

# A premier on gravitational wave lensing

[O'Keeffe, 1965]

**Jose María Ezquiaga**

Niels Bohr Institute

[jose.ezquiaga@nbi.ku.dk](mailto:jose.ezquiaga@nbi.ku.dk)

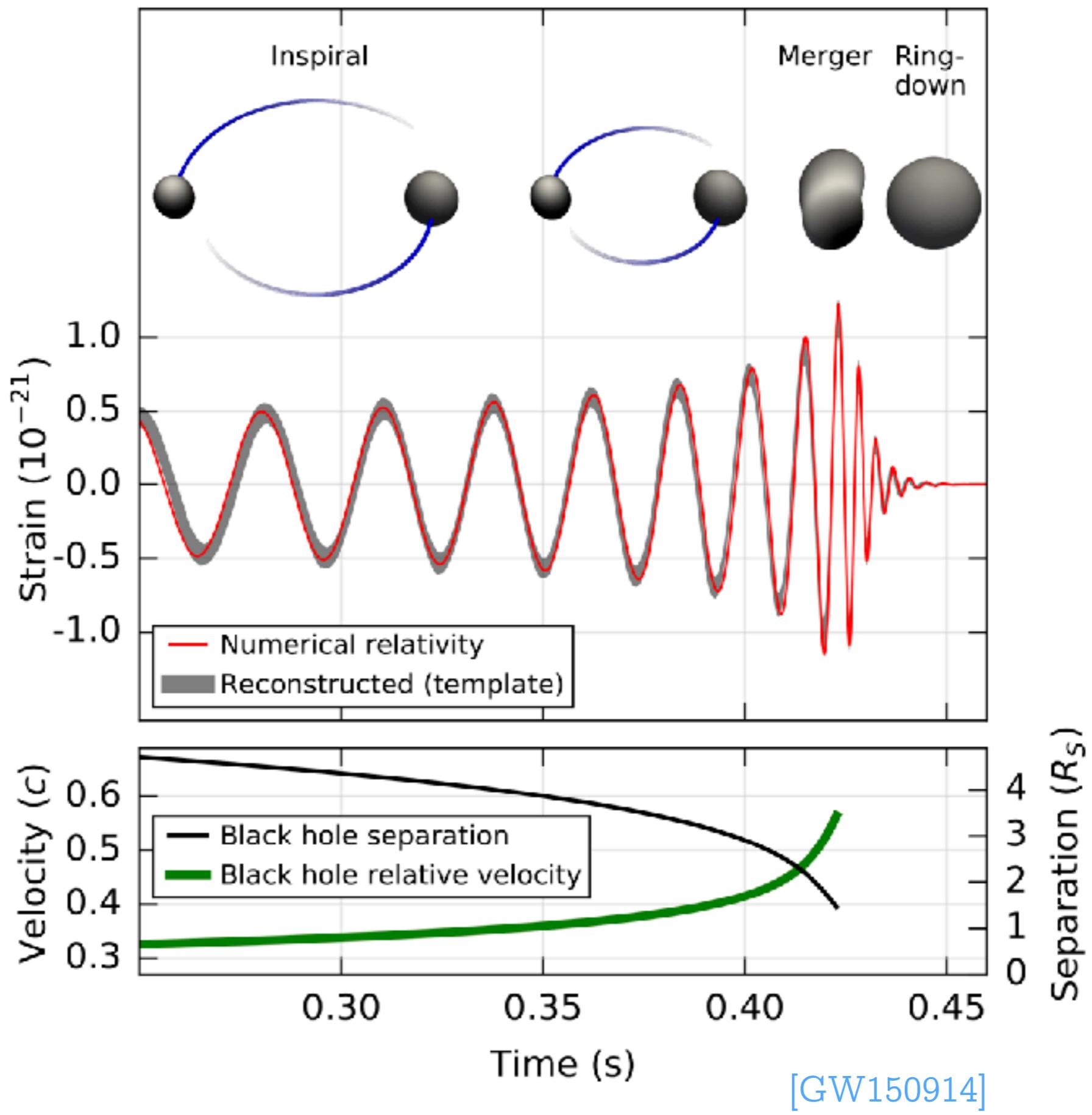
[ezquiaga.github.io](https://ezquiaga.github.io)



VILLUM FONDEN



KØBENHAVNS  
UNIVERSITET



[GW150914]

