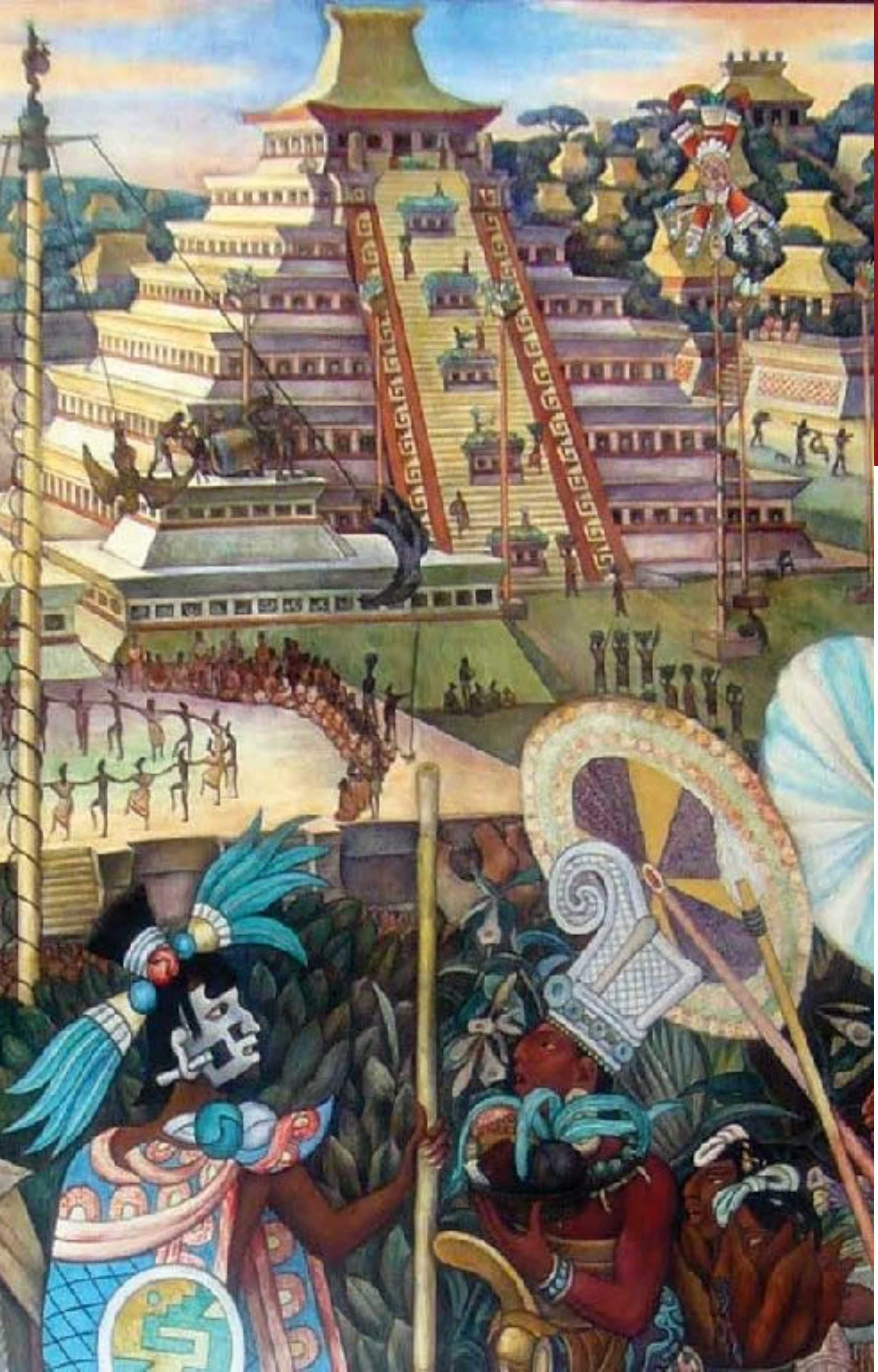


[Diego Rivera]

# Gravitational Waves


**Jose María Ezquiaga**

Niels Bohr Institute

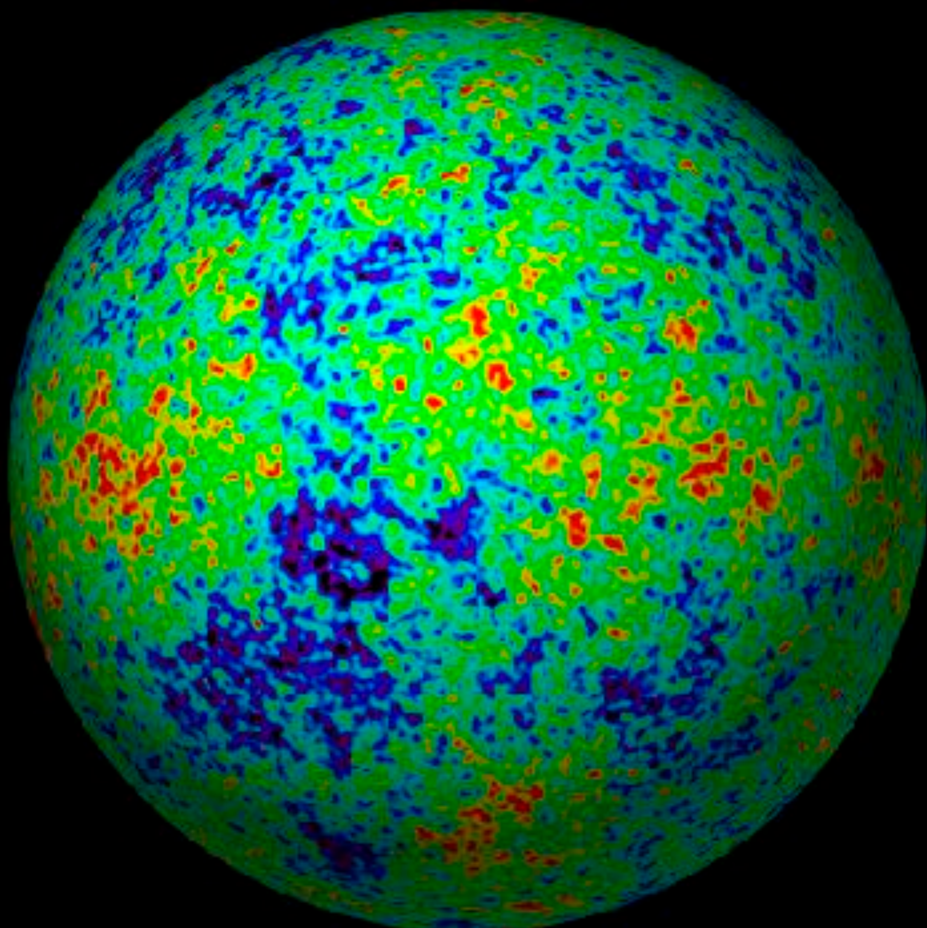
jose.ezquiaga@nbi.ku.dk

ezquiaga.github.io


VILLUM FONDEN

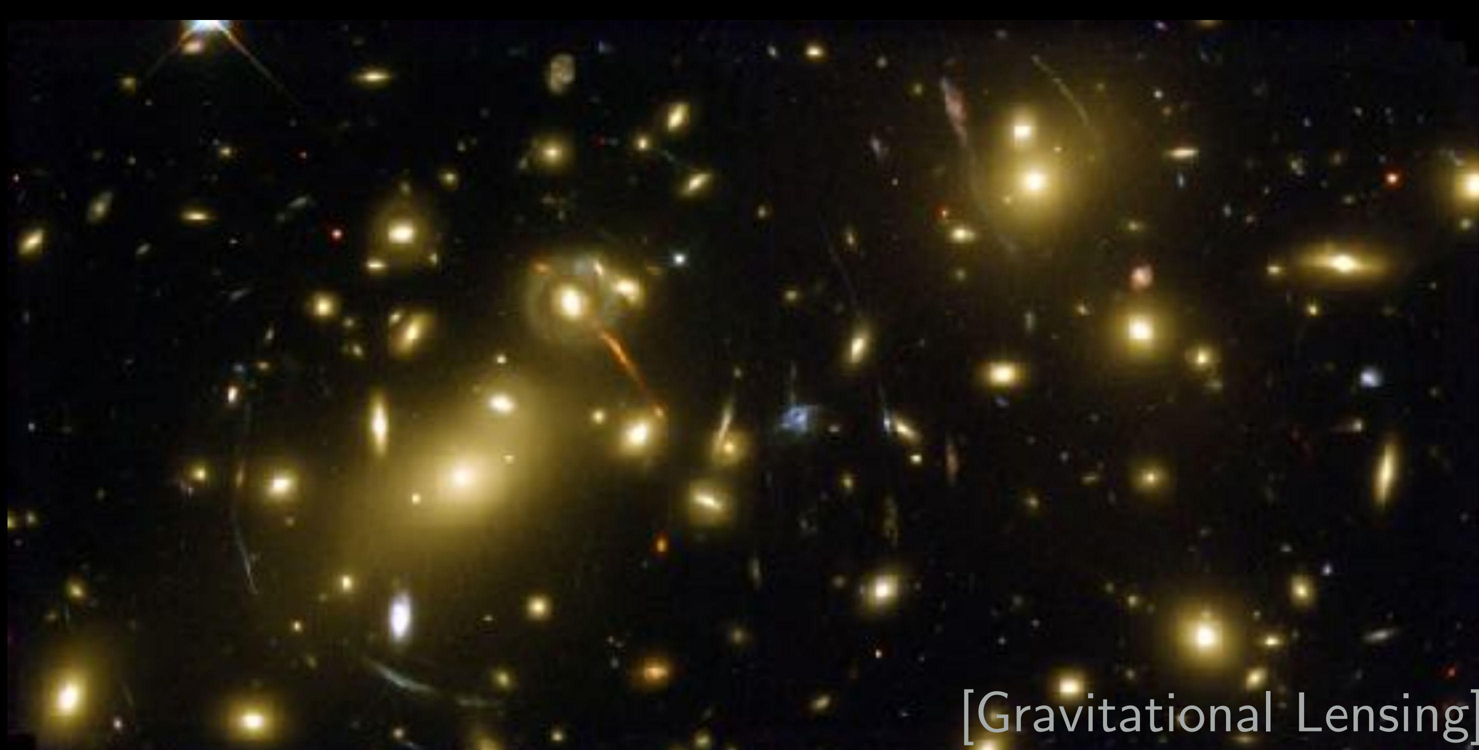
KØBENHAVNS UNIVERSITET

# A plethora of cosmological observations

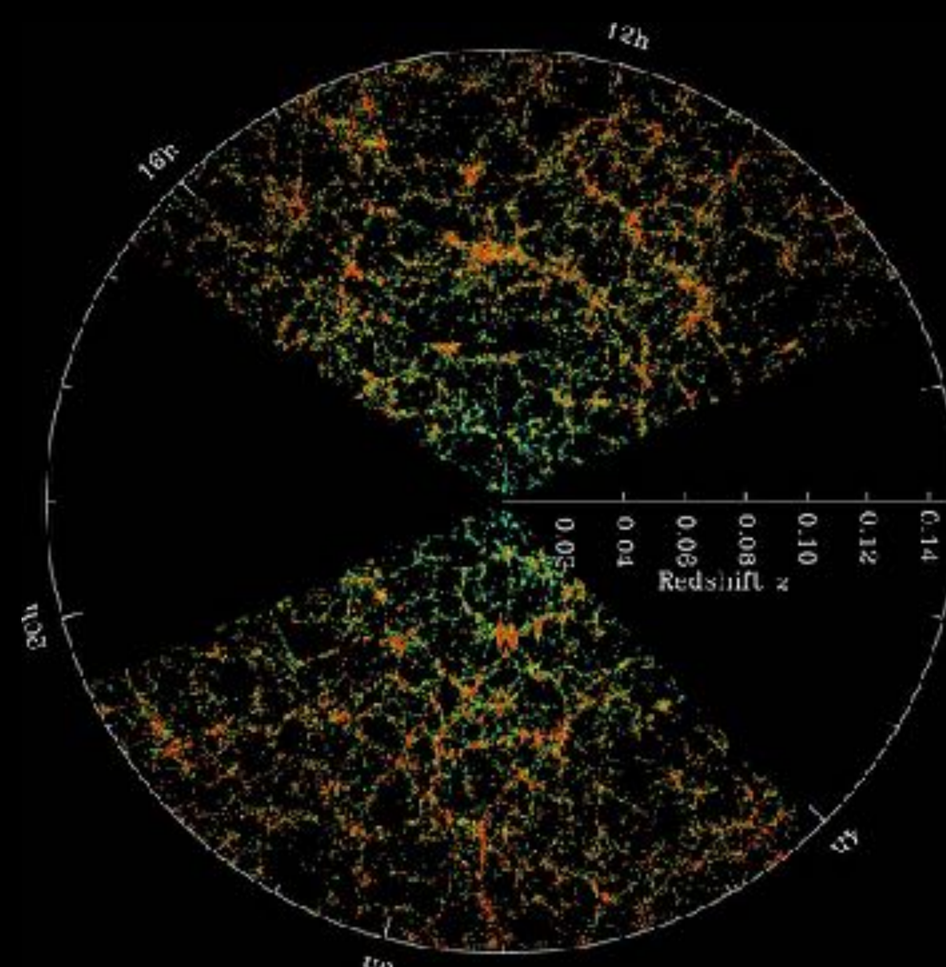


[Cosmic microwave background]

[Supernova]

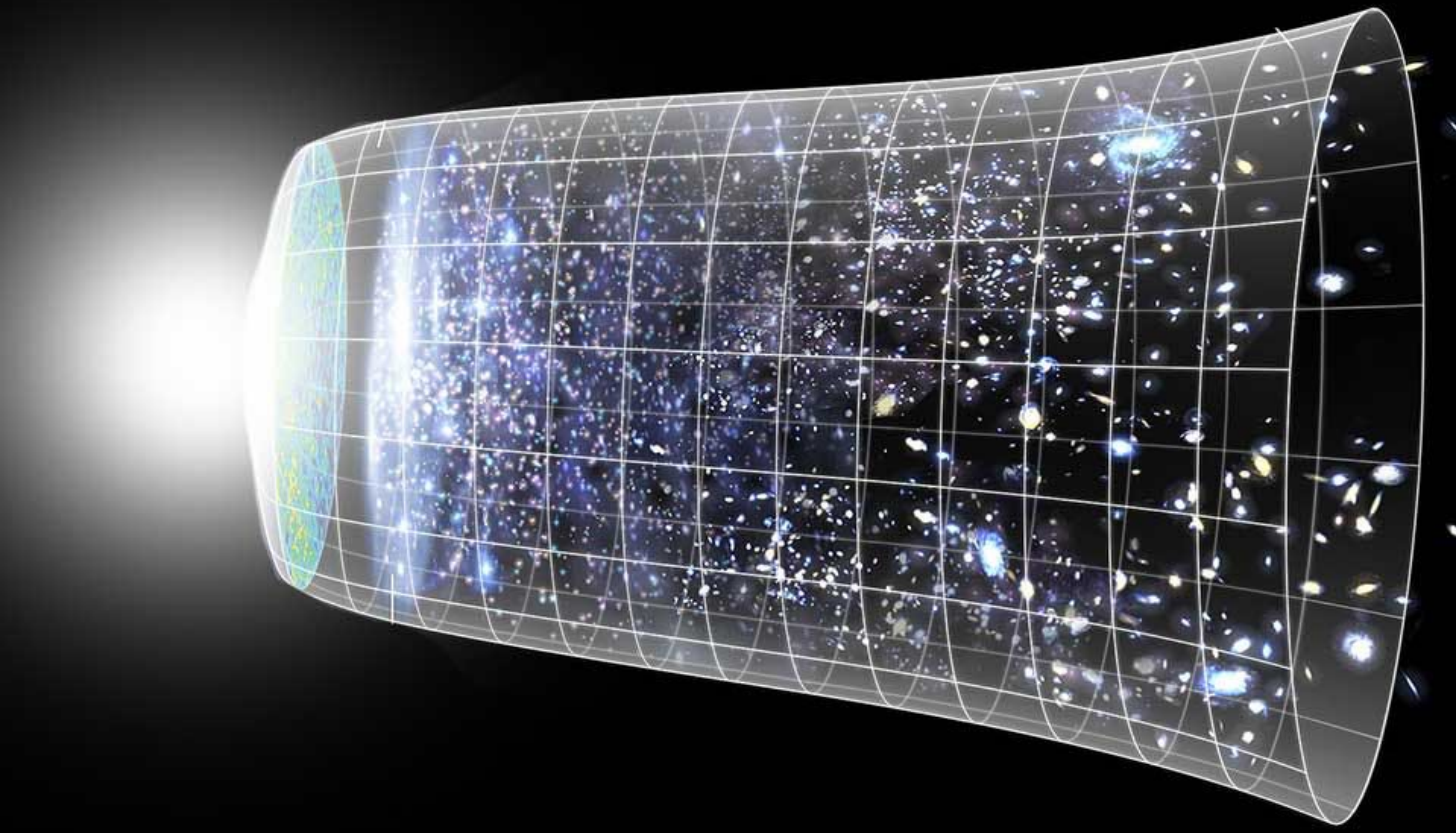


[Gravitational Lensing]



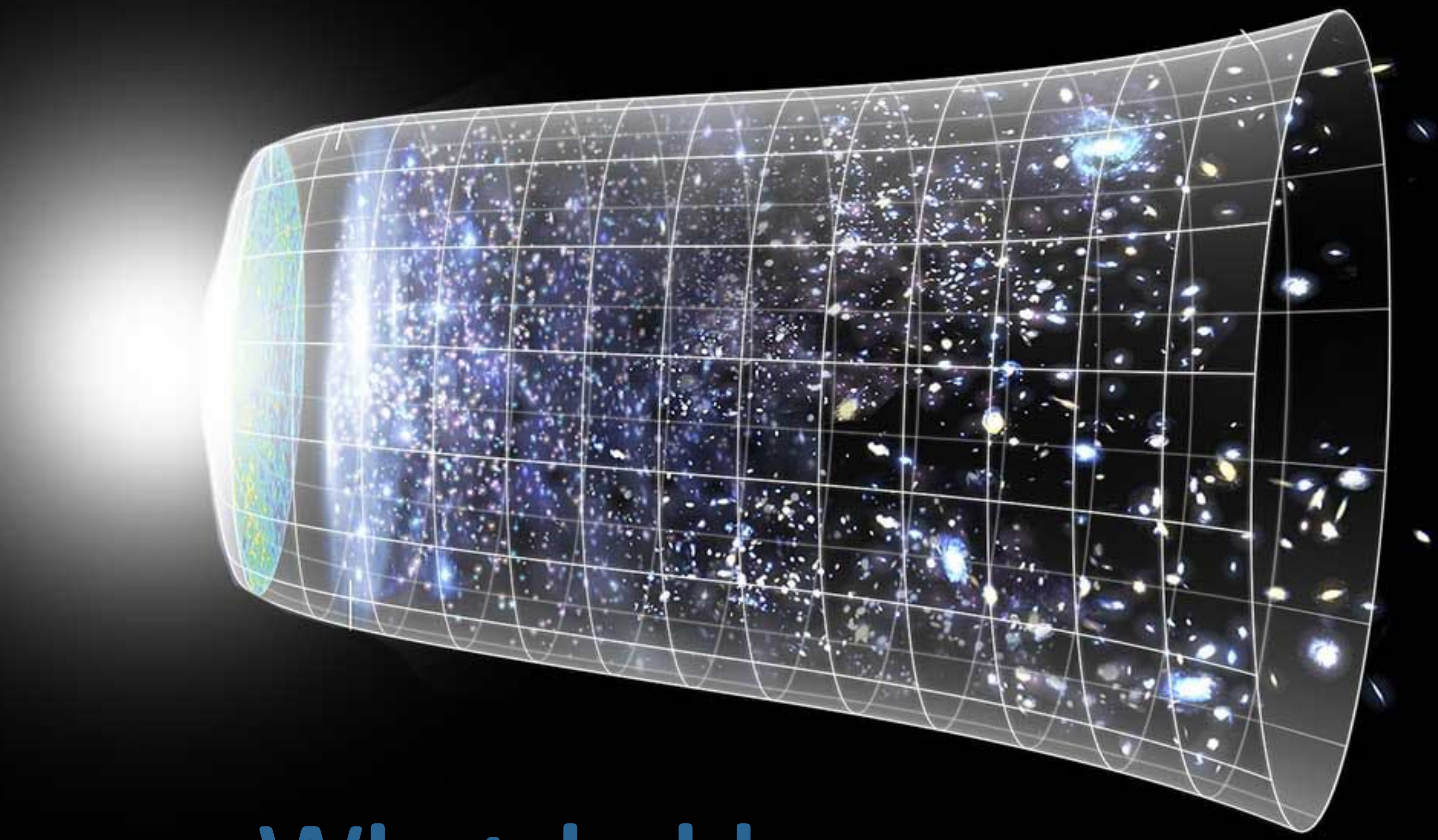

[Cosmic web]

