



Hidden Markov Models

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Examples

Warning about next homework(s)

- Kaggle competition
- Thus, late days not very useful
- Following homework is not computational

Garden Pathing

What is the probability of the sequence “a/Det blue/Adj boat/N”?

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$$\pi_d \beta_{d,a} \theta_{d,a} \beta_{a,blue} \theta_{a,n} \beta_{n,boat} = \quad (1)$$

$$0.3 * 0.2 * 0.4 * 0.3 * 0.5 * 0.3 = 0.00108 \quad (2)$$

Garden Pathing

What is the probability of the sequence “a/Det blue/Adj boat/N”?

$$\pi_d \beta_{d,a} \theta_{d,a} \beta_{a,blue} \theta_{a,n} \beta_{n,boat} = \quad (1)$$

$$0.3 * 0.2 * 0.4 * 0.3 * 0.5 * 0.3 = 0.00108 \quad (2)$$

Example ...

Base case

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1. $\delta_1(a) = -4.6$

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3. $\delta_1(d) = -1.7$

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2. $\delta_1(v) = -5.7$
3. $\delta_1(d) = -1.7$
4. $\delta_1(n) = -4.6$

Example ...

Second position

$$1. \delta_2(a) = \max \left(\underbrace{-5.8}_a, \underbrace{-7.3}_v, \underbrace{-2.6}_d, \underbrace{-7.6}_n \right) + -1.2 = -2.6 + -1.2 = -3.8$$

Example ...

Second position

$$1. \delta_2(a) = \max \left(\underbrace{-5.8}_a, \underbrace{-7.3}_v, \underbrace{-2.6}_d, \underbrace{-7.6}_n \right) + -1.2 = -2.6 + -1.2 = -3.8$$

$$2. \delta_2(v) = \max \left(\underbrace{-6.9}_a, \underbrace{-7.3}_v, \underbrace{-4.7}_d, \underbrace{-4.8}_n \right) + -2.3 = -4.7 + -2.3 = -7.0$$

Example ...

Second position

$$1. \delta_2(a) = \max \left(\underbrace{-5.8}_a, \underbrace{-7.3}_v, \underbrace{-2.6}_d, \underbrace{-7.6}_n \right) + -1.2 = -2.6 + -1.2 = -3.8$$

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$$3. \delta_2(d) = \max \left(\underbrace{-6.9}_a, \underbrace{-6.9}_v, \underbrace{-4.0}_d, \underbrace{-7.6}_n \right) + -3.7 = -4.0 + -3.7 = -7.7$$

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$$1. \delta_2(a) = \max \left(\underbrace{-5.8}_a, \underbrace{-7.3}_v, \underbrace{-2.6}_d, \underbrace{-7.6}_n \right) + -1.2 = -2.6 + -1.2 = -3.8$$

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$$3. \delta_2(d) = \max \left(\underbrace{-6.9}_a, \underbrace{-6.9}_v, \underbrace{-4.0}_d, \underbrace{-7.6}_n \right) + -3.7 = -4.0 + -3.7 = -7.7$$

$$4. \delta_2(n) = \max \left(\underbrace{-5.3}_a, \underbrace{-6.9}_v, \underbrace{-2.5}_d, \underbrace{-6.9}_n \right) + -1.9 = -2.5 + -1.9 = -4.4$$

Example ...

Third position

$$1. \delta_3(a) = \max \left(\underbrace{-5.0}_a, \underbrace{-8.6}_v, \underbrace{-8.6}_d, \underbrace{-7.4}_n \right) + -2.3 = -5.0 + -2.3 = -7.3$$

Example ...

Third position

1. $\delta_3(a) = \max \left(\underbrace{-5.0}_a, \underbrace{-8.6}_v, \underbrace{-8.6}_d, \underbrace{-7.4}_n \right) + -2.3 = -5.0 + -2.3 = -7.3$
2. $\delta_3(v) = \max \left(\underbrace{-6.1}_a, \underbrace{-8.6}_v, \underbrace{-10.7}_d, \underbrace{-4.6}_n \right) + -0.9 = -4.6 + -0.9 = -5.5$

Example ...

Third position

1. $\delta_3(a) = \max \left(\underbrace{-5.0}_a, \underbrace{-8.6}_v, \underbrace{-8.6}_d, \underbrace{-7.4}_n \right) + -2.3 = -5.0 + -2.3 = -7.3$
2. $\delta_3(v) = \max \left(\underbrace{-6.1}_a, \underbrace{-8.6}_v, \underbrace{-10.7}_d, \underbrace{-4.6}_n \right) + -0.9 = -4.6 + -0.9 = -5.5$
3. $\delta_3(d) = \max \left(\underbrace{-6.1}_a, \underbrace{-8.2}_v, \underbrace{-10.0}_d, \underbrace{-7.4}_n \right) + -3.7 = -6.1 + -3.7 = -9.8$

Example ...

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$$2. \delta_3(v) = \max \left(\underbrace{-6.1}_a, \underbrace{-8.6}_v, \underbrace{-10.7}_d, \underbrace{-4.6}_n \right) + -0.9 = -4.6 + -0.9 = -5.5$$

$$3. \delta_3(d) = \max \left(\underbrace{-6.1}_a, \underbrace{-8.2}_v, \underbrace{-10.0}_d, \underbrace{-7.4}_n \right) + -3.7 = -6.1 + -3.7 = -9.8$$

$$4. \delta_3(n) = \max \left(\underbrace{-4.5}_a, \underbrace{-8.2}_v, \underbrace{-8.5}_d, \underbrace{-6.7}_n \right) + -0.9 = -4.5 + -0.9 = -5.4$$

Example ...