Binary black holes

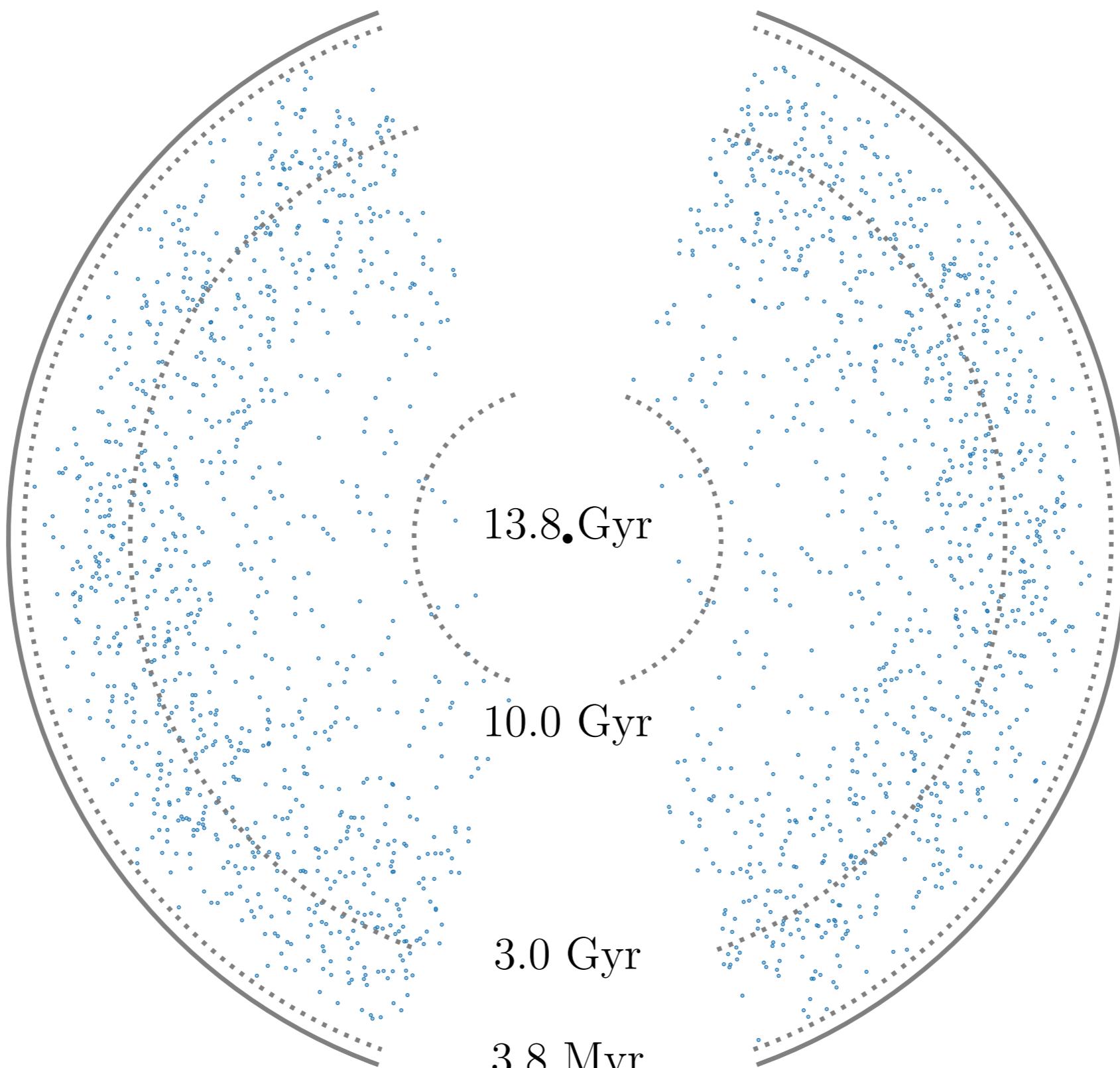
Age of the Universe

13.8 Gyr

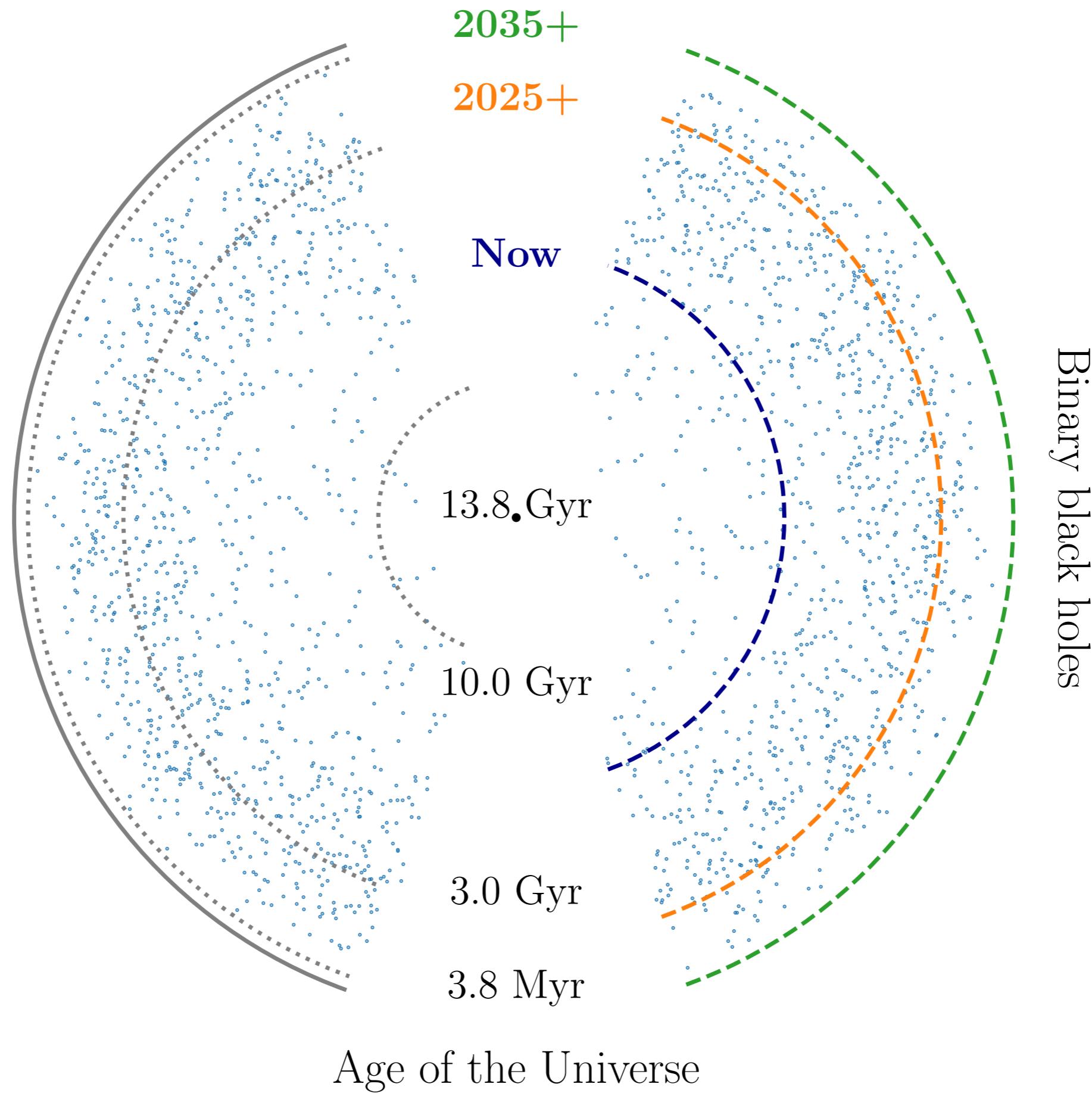
10.0 Gyr

3.0 Gyr

3.8 Myr

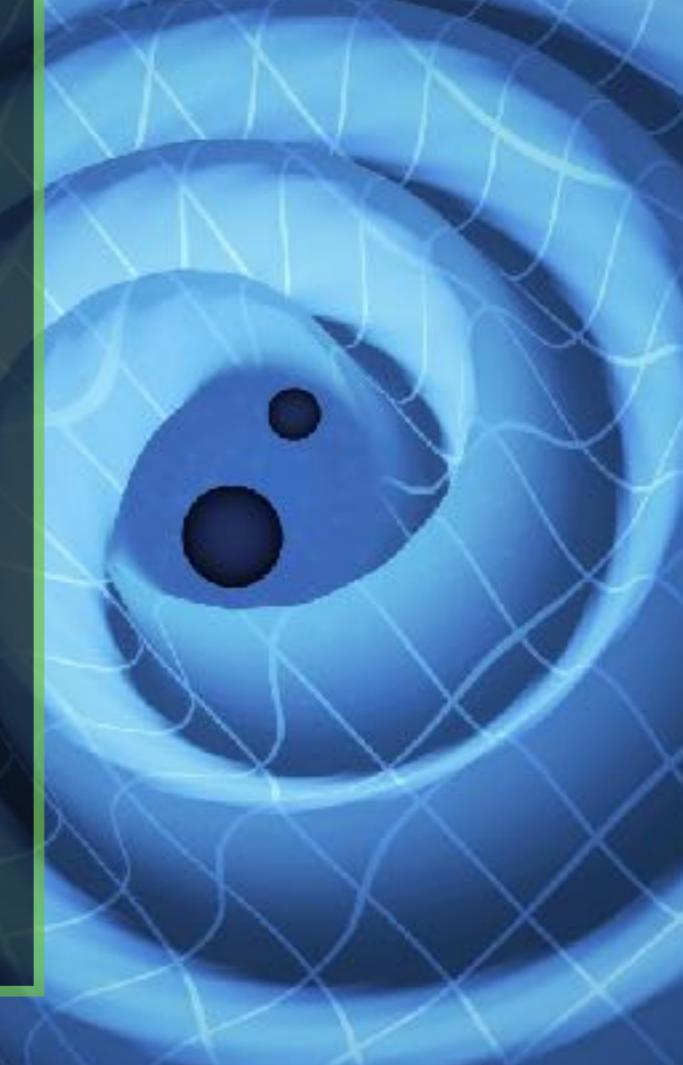


# Gravitational Wave horizons

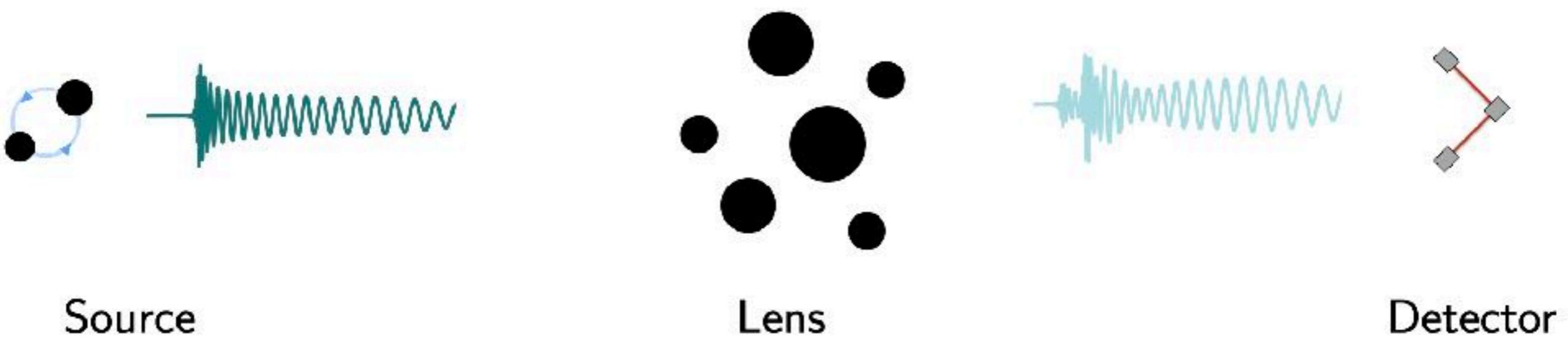
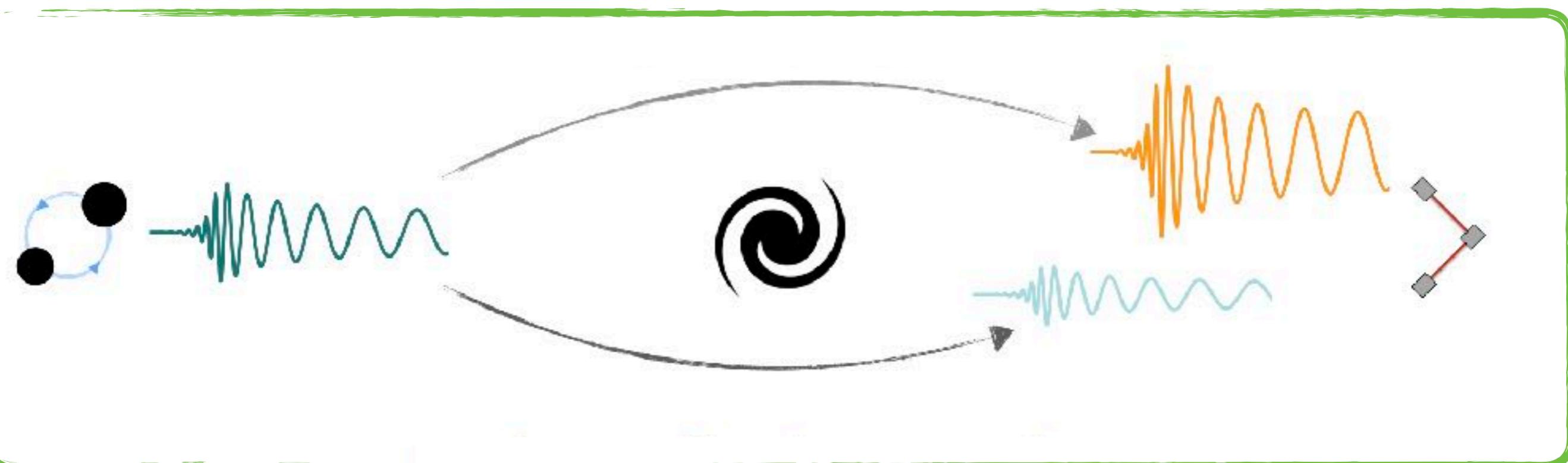


# Gravitational waves

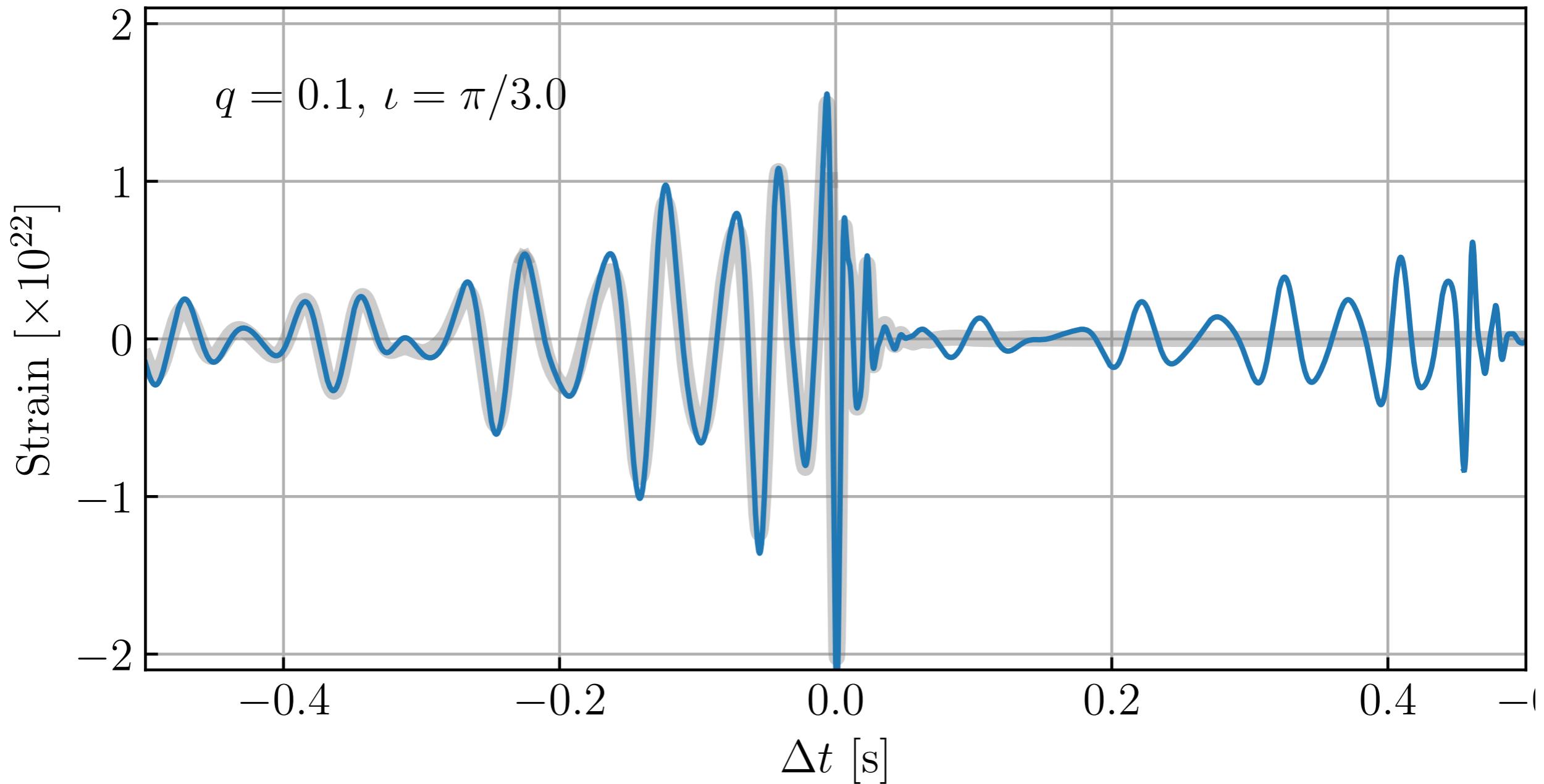
- **Signals from first principles:** general relativity predicts the waveform of a compact binary coalescence
- **Well understood selection effects:** signals can be injected in real noise
- **Powerful population studies:** all sky coverage up to cosmological distances, not affected by medium
- **Wavelengths of astrophysical scale:** frequency from the orbital motion



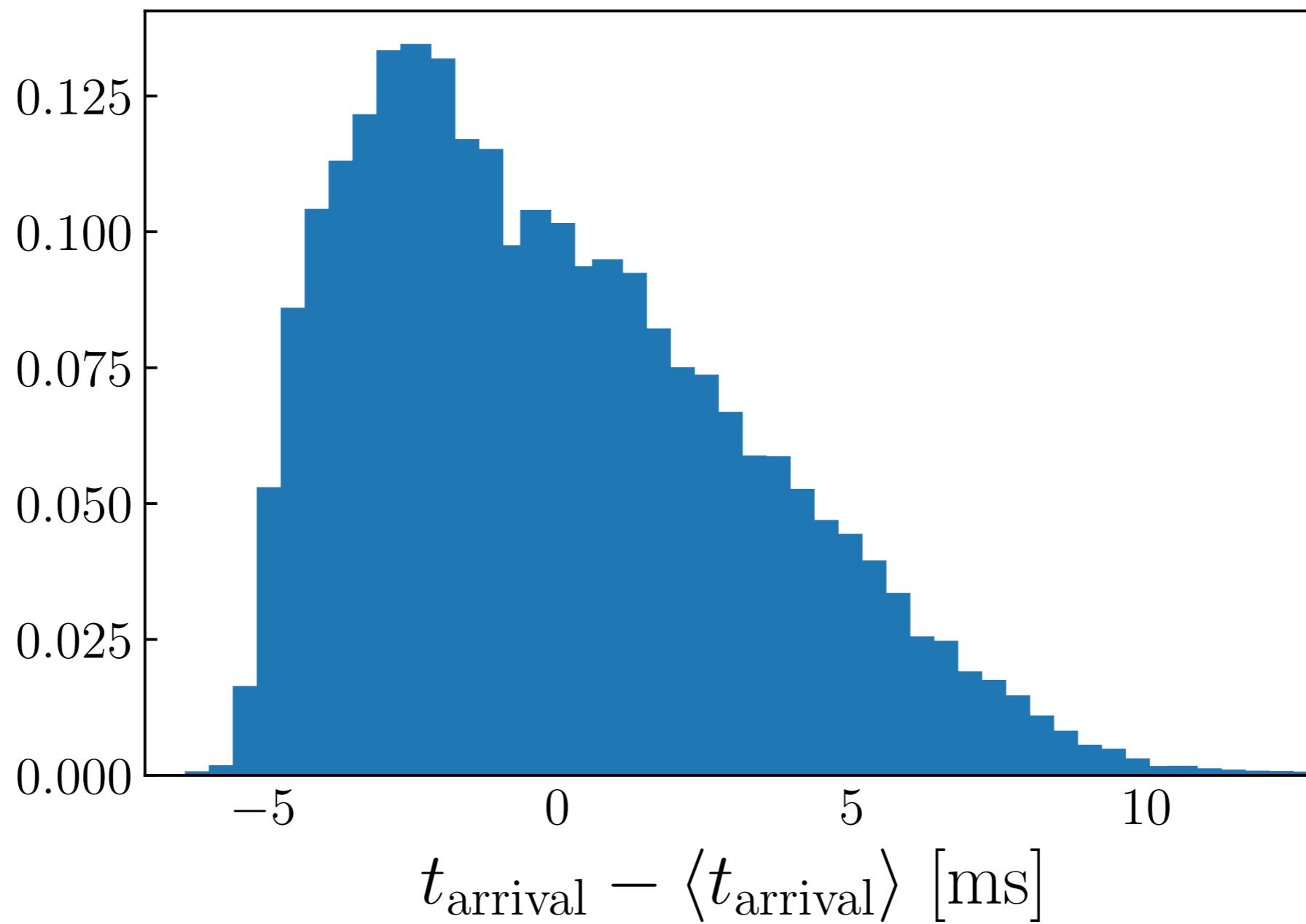
# What lensing?



# Repeated signals

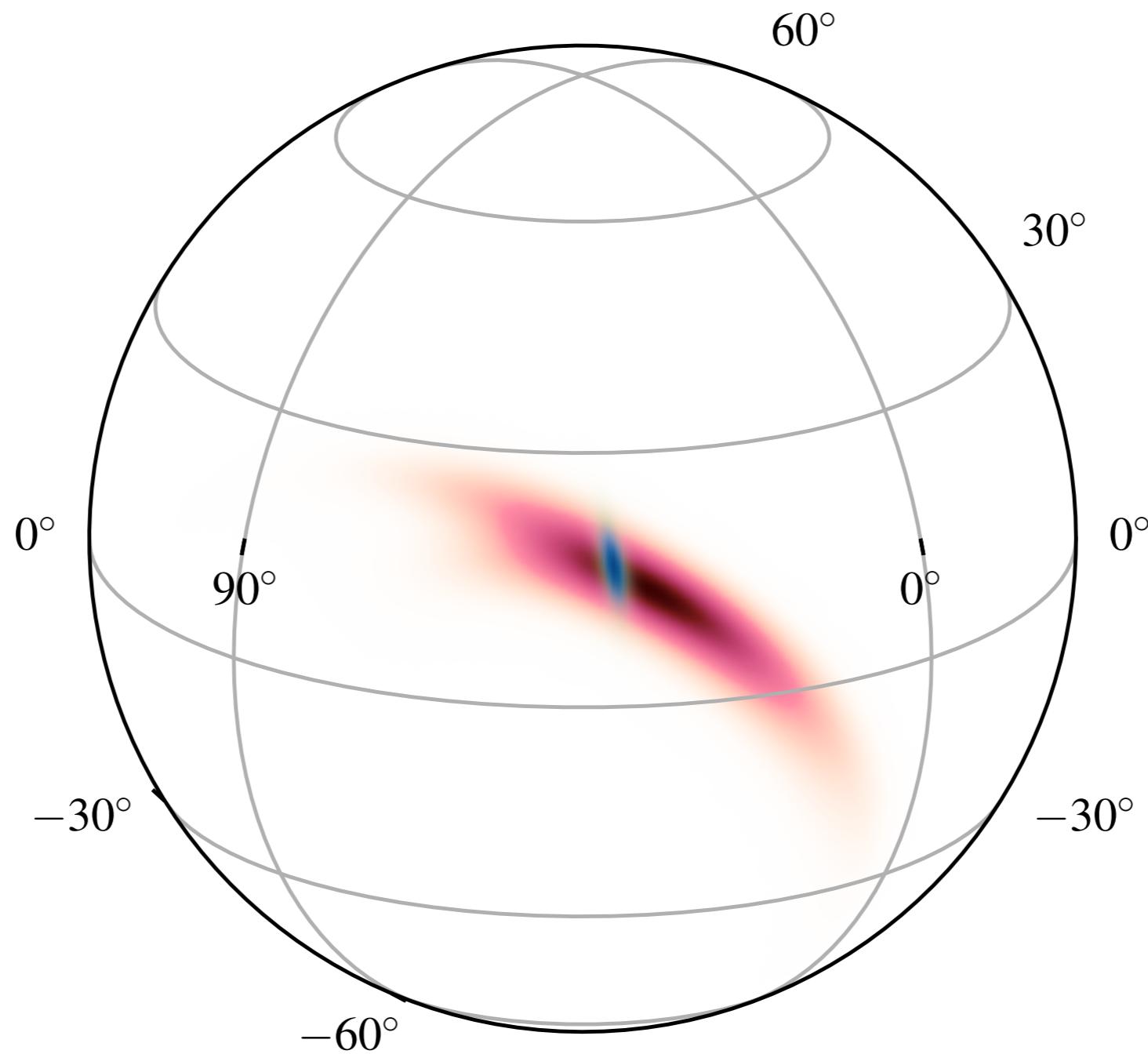


# Precise timing



# Poor sky localization

$$\theta_E \sim 1'' \sqrt{\frac{M}{10^{12} M_\odot}} \sqrt{\frac{1\text{Gpc}}{D}}$$



# Gravitational lensing of gravitational waves

- Repeated signals
- Precise timing
- Poor sky localization
- **Coherent detection of waveform**

# Strong lensing

$$\Delta t_d \cdot \omega \gg 1$$

$$h_L(\omega) = F(\omega, \theta_S) \cdot h(\omega)$$

$$F \approx \sum_j |\mu_j|^{1/2} \exp(i\omega \textcolor{teal}{t}_j - i\pi n_j)$$