# The **standard** cosmological model...



...13.8 billion years of cosmic history

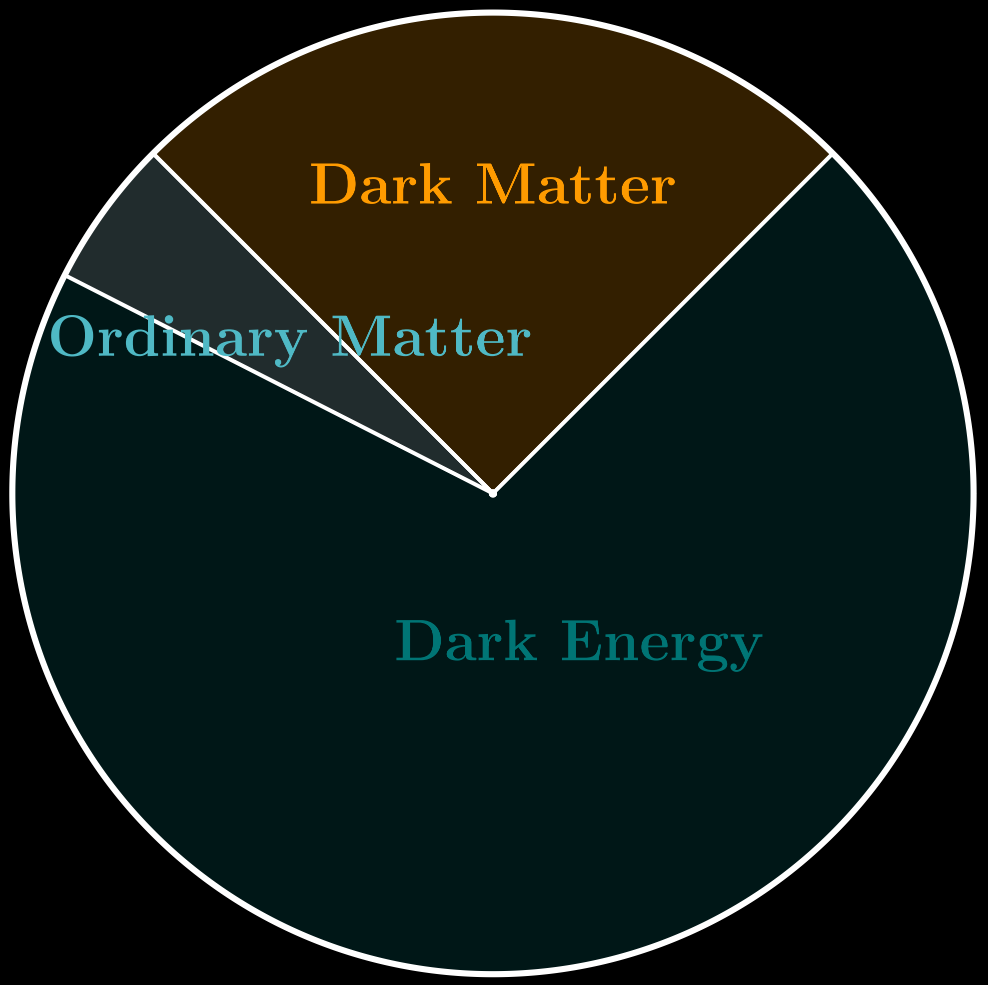
# We **don't** understand the basics of our Universe



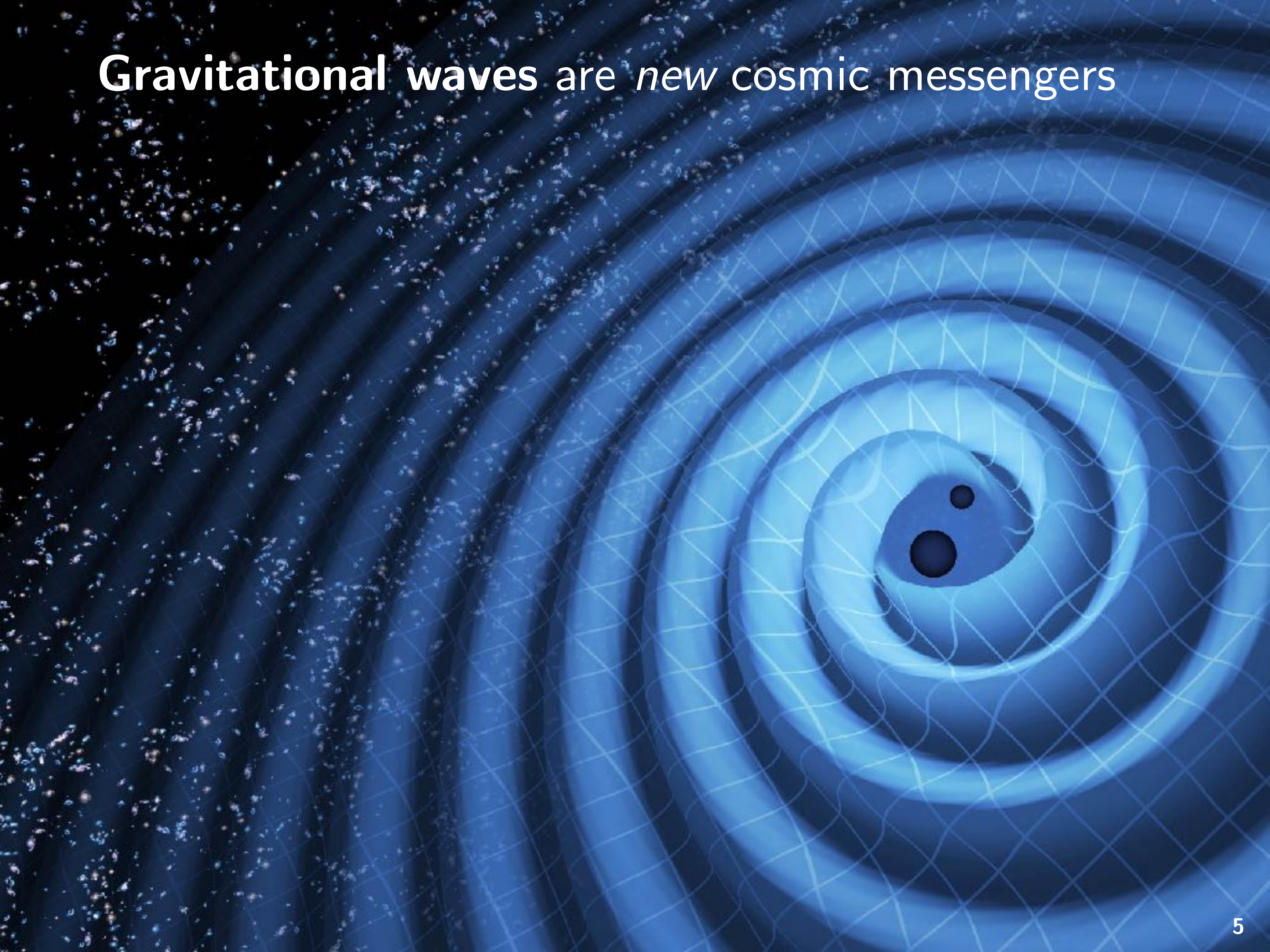
**Why the Universe expands ever faster?**

**What holds galaxies together?**

**Is Einstein gravity valid at cosmological scales?**

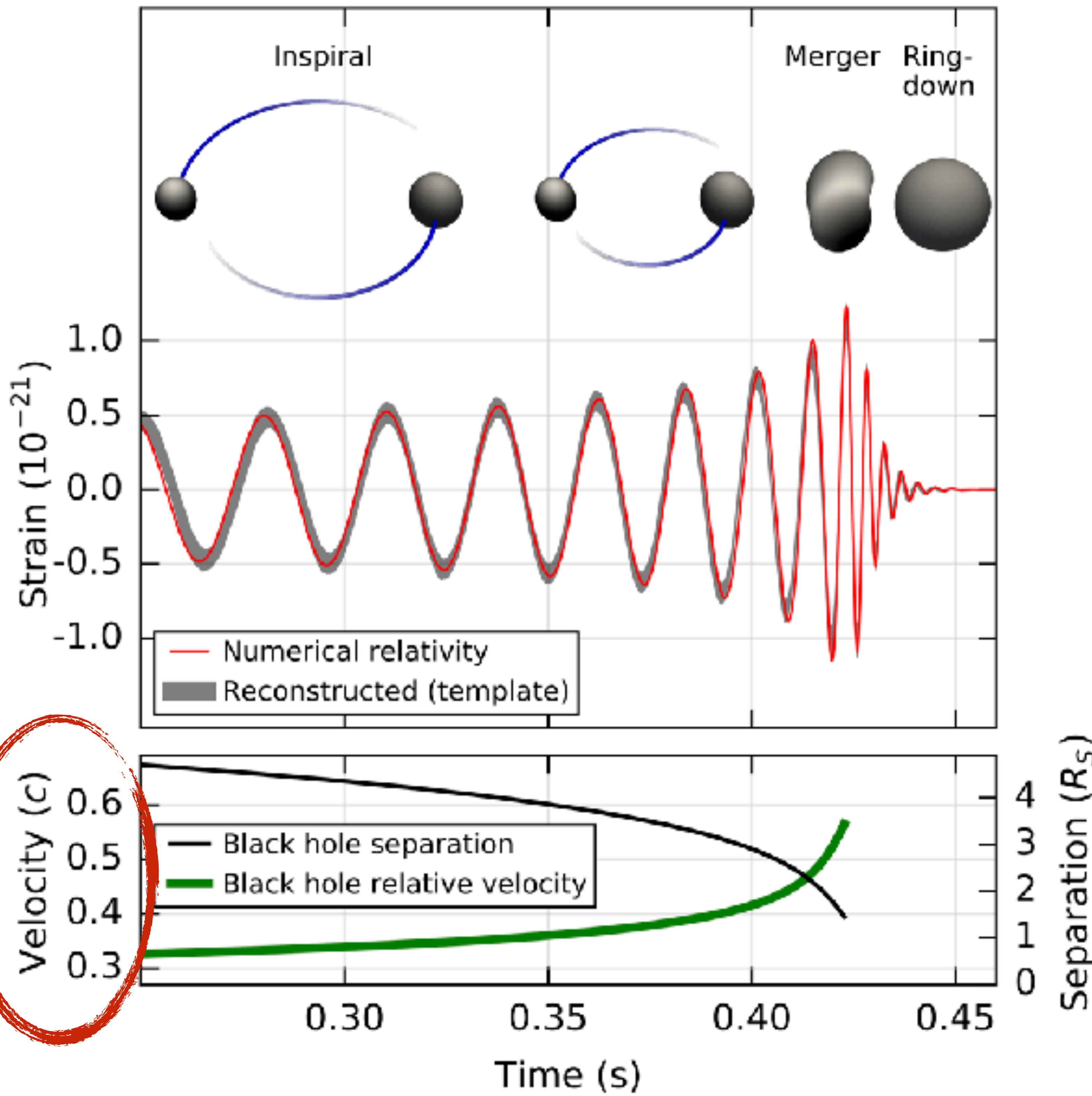


# **Gravitational waves** are *new* cosmic messengers



# Gravitational waves from stellar-mass **binary black holes**

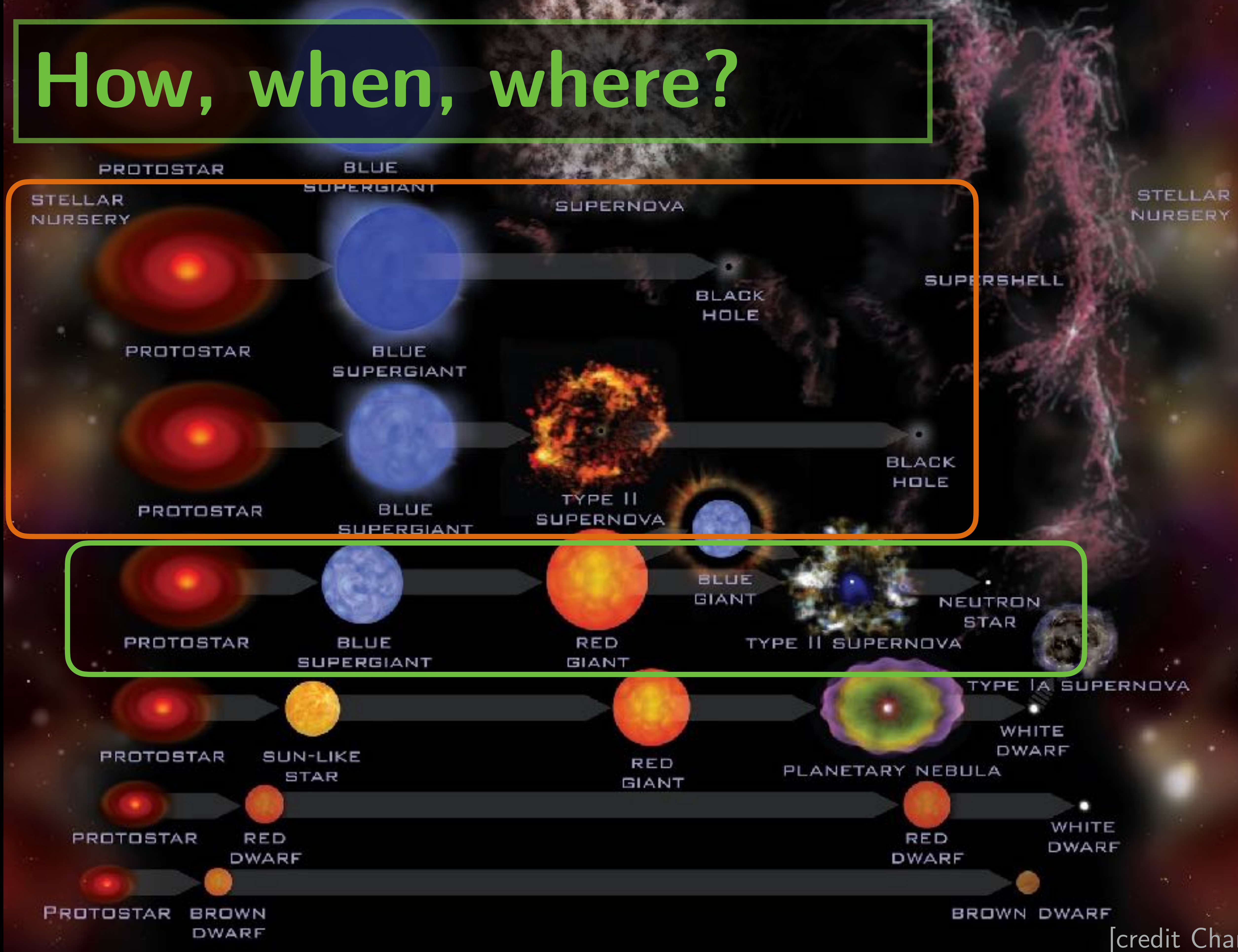
**Strong-field gravity**



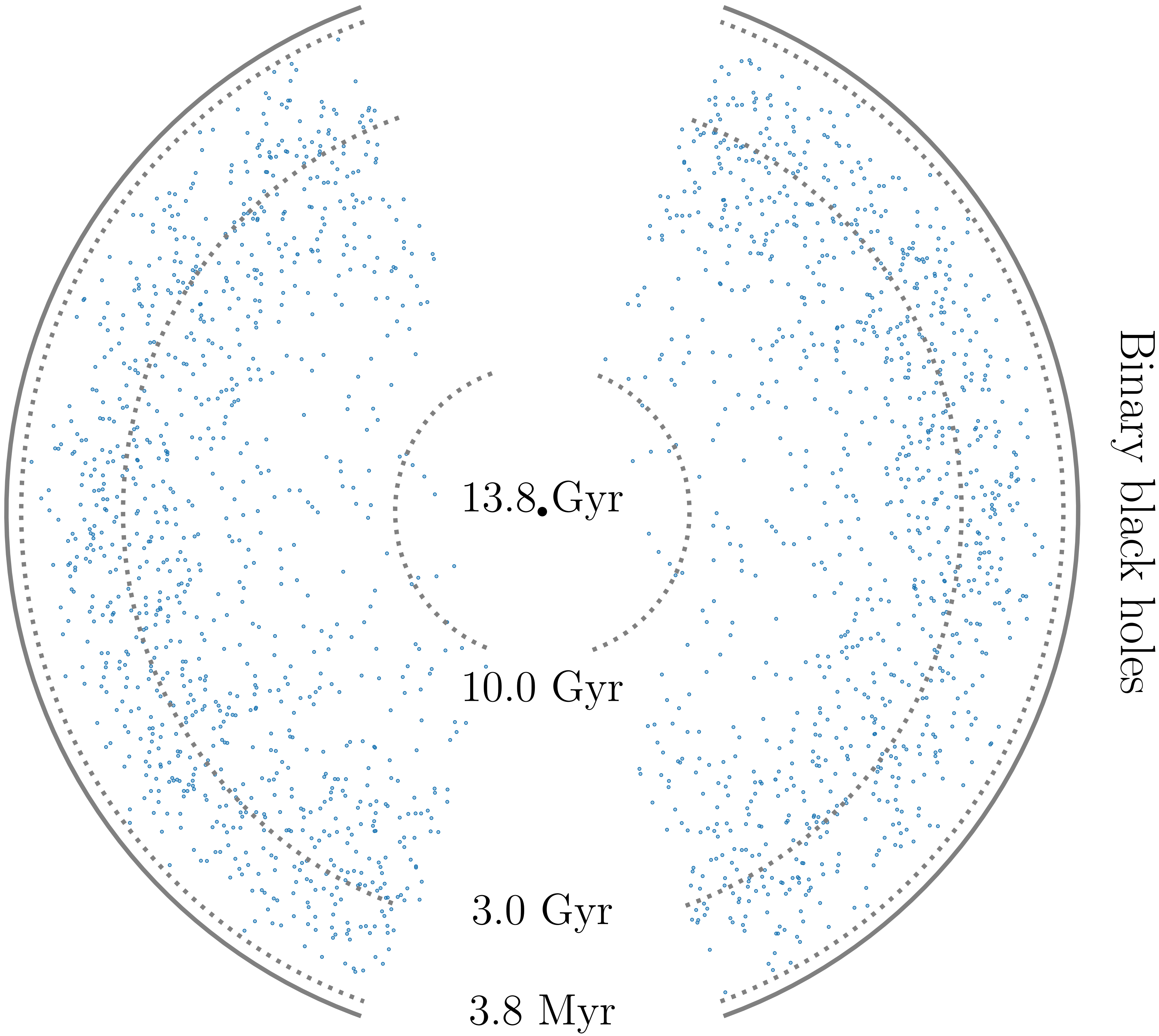

[First detection, GW150914]

