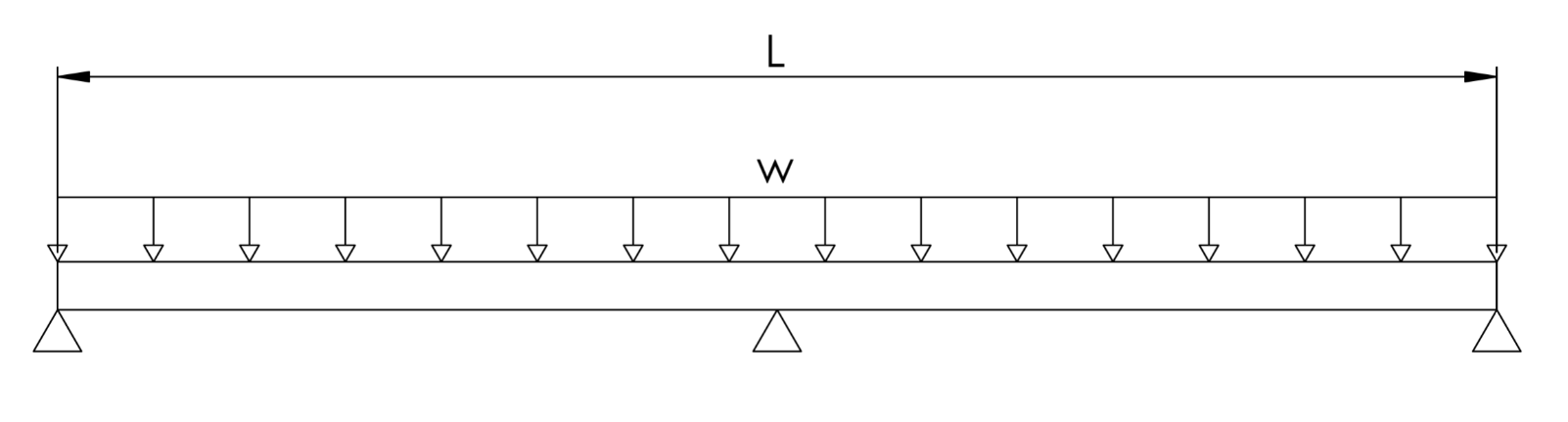
OBJECTIVE: To determine, analytically, the reaction forces *F1*, *F2*, …,*Fn* of a continuous beam of length *L* with *n* equidistant supports subjected to a transversal uniform load *w*.

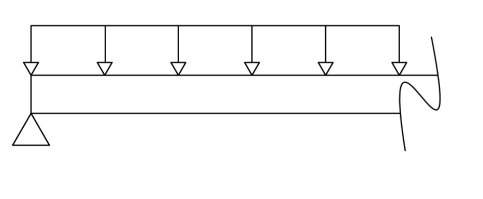
1. Three Supports



Three unknown variables (), two equilibrium equations:

|  |  |
| --- | --- |
|  | (01) |

|  |  |
| --- | --- |
|  | (02) |



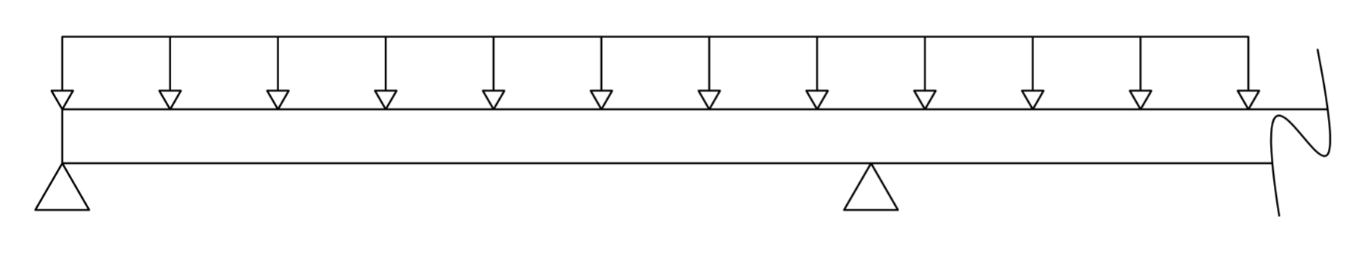
Bending moment (section 01):

Elastic line equation (section 01):

Boundary conditions (section 01):

|  |  |
| --- | --- |
|  | (03) |
|  | (04) |

Five unknown variables () and four equations (2 equilibrium equations and 2 boundary condition equations).



Bending moment (section 02):

Elastic line equation (section 02):

Bounday conditions (section 02):

|  |  |
| --- | --- |
|  | (05) |

Continuity equations (section 01 and 02):

|  |  |
| --- | --- |
|  | (06) |

|  |  |
| --- | --- |
|  | (07) |

Seven unknown variables () e seven equations (2 equilibrium equations, 3 boundary condition equations and 2 continuity equations).

