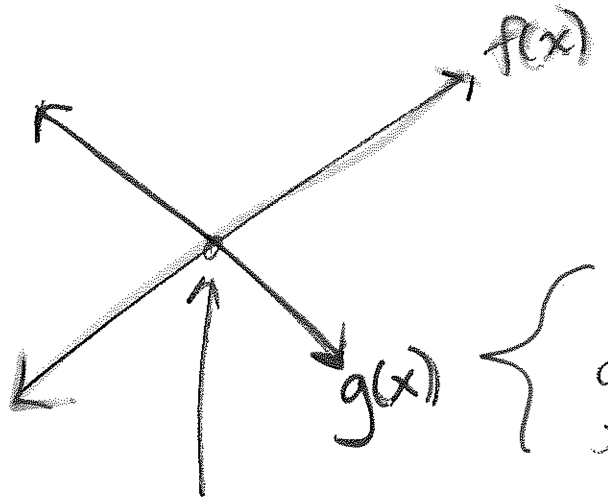
4 – 1 Notes: Solving Systems of Equations by Graphing



the line f(x) represents all solutions to the equation

the line g(x) represents all the solutions to the equation g(x)

this one point is a solution for both!

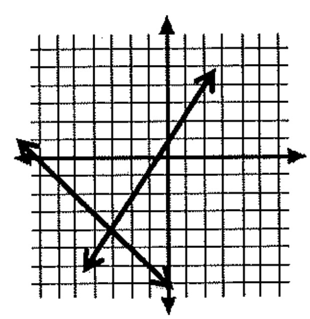
Key Vocabulary

System of Linear Equations:

Solution of a System of Linear Equations:

Example 1) What is the solution to the system shown?

$\boxed{(-3, -4)}$



Example 2: Consider the system. $\begin{cases} y = -2x - 1 \\ x + y = 2 \end{cases}$

a) Is $(-3, 5)$ a solution for the system? How do you know?

plug in for both

$5 = -2(-3) - 1$
$5 = 6 - 1$
$5 = 5$ ✓

$-3 + 5 = 2$
$2 = 2$ ✓

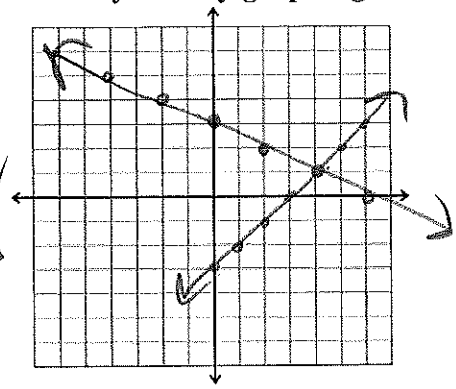
$\boxed{\text{yes}}$

b) Is $(-1, 2)$ a solution for the system? How do you know?

Since (a) works, we already have answer

Examples #3 – 8: Solve each system by graphing.

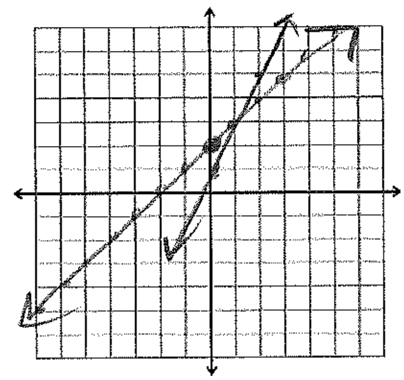
3) $\begin{cases} y = -\frac{1}{2}x + 3 \\ y = x - 3 \end{cases}$



$\boxed{(4,1)}$

4) $\begin{cases} x - y = -2 \\ y = 2x + 1 \end{cases}$

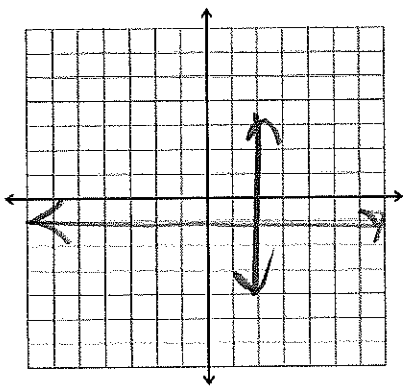
$y = x + 2$
$-y = -x - 2$



$\boxed{(1,3)}$

Check your solution by using substitution with each equation:

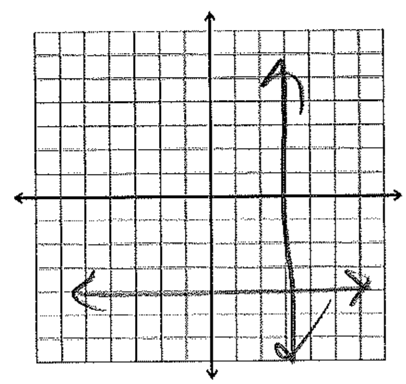
5) $\begin{cases} y = -1 \\ x = 2 \end{cases}$



$\boxed{(2,1)}$

You try #6 – 8!

6) $\begin{cases} x = 3 \\ y = -4 \end{cases}$

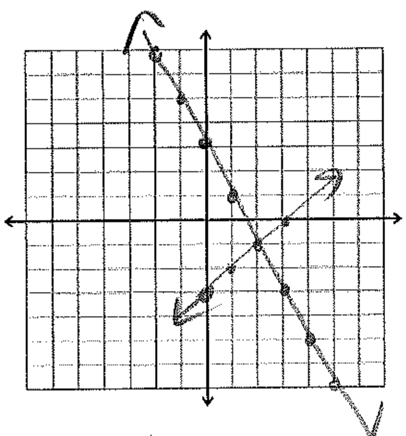


$\boxed{(3,-4)}$

7) $\begin{cases} y = -2x + 3 \\ 3y - 3x = -9 \end{cases}$

$\frac{3y}{3} = \frac{3x}{3} - \frac{9}{3}$

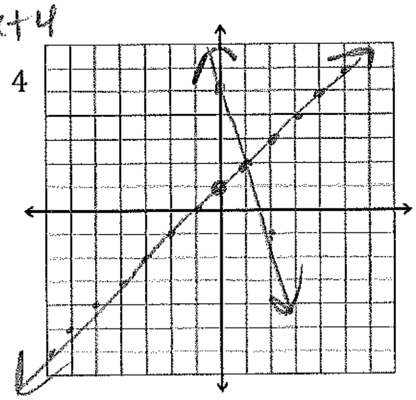
$y = x - 3$



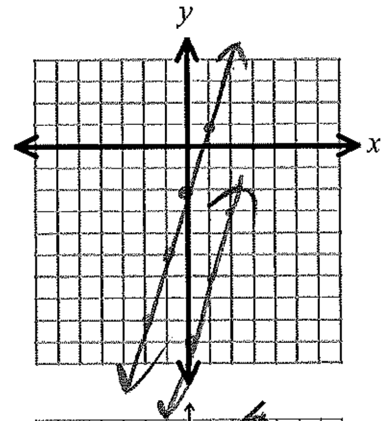
$\boxed{(2,-1)}$

8) $\begin{cases} -4x + 4y = 4 \\ y = 3x + 5 \end{cases}$

$y = x + 1$
$4y = 4x + 4$



$\boxed{(1,2)}$

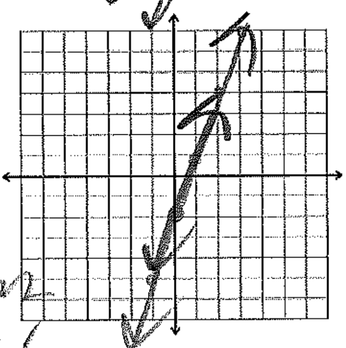


Example 9: Solve the system by graphing: $\begin{cases} y = 3x - 2 \\ 3x - y = 9 \end{cases}$.

