4.2.1 Uniaxial stress

In uniaxial stress state, only the axial stress is non-zero while all other stress components are zero.

Under axial loading condition, the isotropic linear elastic material law gives

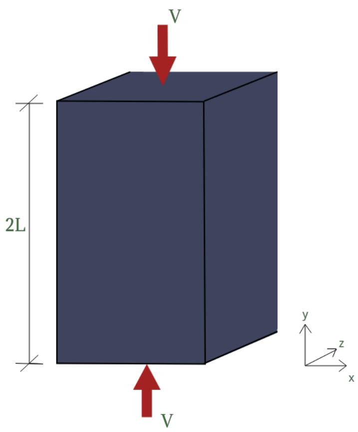
$$\begin{aligned} \epsilon_{axial} &= \frac{1}{E}(\sigma_{axial} - 2\nu\ \sigma_{lateral}) \\ \epsilon_{lateral} &= \frac{1}{E}[(1-\nu)\sigma_{lateral} - \nu\sigma_{axial}] \end{aligned} \tag{4.1}$$

For uniaxial stress, i.e. $\sigma_{lateral} = 0$, Eq. 4.1 gives

$$\begin{aligned} \sigma_{axial} &= E\ \epsilon_{axial} \\ \epsilon_{lateral} &= -\nu\ \epsilon_{axial} \end{aligned} \tag{4.2}$$

Analytical solution

Consider the axial loading condition.



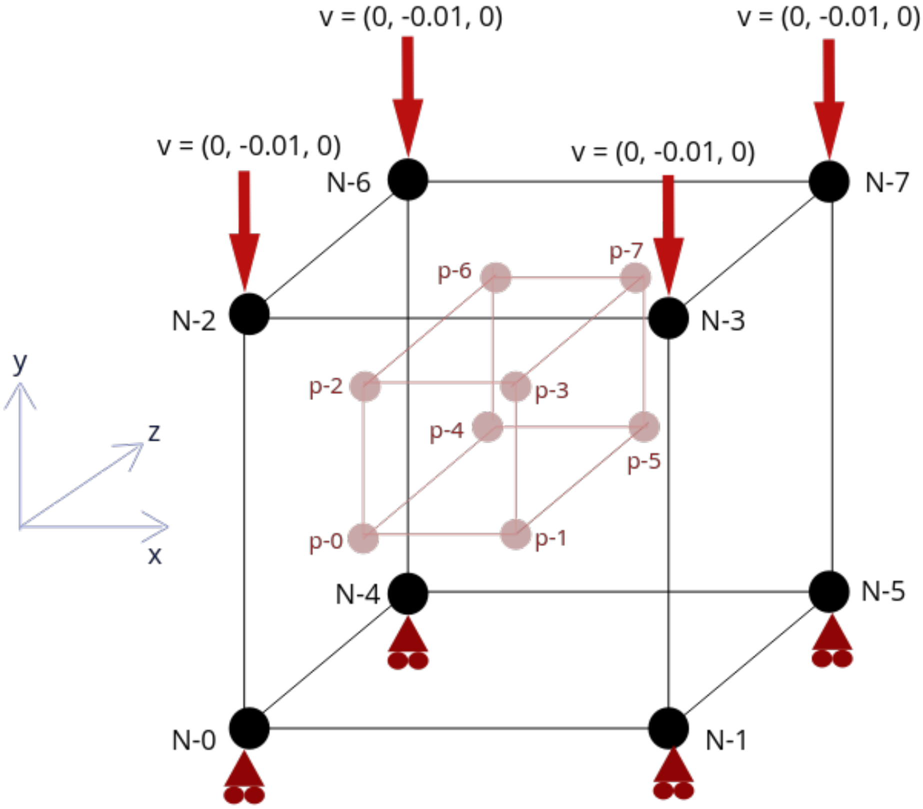
Axial strain and stress at a given time, t can be computed as

$$\begin{aligned} \epsilon_{yy} &= V \times \frac{(t-t_0)}{L} \\ \sigma_{yy} &= E\ \epsilon_{yy} = E \times V \times \frac{(t-t_0)}{L} \end{aligned} \tag{4.3}$$

Lateral strains are

$$\epsilon_{xx} = \epsilon_{zz} = -\nu \times \epsilon_{yy} \tag{4.4}$$

MPM analysis


v = (0, -0.01, 0)
v = (0, -0.01, 0)
v = (0, -0.01, 0)
v = (0, -0.01, 0)
N-6
N-7
N-2
N-3
N-4
N-5
N-0
N-1
p-0
p-1
p-2
p-3
p-4
p-5
p-6
p-7
y
z
x

Mesh	
x-spacing	1m
y-spacing	1m
z-spacing	1m
Particles	
x-spacing	0.5m
y-spacing	0.5m
z-spacing	0.5m
Time	
total analysis time	0.1s
dt	1×10^{-5} s
Material	
material model	ILE
density	1×10^{-6} kg/m^{-3}
E	1000 N/m^{-2}
ν	0.2
gravity	0.0

Solution at 0.1 s	Analytical solution	MPM solution
ϵ_{yy}	-0.001	-0.001
ϵ_{xx}, ϵ_{zz}	0.0002	0.000199892
σ_{yy}	-1.00 N/m^{-2}	-1.00006 N/m^{-2}
σ_{xx}, σ_{zz}	0.00 N/m^{-2}	-0.000149 N/m^{-2}