The linear system can be represented by the following matrices :

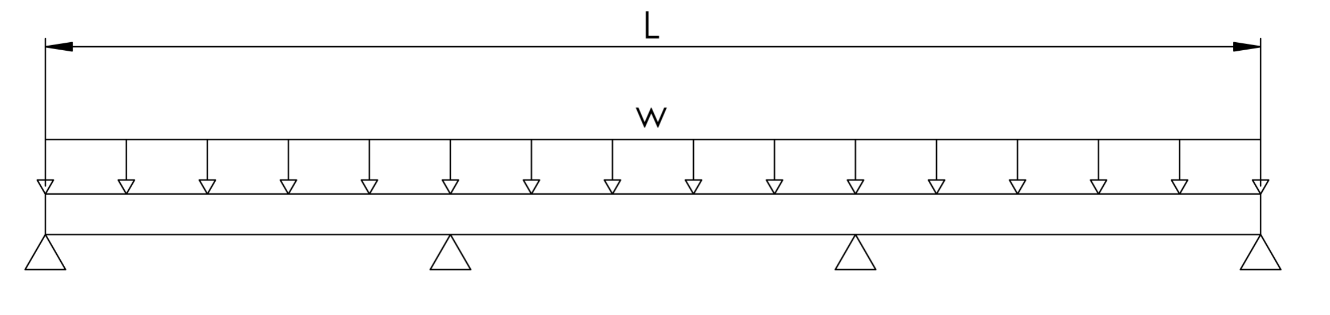
A =

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |

b =

|  |
| --- |
|  |
|  |
|  |
|  |
|  |
| 0 |
| 0 |

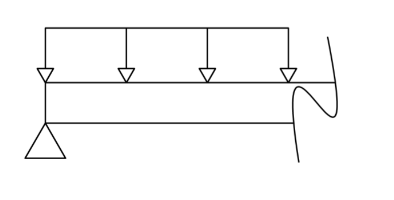
1. Four supports



Four unknown variables (), two equilibrium equations:

|  |  |
| --- | --- |
|  | (01) |

|  |  |
| --- | --- |
|  | (02) |



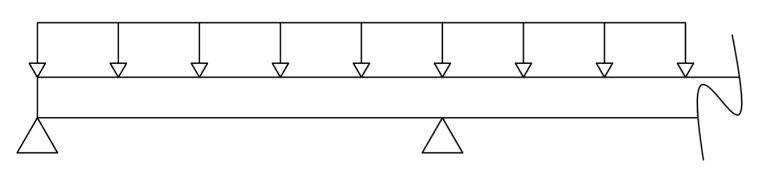
Bending moment (section 01):

Elastic line equation (section 01):

Boundary conditions (section 01):

|  |  |
| --- | --- |
|  | (03) |
|  | (04) |

Five unknown variables () and four equations (2 equilibrium equations and 2 boundary condition equations).



Bending moment (section 02):

Elastic line equation (section 02):

Boundary conditions (section 02):

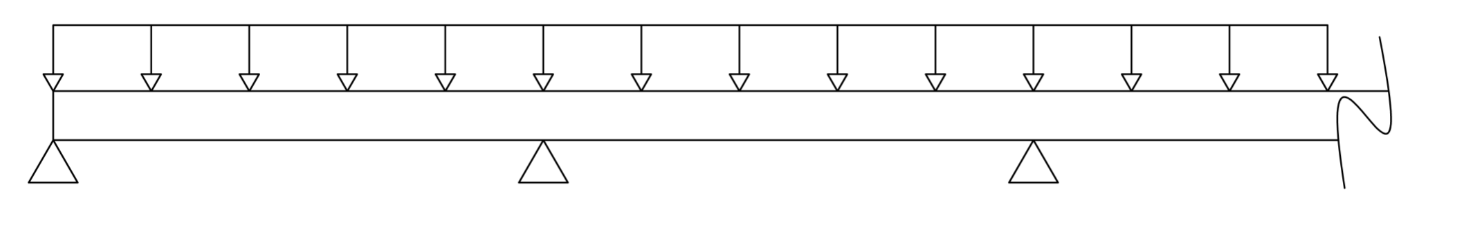
|  |  |
| --- | --- |
|  | (05) |

Continuity equations (sections 01 and 02):

|  |  |
| --- | --- |
|  | (06) |

|  |  |
| --- | --- |
|  | (07) |

Eight unknown variables () and seven equations (2 equilibrium equations, 3 boundary condition equations and 2 continuity equations).



Bending moment (section 03):

Elastic line equation (section 03):

Boundary conditions (section 03):

|  |  |
| --- | --- |
|  | (08) |

Continuity equations (sections 02 and 03):

|  |  |
| --- | --- |
|  | (09) |

|  |  |
| --- | --- |
|  | (10) |

Ten unknown variables () and ten equations (2 equilibrium equations, 4 boundary condition equations and 4 continuity equations).