What do you think the solution is?

$\boxed{\text{no solution}}$

parallel lines
same slope
don't intersect



Example 10: Solve the system by graphing: $\begin{cases} y = 3x - 2 \\ 5y = 15x - 10 \end{cases}$.

What do you think the solution is?

$\frac{5y}{5} = \frac{15x}{5} - \frac{10}{5}$

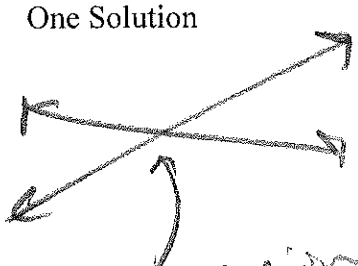
$y = 3x - 2$

same line — overlap
infinitely many solutions

$\boxed{\text{IMS}}$

A system of linear equations can have …

One Solution

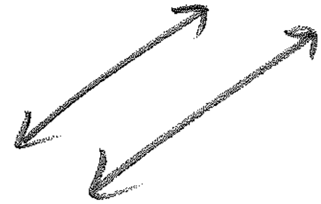


one solution at intersection

Infinitely Many Solutions

same line
overlap

No Solution



parallel lines
same slope

Example 11: A system of two linear equations has infinitely many solutions. What must be true about the equations? Choose all that apply.

a) They are perpendicular.
b) They are the same line.
c) They have the same y-intercept.
d) They are parallel.

Example 12: Which of the following equations will have no solution with $6x + 2y = 8$?

a) $y = -3x + 4$
b) $y = 6x - 5$
c) $y = -3x - 1$
d) $y = 6x + 8$

same slope, different y-intercept

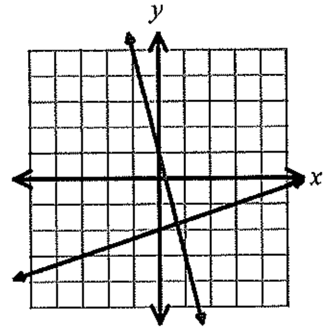
this has same slope but is same line so not parallel

$6x + 2y = 8$

$\frac{2y}{2} = \frac{-6x}{2} + \frac{8}{2}$

$y = -3x + 4$

Example 13: The functions $f(x)$ and $g(x)$ are graphed to the side. Approximate the value of x when $f(x) = g(x)$?



→ about (.7, −1.7)

but only want x so

$\boxed{x \approx .7}$

Reflection: Describe some disadvantages to solving systems by graphing.

– when answers are decimals/fractions

– when #'s are larger than graph

total rate rate

Example 14: A coffee shop sells teas for \$4 each and coffees for \$5 each. If the coffee shop sold 9 drinks for a total of \$40, how many of each type of drink were sold?

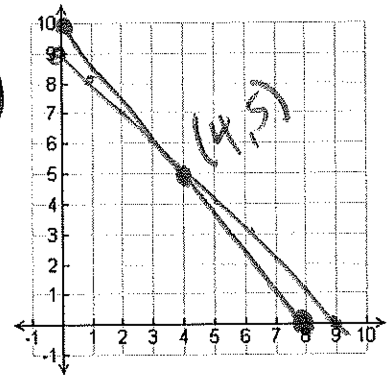
a) Write two equations to model this situation.

$4t + 5c = 40$

$t + 5 = 9$

b) Solve the system by graphing.

tea



coffee

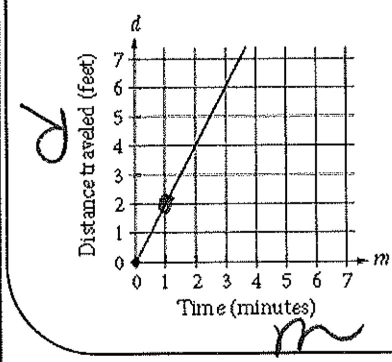
Teas = 5

Coffees = 4

4 – 2 Notes: Solving Systems of Equations by Substitution

Warm-up:

1) The graph shows the distance traveled d, in feet, by a product on a conveyer belt m minutes after the product is placed on the best. Which of the following equations correctly relates d and m?



A) $d = 2m$

B) $d = \frac{1}{2}m$

C) $d = m + 2$

D) $d = 2m + 2$

$d = 2m + 0$

Key Vocabulary

Substitution:

If you know the value of one variable in a system, you can find the value of the other variable by Substituting the known value into one of the equations.

Example 1: Solve the system by using substitution. Check your solution.

$$\begin{cases} 8x + 3y = 25 \\ x = \boxed{2} \end{cases}$$

if we know x is 2, then why not use 2 instead of x???

$8(2) + 3y = 25$

$16 + 3y = 25$

$-16 \qquad -16$

$\Longrightarrow$ $3y = 9$

$y = 3$

$\boxed{(2, 3)}$

Sometimes you don't know a specific numerical value but you do have an equation with one variable isolated. You can substitute the expression into the other equation.

For Examples #2 – 7, solve each system.

2) $\begin{cases} y = \boxed{-2x+4} \\ x + y = -5 \end{cases}$

$x + y = -5$
$x + (-2x+4) = -5$
$x - 2x + 4 = -5$
$-x + 4 = -5$
$-4 \quad -4$
$-x = -9$
$x = 9$

2nd, plug in 9 to x (either equation) to solve for y:
$y = -2x+4$
$y = -2(9)+4$
$y = -18+4$
$y = 22$

$\boxed{(9, 22)}$

3) $\begin{cases} 3x - 2y = 5 \\ y = \boxed{2x-1} \end{cases}$

$3x - 2y = 5$
$3x - 2(2x-1) = 5$
$3x - 4x + 2 = 5$
$-x + 2 = 5$
$-2 \quad -2$
$-x = 3$
$x = -3$

$y = 2x-1$
$y = 2(-3)-1$
$y = 6-1$
$y = -7$
y

$\boxed{(-3, -7)}$

4) $\begin{cases} 6x - 3y = 12 \\ x = \boxed{2y+2} \end{cases}$

$6x - 3y = 12$
$6(2y+2) - 3y = 12$
$12y + 12 - 3y = 12$
$9y + 12 = 12$
$-12 \quad -12$
$9y = 0$
$y = 0$

$x = 2y+2$
$x = 2(0)+2$
$x = 2$

$\boxed{(2, 0)}$

5) $\begin{cases} y = \boxed{x+12} \\ y = -x - 2 \end{cases}$

$x + 12 = -x - 2$
$+x \quad +x$
$2x + 12 = -2$
$-12 \quad -12$
$2x = -14$
$x = -7$

$\boxed{(-7, 5)}$

$y = x + 12$
$y = -7 + 12 = 5$

6) $\begin{cases} y = \boxed{x+1} \\ 2x + 2y = 10 \end{cases}$

$2x + 2y = 10$
$2x + 2(x+1) = 10$
$2x + 2x + 2 = 10$
$4x + 2 = 10$
$-2 \quad -2$
$4x = 8$
$x = 2$

$y = x+1$
$y = 2+1$
$y = 3$

$\boxed{(2,3)}$

7) $\begin{cases} x = y + 12 \\ x = y - 2 \end{cases}$

$y + 12 = y - 2$
$-y \quad -y$
$12 = -2$ false

$\boxed{\text{no solution}}$

Example 8: For the system $\begin{cases} 2x - 3y = 9 \\ 3x = 6y + 3 \end{cases}$, what is the most efficient way to isolate one variable? Solve the system.