Fourth position

$$1. \delta_4(a) = \max \left(\underbrace{-8.5}_a, \underbrace{-7.2}_v, \underbrace{-10.7}_d, \underbrace{-8.4}_n \right) + -3.4 = -7.2 + -3.4 = -10.6$$

Example ...

Fourth position

$$1. \delta_4(a) = \max \left(\underbrace{-8.5}_a, \underbrace{-7.2}_v, \underbrace{-10.7}_d, \underbrace{-8.4}_n \right) + -3.4 = -7.2 + -3.4 = -10.6$$

$$2. \delta_4(v) = \max \left(\underbrace{-9.6}_a, \underbrace{-7.2}_v, \underbrace{-12.8}_d, \underbrace{-5.7}_n \right) + -3.4 = -5.7 + -3.4 = -9.1$$

Example ...

Fourth position

$$1. \delta_4(a) = \max \left(\underbrace{-8.5}_a, \underbrace{-7.2}_v, \underbrace{-10.7}_d, \underbrace{-8.4}_n \right) + -3.4 = -7.2 + -3.4 = -10.6$$

$$2. \delta_4(v) = \max \left(\underbrace{-9.6}_a, \underbrace{-7.2}_v, \underbrace{-12.8}_d, \underbrace{-5.7}_n \right) + -3.4 = -5.7 + -3.4 = -9.1$$

$$3. \delta_4(d) = \max \left(\underbrace{-9.6}_a, \underbrace{-6.8}_v, \underbrace{-12.1}_d, \underbrace{-8.4}_n \right) + -0.5 = -6.8 + -0.5 = -7.3$$

Example ...

Fourth position

$$1. \delta_4(a) = \max \left(\underbrace{-8.5}_a, \underbrace{-7.2}_v, \underbrace{-10.7}_d, \underbrace{-8.4}_n \right) + -3.4 = -7.2 + -3.4 = -10.6$$

$$2. \delta_4(v) = \max \left(\underbrace{-9.6}_a, \underbrace{-7.2}_v, \underbrace{-12.8}_d, \underbrace{-5.7}_n \right) + -3.4 = -5.7 + -3.4 = -9.1$$

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$$4. \delta_4(n) = \max \left(\underbrace{-8.0}_a, \underbrace{-6.8}_v, \underbrace{-10.6}_d, \underbrace{-7.7}_n \right) + -3.4 = -6.8 + -3.4 = -10.2$$

Example ...

Fifth position

$$1. \delta_5(a) = \max\left(\underbrace{-11.8}_a, \underbrace{-10.7}_v, \underbrace{-8.2}_{\mathbf{d}}, \underbrace{-13.2}_n\right) + -2.3 = -8.2 + -2.3 = -11$$

Example ...

Fifth position

$$1. \delta_5(a) = \max \left(\underbrace{-11.8}_a, \underbrace{-10.7}_v, \underbrace{-8.2}_d, \underbrace{-13.2}_n \right) + -2.3 = -8.2 + -2.3 = -11$$

$$2. \delta_5(v) = \max \left(\underbrace{-12.9}_a, \underbrace{-10.7}_v, \underbrace{-10.3}_d, \underbrace{-10.4}_n \right) + -1.6 = -10.3 + -1.6 = -12$$

Example ...

Fifth position

$$1. \delta_5(a) = \max \left(\underbrace{-11.8}_a, \underbrace{-10.7}_v, \underbrace{-8.2}_d, \underbrace{-13.2}_n \right) + -2.3 = -8.2 + -2.3 = -11$$

$$2. \delta_5(v) = \max \left(\underbrace{-12.9}_a, \underbrace{-10.7}_v, \underbrace{-10.3}_d, \underbrace{-10.4}_n \right) + -1.6 = -10.3 + -1.6 = -12$$

$$3. \delta_5(d) = \max \left(\underbrace{-12.9}_a, \underbrace{-10.3}_v, \underbrace{-9.6}_d, \underbrace{-13.2}_n \right) + -3.7 = -9.6 + -3.7 = -13$$

Example ...

Fifth position

$$1. \delta_5(a) = \max \left(\underbrace{-11.8}_a, \underbrace{-10.7}_v, \underbrace{-8.2}_d, \underbrace{-13.2}_n \right) + -2.3 = -8.2 + -2.3 = -11$$

$$2. \delta_5(v) = \max \left(\underbrace{-12.9}_a, \underbrace{-10.7}_v, \underbrace{-10.3}_d, \underbrace{-10.4}_n \right) + -1.6 = -10.3 + -1.6 = -12$$

$$3. \delta_5(d) = \max \left(\underbrace{-12.9}_a, \underbrace{-10.3}_v, \underbrace{-9.6}_d, \underbrace{-13.2}_n \right) + -3.7 = -9.6 + -3.7 = -13$$

$$4. \delta_5(n) = \max \left(\underbrace{-11.3}_a, \underbrace{-10.3}_v, \underbrace{-8.1}_d, \underbrace{-12.5}_n \right) + -1.2 = -8.1 + -1.2 = -9.3$$

Example ...

Reconstruction

Example ...

Reconstruction

For “the old man”, the reconstruction starts with the best part of speech at Position 3, which is noun (-5.4), which has an adjective back pointer, which as a back pointer to determiner. The overall sequence is “The/det old/adj man/n”.

Example ...

Reconstruction

For “the old man”, the reconstruction starts with the best part of speech at Position 3, which is noun (-5.4), which has an adjective back pointer, which as a back pointer to determiner. The overall sequence is “The/det old/adj man/n”.

For “the old man the boats”, the reconstruction starts with the best part of speech at Position 5, which is a noun (-9.3), which leads to the sequence “The/det old/n man/v the/det boats/n”.