Representing the linear equations through matrices:

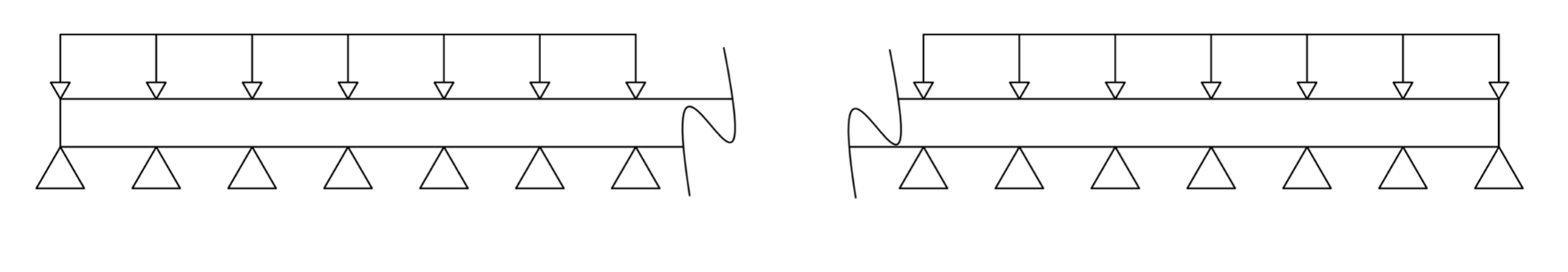
A =

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  | 1 |  |  |  |  | 0 | 0 |
|  |  |  | L |  |  |  |  | 0 | 0 |
|  |  |  | 0 |  |  |  |  | 0 | 0 |
|  |  |  | 0 |  |  |  |  | 0 | 0 |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  | 0 |  |  |  |  |  | 0 |
|  |  |  | 0 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  | 1 | 0 |
|  |  |  |  |  |  |  |  |  |  |

b =

|  |
| --- |
|  |
|  |
|  |
|  |
|  |
| 0 |
| 0 |
|  |
| 0 |
| 0 |

1. N supports (general solution)



Bending moment for three supports:

* Section 01
* Section 02

Bending moment for four supports:

* Section 01
* Section 02
* Section 03

General solution

Bending moment for *n* supports:

* Section j

Integrating the expression to arrive at the elastic line equation yields:

Matrix for three supports:

A =

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |

b =

|  |
| --- |
|  |
|  |
|  |
|  |
|  |
| 0 |
| 0 |

Matrix for four supports:

A =

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  | 1 |  |  |  |  | 0 | 0 |
|  |  |  | L |  |  |  |  | 0 | 0 |
|  |  |  | 0 |  |  |  |  | 0 | 0 |
|  |  |  | 0 |  |  |  |  | 0 | 0 |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  | 0 |  |  |  |  |  | 0 |
|  |  |  | 0 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  | 1 | 0 |
|  |  |  |  |  |  |  |  |  |  |

b =

|  |
| --- |
|  |
|  |
|  |
|  |
|  |
|  |
| 0 |
| 0 |
| 0 |
| 0 |

Matrix for n supports:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| -- |  |  |  | ... |  |  |  |  |  |  | ... |  |  |
| -- |  |  |  | ... | 1 | 1 |  |  |  |  | ... | 0 | 0 |
| -- |  |  |  | ... |  |  |  |  |  |  | ... | 0 | 0 |
| -- |  |  |  | ... | 0 | 0 |  |  |  |  | ... | 0 | 0 |
|  |  |  |  | ... | 0 | 0 |  |  |  |  | ... | 0 | 0 |
|  |  |  |  | ... | 0 |  |  |  |  |  | ... |  |  |
| ⁞ | ⁞ | ⁞ | ⁞ | ... | ⁞ | ⁞ | ⁞ | ⁞ | ⁞ | ⁞ | ... | ⁞ | ⁞ |
|  |  |  |  | ... |  | 0 |  |  |  |  | ... |  |  |
|  |  |  |  | ... | 0 |  |  |  |  |  | ... |  |  |
|  |  |  |  | ... | 0 |  |  |  |  |  | ... |  | 0 |
| ⁞ | ⁞ | ⁞ | ⁞ | ... | ⁞ | ⁞ | ⁞ | ⁞ | ⁞ | ⁞ | ... | ⁞ | ⁞ |
|  |  |  |  | ... |  |  |  |  |  |  | ... |  |  |
|  |  |  |  | ... |  |  |  |  |  |  | ... |  |  |

b =

|  |
| --- |
|  |
|  |
|  |
|  |
|  |
| ⁞ |
|  |
| 0 |
| ⁞ |
| 0 |
| 0 |

The linear system for the general solution is represented by a matrix A of size and a vector *b* of size . The system can be solved through Gauss’s elimination method in order to be as close as possible to an analytical solution.