$\frac{3x}{3} = \frac{6y}{3} + \frac{3}{3}$

$x = \boxed{2y+1}$

$2x - 3y = 9$
$2(2y+1) - 3y = 9$
$4y + 2 - 3y = 9$
$y + 2 = 9$
$y = 7$

$2x - 3y = 9$
$2x - 3(7) = 9$
$2x - 21 = 9$
$+21 \quad +21$
$2x = 30$
$x = 15$

$\boxed{(15, 7)}$

Example 9: Solve the system: $\begin{cases} 5x - y = 8 \\ y = \boxed{5x - 8} \end{cases}$

$5x - y = 8$
$5x - (5x - 8) = 8$
$\cancel{5x - 5x} + 8 = 8$
$8 = 8$
true

$\boxed{\text{IMS}}$

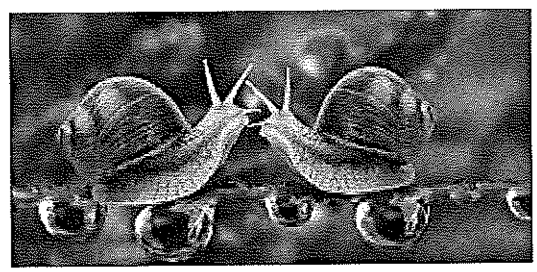
For Examples 10 – 12: Write a system of equations to model each situation, and then solve.

10) Lindsey and Gretchen work at two different hair salons and pay different amounts for their station. Lindsey pays $140 for rent, and $10 per customer that she works on that month. Gretchen only pays $100 for rent, but has to pay $18 per customer. How many customers would it take for them to pay the same amount?

Lindsey $y = 10x + 140$
Gretchen $y = 18x + 100$

$10x + 140 = 18x + 100$
$-18x \quad -18x$
$-8x + 140 = 100$
$-140 \quad -140$
$-8x = -40$
$\boxed{x = 5 \text{ customers}}$

11) Two snails are moving along a branch. (They have a very exciting life!) Snail #1 starts at a position of 15 cm from the start of the branch and moves at 3 cm/min. Snail #2 starts at a position of 9 cm from the start of the branch and moves at 4 cm/min. After how many minutes will they be at the same position? What is their position at that time?



Snail #1 $y = 3x + 15$
snail #2 $y = 4x + 9$

$3x + 15 = 4x + 9$
$-4x \quad -4x$
$-x + 15 = 9$
$-15 \quad -15$
$-x = -6$
$x = 6$

$y = 4x + 9$
$y = 4(6) + 9$
$y = 24 + 9$
$y = 33$

$\boxed{\text{6 minutes, 33 cm}}$

$x = 1 + 2y$

12) Two numbers have a sum of 16. The larger number is one more than two times the smaller number. Find each number.

$x + y = 16$
$x = 1 + 2y$

$1 + 2y + y = 16$
$1 + 3y = 16$
$-1 \quad -1$
$3y = 15$
$y = 5$

$x + y = 16$
$x + 5 = 16$
$x = 11$

$\boxed{11 \ \& \ 5}$

4 – 3 Notes, Day 1: Solving Systems by Using Elimination

Warm-up:

1)

$6x - 5 = 7 + 2x$

$$3x + x + x + x - 3 - 2 = 7 + x + x$$

In the equation above, what is the value of x ?

A) $-\frac{5}{7}$

B) 1

C) $\frac{12}{7}$

D) 3

$6x - 5 = 7 + 2x$
$-2x \quad -2x$
$4x - 5 = 7$
$+5 \quad +5$
$4x = 12$
$x = 3$

2)

$$g(x) = \boxed{2x - 1}$$
$$h(x) = 1 - g(x)$$

The functions g and h are defined above. What is the value of $h(0)$?

A) -2

B) 0

C) 1

D) 2

$h(x) = 1 - (2x - 1)$
$h(x) = 1 - 2x + 1$
$h(x) = -2x + 2$
$h(0) = -2(0) + 2 = 2$

Key Vocabulary

Elimination

The elimination method eliminates (gets rid of) one of the variables when you __+__ the two equations together. Then you will have an equation that you can __solve__.

Steps in solving an equation by elimination:
1) If needed, put both equations in ____________________ form.
2) Find a variable that will be easiest to eliminate.
3) Add vertically.
4) Solve for remaining variable.
5) Substitute that value into an equation to solve for other variable.

Example 1: Solve by elimination.

$$3x + \cancel{2y} = 4$$
$$+\ 5x - \cancel{2y} = 12$$

$8x = 16$

$x = 2$

$\boxed{(2, -1)}$

substitute to get y

$3x + 2y = 4$

$3(2) + 2y = 4$

$6 + 2y = 4$

$-6 \qquad -6$

$2y = -2$

$y = -1$

For #2 – 5: Solve each system by the elimination method.

2) $\begin{cases} x + \cancel{7y} = 13 \\ x - \cancel{7y} = 5 \end{cases}$

$2x = 18$

$x = 9$

$\boxed{(9, \tfrac{4}{7})}$

$x + 7y = 13$

$9 + 7y = 13$

$-9 \qquad -9$

$\cancel{7}y = \frac{4}{7}$

3) $\begin{cases} 3x + 4y = 4 \\ -4y = 16 + 2x \end{cases}$

need to put both into standard form

$3x + \cancel{4y} = 4$

$-2x - \cancel{4y} = 16$

$x = 20$

$\boxed{(20, -14)}$

$3(20) + 4y = 4$

$60 + 4y = 4$

$-60 \qquad -60$

$\frac{4y}{4} = \frac{-56}{4} = -14$

You Try!

4) $\begin{cases} 4x - \cancel{2y} = -2 \\ 3x + \cancel{2y} = -12 \end{cases}$

$7x = -14$

$x = -2$

$\boxed{(-2, -3)}$

$4(-2) - 2y = -2$

$-8 - 2y = -2$

$+8 \qquad +8$

$-2y = 6$

$y = -3$

5) $\begin{cases} 4x - 2y = 7 \\ 2y = 8 + 4x \end{cases}$ $\Rightarrow$

$\cancel{4x} - \cancel{2y} = 7$

$-\cancel{4x} + \cancel{2y} = 8$

$0 + 0 = 15$

false

$\boxed{\text{no solution}}$

6) In the system, $\begin{cases} 3x + 2y = 13 \\ 5x + 2y = 15 \end{cases}$, can you add the two equations to solve for one of the variables? If so, solve the system. If not, what could you do to make a system you could solve?