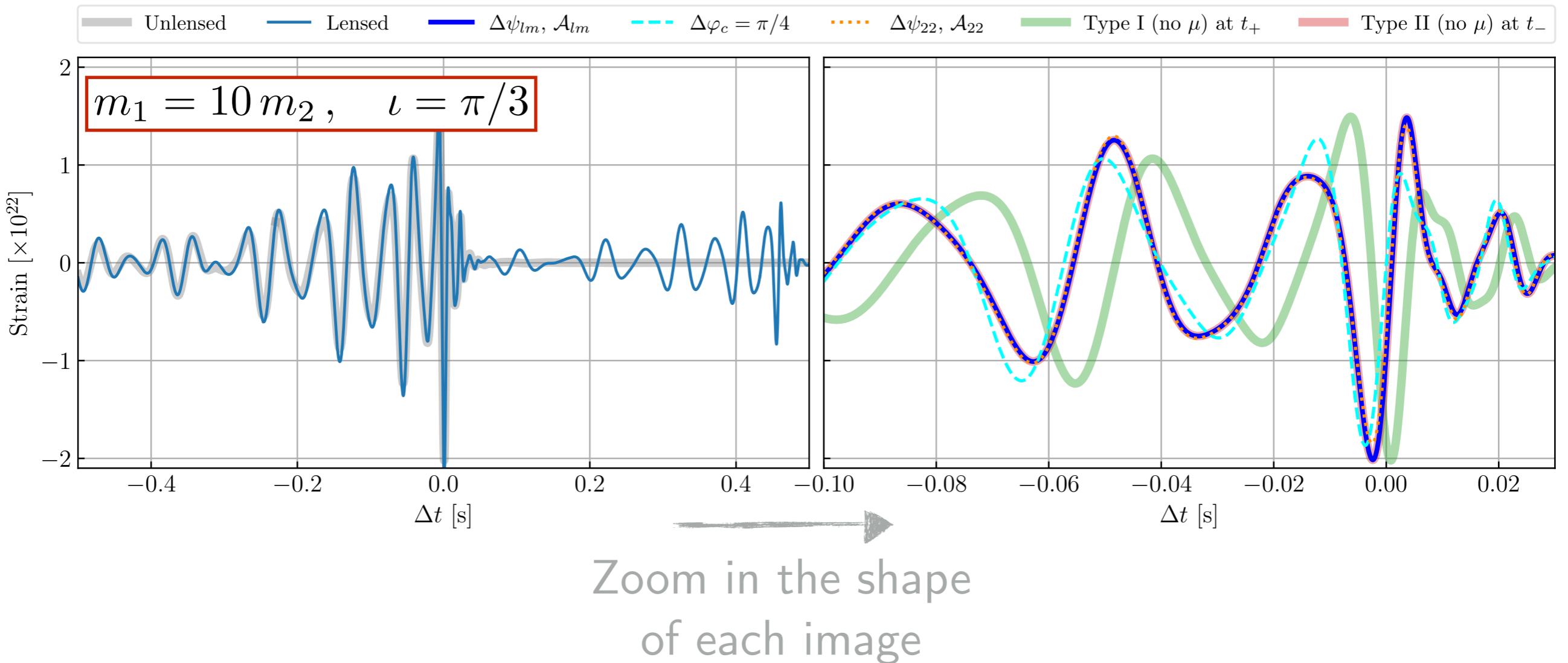
Magnification  
Time delay  
Phase shift

- Each image type (I, II and III) acquire a different phase shift

$$n_j = 0, 1/2, 1$$

# Waveform distortions in type II images

- Lensing imprints *small* but *characteristic* modifications in the signals that cannot be mapped to other astrophysical parameters



# Strong lensing

$$\Delta t_d \cdot \omega \gg 1$$

$$h_L(\omega) = F(\omega, \theta_S) \cdot h(\omega)$$

$$F \approx \sum_j |\mu_j|^{1/2} \exp(i\omega \textcolor{green}{t}_j - i\pi n_j)$$

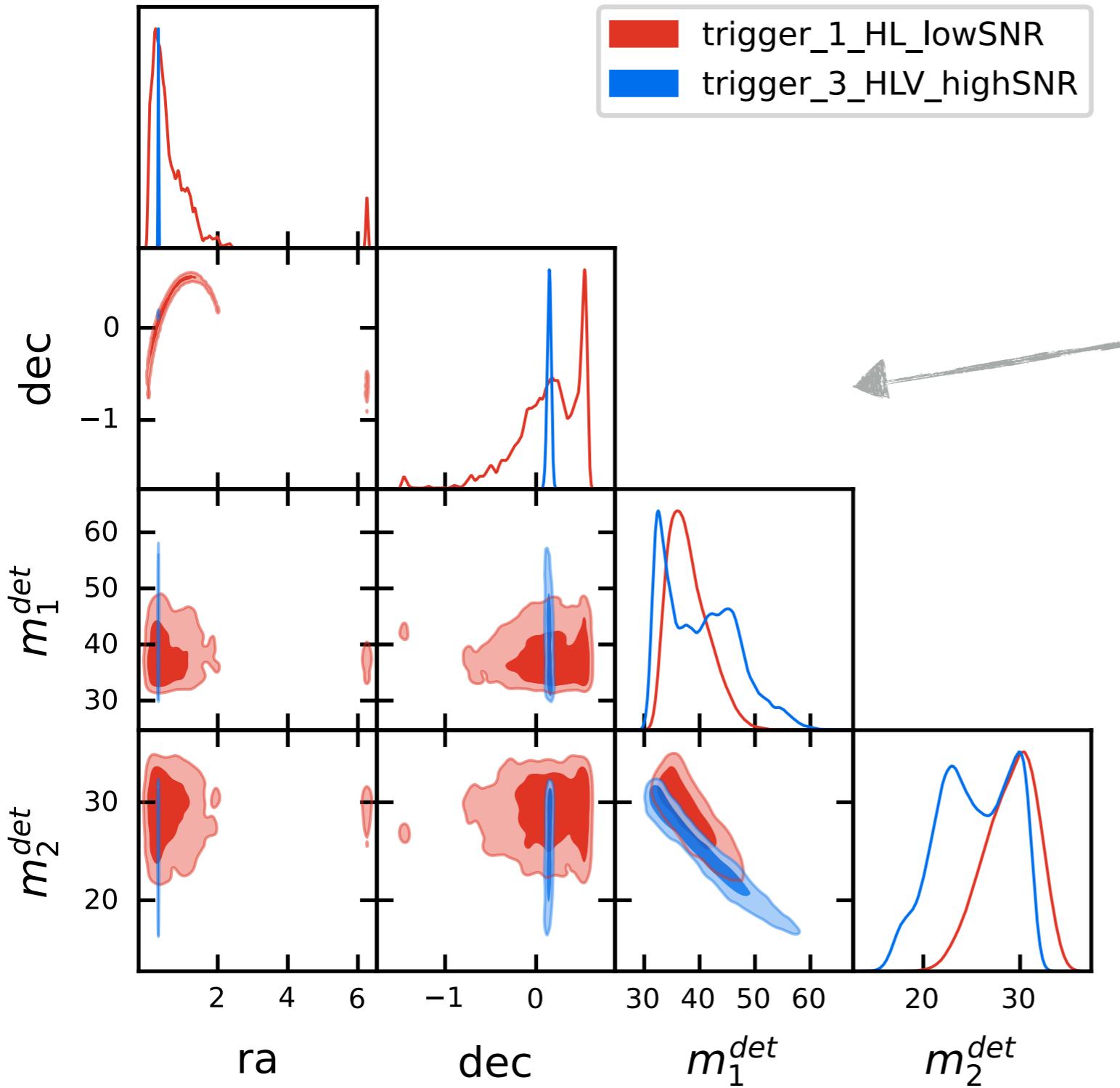
Magnification  
Time delay  
Phase shift

- Each image type (I, II and III) acquire a different phase shift

$$n_j = 0, 1/2, 1$$

- Lensed GWs *can differ* from (unlensed) GR wave-forms
- *Identify* strong lensing with *single image*

# Searching for strongly lensed GWs



# Lensing or luck?

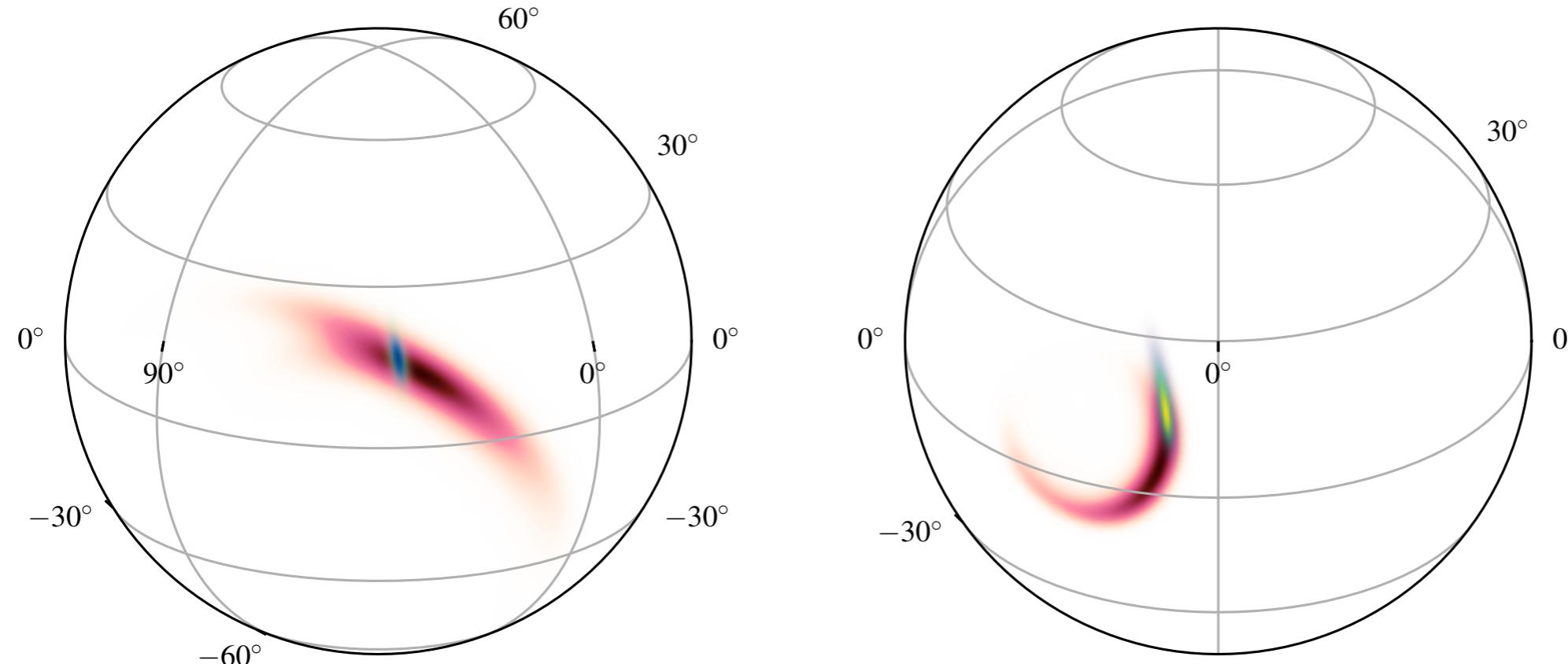


$$N_{\text{false alarm}} \sim N^2$$

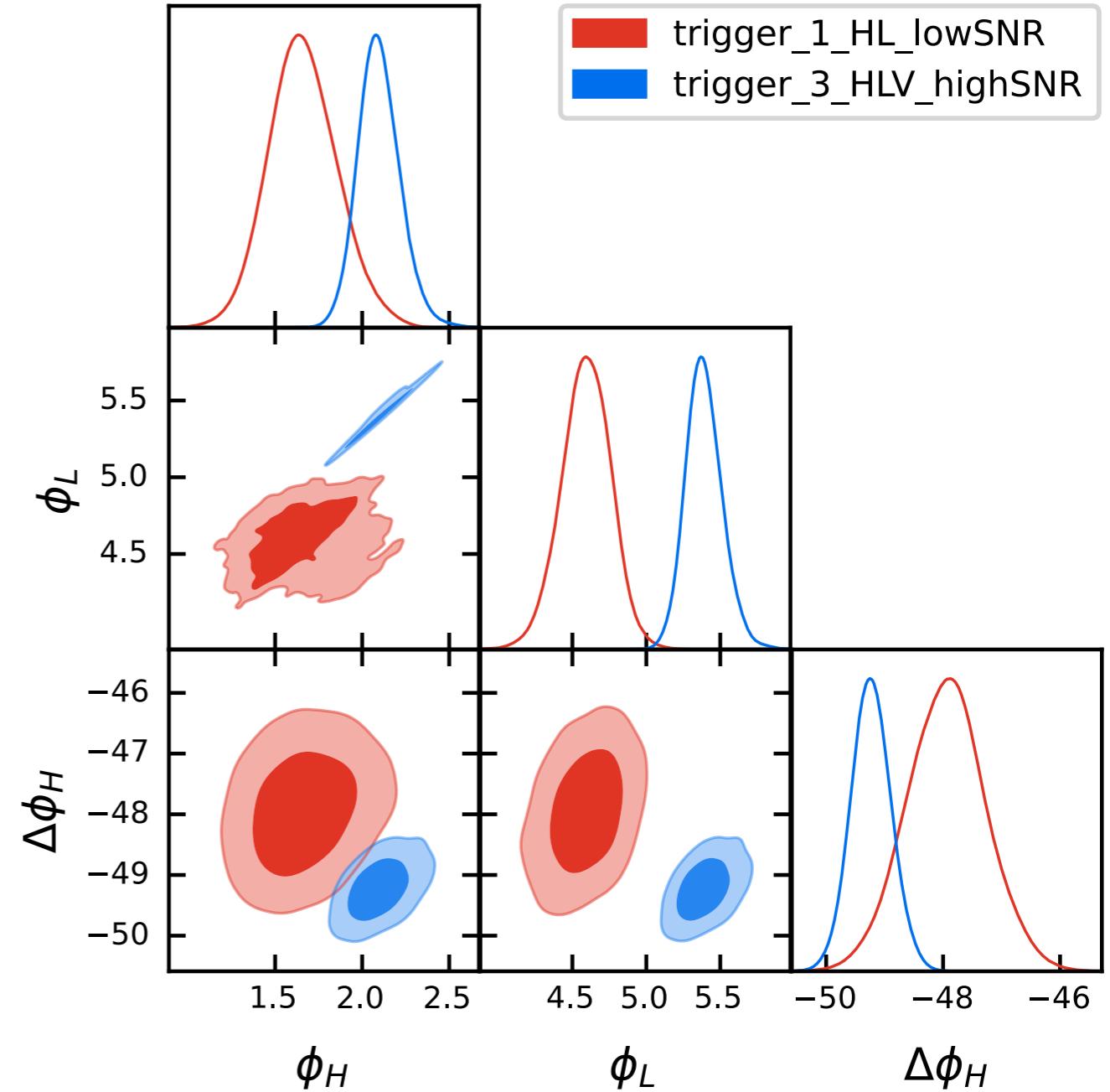
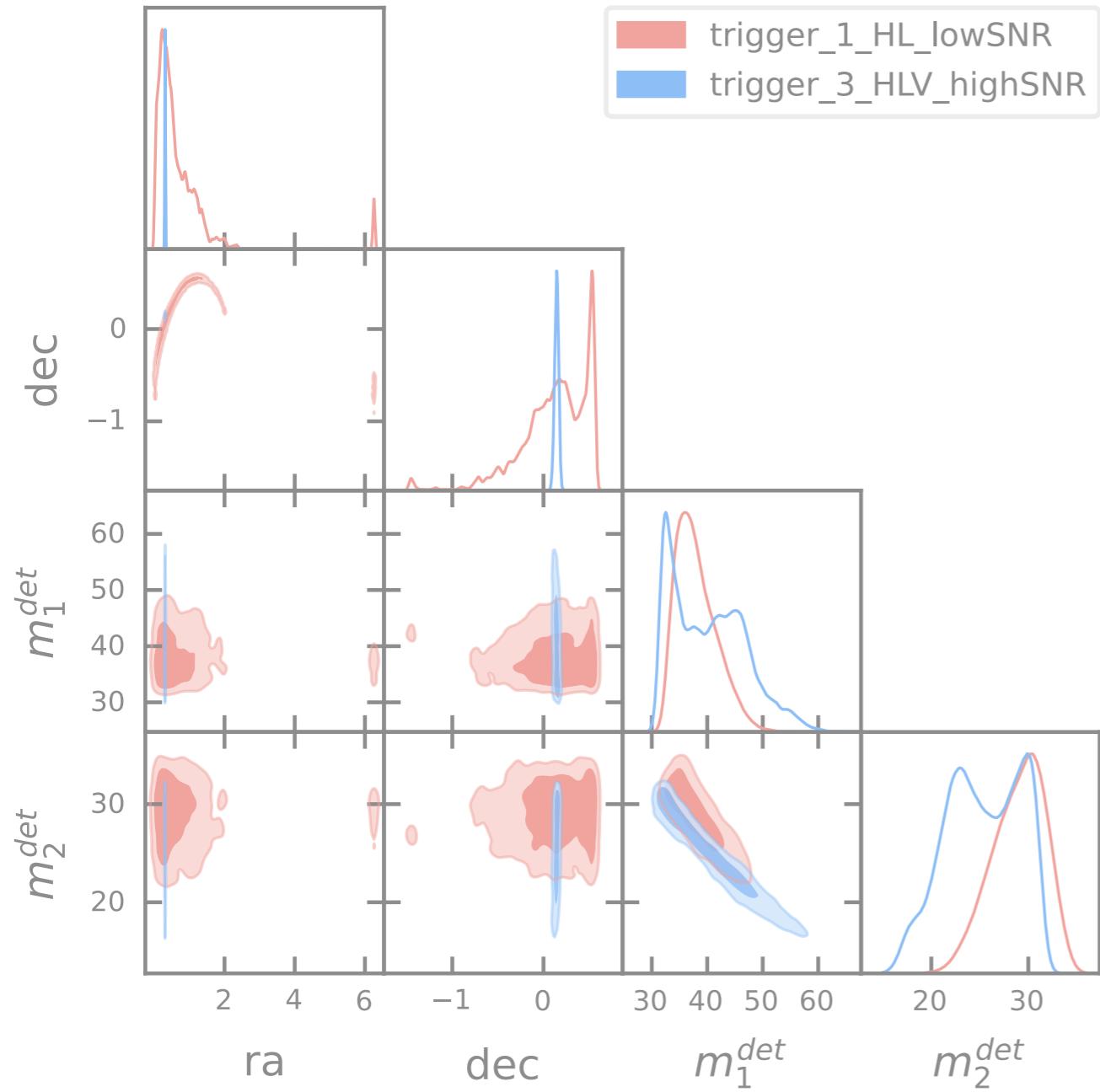
█ Lensed Image 1  
█ Lensed Image 2