# Stellar evolution

## How, when, where?

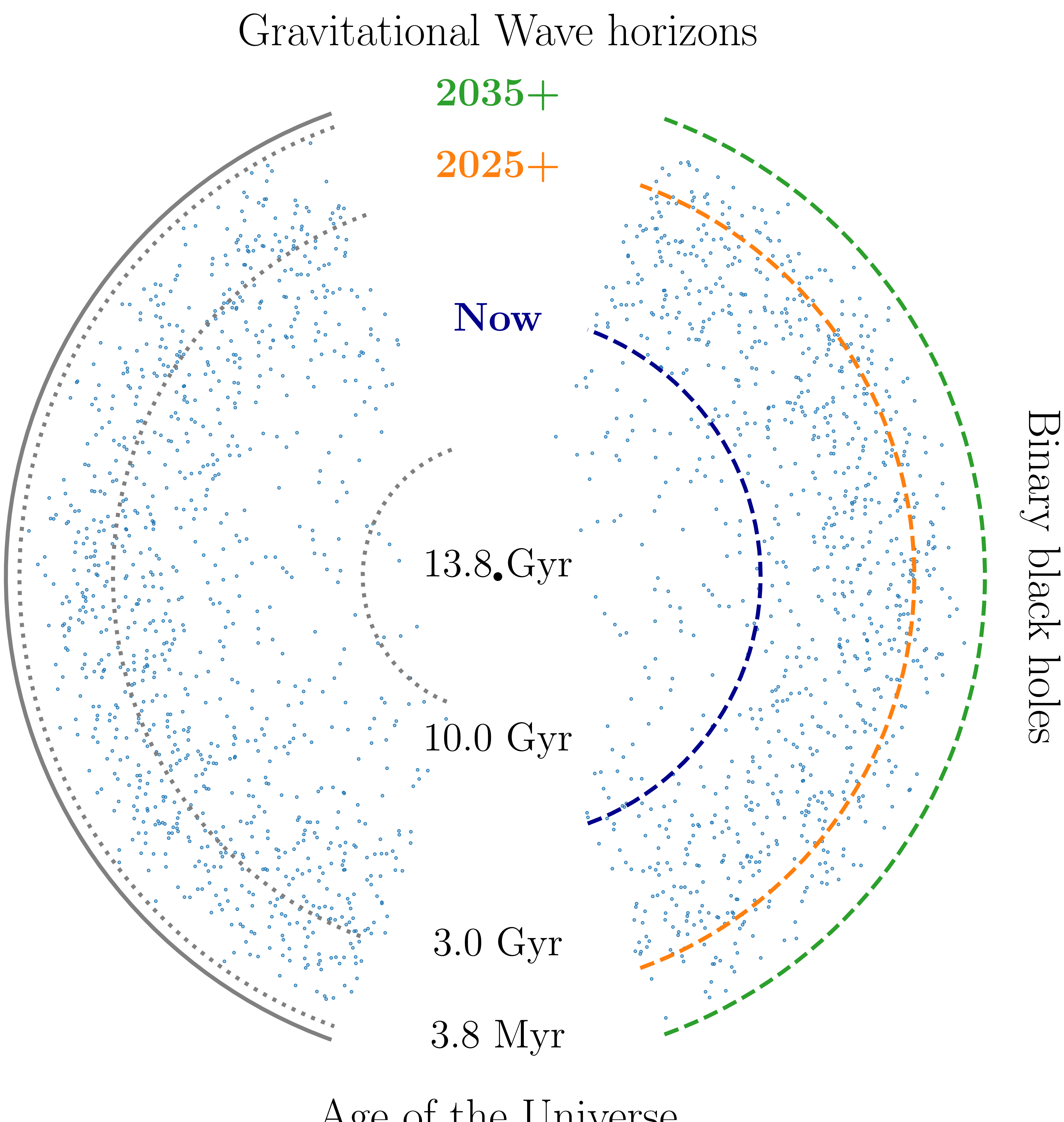



[credit Chandra]

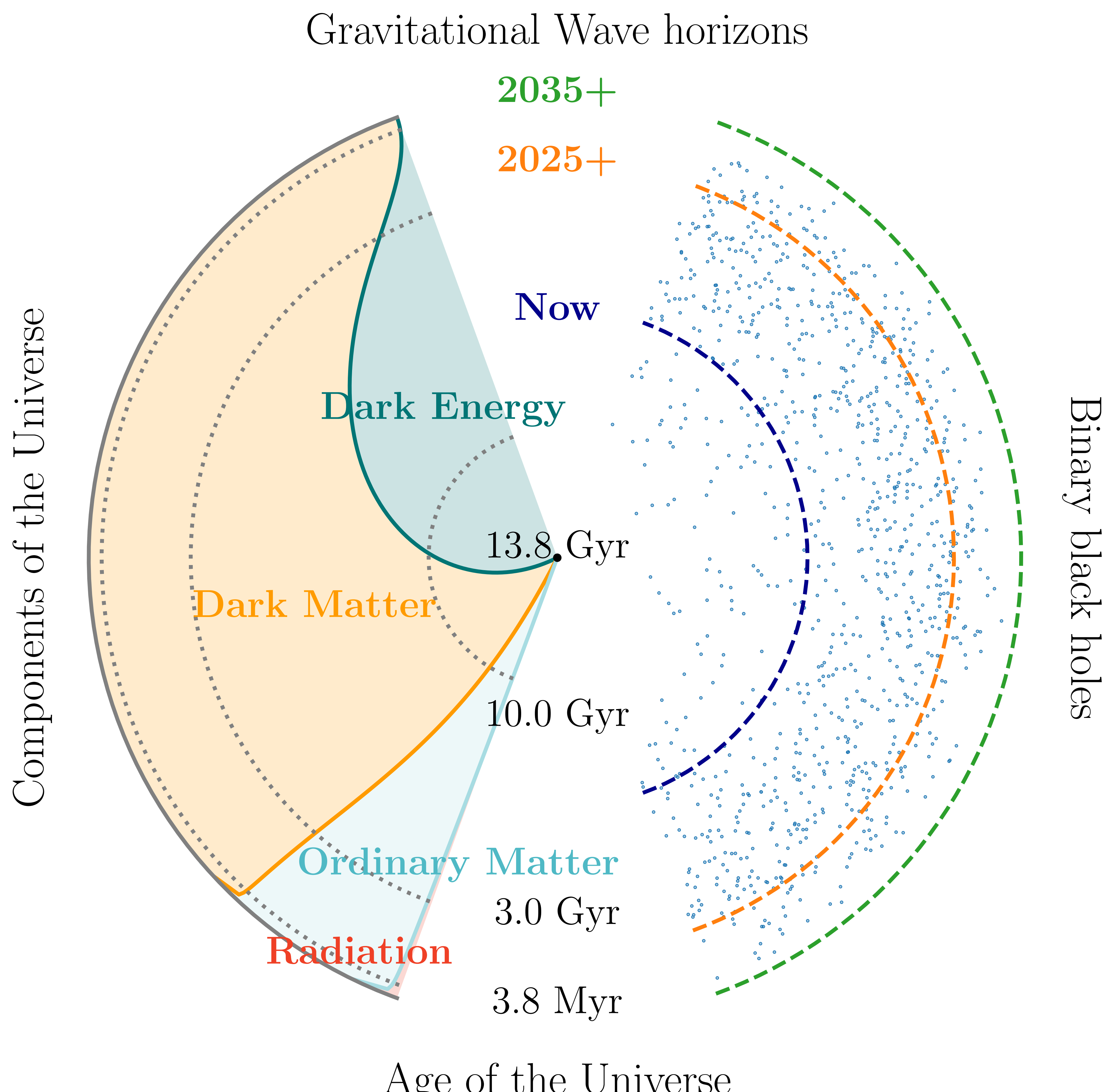



Age of the Universe

*stellar mass binary black holes



Gravitational Wave horizons
2035+
2025+
Now
13.8 Gyr
10.0 Gyr
3.0 Gyr
3.8 Myr
Binary black holes
Age of the Universe


Gravitational Wave horizons
2035+
2025+
Now
Components of the Universe
Dark Energy
13.8 Gyr
Dark Matter
10.0 Gyr
Ordinary Matter
3.0 Gyr
Radiation
3.8 Myr
Binary black holes
Age of the Universe

# The plan

**0.** Motivation: gravity, astrophysics, cosmology

**1.** A *crash-course* on gravitational waves

linearized Einstein's equations, quadrupole formula, compact binary coalescences

**2.** The new era of gravitational-wave astronomy

detectors, matched-filtering, data analysis, current observations, next generation detectors

**3.** *Standard siren* cosmology

bright, dark and spectral sirens, status and future prospects

**4.** Gravitational wave *lensing*

lensing regimes (geometric/wave optics), current search efforts, science case

# The plan - *practicalities*

- Please ask *questions*! (during and after the lectures)
- The goal of these lectures is to give an *overview* of gravitational wave astronomy and its application to cosmology
  - I will avoid technical derivations. Focus on compact binaries
  - There are many slides. No need to cover them all!
- Detailed derivations can be found in my lecture notes: ezquiaga.github.io/lectures/Lecture_Notes
  - Also references to seminal papers and books
- The slides contain references [in brackets] with links to papers/sources
  - QR code linking to the slides
- *Remember*, please ask *questions*!

(during and after the lectures)

# 1. A crash-course on gravitational waves

# Gravitational waves in flat space

- Perturbations around Minkowski

$$g_{\mu\nu}(t,\vec{x}) = \eta_{\mu\nu} + h_{\mu\nu}(t,\vec{x})$$

$$|h_{\mu\nu}(t,\vec{x})| \ll 1$$

- Einstein field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- Gravitational wave propagation

$$\boxed{\Box h_{\mu\nu} = -16\pi G \left( T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T \right)}$$

# Gravitational wave properties

- Wave equation in vacuum $\quad \Box h_{\mu\nu} = 0$

- Wave ansatz $\quad h_{\mu\nu}(x) = \mathrm{Re}\left[A_{\mu\nu}(x)e^{i\theta(x)}\right] \qquad k_\mu \equiv \partial_\mu \theta$

$$A_{\mu\nu} \equiv A\epsilon_{\mu\nu}$$

- Highly oscillatory phase: $\quad \theta \to \theta/\varepsilon$

- Leading order: *gravitational wave follow null geodesics*

$$\eta_{\mu\nu}k^\mu k^\nu = 0$$

- Next to Leading order: *gravitons conserved + parallel transport*

$$\nabla^\mu(A^2 k_\mu) = 0 \qquad k^\alpha \nabla_\alpha \epsilon_{\mu\nu} = 0$$

# Gravitational wave polarizations

- Counting degrees of freedom:

Symmetric 4D tensor $\epsilon_{\mu\nu} = \epsilon_{\nu\mu}$ : **10**

Lorenz gauge $\nabla^{\mu} h_{\mu\nu} = 0$: **10 - 4 = 6**

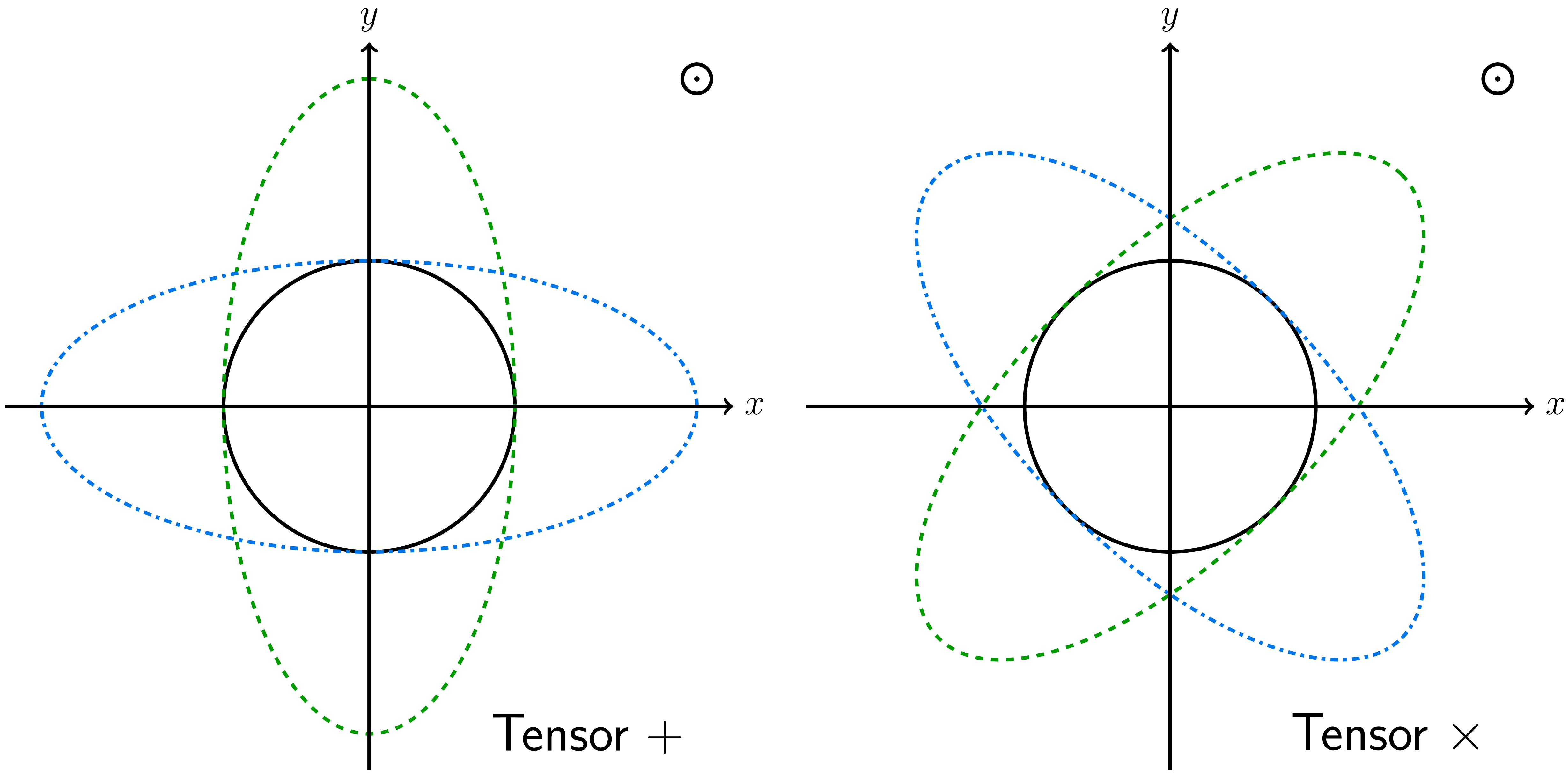
Residual gauge $\epsilon_{0\mu} = 0$ : **10 - 4 - 4 = 2**

- Polarization decomposition:

$$\boxed{\epsilon_{\mu\nu}(x) = \epsilon_{+}(x)\hat{\epsilon}^{+}_{\mu\nu} + \epsilon_{\times}(x)\hat{\epsilon}^{\times}_{\mu\nu}}$$

$$\epsilon_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \epsilon_{+} & \epsilon_{\times} & 0 \\ 0 & \epsilon_{\times} & -\epsilon_{+} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

# Gravitational Wave Polarizations




— $\omega t = 0, \pi$ - - - $\omega t = \pi/2$ ······ $\omega t = 3\pi/2$

# Gravitational waves in curved space

- Perturbations around curved background

$$g_{\mu\nu} = g^{\mathrm{B}}_{\mu\nu} + h_{\mu\nu}$$

- Definition is not unique, short-wave approx.

$$\lambda_{\mathrm{gw}} \ll L_{\mathrm{B}} \sim |R^{\mathrm{B}}_{\alpha\beta\gamma\rho}|^{-1/2}$$

- We can fix the transverse-traceless gauge in vacuum $\nabla^{\mu} h_{\mu\nu} = h = 0$

- Wave equation

$$\boxed{\Box h_{\mu\nu} + 2R^{\mathrm{B}}_{\mu\alpha\nu\beta} h^{\alpha\beta} = 0}$$

*New interactions!* $\partial g^{\mathrm{B}}_{\mu\nu}$ $\partial\partial g^{\mathrm{B}}_{\mu\nu}$

# Gravitational waves in cosmology

- Perturbations around homogeneous and isotropic backgrounds

$$g_{\mu\nu} = g_{\mu\nu}^{\rm FLRW} + h_{\mu\nu}$$

- GWs unambiguously defined + scalar-vector-tensor decomposition

- Wave equation in vacuum

$$\Box^{\rm FLRW} h_{ij} + 2R^{\rm FLRW}_{ijkl} h^{jl} = 0$$

$$\downarrow$$

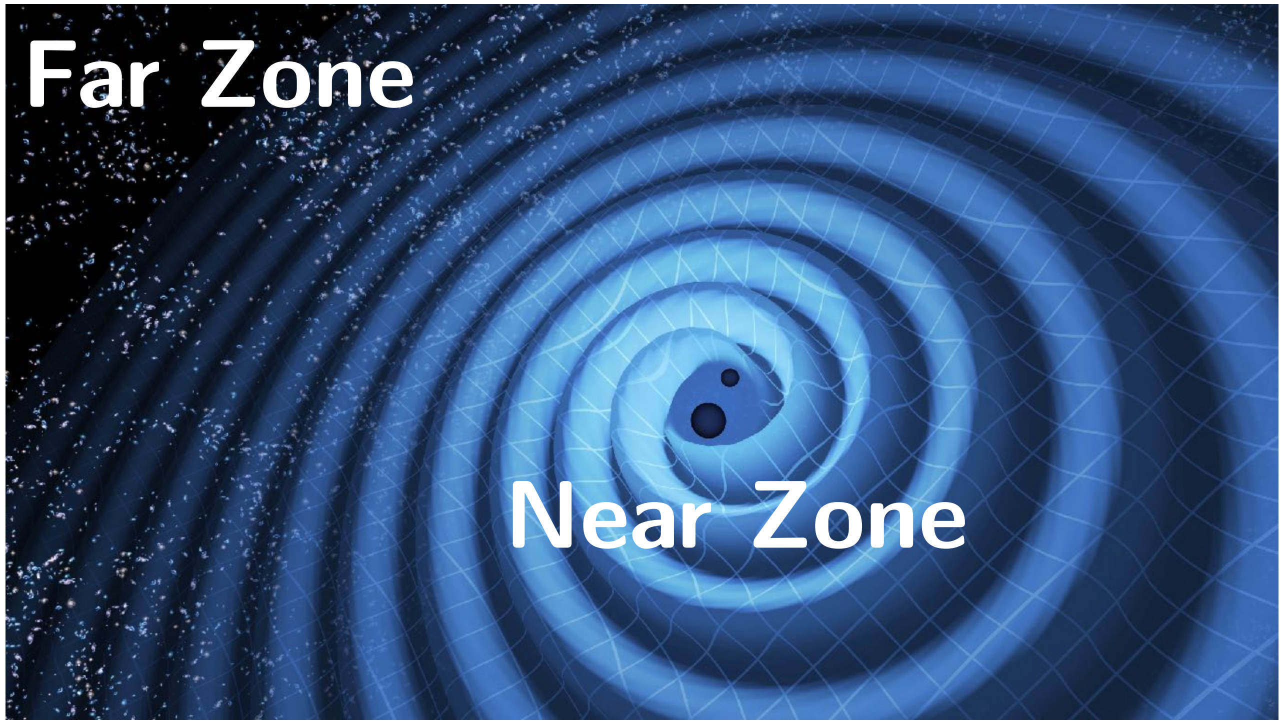
$$h''_{ij} + 2\mathcal{H}h'_{ij} + \nabla^2 h_{ij} = 0$$

$$\downarrow$$

$$h_{ij}(\eta, \mathbf{x}) \simeq \frac{1}{a(\eta)} h_{ij}^{\rm flat}(\eta, \mathbf{x})$$

# Gravitational wave generation

- Different regimes




- Rewriting the field equations:

$$\Box \bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu} + \mathcal{O}(h^2) \equiv -16\pi G \tau_{\mu\nu}$$

- Green's function solution:

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} g_{\mu\nu} h$$

$$\boxed{\bar{h}_{\mu\nu}(t,\vec{x}) = 4G \int d^3x' \, \frac{\tau_{\mu\nu}(t - |\vec{x}-\vec{x}'|, \vec{x}')}{|\vec{x}-\vec{x}'|}}$$

# Quadrupole formula

- Far zone solution: *expand large distances*
- Near zone solution: *expand small velocities v/c*
- Leading Newtonian limit: *match near and far zone solutions*

$$\boxed{h_{ij}^{TT}(t,\vec{x}) = \frac{2G}{c^4 r}\frac{d^2 Q_{ij}^{TT}(t-r/c)}{dt^2}}$$

