Exercises for ICTP School 2025

Exercise 1: Gravitational wave properties

The propagation of gravitational waves is a curved background is described by

$$\Box\bar{h}_{\mu\nu} + 2\bar{R}_{\alpha\mu\beta\nu}\bar{h}^{\alpha\beta} = 0\,. \tag{0.0.1}$$

Using the plane wave ansatz

$$h_{\mu\nu}(x) = \mathrm{Re}\left[\left(A_{\mu\nu}(x) + \varepsilon A^{(1)}_{\mu\nu}(x) + \varepsilon^2 A^{(2)}_{\mu\nu}(x) + \cdots\right) e^{i\theta(x)/\varepsilon}\right]\,, \tag{0.0.2}$$

explicitly derive that:

1. GWs propagate in null geodesics,
2. the number of gravitons is preserved,
3. the polarization tensor is orthogonal to the propagation direction and parallel transported.

Bibliography