█ Event 12  
█ Event 89

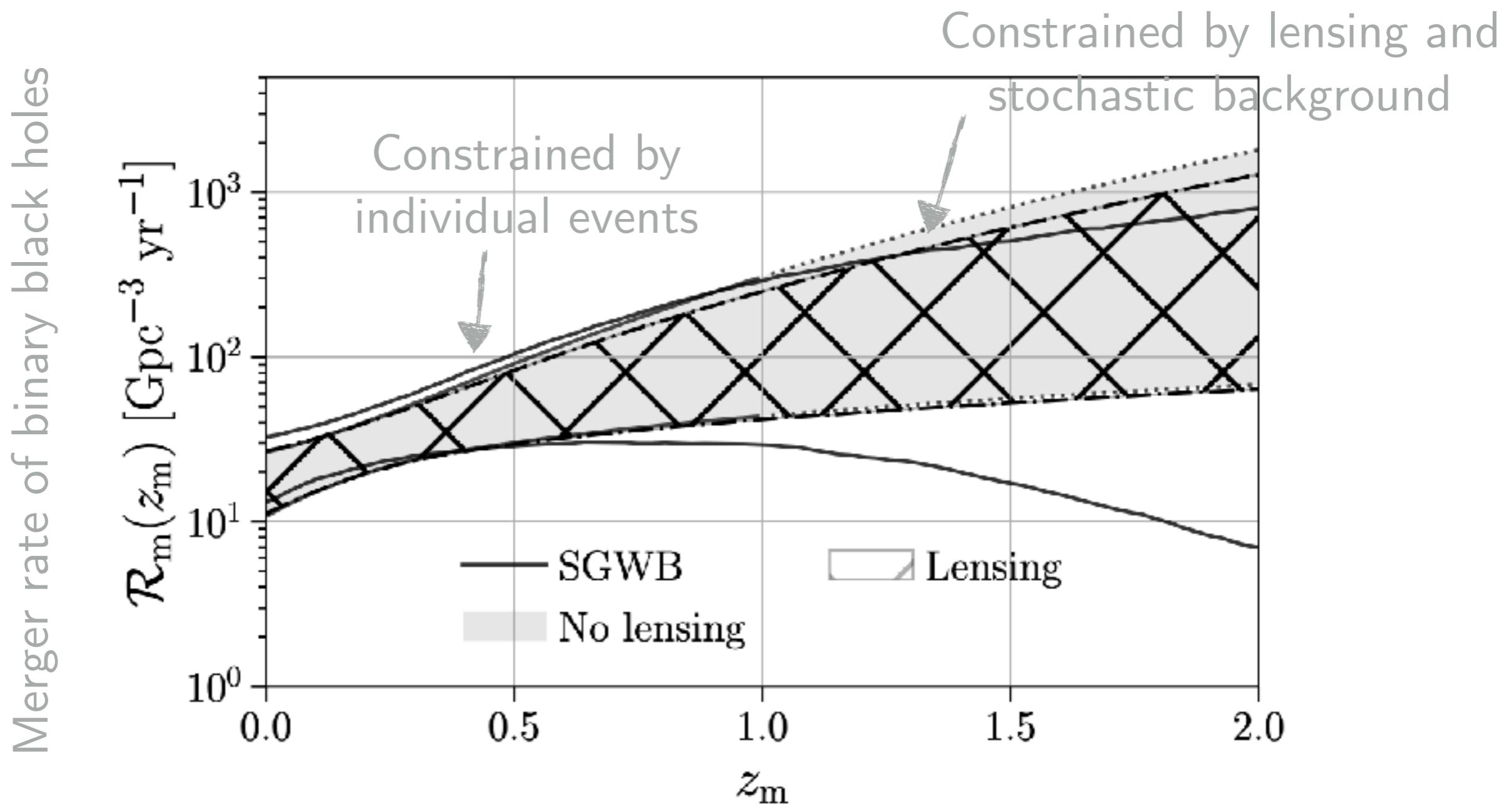
Mesut Çalışkan  
(JHU)



# Fight false alarms: phase consistency



# LVK: no evidence of strong lensing so far...



LVC (incl. Ezquiaga); *Search GW lensing O3a* (ApJ, [arXiv 2105.06384](https://arxiv.org/abs/2105.06384), [science summary](#))

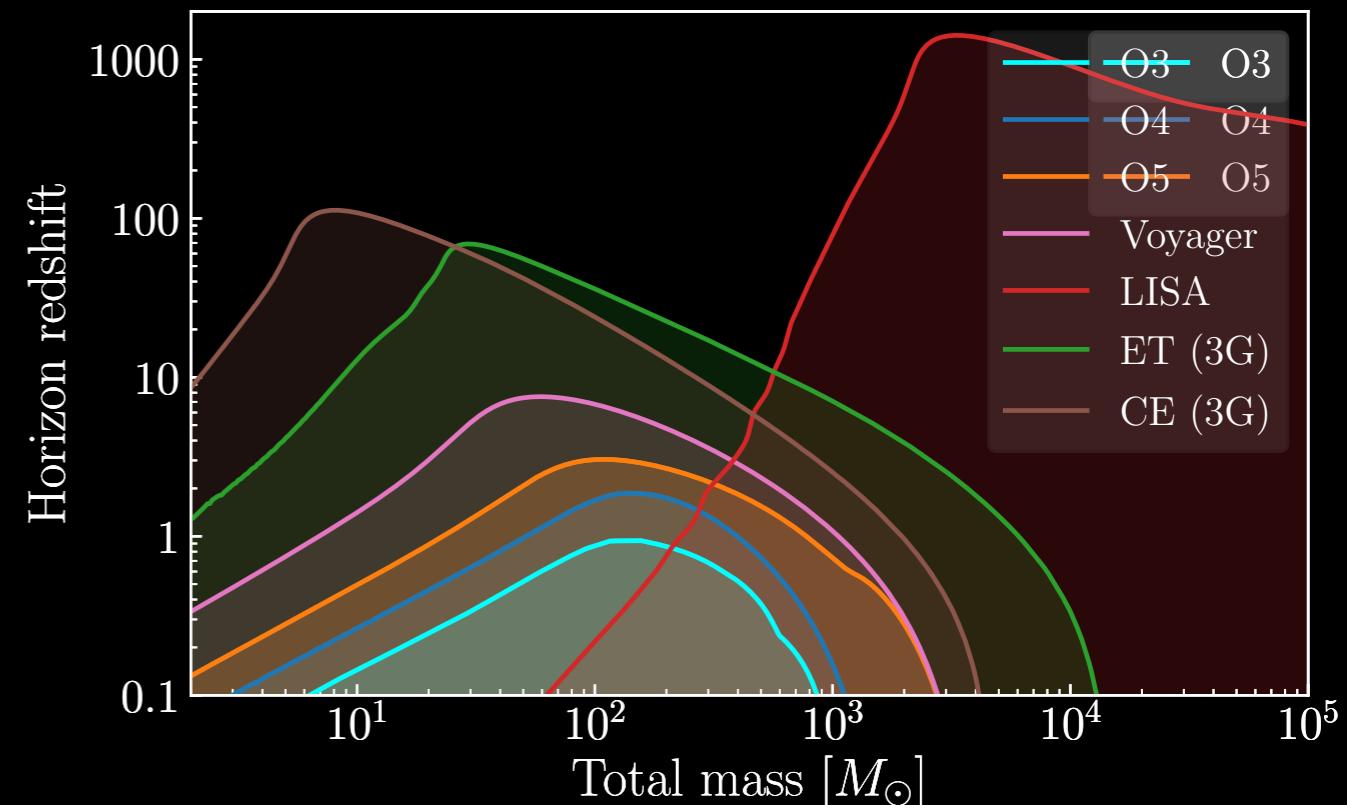
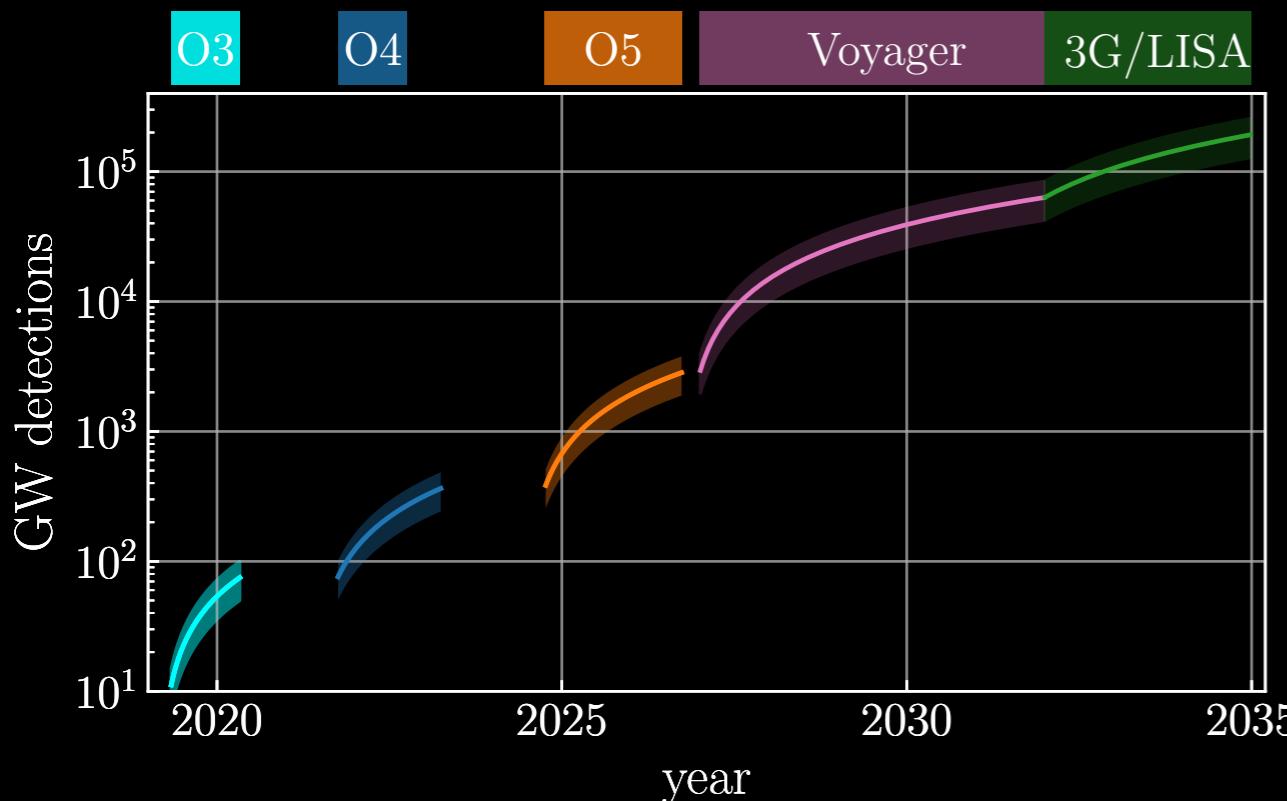
LVK (incl. Ezquiaga); *Search GW lensing full O3* ([arXiv 2304.08393](https://arxiv.org/abs/2304.08393), [science summary](#))