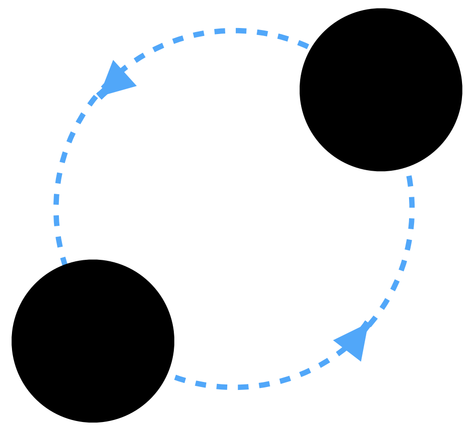
*Amplitude scales inversely with distance*

*Gravitational waves sourced by accelerated quadrupole moment*

$$Q^{ij} \equiv \int d^3x \tau^{00}(x)\left(x^i x^j - \frac{1}{3}r^2\delta^{ij}\right)$$

# Compact binary coalescence

- At leading order in post-Newtonian expansion



$$h_+(t) = h_c \left(\frac{1+\cos^2\iota}{2}\right) \cos\left[\Phi(t)\right]$$

$$h_\times(t) = h_c \cos\iota \, \sin\left[\Phi(t)\right]$$

$$\mathcal{M}_c = (m_1 m_2)^{3/5}/(m_1+m_2)^{1/5}$$

- Amplitude

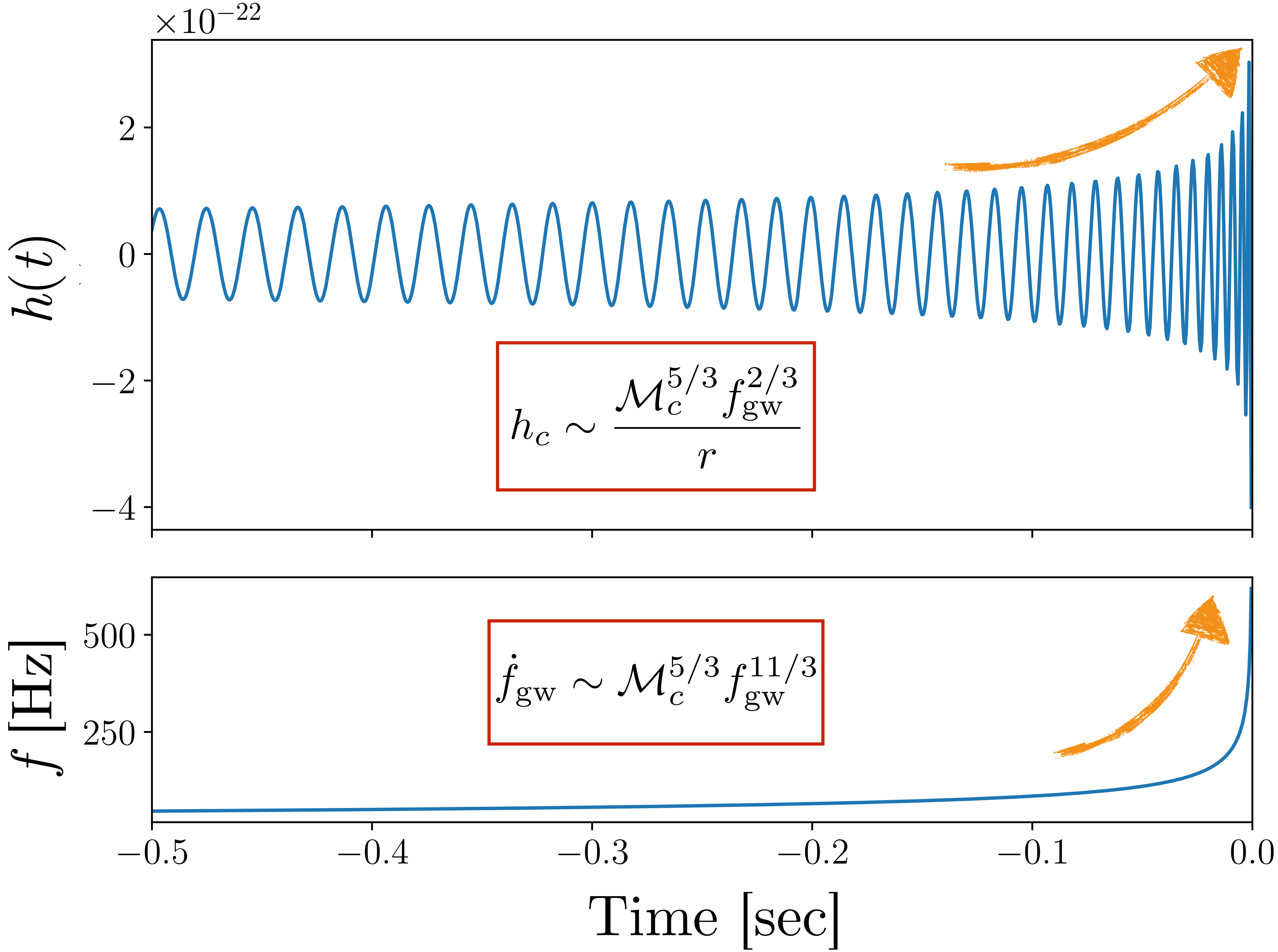
$$\boxed{h_c \sim \frac{\mathcal{M}_c^{5/3} f_{\mathrm{gw}}^{2/3}}{r}}$$

- Frequency

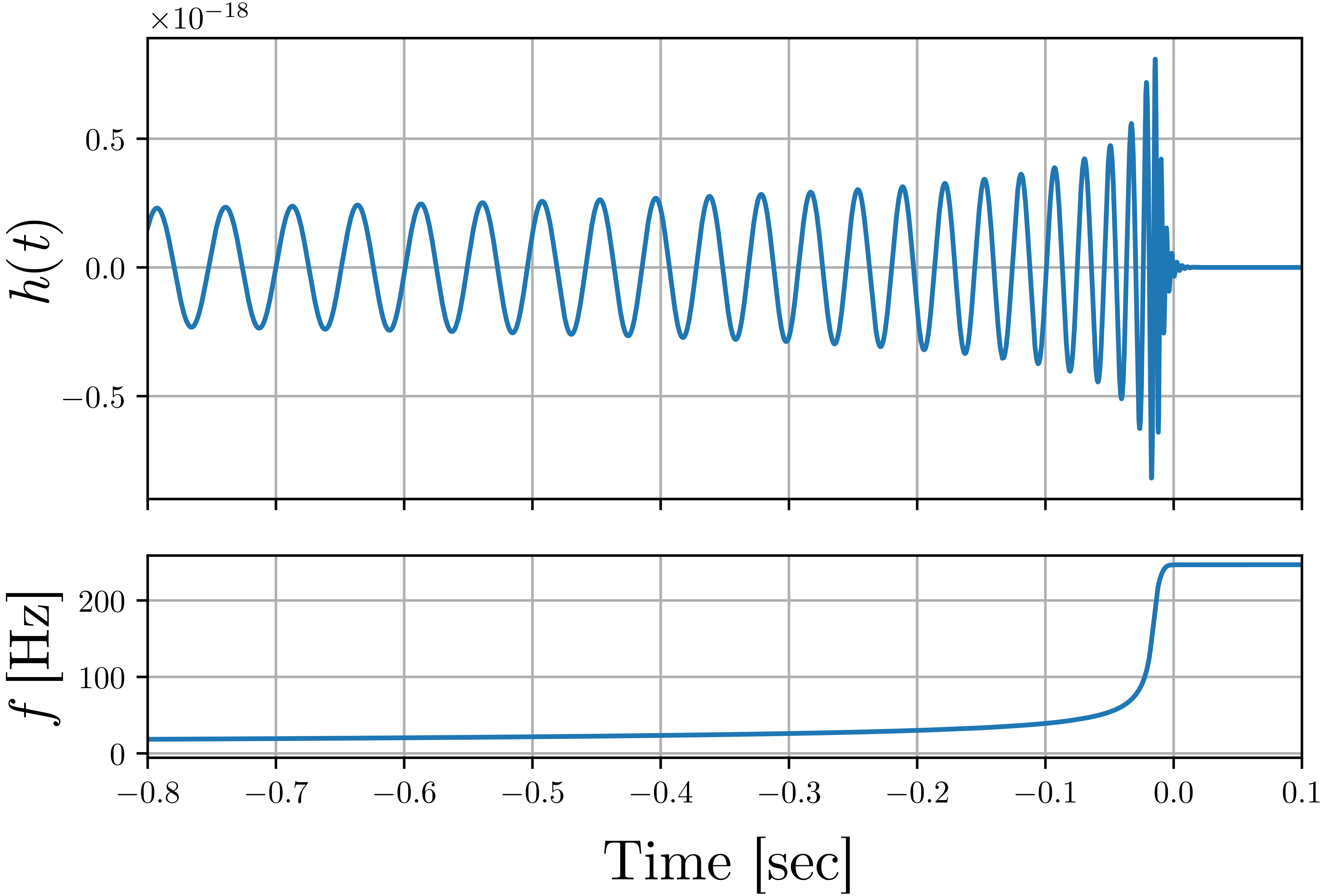
$$\boxed{\dot{f}_{\mathrm{gw}} \sim \mathcal{M}_c^{5/3} f_{\mathrm{gw}}^{11/3}}$$

$$f_{\mathrm{gw}} \sim d\Phi/dt$$

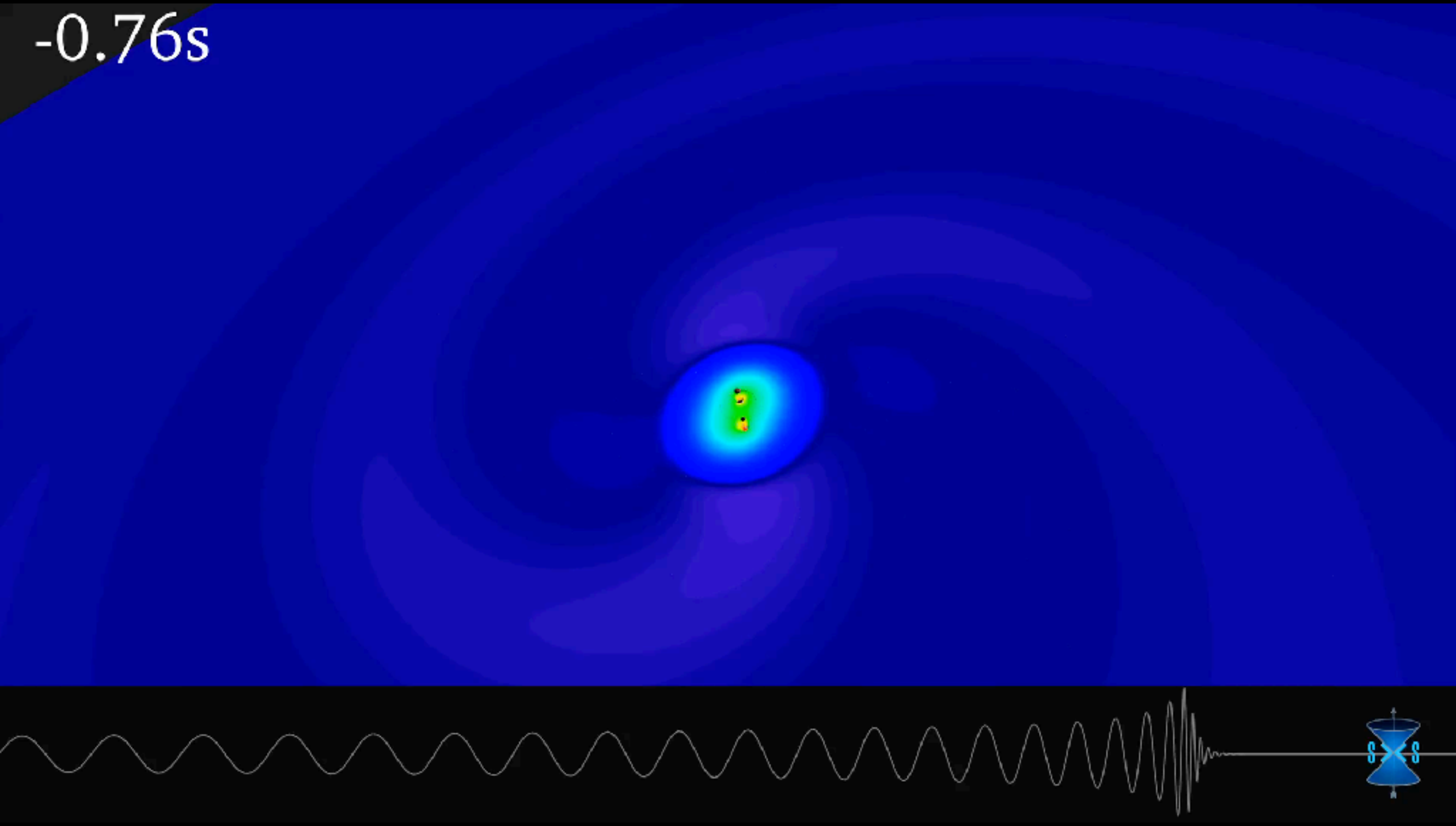
# Inspiral - the "chirp"



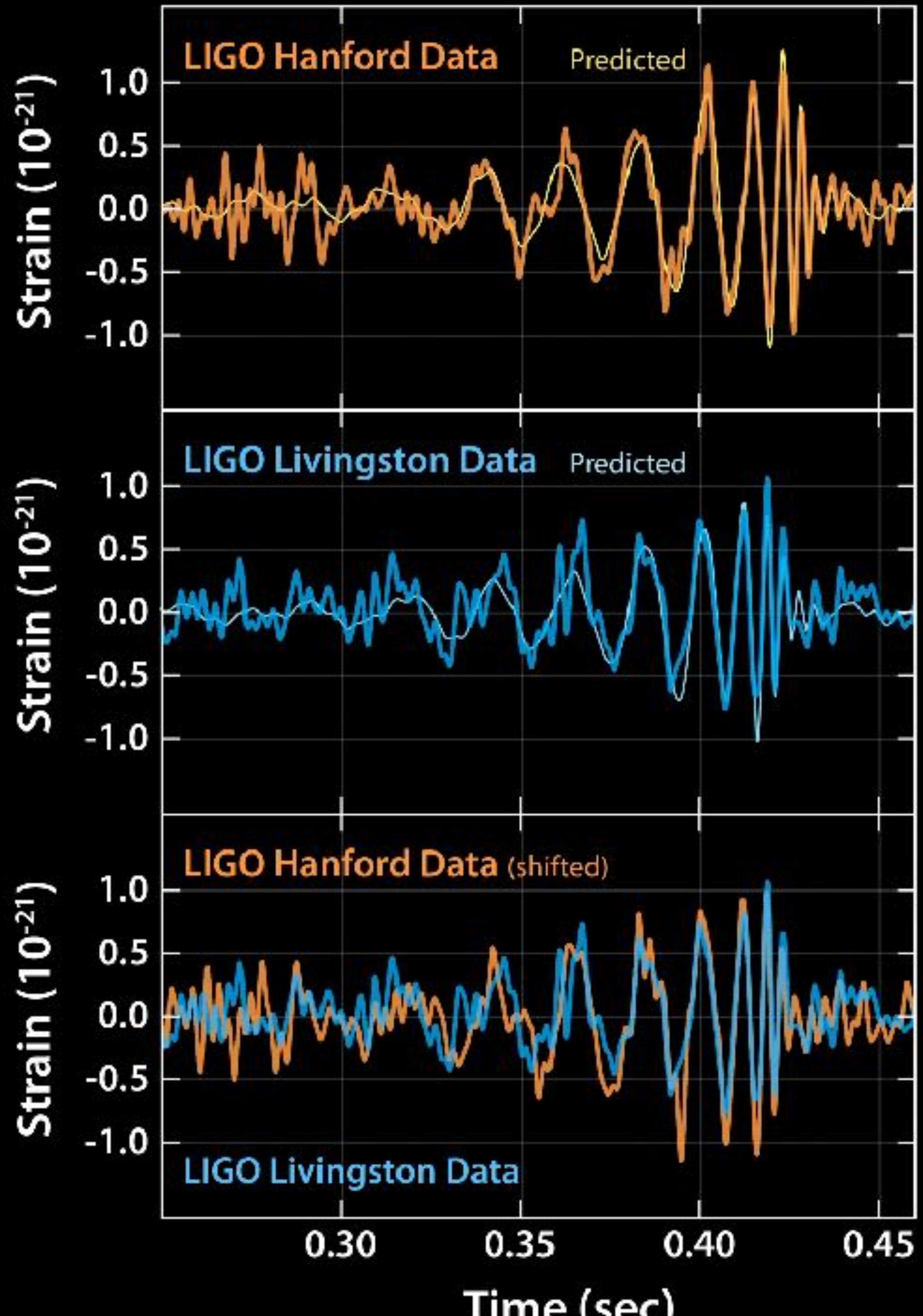
# Inspiral-Merger-Ringdown

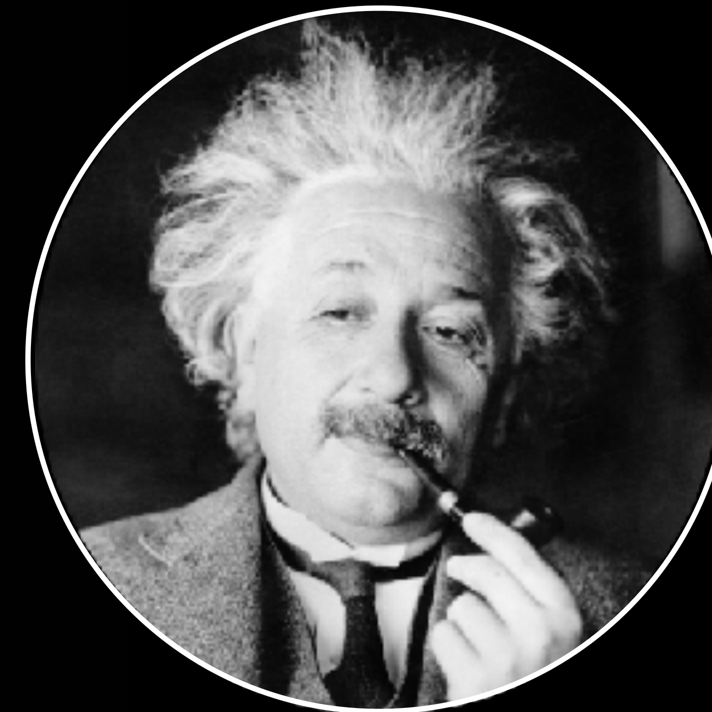


# Numerical simulation of a binary black hole merger




[Credit: SxS Collaboration]


LIGO Hanford Data
Predicted
LIGO Livingston Data
Predicted
LIGO Hanford Data (shifted)
LIGO Livingston Data
Strain ($10^{-21}$)
1.0
0.5
0.0
-0.5
-1.0
0.30
0.35
0.40
0.45
Time (sec)



# Cosmological compact binary coalescence

- Compact binaries at cosmological distances

$$h_c(t_{\rm obs}) \sim \frac{\mathcal{M}_c^{5/3} f_{\rm gw}^{2/3}}{a(t_{\rm obs})\, r}$$

$$f_{\rm gw} = (1+z) f_{\rm obs}$$

$$\mathcal{M}_z = (1+z)\mathcal{M}_c$$

$$\boxed{h_c(t_{\rm obs}) \sim \frac{\mathcal{M}_z^{5/3} f_{\rm obs}^{2/3}}{d_{\rm L}^{\rm gw}}}$$

$$\boxed{d_{\rm L}^{\rm gw} = d_{\rm L}^{\rm em} = a_0(1+z)\int_0^{z_{\rm src}} \frac{dz}{H(z)}}$$