$3x + 2y = 13$

$5x + 2y = 15$

$8 + 4y = 28$

never do this!

It's garbage

Sometimes the original system does not have opposite __terms__. You can change any equation by __multiplying__ it by a negative one (or any other number) to make opposite __terms__.

For #7 – 10, solve each system.

7) $5x - 3y = 19$
$-(5x + 4y = 5)$

want to be $-5x$, so I'll multiply all by $-$

$5x - 3y = 19$
$-5x - 4y = -5$

$-7y = 14$
$y = -2$

$5x - 3(-2) = 19$
$5x + 6 = 19$
$\quad -6 \quad -6$
$\frac{5x}{5} = \frac{13}{5}$

$\boxed{\left(\frac{13}{5}, -2\right)}$

8) $x - 3y = 7$
$3y = -23 - x$

$\Rightarrow$

$x - \cancel{3y} = 7$
$x + \cancel{3y} = -23$

$2x = -16$
$x = -8$

$-8 - 3y = 7$
$+8 \quad\quad +8$
$\frac{-3y}{-3} = \frac{15}{-3}$
$y = -5$

$\boxed{(-8, -5)}$

9) $x + y = 3$
$-(x + y = 5)$

$\cancel{x} + \cancel{y} = 3$
$-\cancel{x} - \cancel{y} = -5$

$0 + 0 = -2$

$\boxed{\text{no solution}}$

10) $(0.25x - 0.05y = 1)\,100$
$(0.25x + 0.1y = 2.5)\,100$

$25x - 5y = 100$
$-(25x + 10y = 250)$

$\cancel{25x} - 5y = 100$
$-\cancel{25x} - 10y = -250$

$\frac{-15y}{-15} = \frac{-150}{-15}$

$y = 10$

$\boxed{(6, 10)}$

$25x - 5(10) = 100$
$25x - 50 = 100$
$\frac{25x}{25} = \frac{150}{25}$
$x = 6$

11) The Spanish club sells food at sporting events. At the football game they charge \$3 for the popcorn and \$1 for the sodas. They made \$75 at the football game. At the track meet they sold the popcorn for \$2 and the sodas for \$1. They made \$55 at the track meet. How many bags of popcorn and sodas did they sell, if they sold the same at both games?

football: $3p + 1s = 75$
track: $2p + 1s = 55$

$\Rightarrow$ $3p + \cancel{s} = 75$
$-2p - \cancel{s} = -55$
$p = 20$

$3(20) + s = 75$
$60 + s = 75$
$-60 \quad -60$
$s = 15$

$\boxed{\text{20 popcorns, 15 sodas}}$

Reflect: Explain how you know whether to use elimination versus substitution to solve a system of equations?

$Ax + By = C$
$Ax + By = C$
elimination

$x = \boxed{\quad}$
$Ax + By = C$
Substitution

Example 12: Determine which method of solving is easiest for each system. Write "Elimination" or "Substitution." **Do not solve the systems.**

a) $\begin{cases} 4x - 3y = 9 \\ 7x + 3y = 2 \end{cases}$ E

b) $\begin{cases} y = 6x - 3 \\ x = 2y \end{cases}$ S

Example 13: Solve the system below by using substitution. Then solve it again by using elimination. Which method do you prefer? Why?

$\begin{cases} 2x = 12 + 2y \\ 2x + 2y = 48 \end{cases}$ either

$2x - 2y = 12$
$2x + 2y = 48$
elimin

$x = \boxed{6 + y}$
$2x + 2y = 48$
Substitution

$\begin{cases} 2x = 12 + 2y \\ 2x + 2y = 48 \end{cases}$ either

4 – 3 Notes, Day 2: More Solving Systems with Elimination

Warm-up:

1) $(a + 2b + 3c) - (4a + 6b - 5c)$ is equivalent to:

A. $-4a - 8b - 2c$
B. $-4a - 4b + 8c$
C. $-3a + 8b - 2c$
D. $-3a - 4b - 2c$
E. $-3a - 4b + 8c$

2) As part of a lesson on motion, students observed a cart rolling at a constant rate along a straight line. As shown in the chart below, they recorded the distance, y feet, of the cart from a reference point at 1-second intervals from $t = 0$ seconds to $t = 5$ seconds.

t	0	1	2	3	4	5
y	14	19	24	29	34	39

Which of the following equations represents this data?

F. $y = t + 14$
G. $y = 5t + 9$
H. $y = 5t + 14$
J. $y = 14t + 5$
K. $y = 19t$

Key Vocabulary

Elimination with Multiplication

Often the equations in a system aren't easy to graph, or aren't easy to use the substitution method and won't eliminate a variable if you just add the equations. We can use ________________________ with one or both equations so that one variable will eliminate.

Example 1: $\begin{cases} 3x - 4y = 10 \\ x + 2y = 0 \end{cases}$

$2(x + 2y = 0)$

want to be +4y so I'll mult whole equation by 2

$3x - \cancel{4y} = 10$
$2x + \cancel{4y} = 0$

$5x = 10$
$x = 2$

$\boxed{(2, -1)}$

$2 + 2y = 0$
$-2 \quad -2$
$2y = -2$
$y = -1$

Steps:
1) If needed, write each equation in standard form.
2) Decide which variable you would like to eliminate.
3) Multiply one or both equations by a constant in order to create opposite coefficients for the x and/or y terms.
4) Add vertically.
5) Solve for one variable.
6) Use substitution and solve for the other variable.

Solve the systems:

2) $\begin{cases} 2x = y + 10 \\ x + 2y = 5 \end{cases}$

$2x - y = 10$
$-2(x + 2y = 5)$

$2x - y = 10$
$-\cancel{2x} - 4y = -10$

$-5y = 0$
$y = 0$

$\boxed{(5, 0)}$

$2x = 0 + 10$
$2x = 10$
$x = 5$

3) $\begin{cases} 2x - 7y = 20 \\ 5x + 8y = -1 \end{cases}$ (-5) (2)

$-\cancel{10x} + 35y = -100$
$\cancel{10x} + 16y = -2$

$51y = -102$
$y = -2$

$\boxed{(3, -2)}$

$2x - 7(-2) = 20$
$2x + 14 = 20$
$-14 \quad -14$
$2x = 6$
$x = 3$

4) $\begin{cases} 2x - 3y = 5 \\ 5x - 6y = 12 \end{cases}$ (-2)

$-4x + \cancel{6y} = -10$
$5x - \cancel{6y} = 12$
$x = 2$

$\boxed{(2, -1/3)}$

$2(2) - 3y = 5$
$4 - 3y = 5$
$-4 \quad -4$
$-3y = 1$
$y = -\frac{1}{3}$

5) $\begin{cases} -6x + 2y = 12 \\ -3x + y = 6 \end{cases}$ (-2)

$-6x + 2y = 12$
$6x - 2y = 12$

$0 + 0 = 0$

$\boxed{\text{IMS}}$

Solve the systems:

6) $\begin{cases} 3x = 4y - 5 \\ -6x + 8y = 2 \end{cases}$

$(3x - 4y = -5)(2)$
$-6x + 8y = 2$

$6x - 8y = -10$
$-6x + 8y = 2$
$0 + 0 = -8$

$\boxed{\text{no solution}}$

7) $\begin{cases} x = \boxed{-y + 4} \\ .10x + .05y = 0.35 \end{cases}$ (10)

$10x + 5y = 35$

$10(-y+4) + 5y = 35$
$-10y + 40 + 5y = 35$
$-5y + 40 = 35$
$-40 \quad -40$
$-5y = -5$
$y = 1$

$x = -1 + 4 = 3$

$\boxed{(3, 1)}$

Reflect: Describe the difference between systems with no solution and those with infinitely many solutions? How can you tell which is which?