# Looking ahead

**Now**

**2G:** current generation ground-based GW detectors

**3G:** next generation ground-based GW detectors



Approx. **100**  
events typically at  
 $z < 0.6$

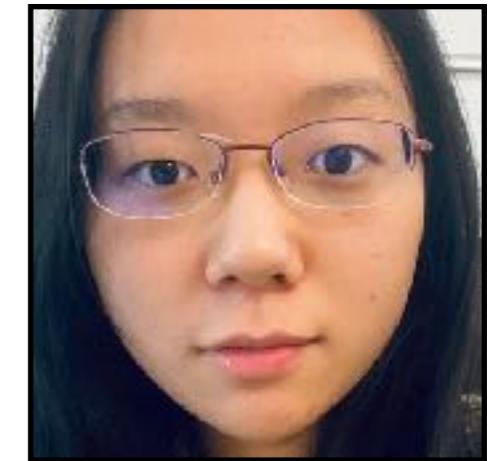


**1000s / year**  
with some  $z > 1$



**100,000s / year**  
with most  $z > 1$

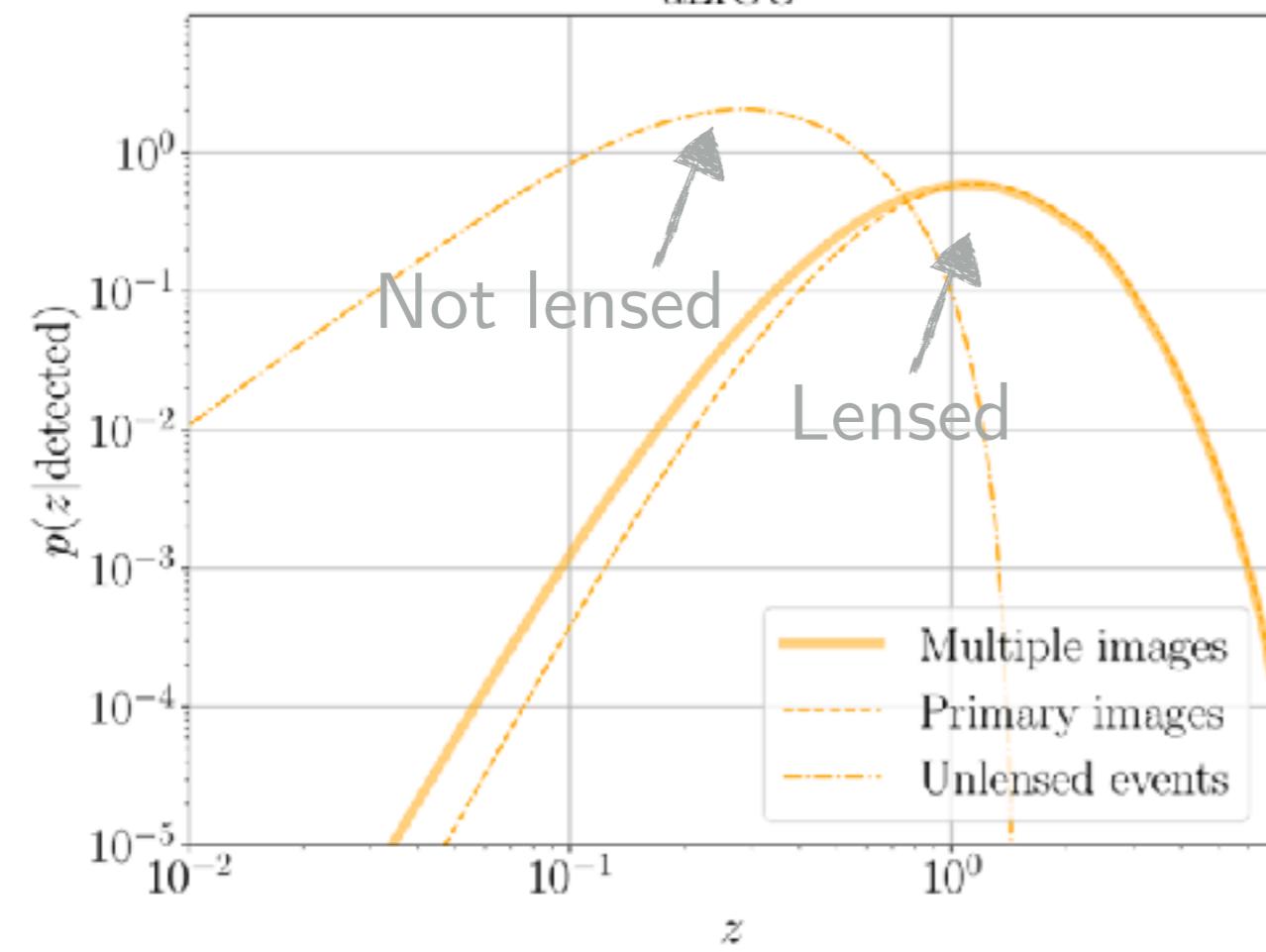
# Probing source and lens populations



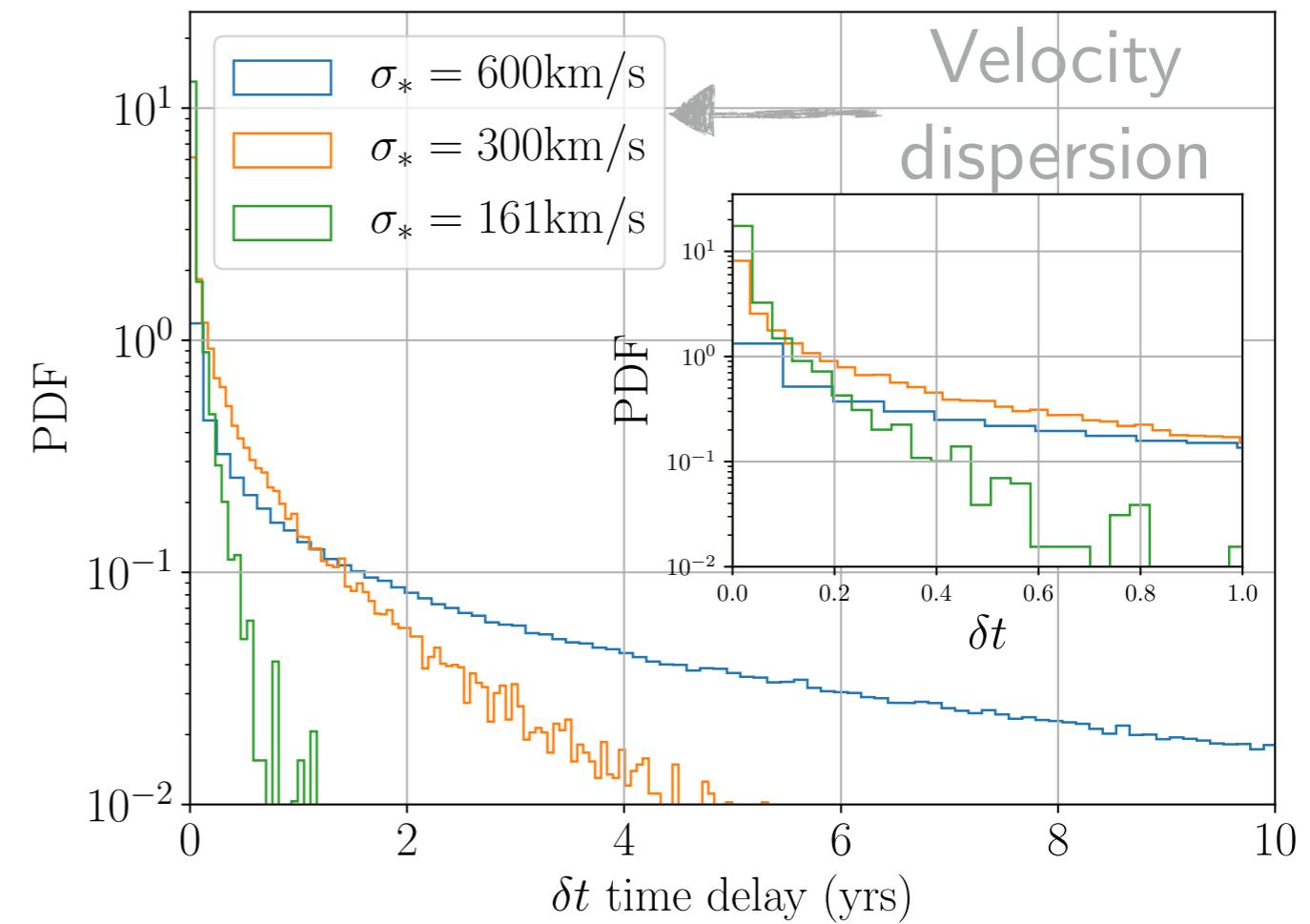
Fei Xu  
(UChicago)

Detected populations

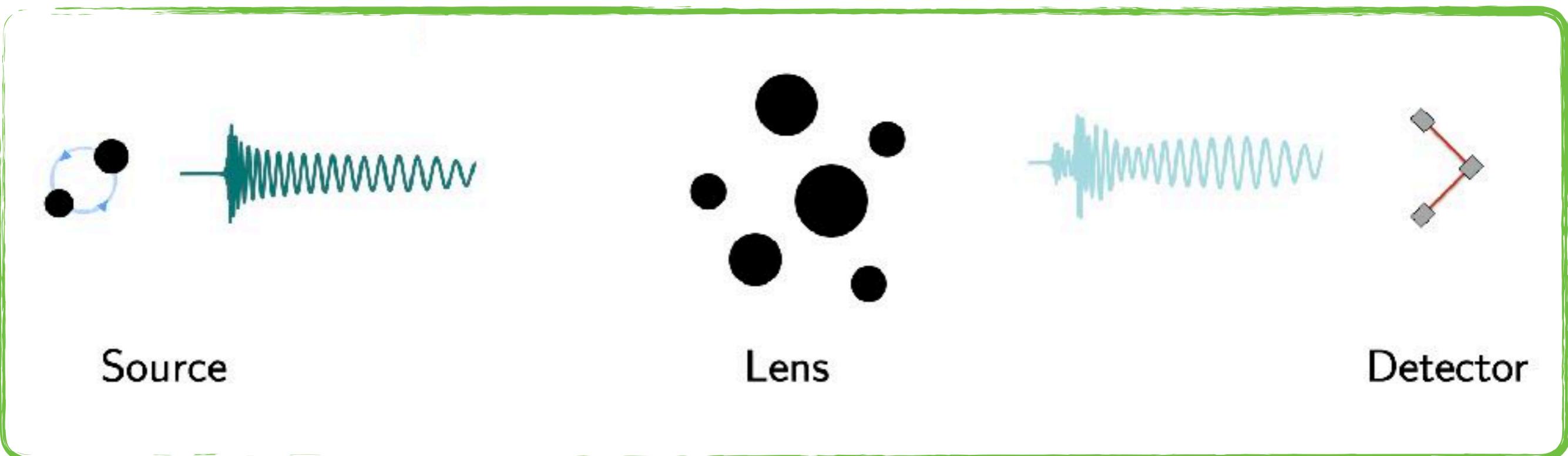
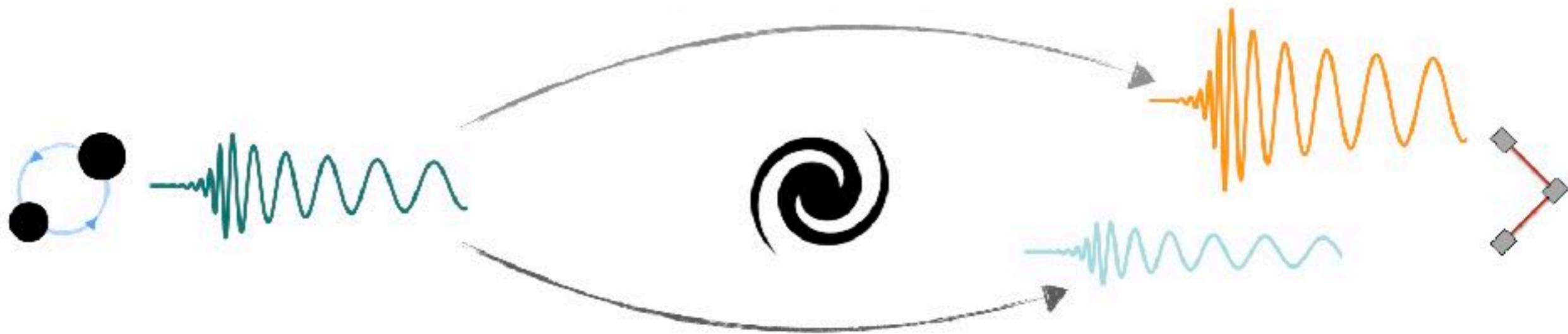
aLIGO