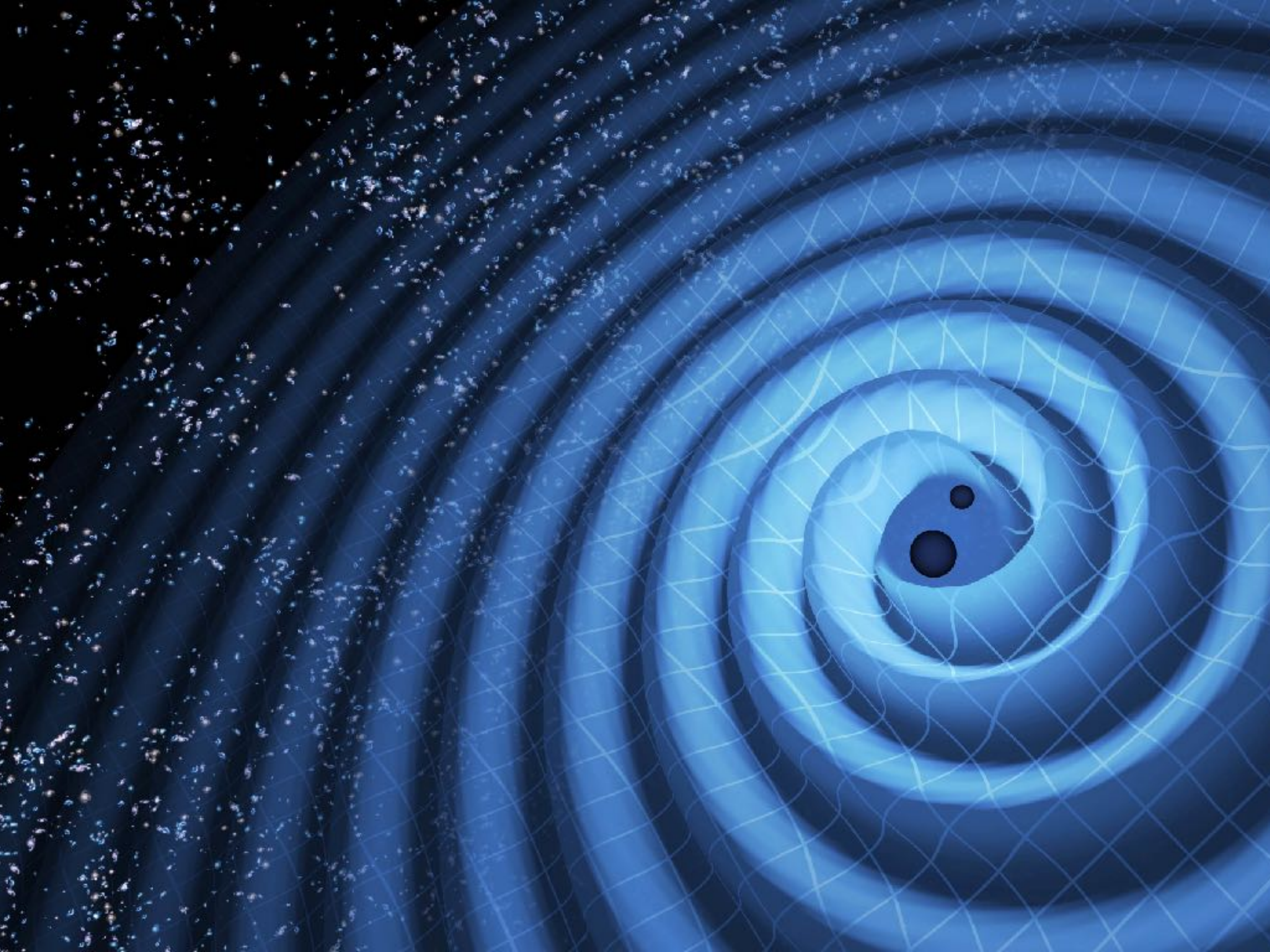
*GW's amplitude scale with the inverse of the luminosity distance!*

*Their amplitude is sensitive to the expansion rate of the Universe!*

# 1. Key takeaways

- Gravitational waves are *linear perturbations* of space-time that propagate across the Universe
- They propagate along *null geodesics* and carry only *two polarizations*
- Gravitational waves are sourced by the *second time derivative* of the *quadrupole moment*
- Compact binary coalescences produce sizable gravitational waves with a *chirping* waveform
- On a cosmological background, amplitude scales inversely with the *luminosity distance*

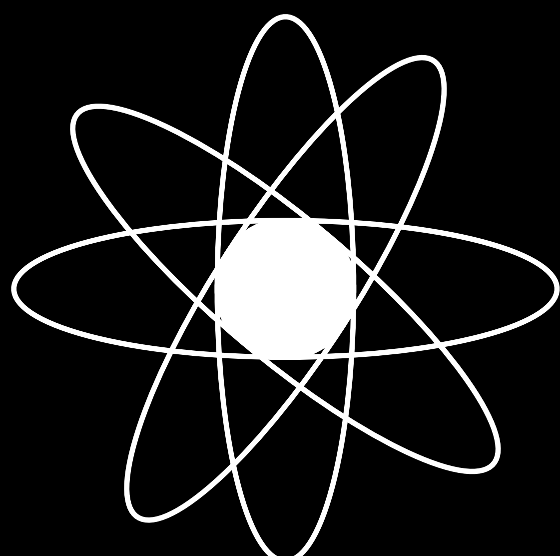
# 2. The new era of gravitational-wave astronomy



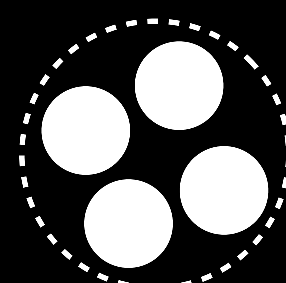
[Credit: R. Hurt, Caltech/MIT/LIGO Lab]

The variation in the distance is minuscule

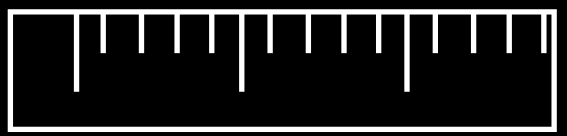
0.000000000000000001 meters



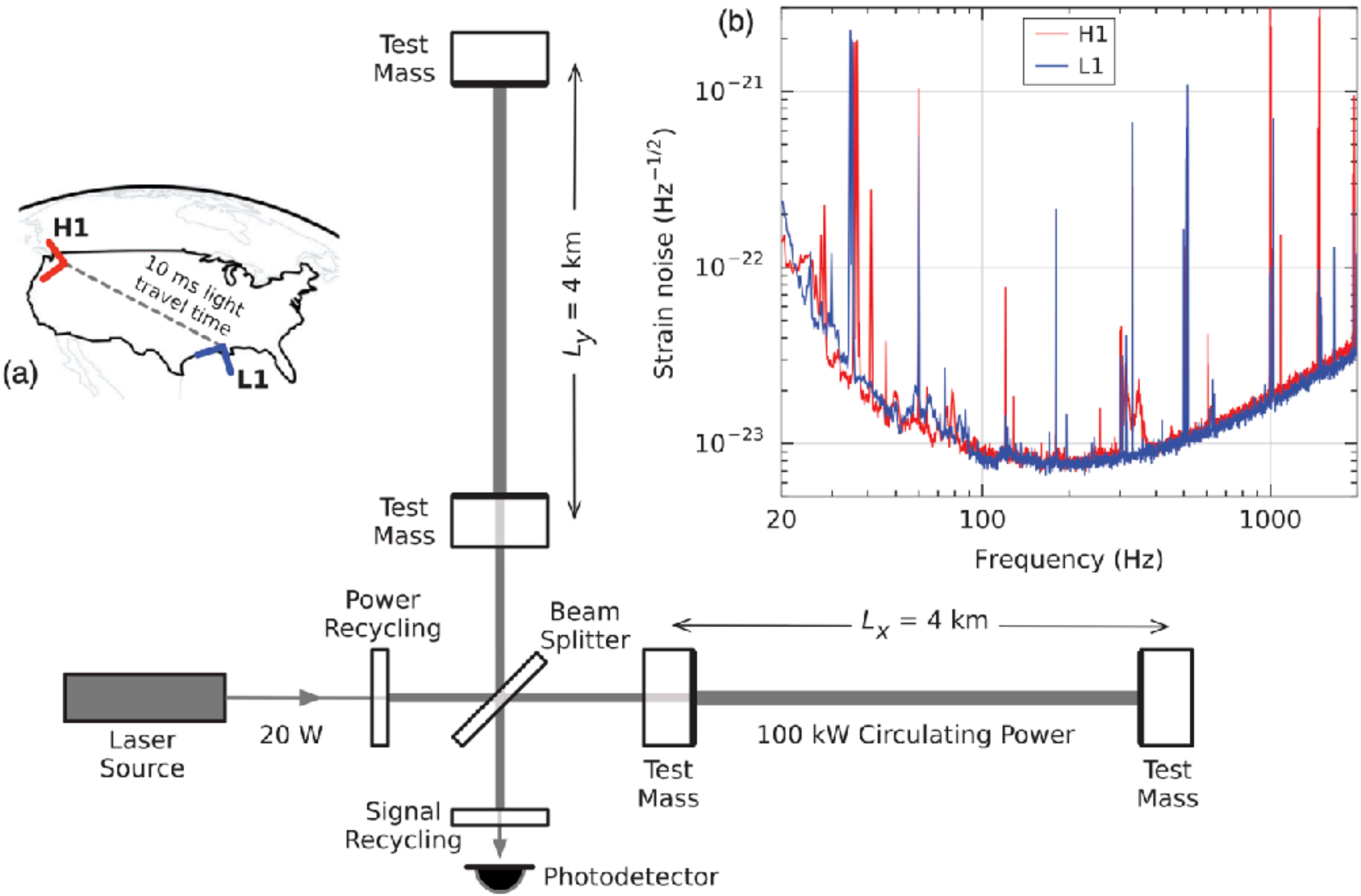
atom: $10^{-10}$ meters



nucleus: $10^{-15}$ meters



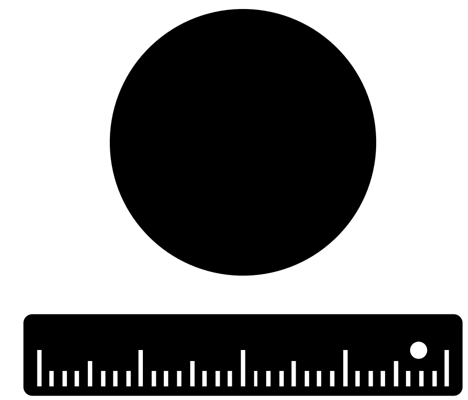
GW effect: $10^{-18}$ meters




[GW150914 PRL]

# Tuned for detecting compact objects

$$f \sim \frac{1}{2\pi}\frac{1}{2t_{\mathrm{Sch}}} \sim 800\mathrm{Hz}\left(\frac{10M_\odot}{M}\right)$$

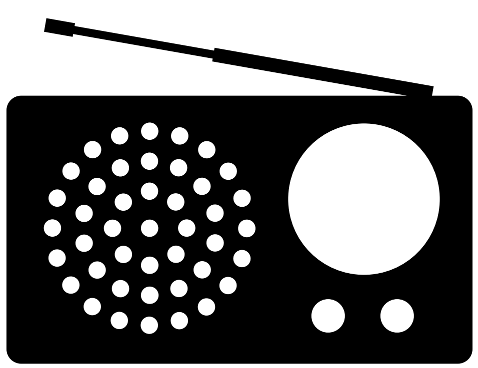


$$h \sim \mathcal{O}(1)\cdot\frac{r_{\mathrm{Sch}}}{r} \sim 10^{-23}\left(\frac{1\mathrm{Gpc}}{r}\right)\left(\frac{M}{10M_\odot}\right)$$

$$r_{\mathrm{Sch}} = 2GM/c^2$$

Coincides audible frequencies

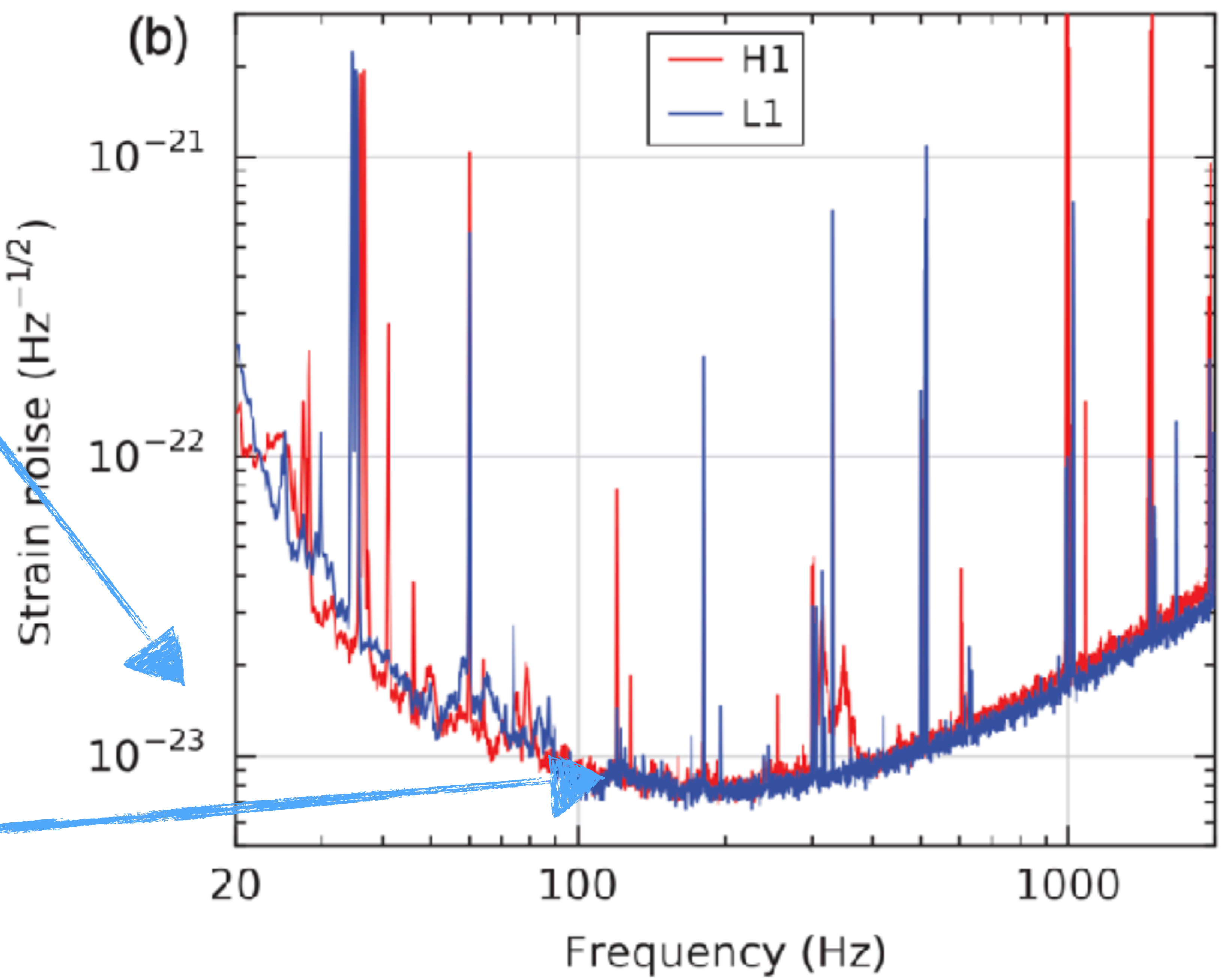
Cosmological distance



$$\frac{1\mathrm{Gpc}}{c} \sim 3\mathrm{Gyr} \sim 0.2t_{\mathrm{Uni}}$$

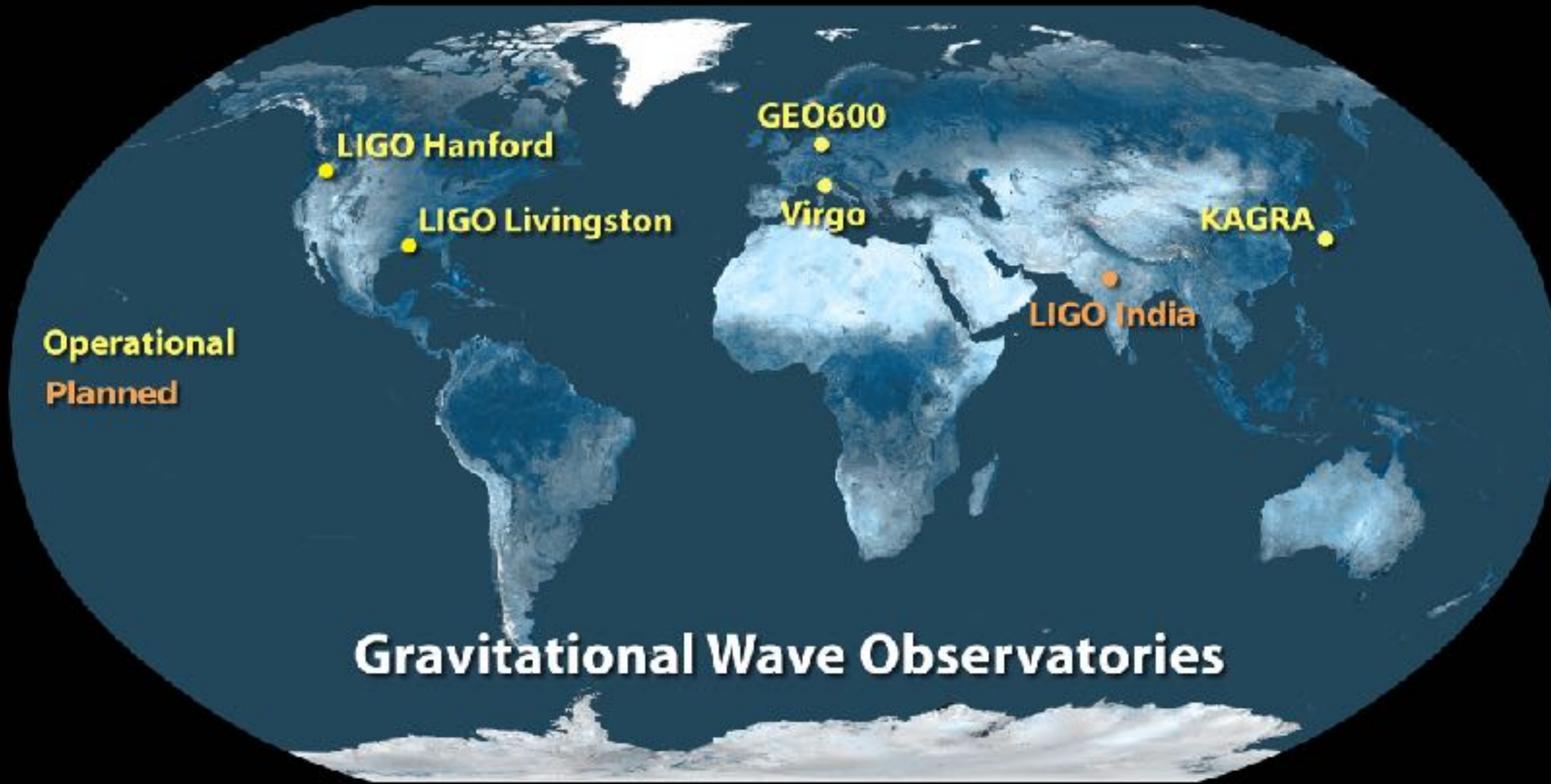
# Tuned for detecting compact objects

$$h \sim 10^{-23} \left(\frac{1\text{Gpc}}{r}\right) \left(\frac{M}{10 M_\odot}\right)$$




$$f \sim 800\text{Hz} \left(\frac{10 M_\odot}{M}\right)$$

# The era of gravitational wave astronomy is **here**!




[Hanford, US]

[Livingston, US]

[Virgo, Italy]

[KAGRA, Japan]

# Gravitational wave detectors

- Detectors are defined by their *noise, n(t)*
- Some simplifying assumptions:

Stationary: $R(\tau) \equiv \langle n(t) n(t+\tau) \rangle$

Ergodic: $\langle n \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} n(t) dt$

Zero-mean: $\langle n(t) \rangle = 0$

Gaussian: $\langle \tilde{n}^*(f) \tilde{n}(f') \rangle = \frac{1}{2} S_n(f) \delta(f - f')$

