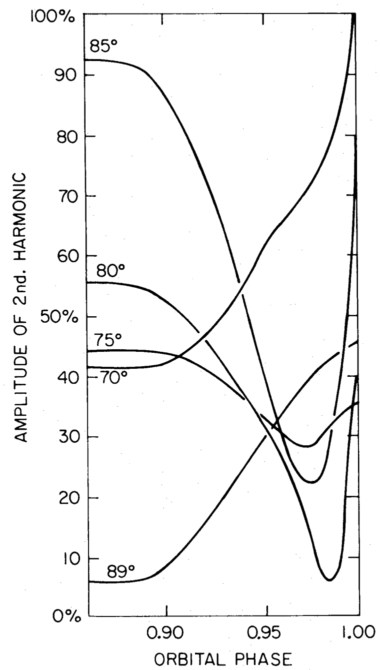

FIG. 10.—Sinusoidality of the oscillations as a function of binary phase, for models with $f_0 = 0.1\%$, and different inclination angles. Plotted is the ratio of amplitudes of the second harmonic and the fundamental in percentages. Note that the second harmonic decreases in relative strength during eclipse for $i = 75°$.

values of $i \approx 75°$. However, the same comments about asymmetry and eclipse centering that we mentioned for Figures 4 and 5 should be recalled.

In Figure 8 the oscillation amplitude of the model with $f_0 = 0.1\%$ is compared to the observations of UX UMa. The poor quality of the data makes a hard confrontation impossible here, but the large peaks shown in the 85° curve, or the drop to zero in the 89° curve, do not seem present in the data. The 75° curve again appears to produce the best fit. For slightly different values of f_0, the amplitude of the oscillation would be different during eclipse, as shown in Figure 9. Here we have plotted the oscillation amplitude for a model with f_0 values between 0.05% and 0.4% at $i = 75°$. The phase shift curve is not strongly affected by these small variations in f_0, but would be if f_0 varied over a wider range. For $f_0 < 0.02$ the direct light from the white dwarf becomes too faint, and the phase-shift curves become similar to the ones shown in Figure 4.

Figure 10 summarizes the strength of the second harmonic in a system with $f_0 \approx 0.1\%$ and different inclination angles, as a function of binary phase. In some cases (e.g., at 85°) the second harmonic can be present prominently, although this is also dependent on the choice we have made for f_0. The model at 75°, which fits the observations of amplitude and phase shift of the fundamental best, produces a second harmonic which is at least a factor of 2 smaller in amplitude than the fundamental and does not grow larger in the course of eclipse. If correct, this model suggests that the second harmonic in UX UMa may be detectable with only a modest improvement over presently available data.

c) Discussion

Although the ability of the model to reproduce the eclipse-related phase-shift observations of DQ Her and UX UMa seems impressive, we remind the reader that the model is still very primitive in several respects. Perhaps the most serious shortcoming is that the disk is assumed to be axisymmetric. We have mentioned that this assumption is the likely cause of several differences with the observations of amplitude and phase shift of the fundamental, shown in Figures 4, 5, 7, and 8. It is also almost certain to affect the quality of the sinusoid: nonaxisymmetric disks produce a complicated mix of higher harmonics in the reflection process.

Another important simplification was our assumption that the source of the oscillations was a single small spot on the equator of the rotating white dwarf. One may argue that if the emission spot is a magnetic pole, both poles should be considered, and there is no compelling reason to place the poles on the equator. On the other hand, the emission spot in UX UMa is not likely to be a magnetic pole, so the assumed geometry of the spot is almost certainly far from the truth for that system. Our attitude in this first paper on eclipse-related phase shifts has been to stay away from questions about the emission spot, to use the simplest possible model for it, and concentrate on the reflection process by the disk. In a subsequent paper we will investigate the more detailed properties and geometry of the emission spot in DQ Her, including the closely related question whether the fundamental rotation period is 71 s or 142 s. Finally, in a third paper, we will study the properties of the emission spot in UX UMa (and other systems with a variable oscillation period), to see whether any clues about the nature of the rapid oscillations can be found.