$0 + 0 = 0$ — IMS

$0 + 0 = \#$ — no sol.

Example 8: Josie owns a nail shop that charges $12 for a manicure and $20 for a pedicure. Her cousin owns a shop and charges $16 for a manicure and $30 for a pedicure. On Monday they compared how much they made. Josie made $520 and her cousin made $760. If they both sold the same amount of pedicures and manicures, how many pedicures and manicures did they each sell?

Josie $(12m + 20p = 520)(-3)$
cousin $(16m + 30p = 760)(2)$

$-36m - \cancel{60p} = -1560$
$32m + \cancel{60p} = 1520$
$-4m = -40$
$m = 10$

$12(10) + 20p = 520$
$120 + 20p = 520$
$-120 \quad -120$
$20p = 400$
$p = 20$

$\boxed{\begin{array}{l}\text{mani } \$10 \\ \text{pedi } \$20\end{array}}$

Example 9: A store sells guitars and basses. In one day, a total of 5 instruments were sold. If guitars sell for \$200 each and basses sell for \$150 each, and the total cost was \$900, how many of each type were sold?

$200g + 150b = 900$

$-200(g + b = 5)$

$\boxed{\text{2 basses, 3 guitars}}$

$200\cancel{g} + 150b = 900$

$-200\cancel{g} - 200b = -1000$

$-50b = -100$

$b = 2$

Example 10: Susan is buying black and green olives from the olive bar for her party. She buys 4 lb of olives. Black olives cost \$3.00 a pound. Green olives cost \$5.00 a pound. She spends \$15.50. How many pounds of each type of olive does she buy?

$3b + 5g = 15.50$

$-3(b + g = 4)$

$\boxed{\text{1.75 lbs green, 2.25 lbs black}}$

$3\cancel{b} + 5g = 15.50$

$-3\cancel{b} - 3g = -12$

$2g = 3.50$

$g = 1.75$

Example 11: Which equation would make this system have an infinite number of solutions? Choose all that apply. $\begin{cases} y = x + 2 \\ \_\_\_\_\_\_ \end{cases}$

same line

A) $2y = 4x + 8$

$\frac{2y}{2} = \frac{4x}{2} + \frac{8}{2}$

$y = 2x + 4$

B) $y - x = 3$

$x = x + 3$

C) $5x - 5y = 10$

$\frac{-5y}{-5} = \frac{-5x}{-5} + \frac{10}{-5}$

$y = x - 2$

D) $-4x + 4y = 12$

$\frac{4y}{4} = \frac{4x}{4} + \frac{12}{4}$

$y = x + 3$

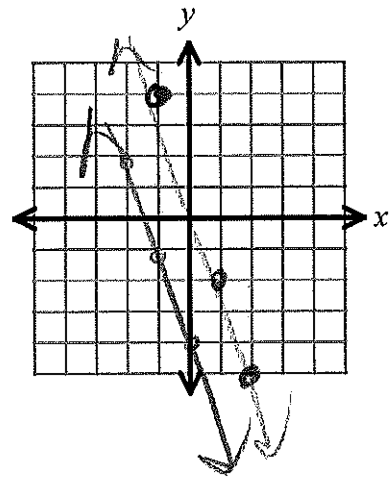
$\boxed{\text{none of these?}}$

Example 12: What is the solution to the system below, where the first line is written as an equation, and the second line is given as a set of ordered pairs?

<table>
<tr><th>Line 1</th><th>Line 2</th></tr>
<tr><td>$y = -3x - 4$</td><td><table><tr><th>x</th><th>y</th></tr><tr><td>-1</td><td>4</td></tr><tr><td>1</td><td>-2</td></tr><tr><td>2</td><td>-5</td></tr></table></td></tr>
</table>

graph

graph (plot points)



$\boxed{\text{no solution}}$

4 – 4 Notes: Graphing Linear Inequalities

Warm-up:

1) The equations below are linear equations of a system where $a, b,$ and c are positive integers.

$$ay + bx = c$$
$$ay + bx = c$$

} same line, IMS

Which statement below is true at least once for such a system of equations? Choose all that apply.

~~A) The two lines are parallel.~~
B) The system represents a single line.
~~C) The two lines intersect at one point.~~
~~D) There is no solution.~~
E) There are infinitely many solutions.
~~F) There is one solution.~~

2) Which statement below shows the solution of the inequality for y? $x - y < 3$

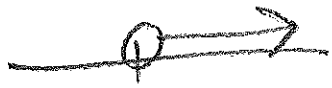
A) $y < x - 3$
B) $y > x + 3$
C) $y < -x - 3$
D) $y > x - 3$

$-x \quad\quad -x$

$\frac{-y}{-1} < \frac{-x}{-1} + \frac{3}{-1}$

$y > x - 3$

$x > 5$



5

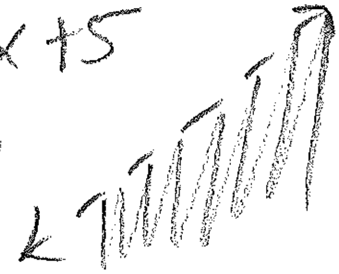
open circle since >

$x \le 18$

18

$y \le x + 5$

dotted



shade below

$y \ge x - 10$

solid line since $\ge$

shade above

Key Vocabulary

Linear Inequality

Example 1: Which ordered pair is **not** a solution of $x - 3y \le 6$? Choose all that apply.

A) (0,0)

$x - 3y \le 6$?

$0 - 3(0) \le 6$

$0 \le 6$ $\boxed{\text{yes}}$

B) (6,-1)

$6 - 3(-1) \le 6$?

$6 + 3 \le 6$?

$\boxed{\text{no}}$

C) $\left(4, -\frac{5}{3}\right)$

$4 - 3\left(-\frac{5}{3}\right) \le 6$?

$4 + 5 \le 6$?

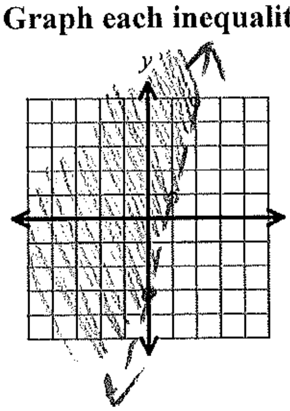
$\boxed{\text{no}}$

Graphing a linear inequality in two variables:

For examples 2 – 9: Graph each inequality.