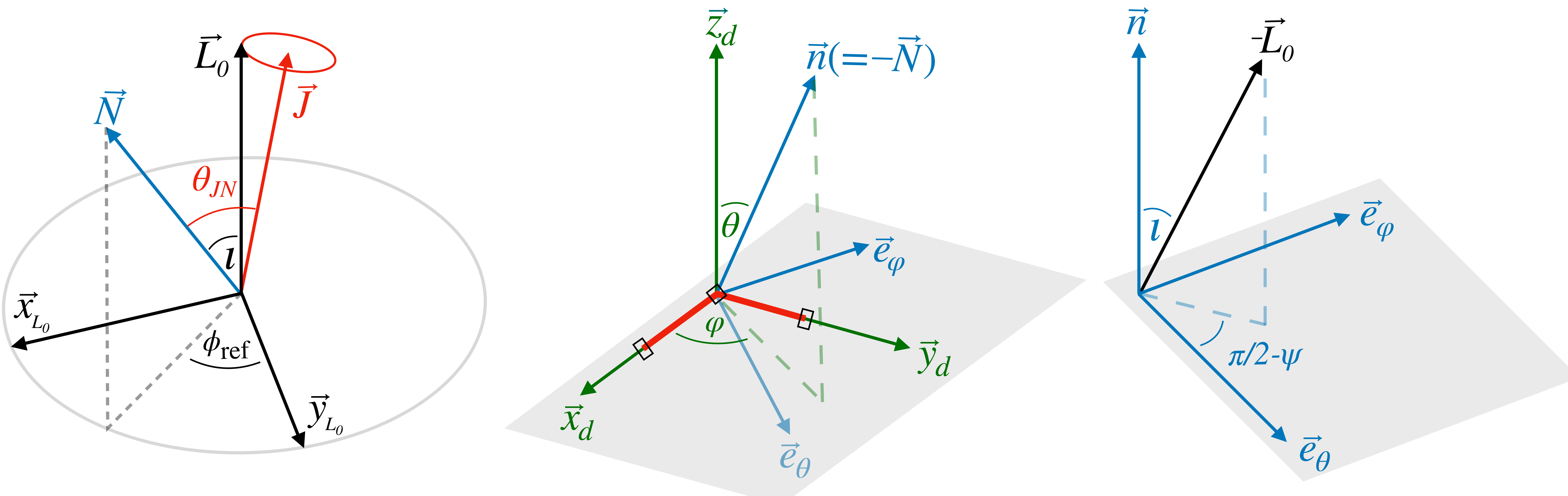
- Probability of noise realization *n(t)*

$$\boxed{p_n[n(t)] \propto \exp\left[-2 \int_0^\infty \frac{|\tilde{n}(f)|^2}{S_n(f)} df\right]}$$

# Gravitational wave detectors

- Detectors are also defined by their *antenna response*

$$h(t) = h_+(t)F_+(\hat{n}) + h_\times(t)F_\times(\hat{n})$$

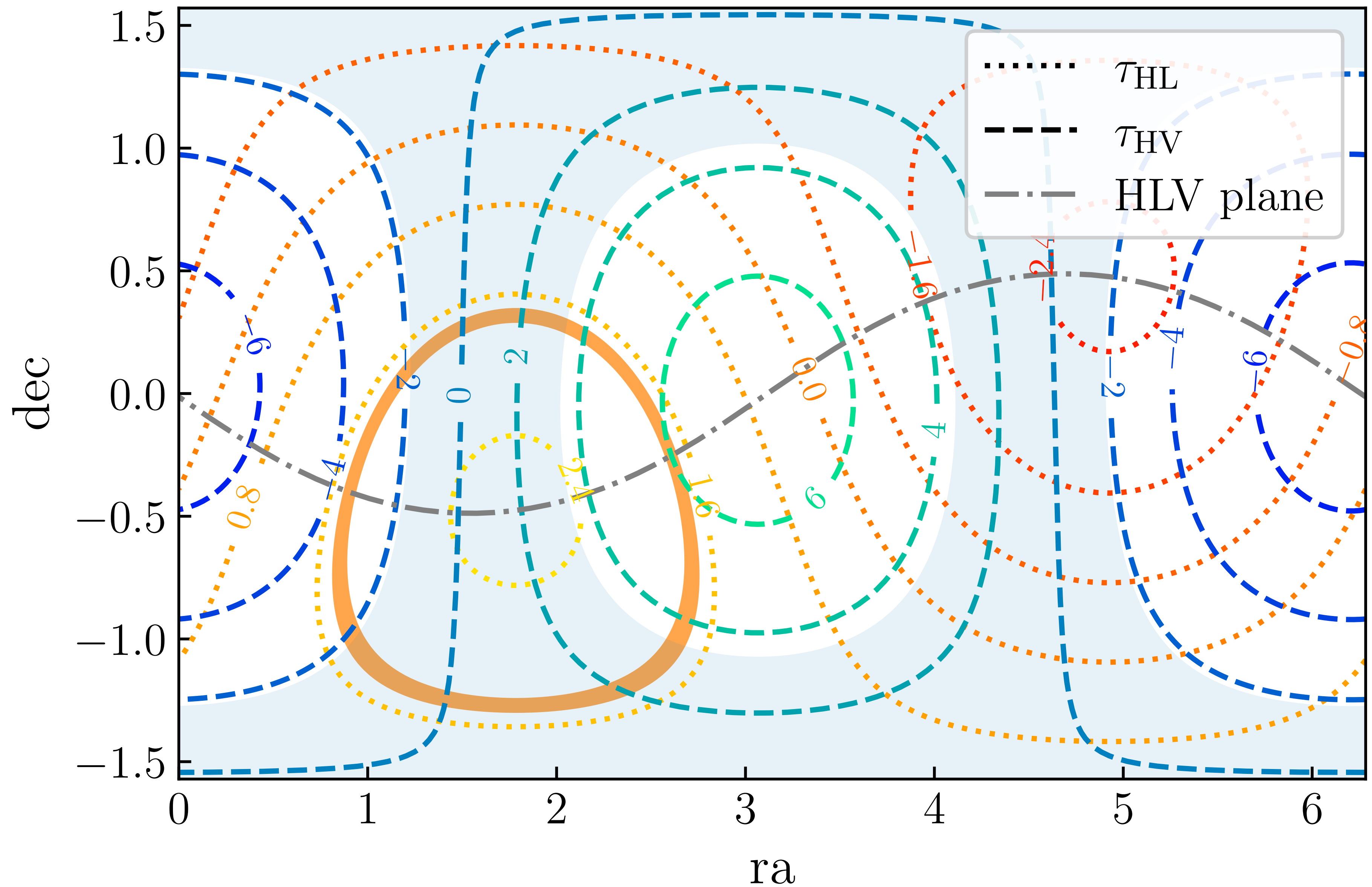



(a) Source (b) Earth detector (c) Sky

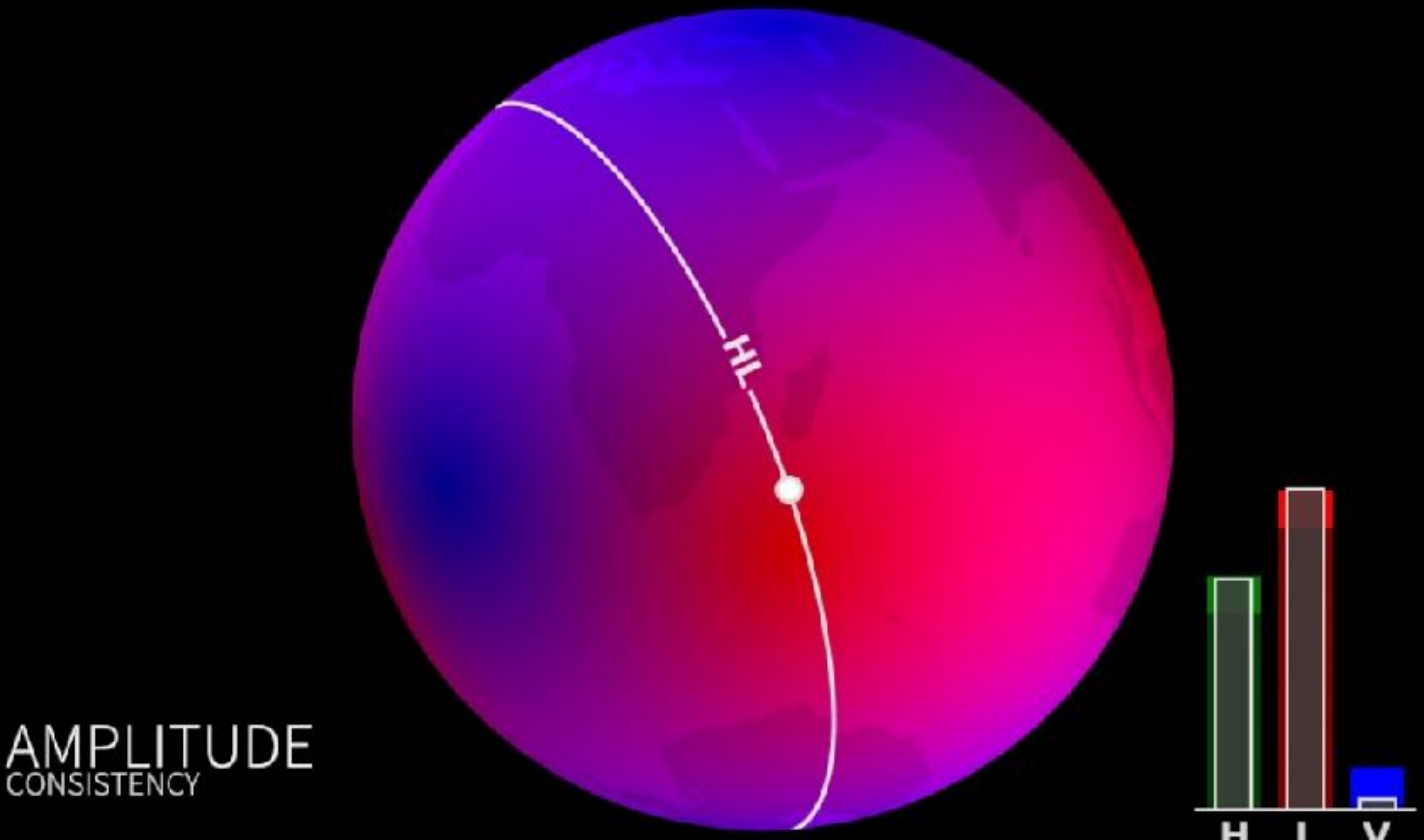
# Sky localization

- The arrival time difference between two detectors defines a ring in the sky

$$\Delta t_{d_1 d_2} = \vec{n} \cdot \vec{r}_{d_1 d_2} / c$$



# Sky localization




[Credit: Leo Singer]

# Matched-filtering

- The data stream: $$d(t) = s(t) + n(t)$$
- Filter data: $$\hat{d} = \int_{-\infty}^{\infty} dt\, d(t) K(t)$$
- Signal to noise: $$S/N = \frac{\int_{-\infty}^{\infty} df\, \tilde{s}(f) \tilde{K}^*(f)}{\sqrt{\int_{-\infty}^{\infty} df\, \frac{1}{2} S_n(f) |\tilde{K}(f)|^2}}$$
- Define noise weighted diner product

$$(a|b) \equiv \mathrm{Re}\left[\int_{-\infty}^{\infty} \frac{\tilde{a}^*(f)\tilde{b}(f)}{S_n(f)/2}\right] = 4\mathrm{Re}\left[\int_{0}^{\infty} \frac{\tilde{a}^*(f)\tilde{b}(f)}{S_n(f)}\right],$$

- Rewrite S/N: $$\boxed{S/N = \frac{(u|s)}{\sqrt{(u|u)}} \qquad \tilde{u}(f) = \frac{1}{2} S_n(f) \tilde{K}(f)}$$

# Matched-filtering

- Signal to noise: $$S/N = \frac{(u|s)}{\sqrt{(u|u)}}$$

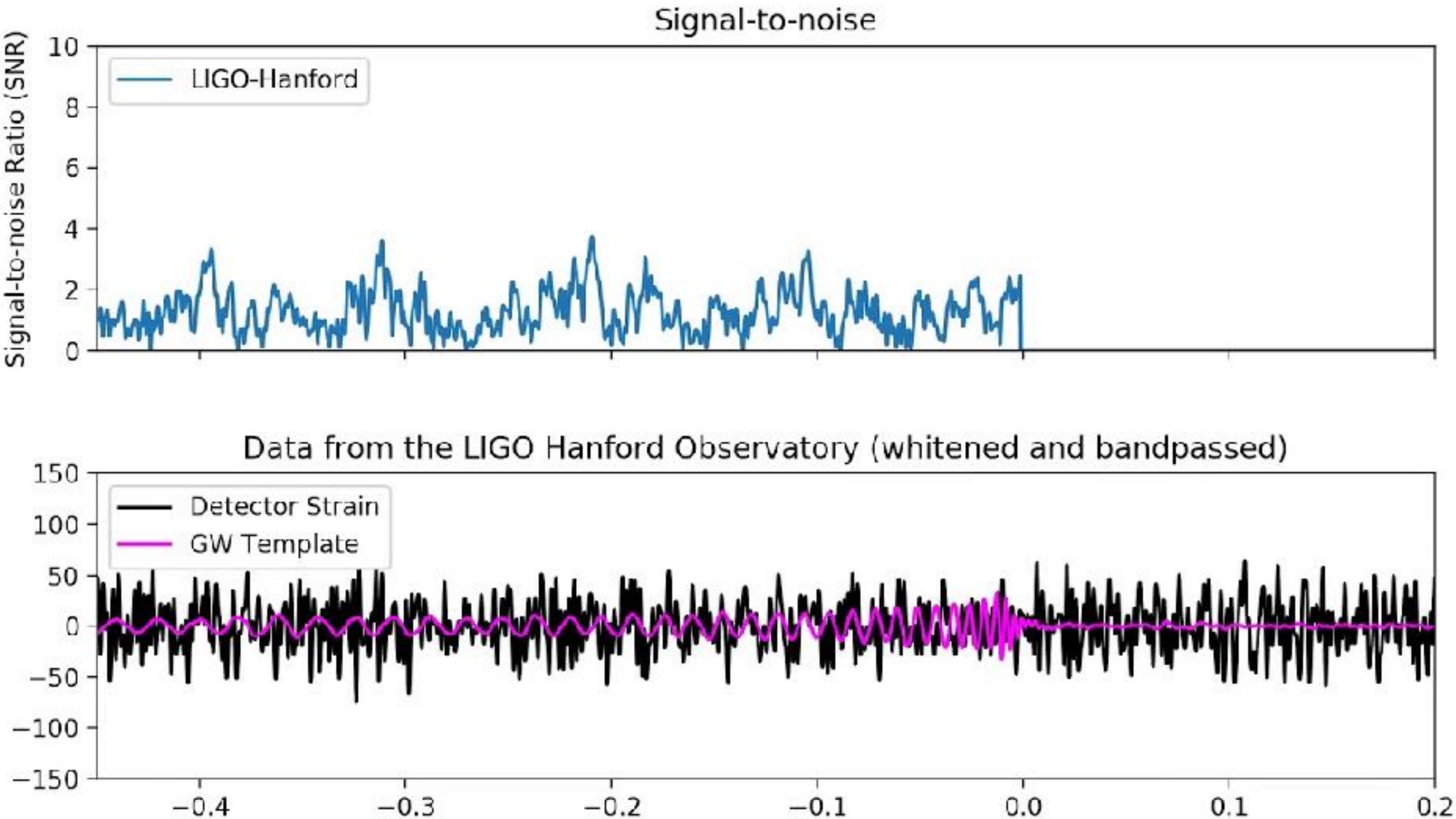
- Optimal filter when *u* is parallel to *s*

$$\tilde{K}(f) \propto \frac{\tilde{s}(f)}{S_n(f)}$$

- Optimal signal-to-noise ratio

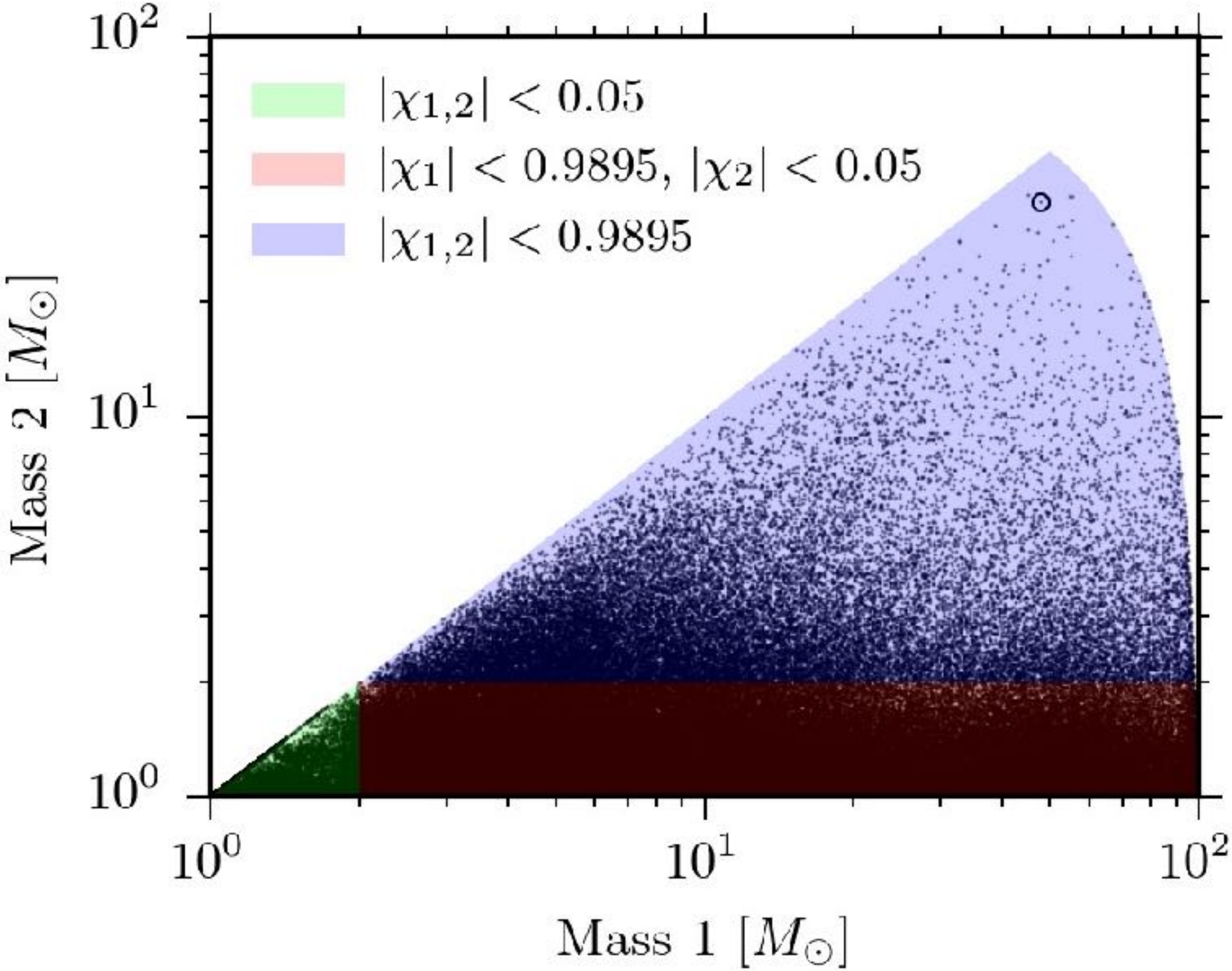
$$\boxed{\rho^2_{\mathrm{opt}} = (h|h) = 4\mathrm{Re}\left[\int_0^\infty \frac{|\tilde{h}(f)|^2}{S_n(f)}\right]}$$