Time delay distributions

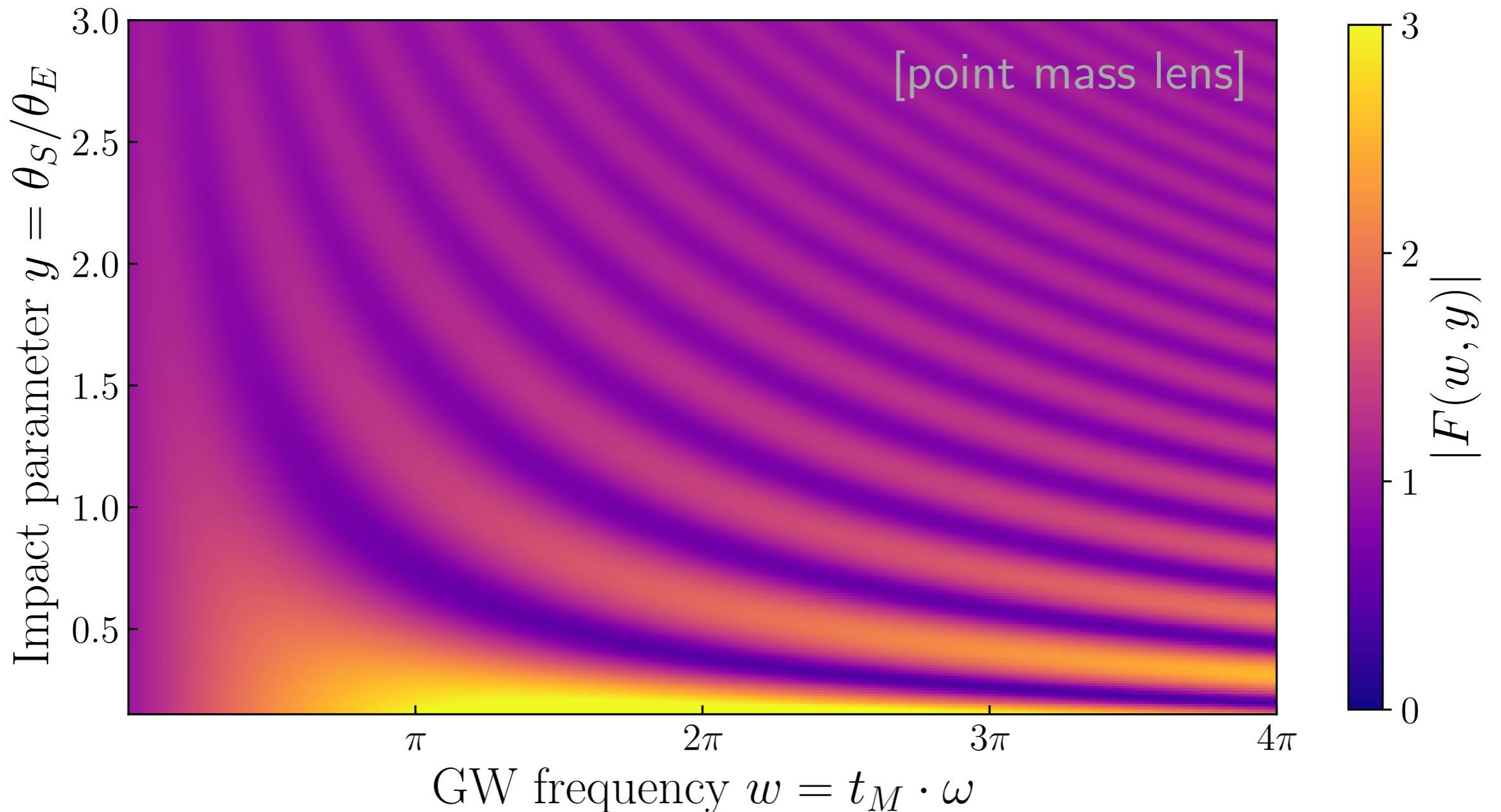


# What lensing?



# Wave effects:

$$F(w, \vec{y}) = \frac{w}{2\pi i} \int d^2x \exp [iwT(\vec{x}, \vec{y})]$$

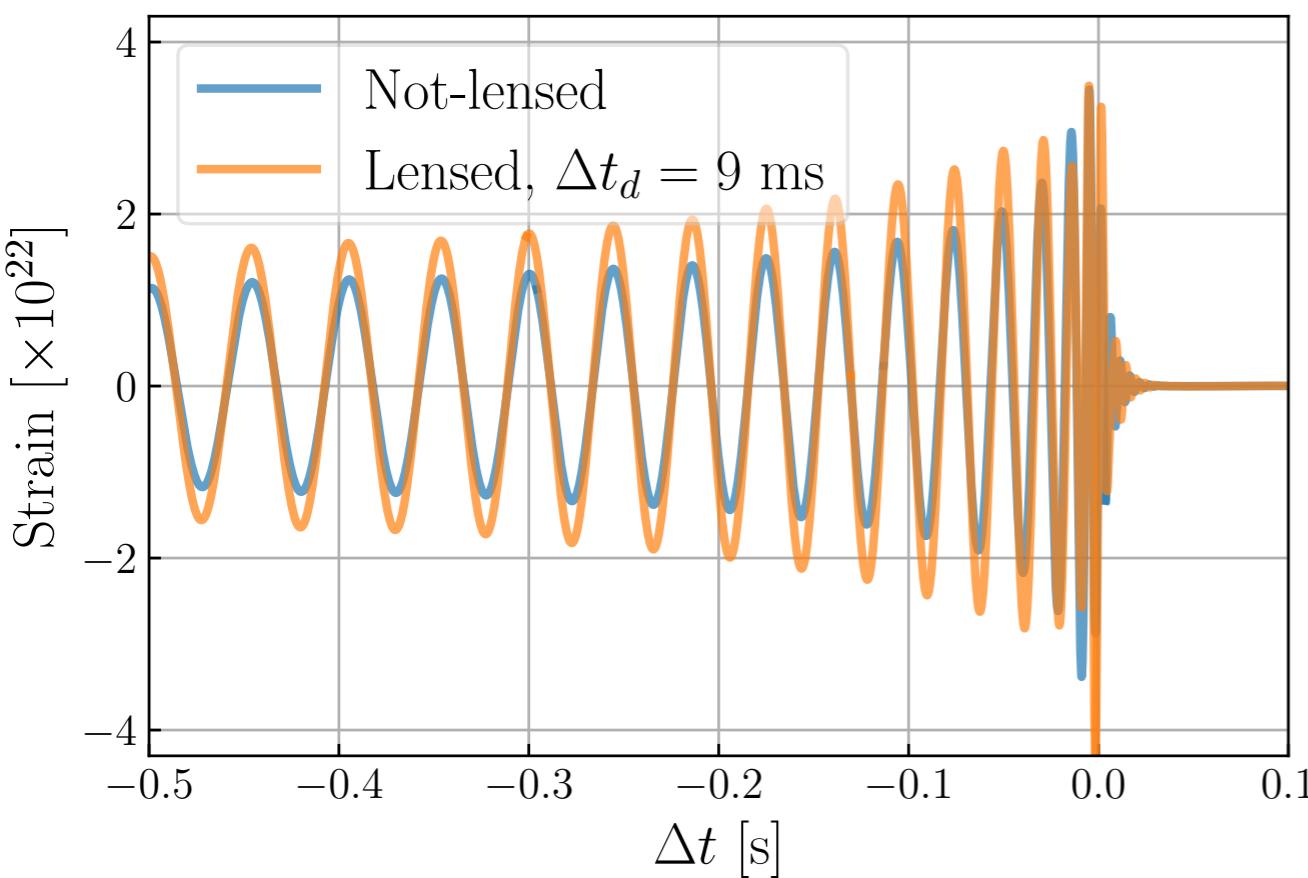


[point mass lens]

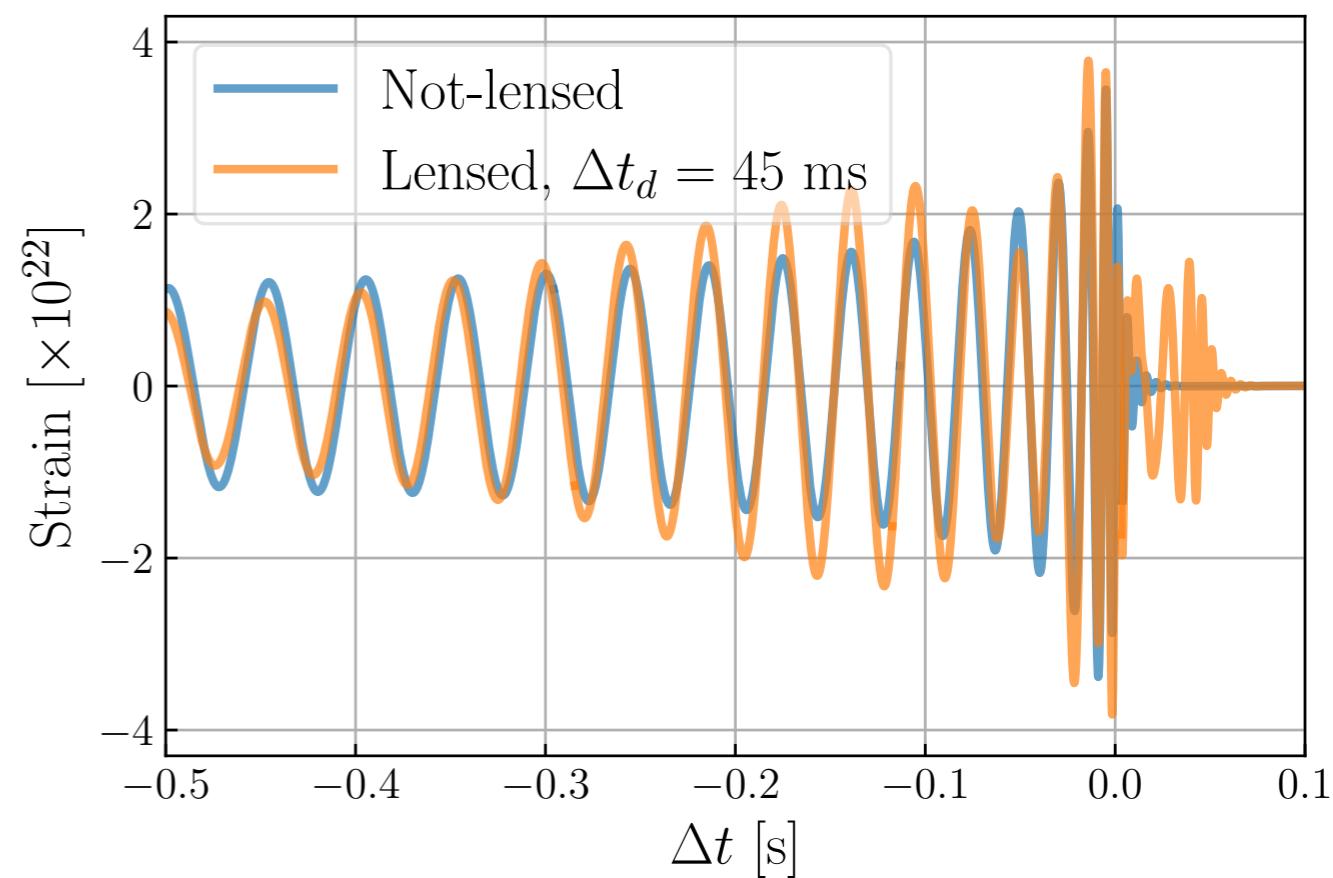
# Wave effects:

$$\Delta t_d(y=1) \simeq 4 \left( \frac{(1+z_L)M_L}{100M_\odot} \right) \text{ ms}$$

Diffraction

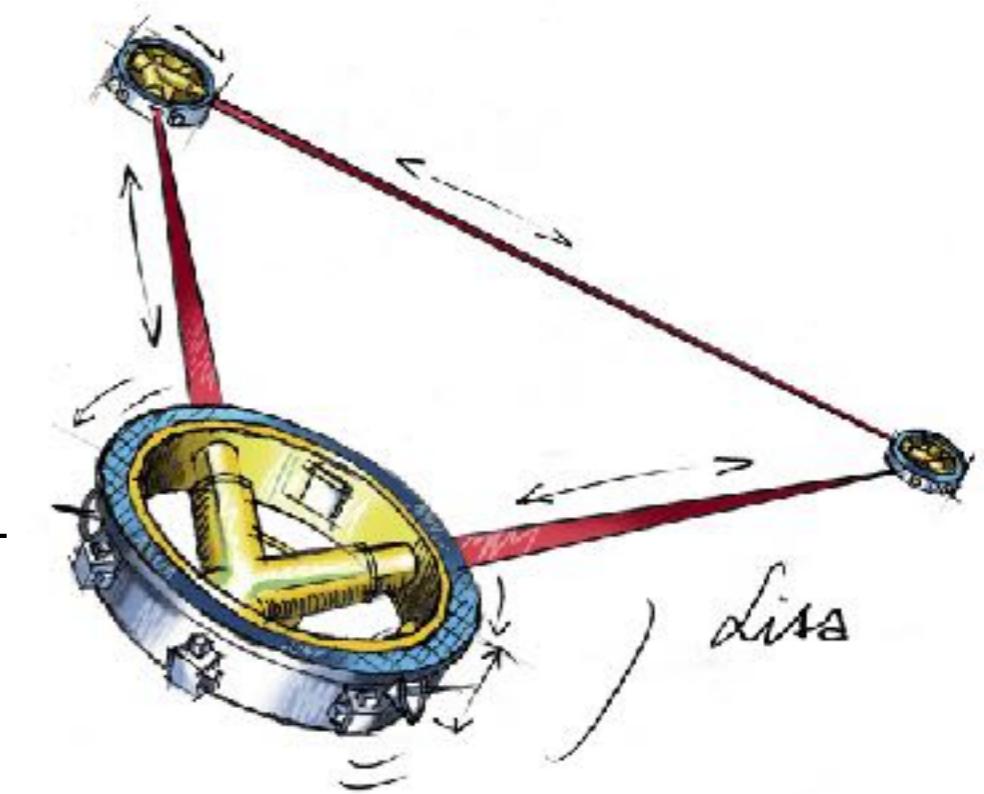
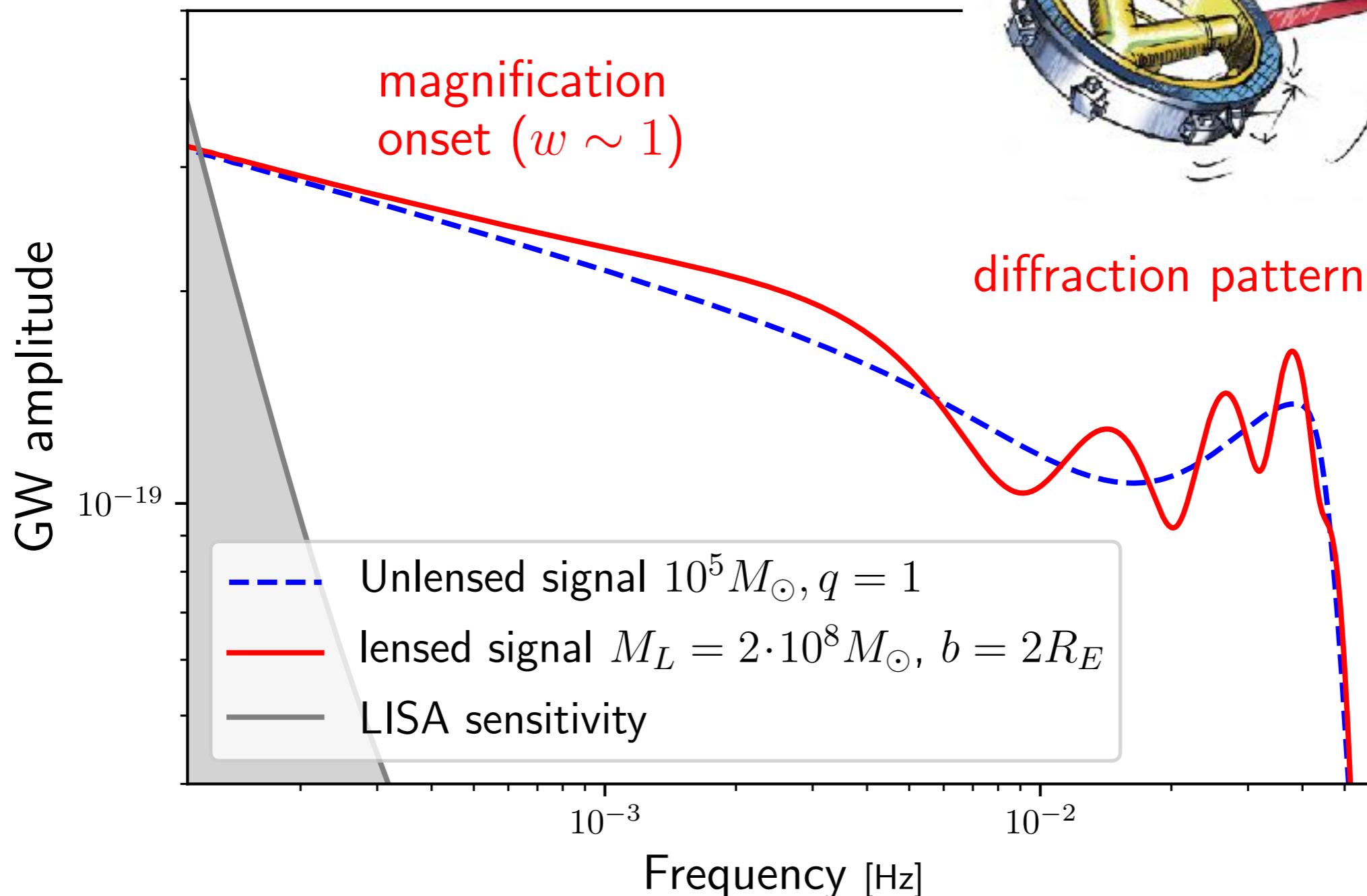