2) $y > 4x - 3$

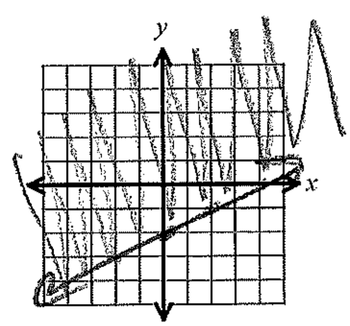
dotted



3) $x - 2y \le 4$

$\frac{-2y}{-2} \le \frac{-x + 4}{-2}$

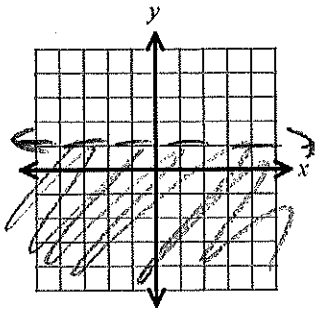
$y \ge \frac{1}{2}x - 2$



4) $5y < -5$

$\frac{5y}{5} < \frac{-5}{5}$

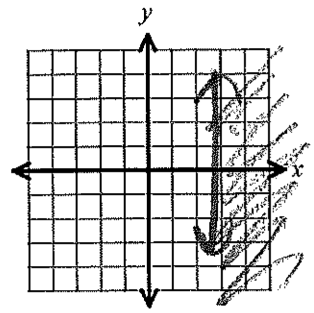
$y < -1$



5) $-3x \leq -9$

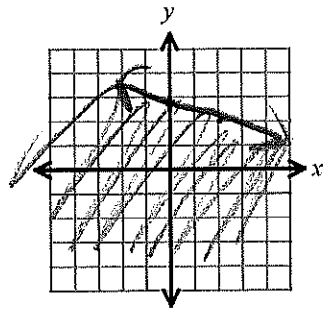
$\frac{-3x}{-3} \geq \frac{-9}{-3}$

$x \geq 3$



You try!

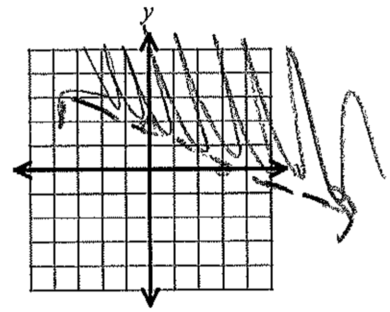
6) $y \leq -\frac{1}{2}x + 3$



7) $2x + 3y > 6$

$\frac{3y}{3} > \frac{-2x}{3} + \frac{6}{3}$

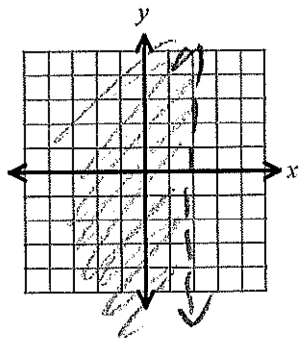
$y > -\frac{2}{3}x + 2$



8) $2x < 4$

$\frac{2x}{2} < \frac{4}{2}$

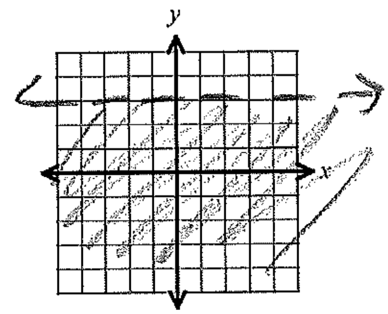
$x < 2$



9) $-4y > 12$

$\frac{-4y}{-4} < \frac{12}{-4}$

$y < -3$



For examples 10 – 13: Match each linear inequality to its graph below.

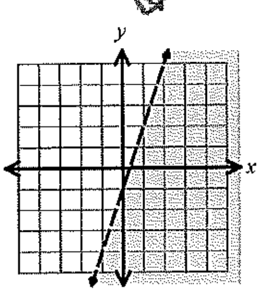
10) $y \geq 3x - 1$ 11) $y < 3x - 1$ 12) $-y > 3x - 1$ 13) $-3(y - 1) \geq 9x$

$y < -3x + 1$

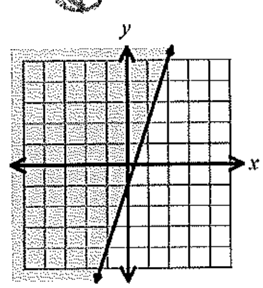
$-3y + 3 \geq 9x$

$\frac{-3y}{-3} \geq \frac{9x}{-3} - \frac{3}{-3}$

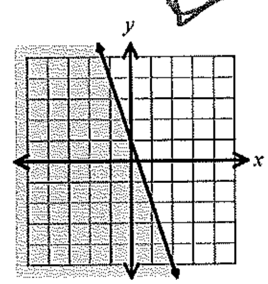
A)



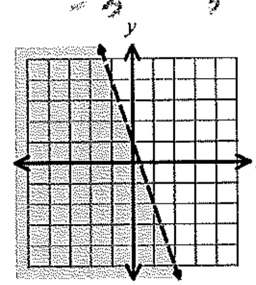
B)



C)



D)



$y \leq -3x + 1$

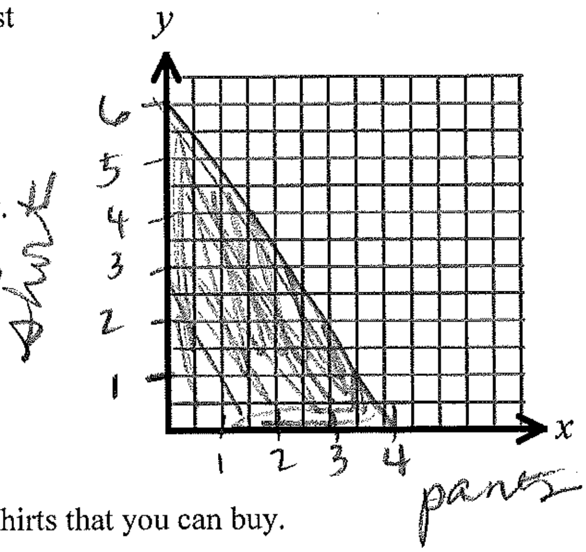
Example 14: You are going clothes shopping and can spend at most \$120 on clothes. It costs \$30 for a pair of pants and \$20 for a shirt. Let x represent the number of pants you can buy. Let y represent the number of shirts you can buy.

a.) Write and graph an inequality that describes the different number of pants and shirts you can buy.

$30p + 20s \leq 120$

$p = 4$

$s = 6$



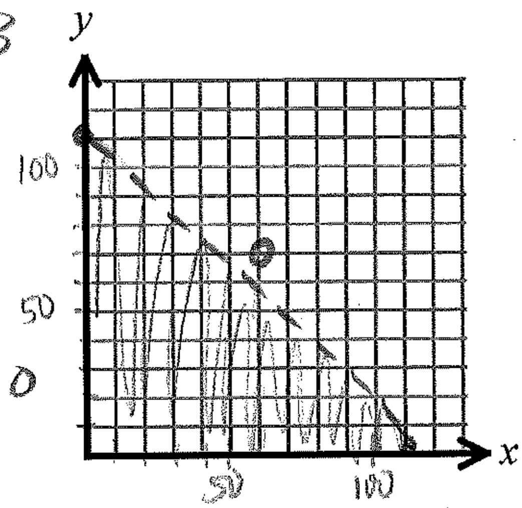
b.) Give three possible combinations of pants and shirts that you can buy.