IV. CONCLUSIONS

Numerical simulations of the behavior of the rapid oscillations during eclipse in DQ Her and UX UMa led to the following results:

1. The essential characteristics of the eclipse-related phase shifts were reproduced quite well for both systems by a simple model, seen at two different inclination angles.
2. This model used a *progradely* rotating emission spot on the surface of the white dwarf, showing that the UX UMa data do not necessarily require a retrogradely rotating oscillation source.
3. No claim is made that the success of the fits indicates that the source of the rapid oscillations is a spot on the surface of the rotating white dwarf. Equally good fits may result if the source of the oscillation were an orbiting blob of hot matter at the disk's inner edge, or even a *progradely* moving nonradial oscillation of the white dwarf.
4. The model fits the DQ Her data particularly well for inclination angles so close to 90° that the entire front half of the disk (including the white dwarf) was obscured by the outer edge of the disk. This occurred at $i \approx 89°$ in the presented model, but might occur at somewhat lower values for i if the disk had a thickened outer rim.
5. The suggested permanent obscuration of the white dwarf by the disk in DQ Her provides a natural explanation for the repeated failure to detect any X-rays from this system.
6. The model fits the UX UMa data well, if $\sim 0.1\%$ of the oscillating light from the emission spot on the white dwarf lies in the optical band, and can be seen directly. This direct light will not affect the agreement of the model with DQ Her observations if the white dwarf in that system is indeed permanently obscured. For UX UMa an inclination angle near 75° seems to fit best.
7. Oscillations from the model were generally found to be fairly good sinusoids. For DQ Her the observed amplitude of the second harmonic ($\sim 4\%$) outside of eclipse roughly agreed with the model-produced value. For UX UMa the model predicts the presence of a second harmonic with amplitude between 25% and 50% of the fundamental and varying during eclipse.
8. The model predicts variations in the amplitude of the fundamental and second harmonic for UX UMa, which have a distinct signature. Observational determination of these variations would be very useful.
9. The model also predicts that if a wavelength-dependent phase shift could be observed for UX UMa, it would occur in the direction opposite to that in DQ Her.
10. Other systems which show rapid oscillations and eclipses may provide further tests of the model. Two of them (HT Cas and Z Cha) are currently under investigation by the author.

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REFERENCES

Alpar, M. A. 1979, *M.N.R.A.S.*, **189**, 305.
Bath, G. T. 1973, *Nature Phys. Sci.*, **246**, 84.
Chanan, G. A., Nelson, J. E., and Margon, B. 1978, *Ap. J.*, **226**, 963.
Chester, T. J. 1979, *Ap. J.*, **230**, 169.
Herbst, W., Hesser, J. E., and Ostriker, J. P. 1974, *Ap. J.*, **193**, 679.
Hutchings, J. B., Cowley, A. P., and Crampton, D. 1979, *Ap. J.*, **232**, 500.
Kiplinger, A. L., and Nather, R. E. 1975, *Nature*, **255**, 125.
Nather, R. E., and Robinson, E. L. 1974, *Ap. J.*, **190**, 637.
Nelson, M. R., and Olson, E. C. 1976, *Ap. J.*, **207**, 195.
Novikov, I. D., and Thorne, K. S. 1973, *Les Houches Lectures: 1972, Black Holes*, ed. B. DeWitt and C. DeWitt (New York: Gordon and Breach).
Paczyński, B. 1971, *Ann. Rev. Astr. Ap.*, **9**, 183.

Papaloizou, J., and Pringle, J. E. 1977, *M.N.R.A.S.*, **181**, 441.
Patterson, J., Robinson, E. L., and Nather, R. E. 1978, *Ap. J.*, **224**, 570.
Rees, M. J. 1974, *M.N.R.A.S.*, note added in proof, **166**, 113.
Shakura, N. I., and Sunyaev, R. A. 1973, *Astr. Ap.*, **24**, 337.
Tyson, J. A., Nelson, J. E., and King, R. A. 1978, preprint.
Walker, M. F. 1956, *Ap. J.*, **123**, 68.
Warner, B., and Robinson, E. L. 1972, *Nature Phys. Sci.*, **239**, 2.

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