# Matched-filtering



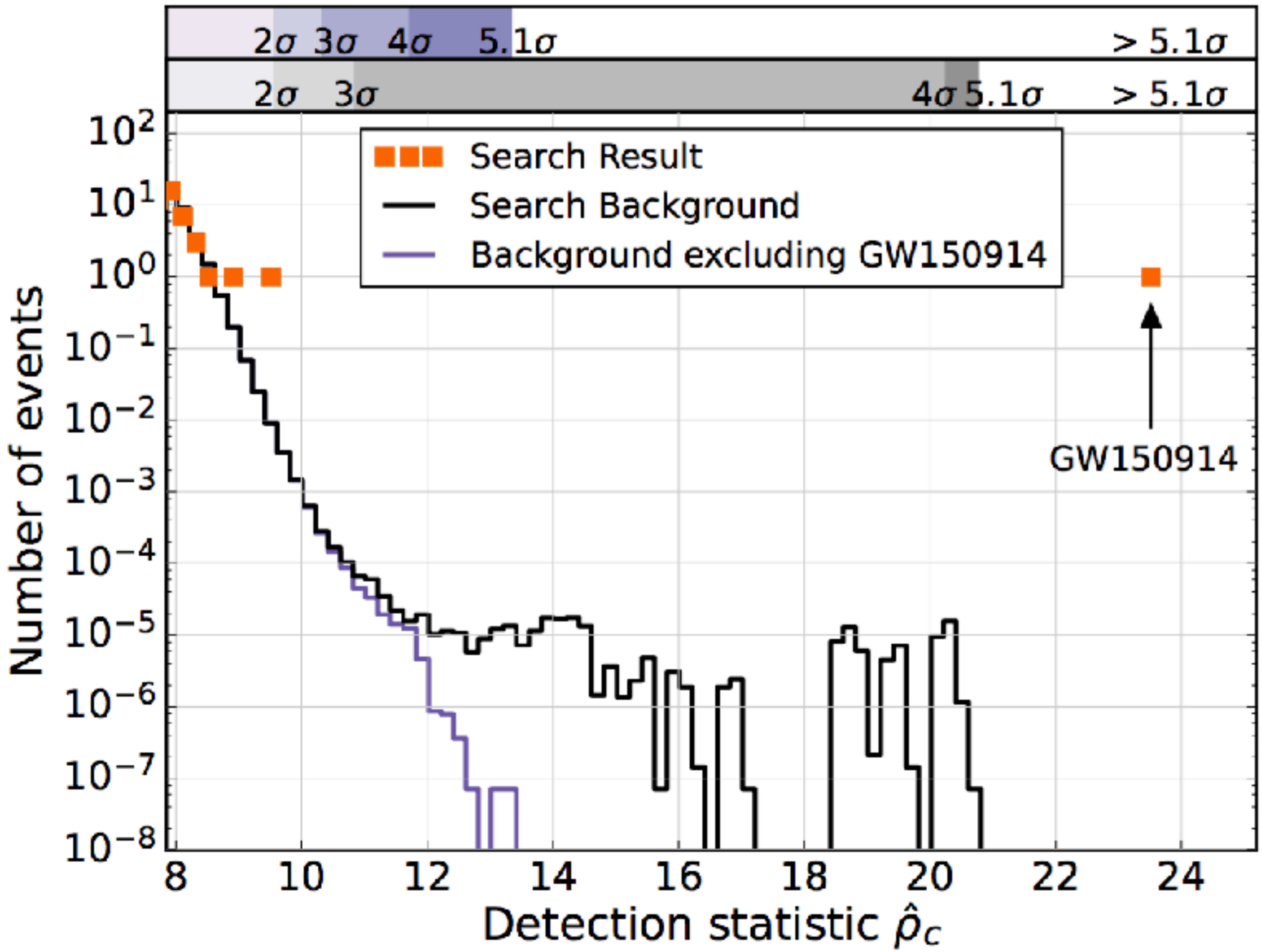

[Credit: Alex Nitz]

# Matched-filtering




[Credit: LVK]

# Detection statistics




[Credit: LVK]

# Parameter estimation

- If we subtract the the right signal to the data, we should recover the noise
$$n(t) = d(t) - s(t)$$

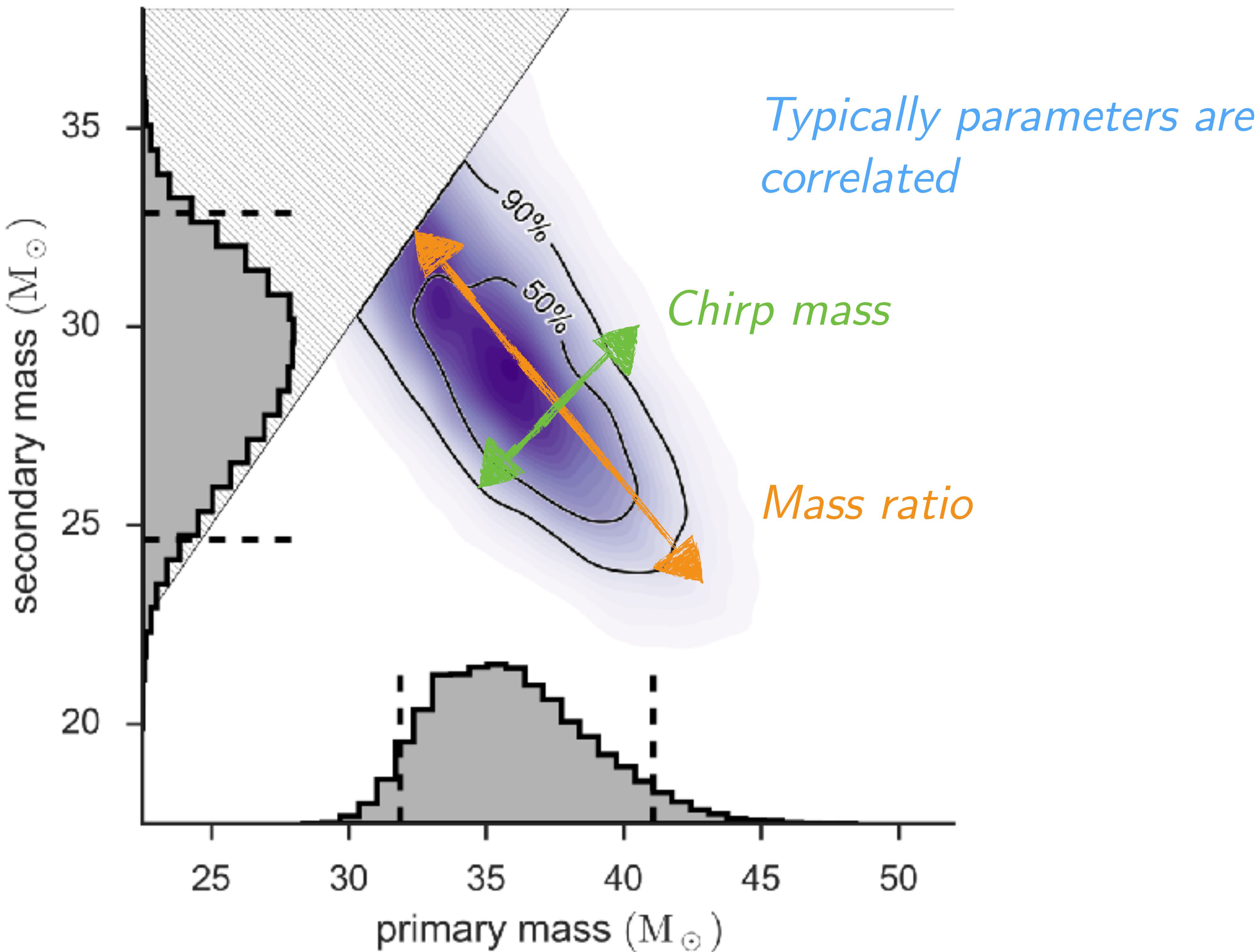
- Assuming Gaussian noise, the likelihood of the data is

$$\begin{aligned}\Lambda(d|\theta) &\propto \exp\left[-\frac{1}{2}(d-h(\theta)|d-h(\theta))\right]\\ &= \exp\left[(d|h(\theta)) - \frac{1}{2}(h(\theta)|h(\theta)) - \frac{1}{2}(d|d)\right]\end{aligned}$$

- The posterior distribution of a parameter

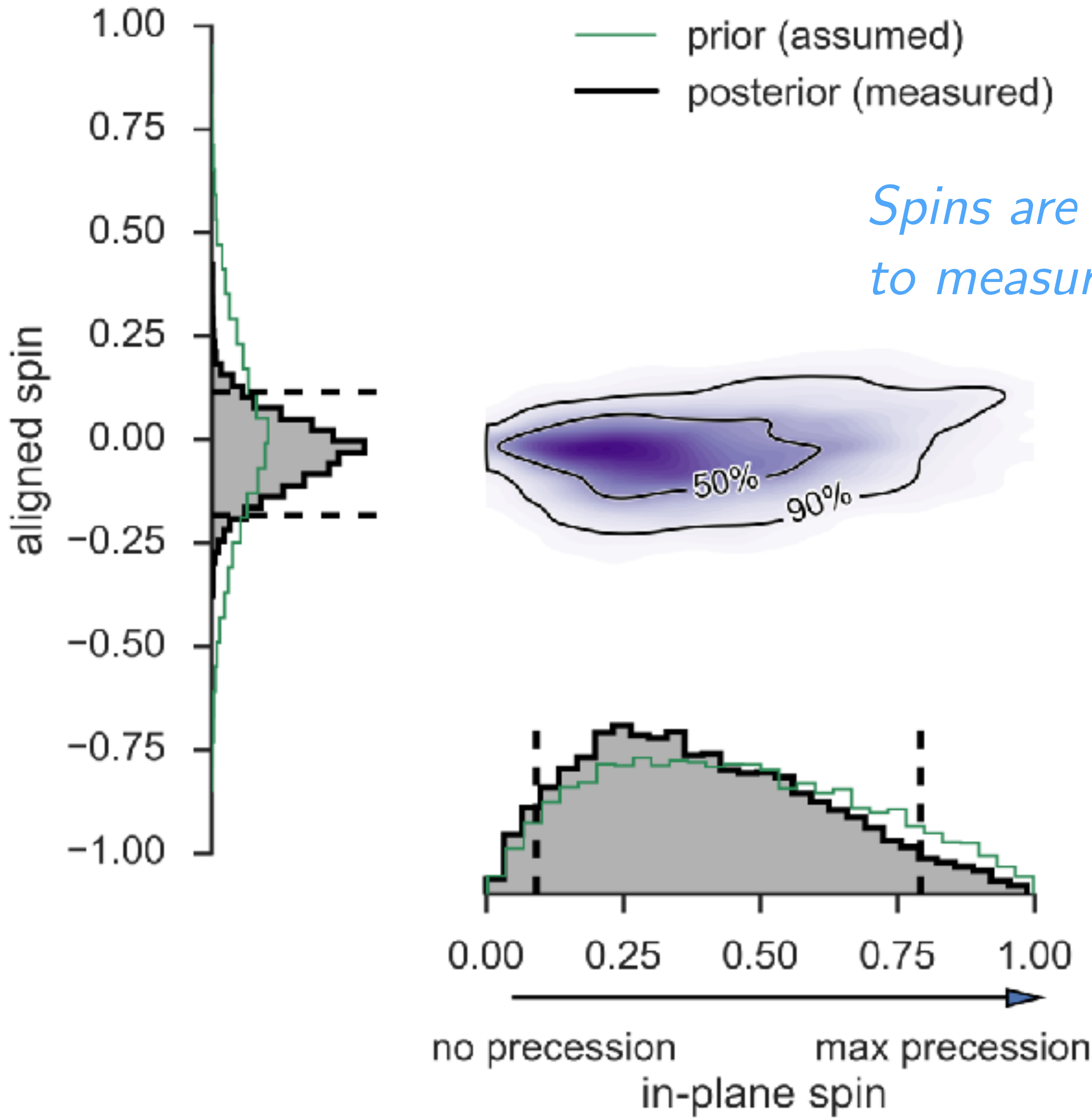
$$\boxed{p(\theta|d) \propto p(\theta)\exp\left[(d|h(\theta)) - \frac{1}{2}(h(\theta)|h(\theta))\right]}$$

# Parameter estimation




[Credit: LVK]

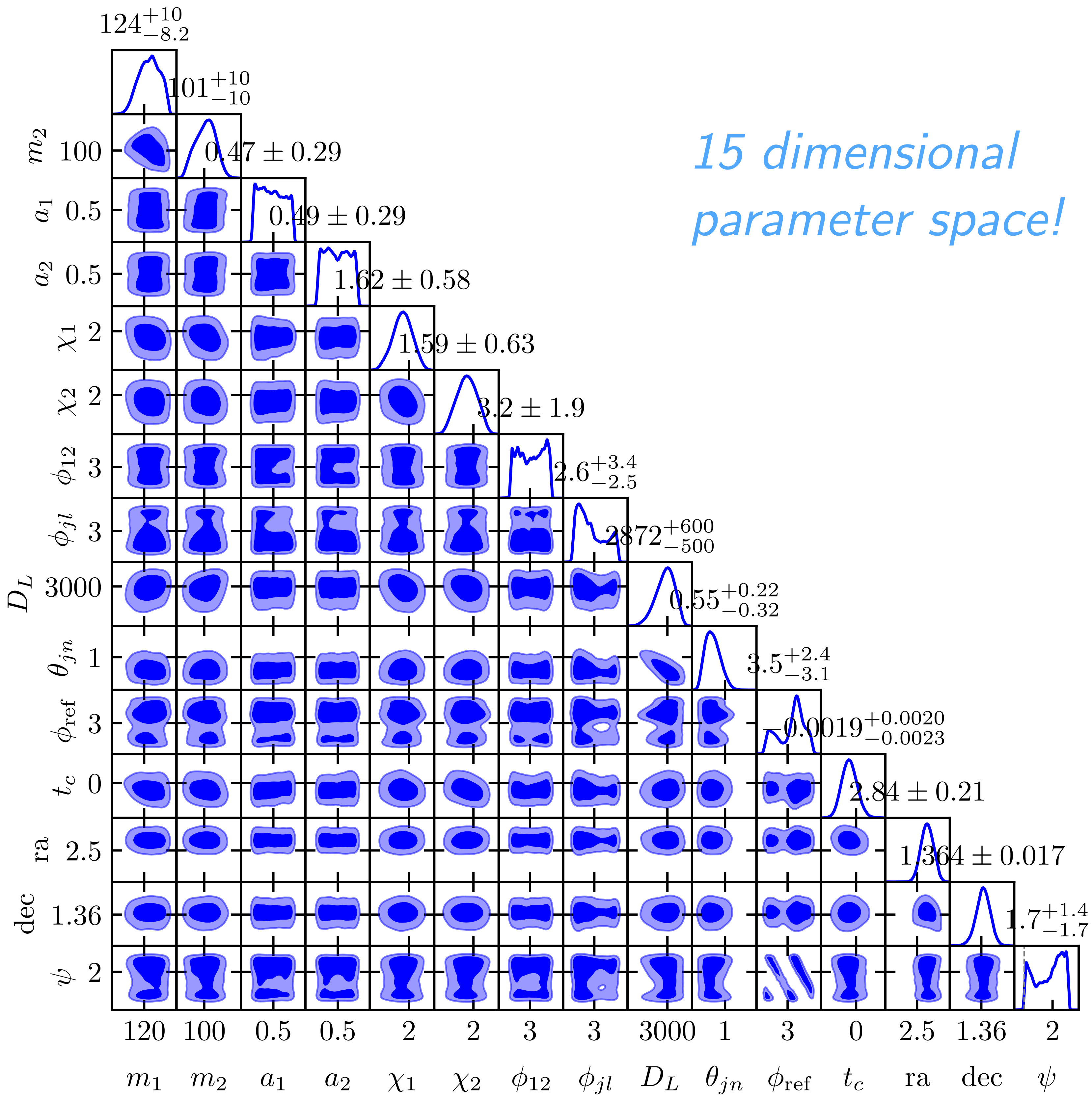
# Parameter estimation




*Spins are more difficult to measure*

[Credit: LVK]

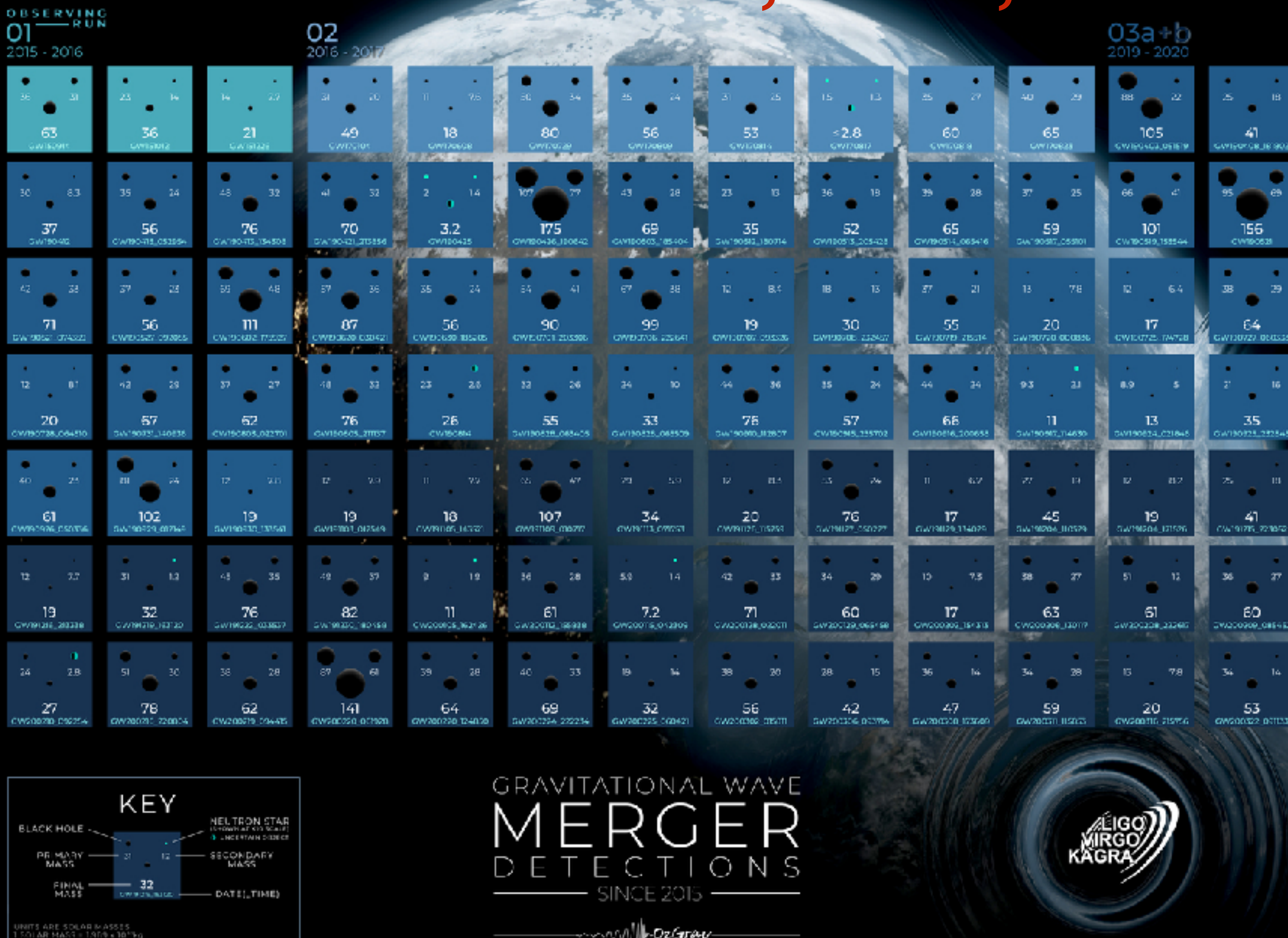
# Parameter estimation




*15 dimensional parameter space!*

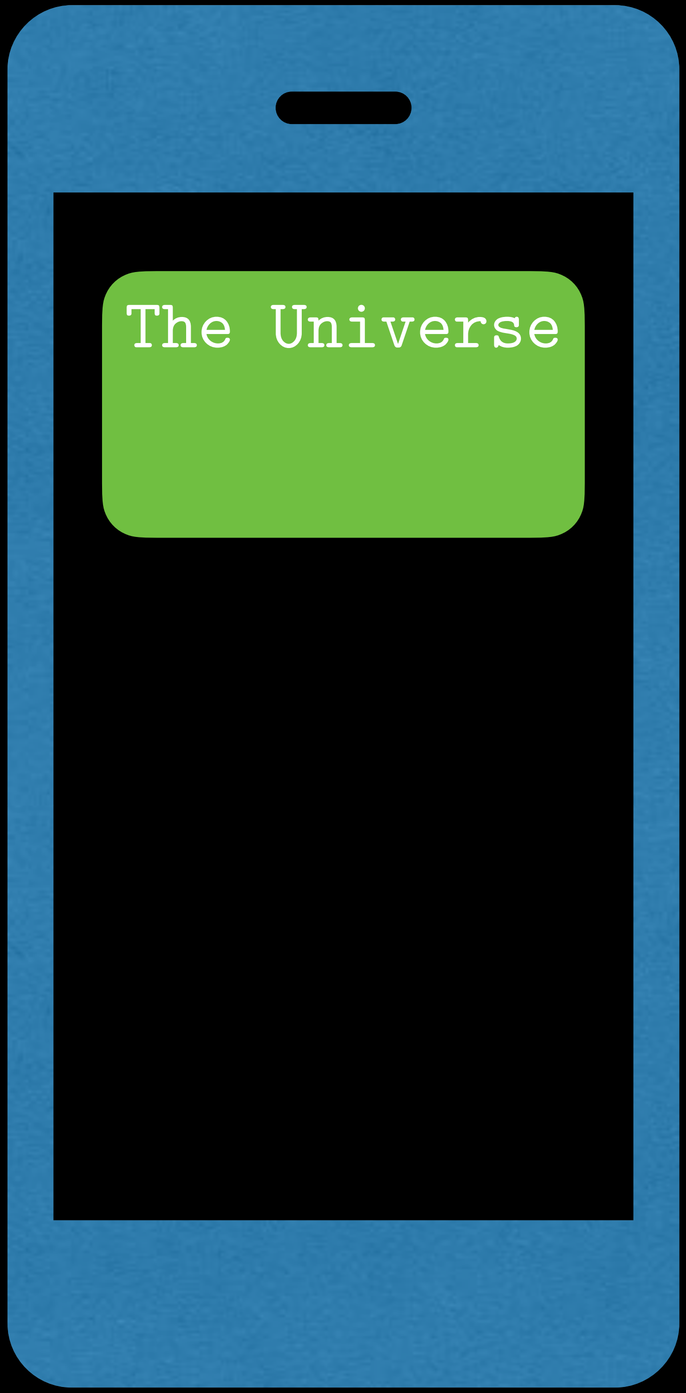
The era of gravitational wave astronomy is **here**!