1 pants 1 shirt   0 and 0   2 and 3

Examples 15: A financial advisor suggests that if a person is an aggressive investor, the percent y of money that the person invests in stocks should be greater than the difference of 110 and the person's age x.

a) Write and graph a linear inequality that relates the percent of money invested in stocks to an aggressive investors age.

$y > 110 - x \Rightarrow y > -x + 110$



b) If an aggressive investor is 60 years old, should the investor put 70% of her investments into stocks? Explain your answer.

no

4 – 5 Notes: Systems of Linear Inequalities

Warm-up:

1) Which of the following expressions has an even integer value for all integers a and c?

   a) $8a + 2ac$
   b) $3a + 3c$
   c) $2a + c$
   d) $a + 2c$
   e) $ac + a^2$

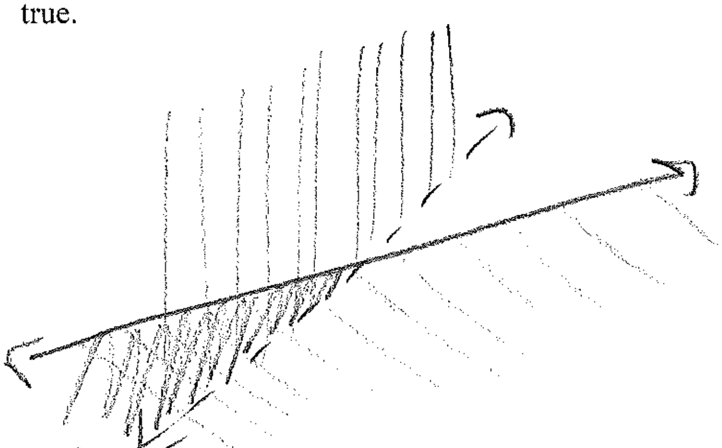
2) For what value of a would the following system of equations have an infinite number of solutions?

$$-3\,(2x - y = 8) \Rightarrow -6x + 3y$$
$$6x - 3y = 4a$$

A. 2
B. 6
C. 8
D. 24
E. 32

Key Vocabulary

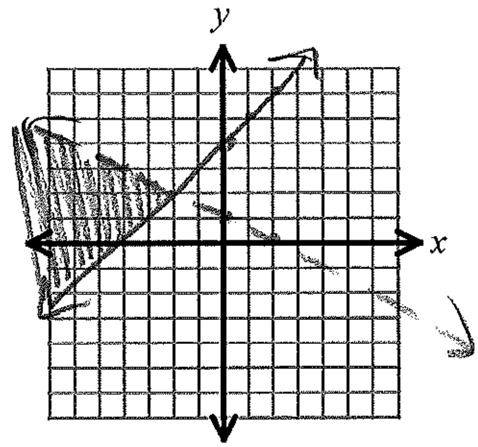
System of Linear Inequalities: A system of linear inequalities consists of two or more linear inequalities that have the same variables. The Solution of a system of linear inequalities are all the ordered pair that make all the inequalities in the system true.



this area is the solution ~~set~~ because it's where they are both shaded

Example 1) Solve the system of inequalities by graphing.

$\begin{cases} y > -\frac{1}{2}x + 1 \\ y \leq x + 4 \end{cases}$ shade below / " above



Reflect: Which of the following ordered pairs are solutions to the previous system? How can you tell?

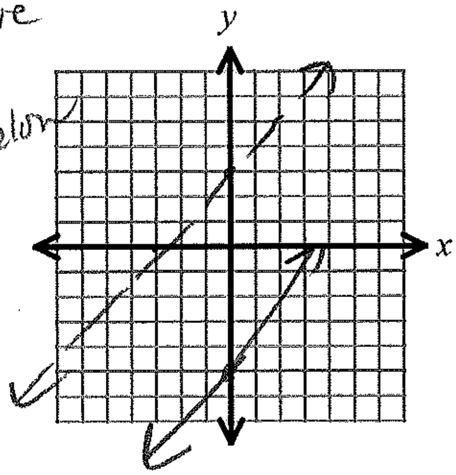
(0, 0)	(2, 3)	(-4, 2)	(-2, 4)	(2, 0)	(5, 0)	(0, 4)
no	no		no	no	no	no

For Examples 2 – 5: Graph each system of linear inequalities

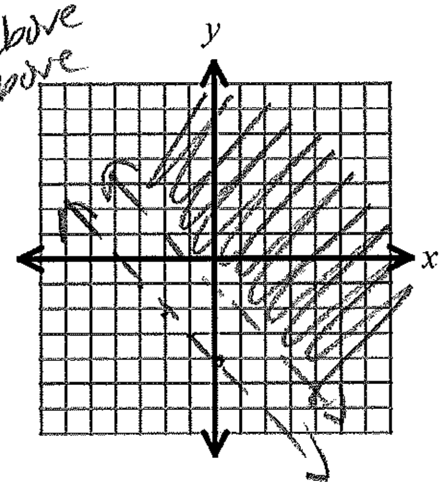
2) $\begin{cases} y > x + 3 \\ -x + y \leq -5 \end{cases}$ above

$y \leq x - 5$ below

no sol



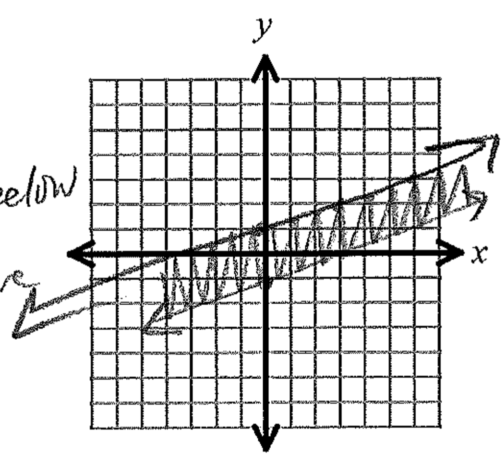
3) $\begin{cases} y > -x - 4 \\ y > -x - 1 \end{cases}$ above / above



4) $\begin{cases} -\frac{1}{3}x + y \leq 1 \\ \frac{3y}{3} \geq \frac{x}{3} - \frac{3}{3} \end{cases}$

$y \leq \frac{1}{3}x + 1$ below

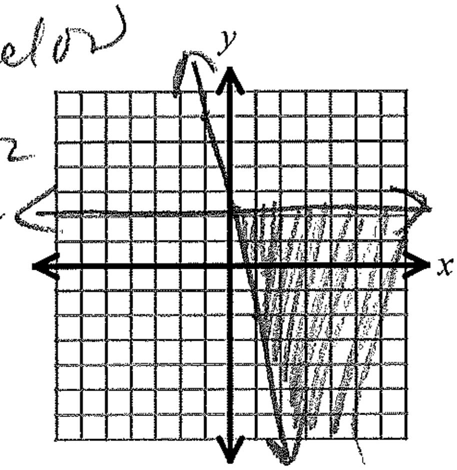
$y \geq \frac{1}{3}x - 1$ above



5) $\begin{cases} y \geq -2 \\ 4x + y \geq 2 \end{cases}$ below

$y \geq -4x + 2$

above

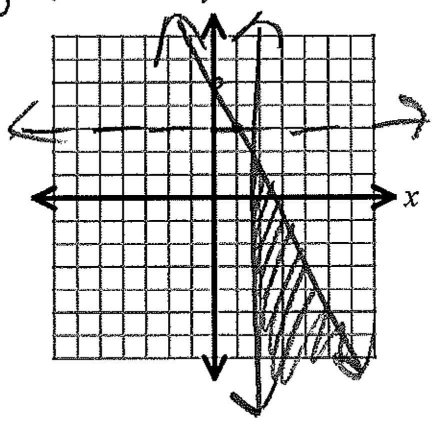


For #6 – 7, graph the system of inequalities.