Interference

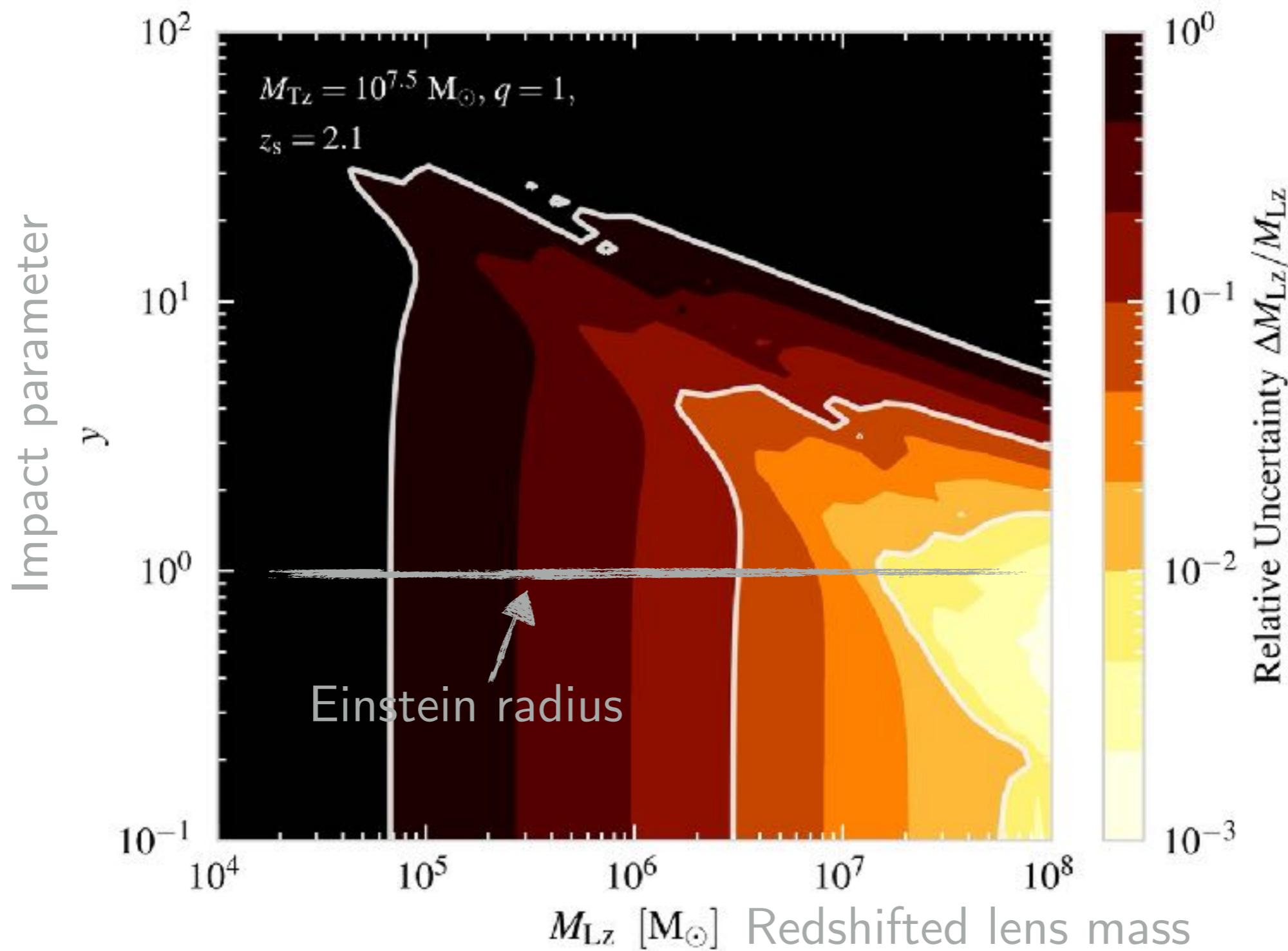


Probing compact objects ([Dai et al.'18](#), [Diego'19](#), [Tambalo et al.'22](#), ...), strong lensing + microlensing ([Seo et al.'21](#), [Mena et al.'22](#), ...), breaking mass-sheet degeneracy ([Cremonese, Ezquiaga, Salzano'21](#)), solving diffraction integral ([Feldbrugge&Turok'20](#), [Tambalo et al.'22](#))

# Wave effects: LISA



# Increased optical depth

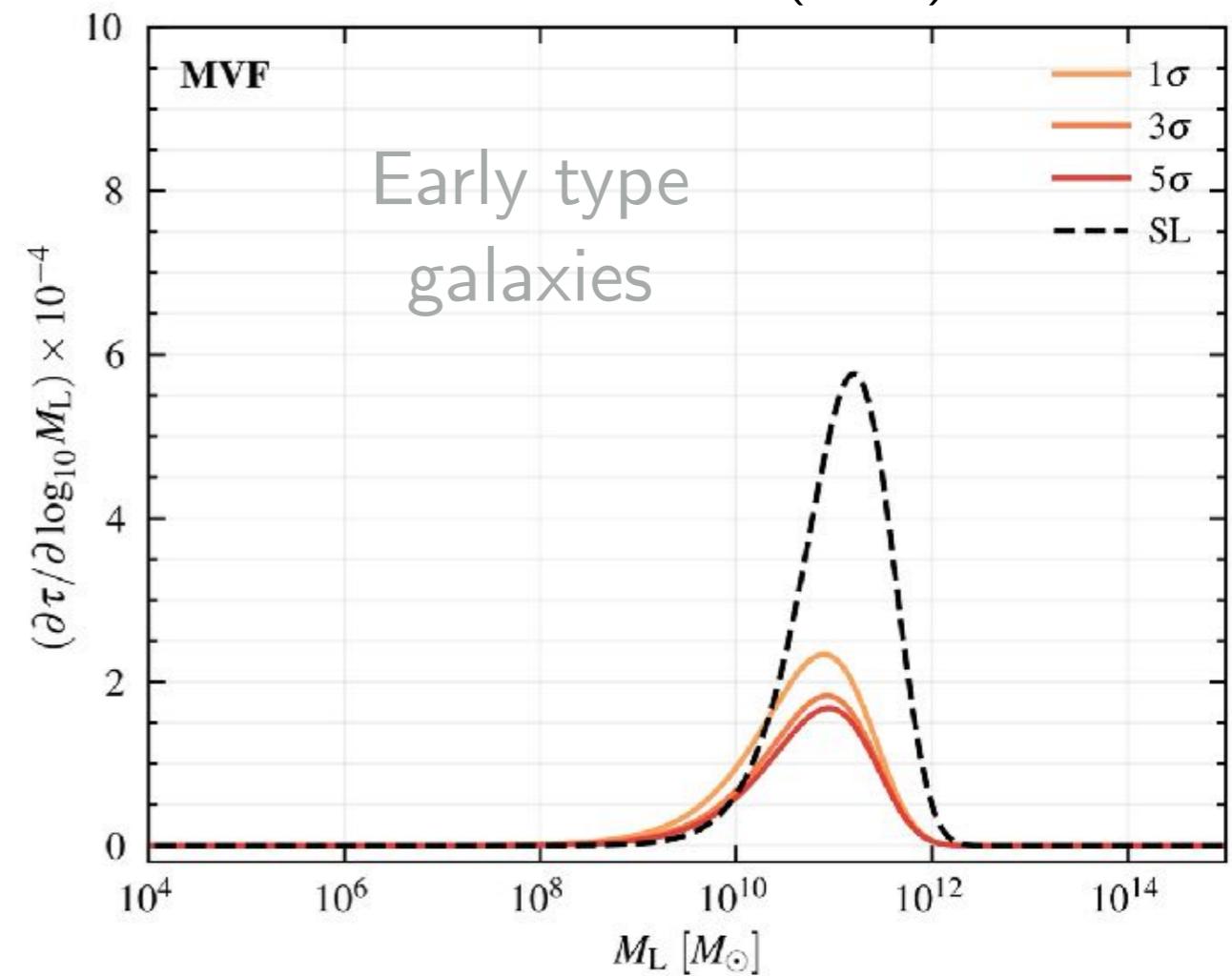
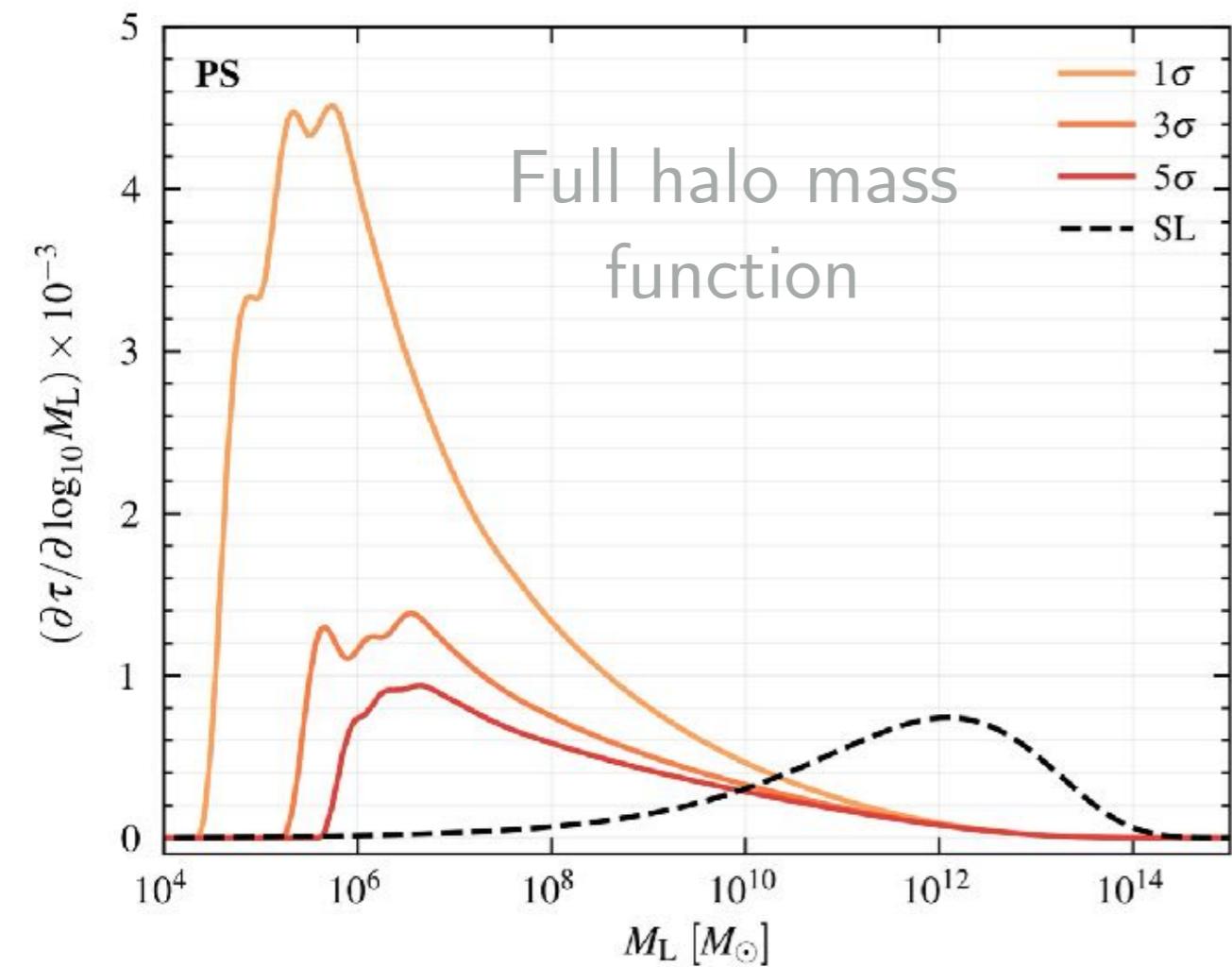


# Probing dark matter sub-halos

- Probability of wave optics lensing:  
few percent and larger than strong lensing

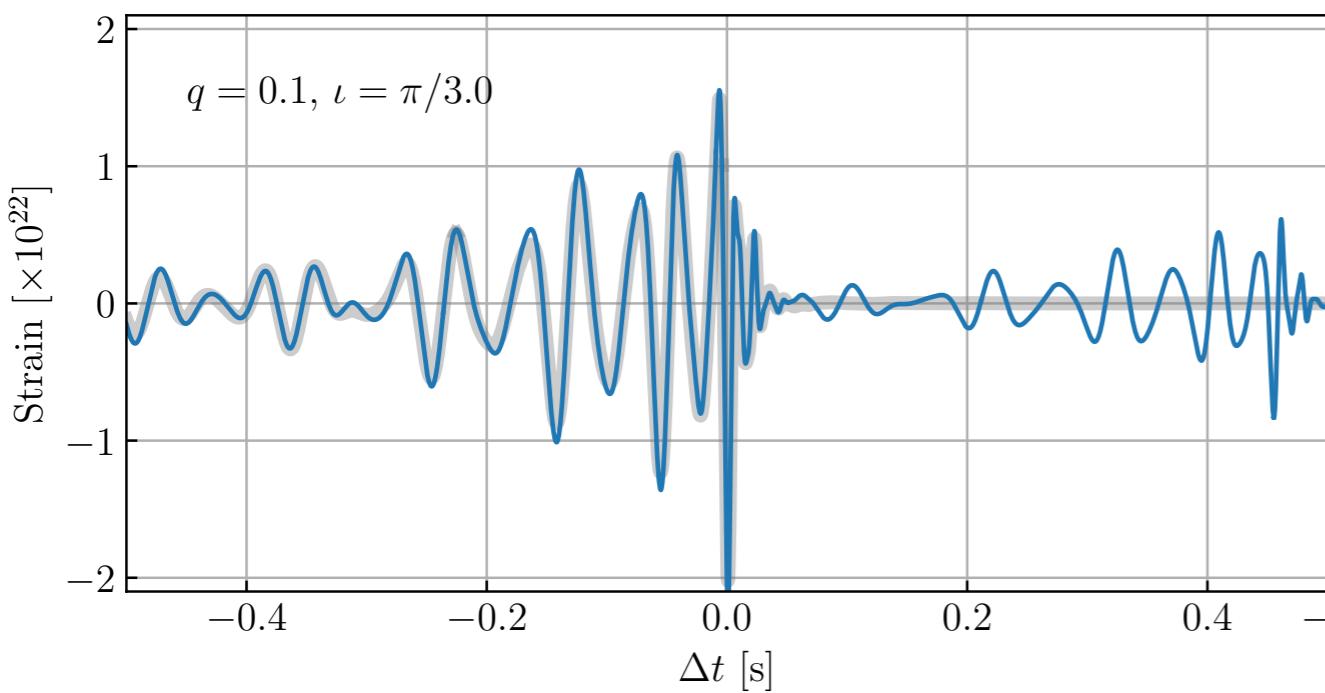
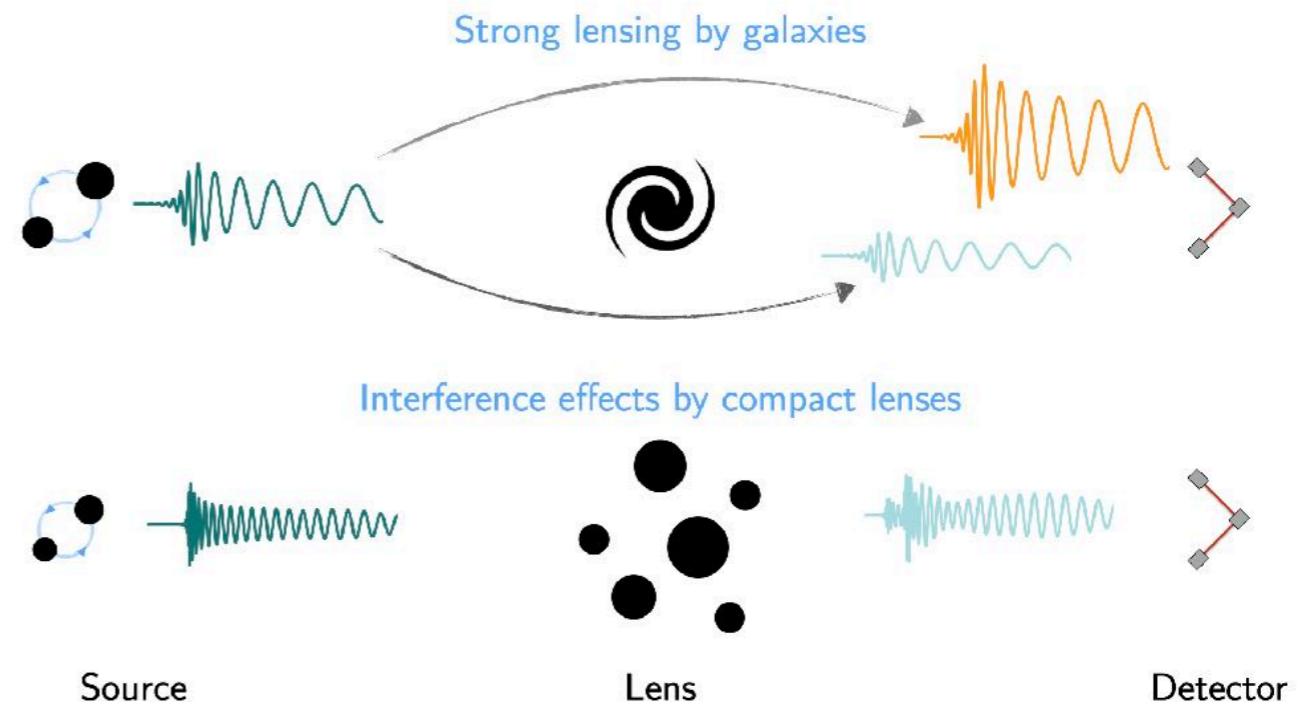
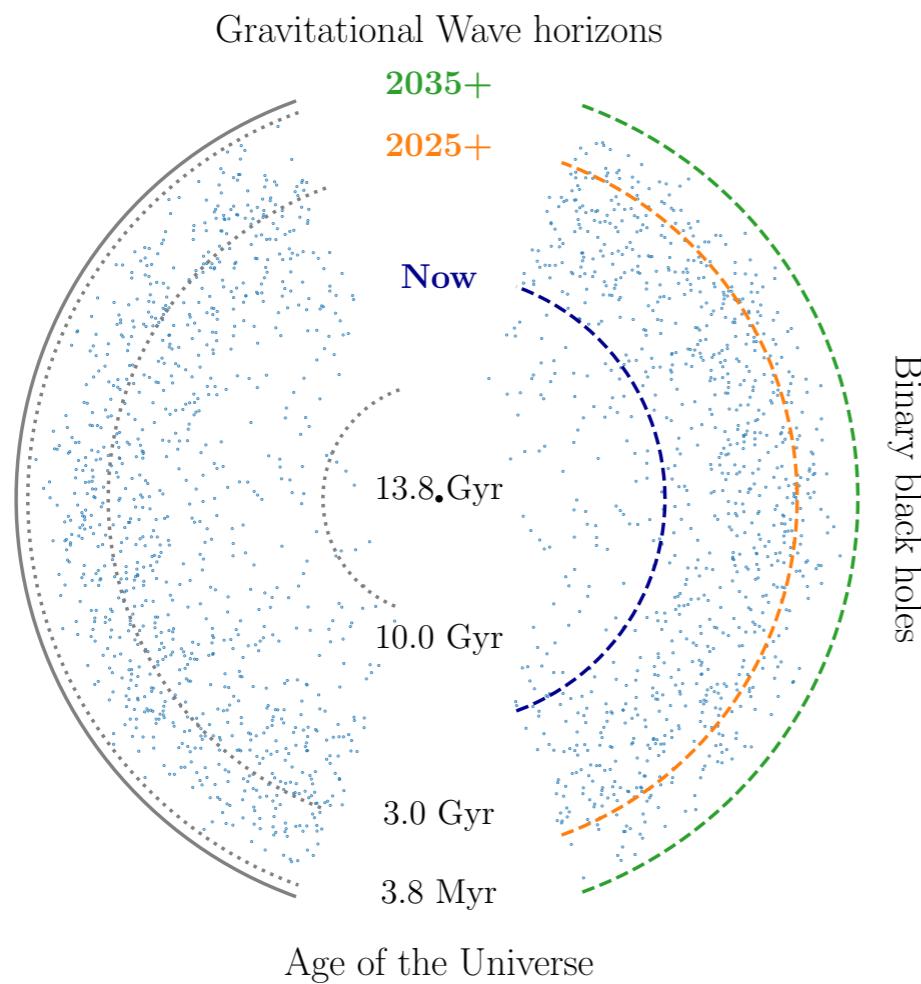