# ~100 events: BBH, BNS, NSBH



# O4 is happening!

https://gracedb.ligo.org/superevents/public/O4/#



GraceDB Public Alerts Latest Search Documentation Login

Please log in to view full database contents.

# LIGO/Virgo/KAGRA Public Alerts

- More details about public alerts are provided in the LIGO/Virgo/KAGRA Alerts User Guide.
- Retractions are marked in red. Retraction means that the candidate was manually vetted and is no longer considered a candidate of interest.
- Less-significant events are marked in grey, and are not manually vetted. Consult the LVK Alerts User Guide for more information on significance in O4.
- Less-significant events are not shown by default. Press "**Show All Public Events**" to show significant and less-significant events.

---

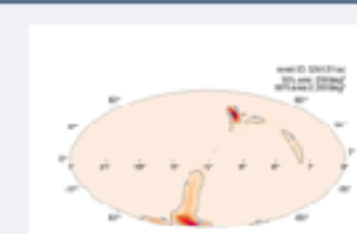
## O4 Significant Detection Candidates: **167** (186 Total - 19 Retracted)

## O4 Low Significance Detection Candidates: **2839** (Total)

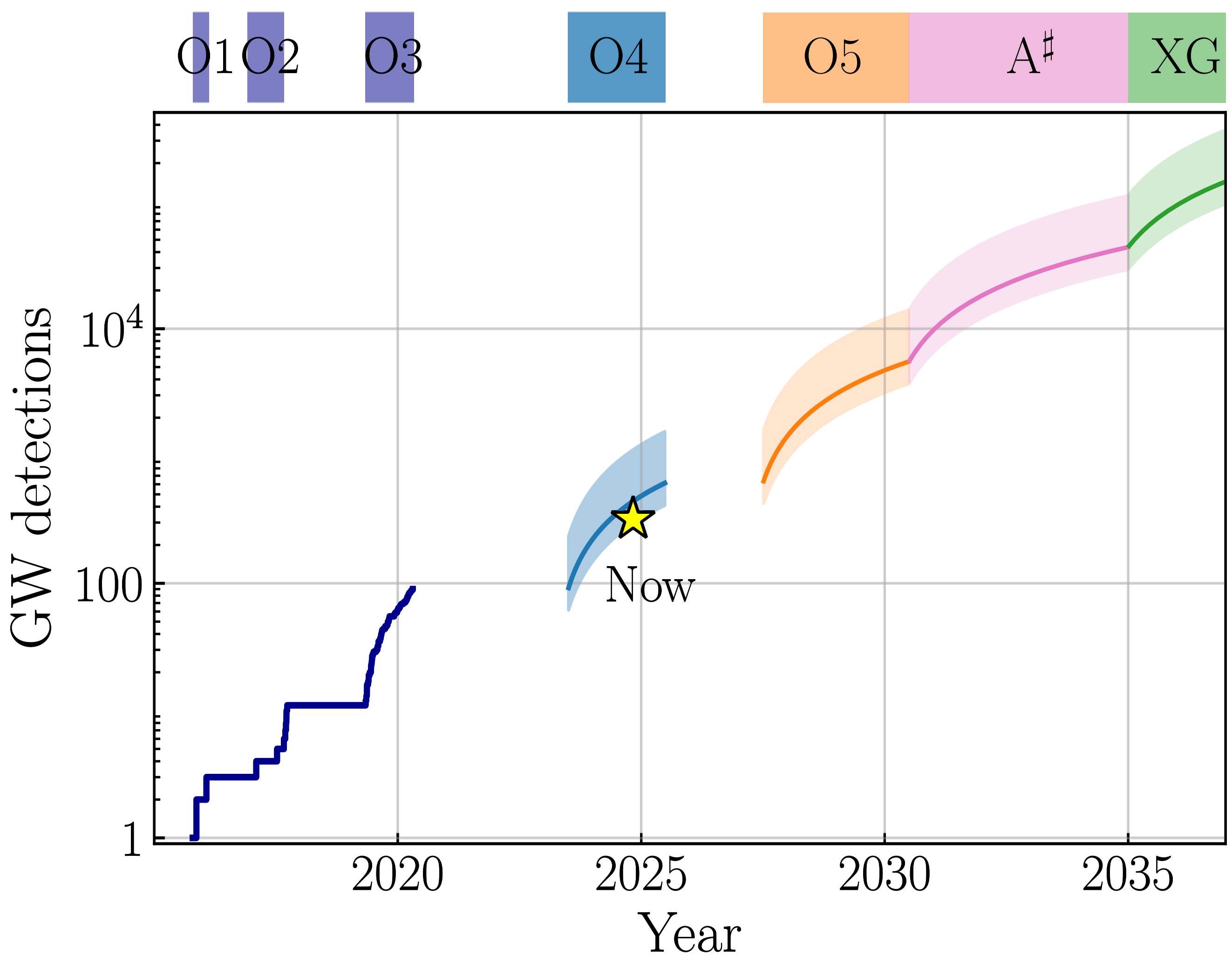
Show All Public Events

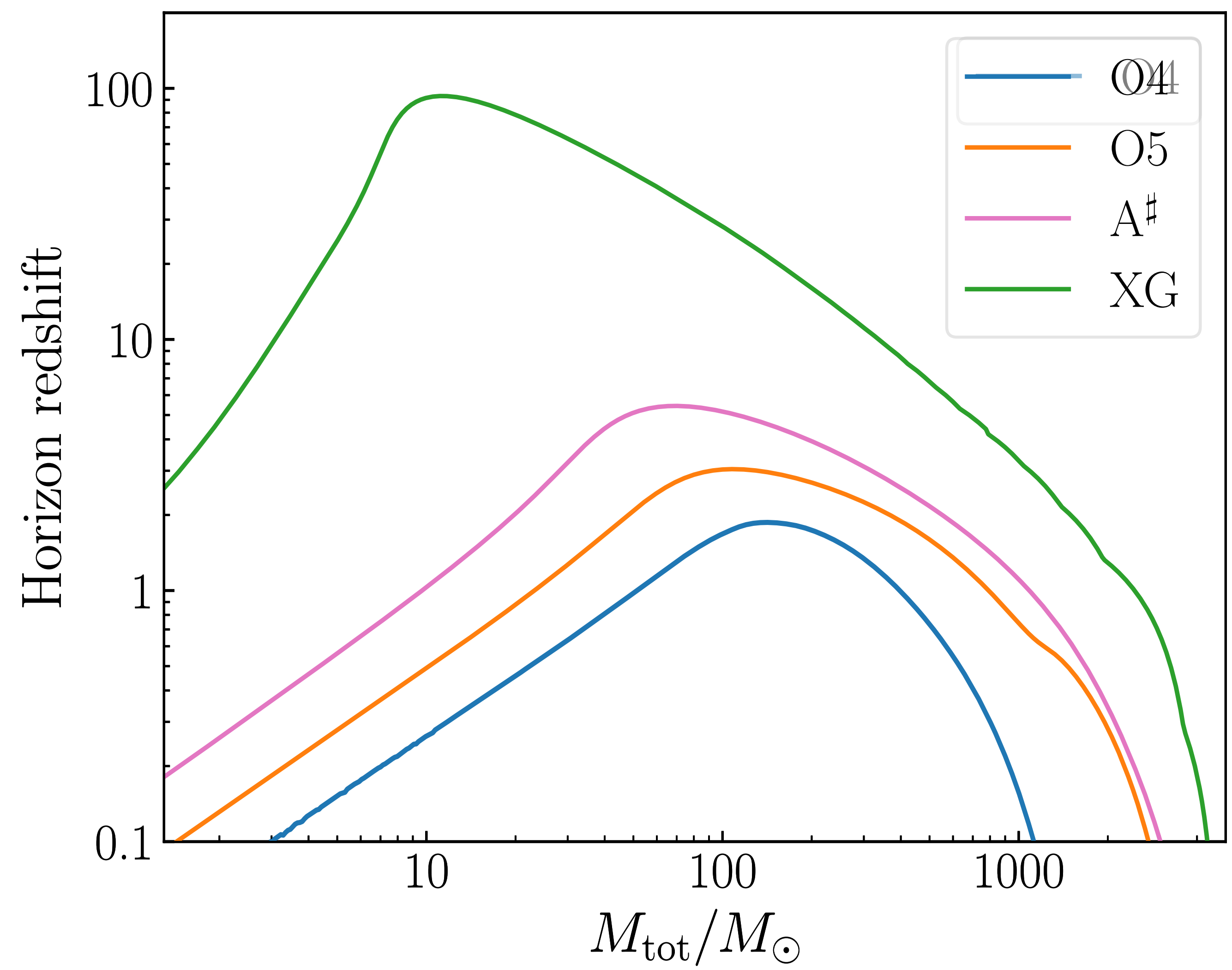
Page 1 of 13. next last »

SORT: EVENT ID (A-Z) ▼

| Event ID | Possible Source (Probability) | Significant | UTC | GCN | Location |
|---|---|---|---|---|---|
| S241201ac | BBH (97%), Terrestrial (3%) | Yes | Dec. 1, 2024 05:57:58 UTC | GCN Circular Query<br>Notices \| VOE |  |

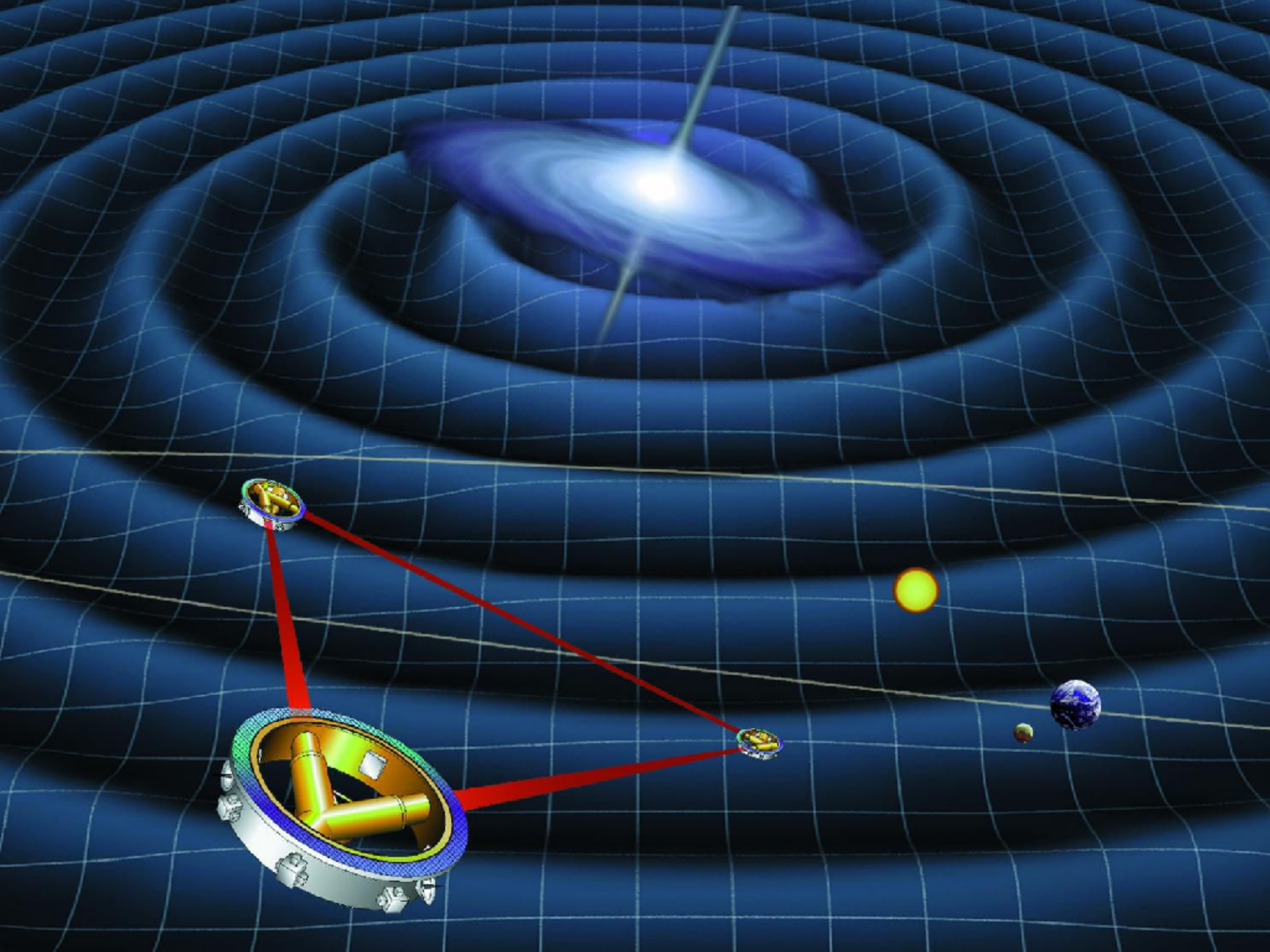
# The **future**: "big data" & distant Universe

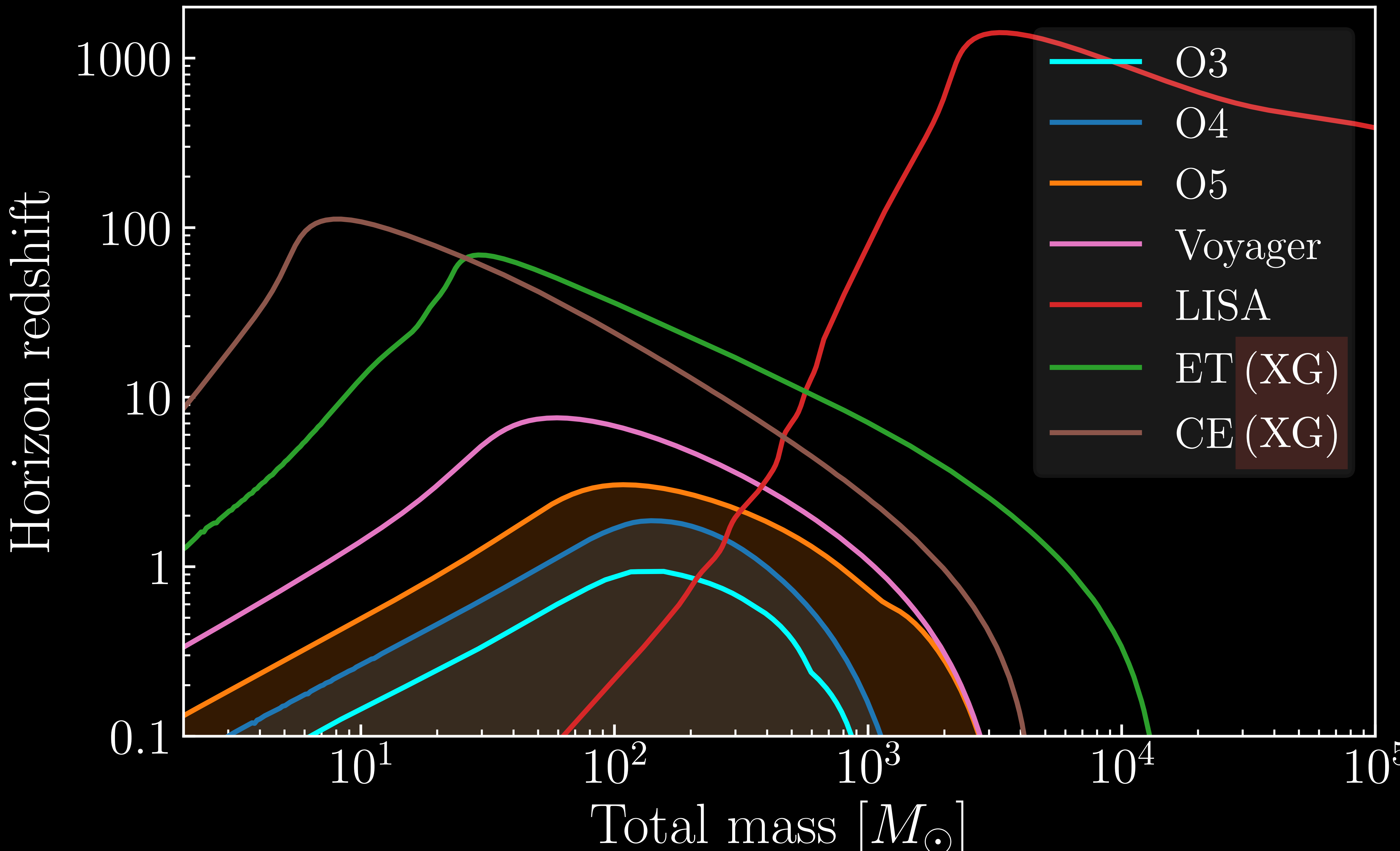



[XG = next-generation detector = Cosmic Explorer / Einstein Telescope]

[Chen, **Ezquiaga** & Gupta (CQG'24)]





1000
100
10
1
0.1
$10^1$
$10^2$
$10^3$
$10^4$
$10^5$
Total mass $[M_\odot]$
Horizon redshift
O3
O4
O5
Voyager
LISA
ET (XG)
CE (XG)

# 2. Key takeaways

- Gravitational waves detectors are describes by their *noise* and *antenna pattern* function

- The *optimal signal to noise* is given when the filter matches the signal

- Data stream can be *matched filtered* using a template bank. An event is found when it cannot be explained by noise background

- Once an event is detected, we can infer the parameters. This is a *15D* parameter space

- Almost *300* significant candidates since the first observation. Many more to come in the future!