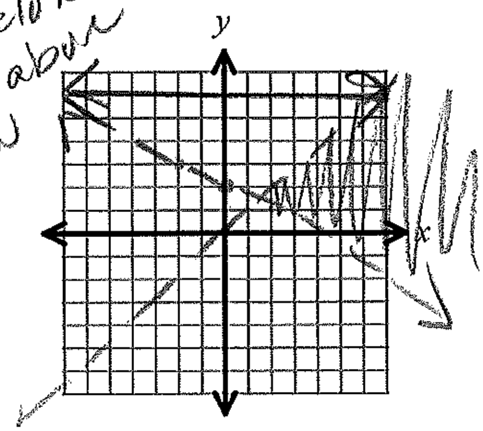
6) $\begin{cases} x \geq -2 \\ 6x + 3y \leq 15 \\ y < 3 \end{cases}$ right

$\frac{3y}{3} \leq \frac{-6x+15}{3}$

$y \leq -2x + 5$ below



7) $\begin{cases} y \leq 6 \\ y > -\frac{1}{2}x + 2 \\ y < x \end{cases}$ below, above, below



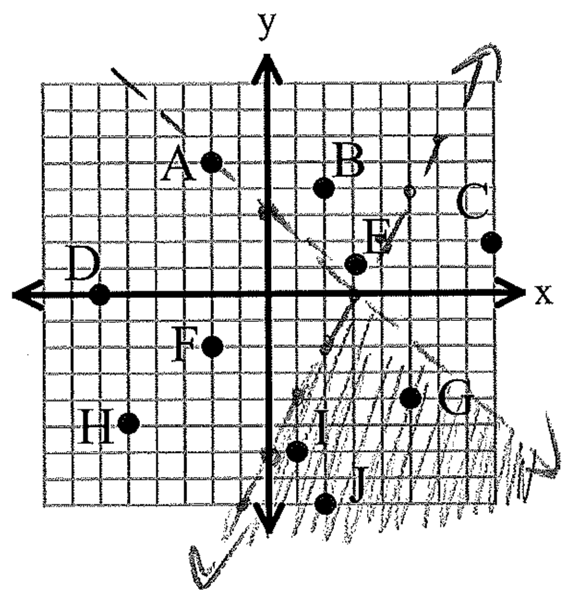
Example 8: Given the system of inequalities shown below, determine all the points that are solutions to this system of inequalities.

$\begin{cases} x + y < 3 \\ 2x - y > 6 \end{cases}$ $y < -x + 3$ below

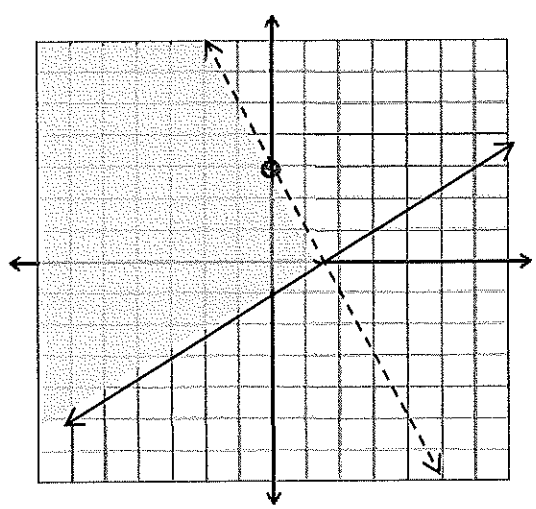
$\frac{-y}{-1} > \frac{-2x}{-1} + \frac{6}{-1}$

$y < 2x - 6$ below

$\boxed{G\ I\ J}$



Example 9: The solution set of a system of inequalities is shown in the graph as a shaded region. The equations of the boundaries are $y = \frac{2}{3}x - 1$ and $-2x - y = -3$. Write a system of linear inequalities that could represent the solution.



$y = \frac{2}{3}x - 1$ above, solid

$-2x - y = -3$ below, dotted

$$\boxed{\begin{cases} y \geq \frac{2}{3}x - 1 \\ -2x - y > -3 \end{cases}}$$

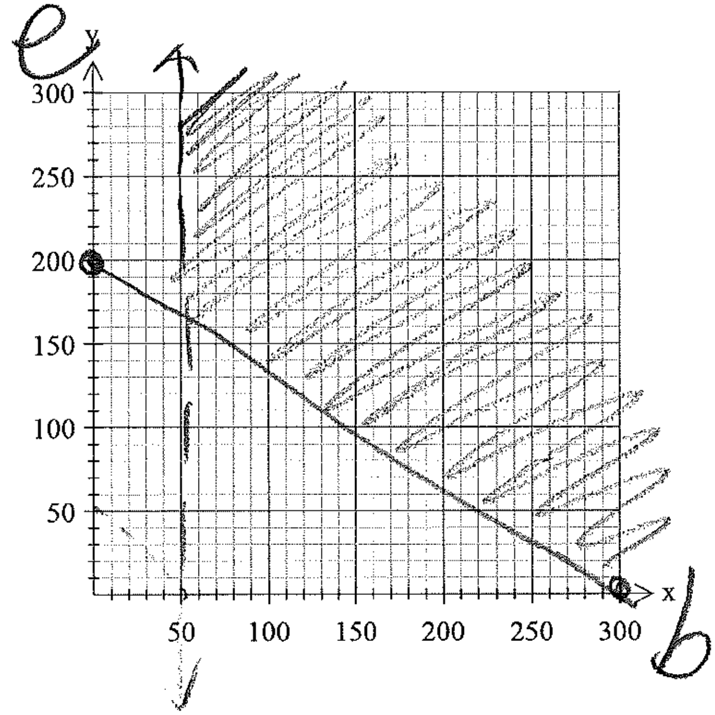
Example 10: Penelope is selling bracelets and earrings to make money for summer vacation. The bracelets cost \$2 and earrings cost \$3. She needs to make at least \$600. Penelope knows that she will sell more than 50 bracelets. Use x for # of bracelets and y for # of earrings.

a) Write a system of inequalities to represent this situation.

$2b + 3e \geq 600$

$b > 50$

$b > 0 \quad e > 0$



b) Graph the two inequalities and shade the intersection.

$b = 300$

$e = 200$

c) Name three combinations of bracelets and earrings that Penelope could see.

(200, 200)

above above

Example 11) If the system of inequalities $y \geq 2x + 1$ and $y > \frac{1}{2}x - 1$ is graphed, which quadrant contains no solution to the system?

A) Quadrant II
B) Quadrant IV
C) Quadrant III
D) There are solutions in all four quadrants.