Mesut Çalışkan  
(JHU)

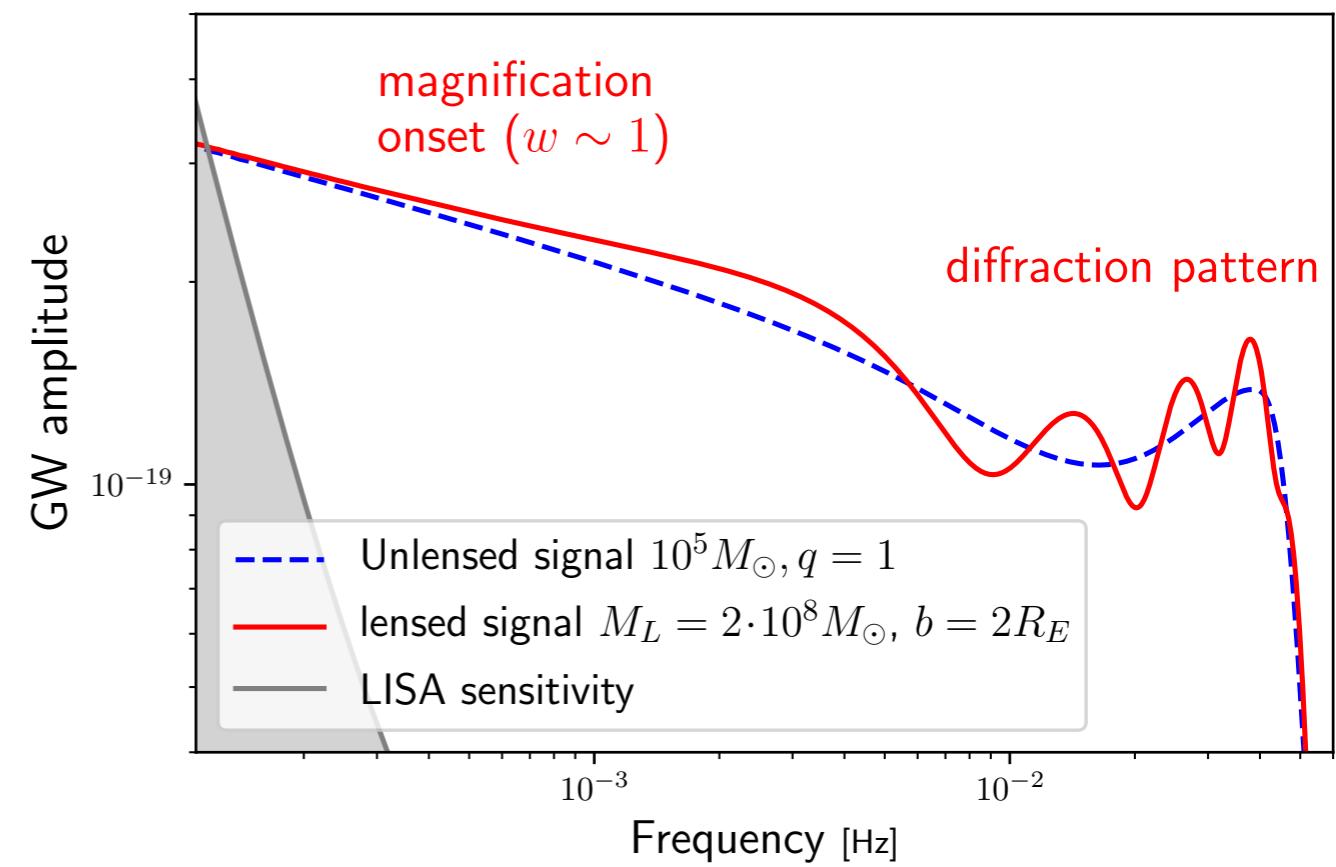


# Synergies

- Cross correlate with strong lensing catalogs (I'd love to know where to find them and how to use them!)
- Targeted follow-ups for promising candidates
- Joint lensing inference to improve cosmology/gravity tests
- Constraints on optical depth combining different probes



**Multiple images, type II distortion,  
expected soon!**



**Wave effects, PBH, sub-halos**



Medfinansieret af Den Europæiske  
Unions Connecting Europe-facilitet

VILLUM FONDEN

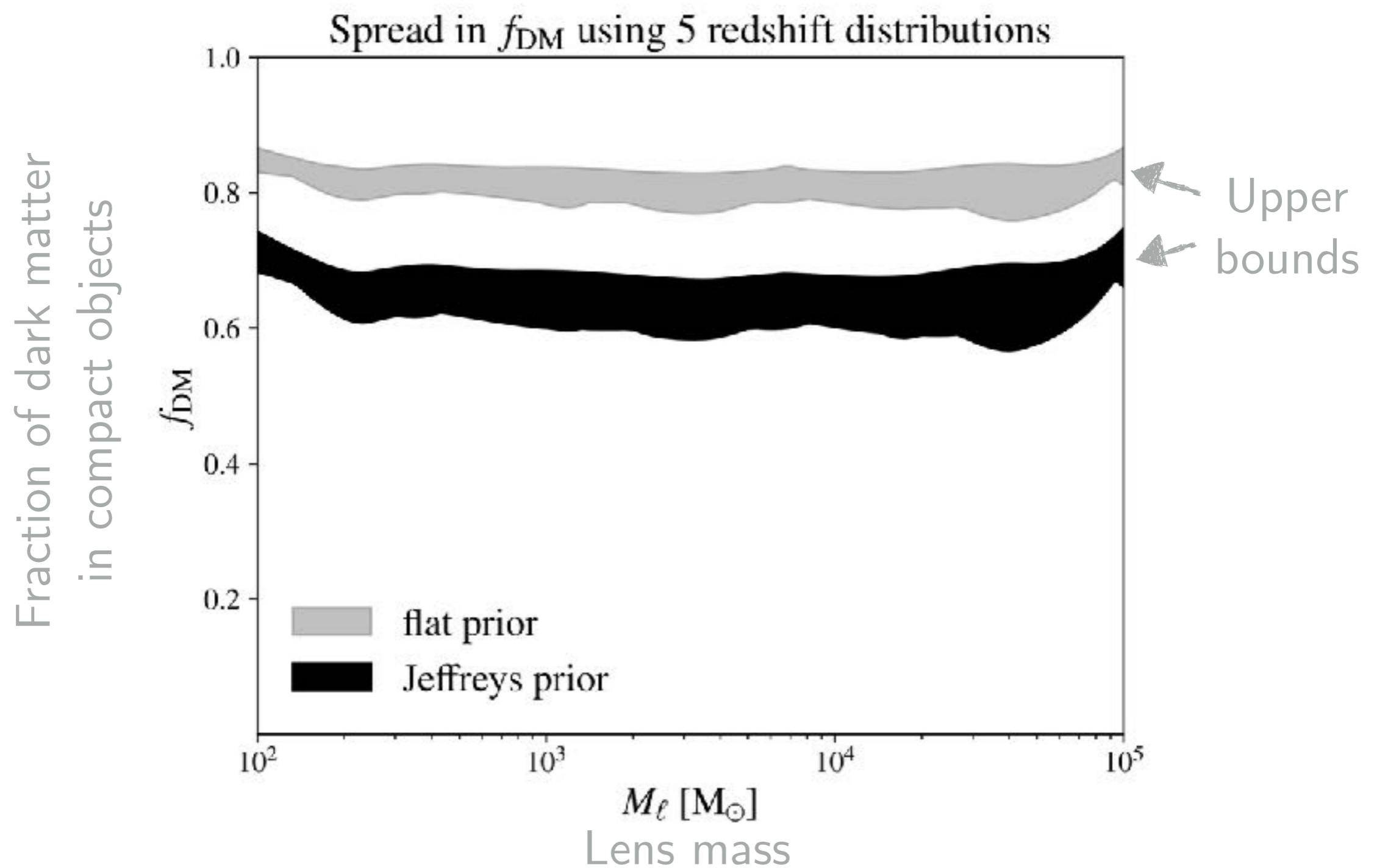


# Join us!

[ezquiaga.github.io/joinus](http://ezquiaga.github.io/joinus)



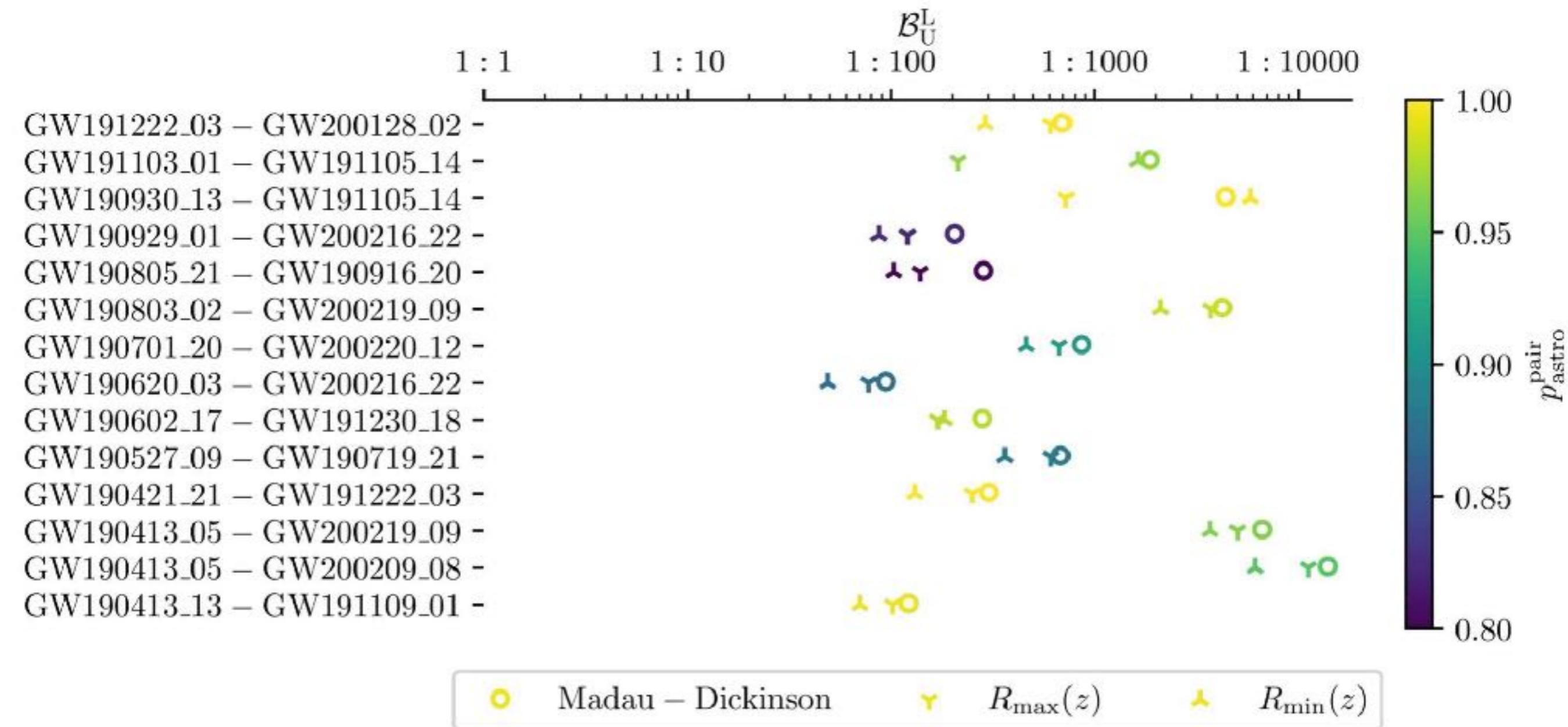
# LVK: no evidence of wave effects so far...



LVC (incl. Ezquiaga); Search GW lensing O3a (ApJ, [arXiv 2105.06384](#), [science summary](#))

LVK (incl. Ezquiaga); Search GW lensing full O3 ([arXiv 2304.08393](#), [science summary](#))

# LVK: no evidence of strong lensing so far...



LVC (incl. Ezquiaga); *Search GW lensing O3a* (ApJ, [arXiv 2105.06384](#), [science summary](#))

LVK (incl. Ezquiaga); *Search GW lensing full O3* ([arXiv 2304.08393](#), [science summary](#))