$\boxed{2.}$ $f(x,y,z) = x^4+y^4+z^4$ $\quad M: x+y+z=1$ Yauheni Zviazdov 1

$F(x,y,z,\lambda) = x^4+y^4+z^4+\lambda(x+y+z-1)$

Hranice

$\operatorname{grad} F = (4x^3+\lambda,\ 4y^3+\lambda,\ 4z^3+\lambda,\ x+y+z-1)$

$$\begin{cases} 4x^3+\lambda=0 \\ 4y^3+\lambda=0 \\ 4z^3+\lambda=0 \\ x+y+z-1=0 \end{cases} \quad \begin{cases} \lambda=-4x^3 \\ \lambda=-4y^3 \\ \lambda=-4z^3 \\ x+y+z-1=0 \end{cases} \quad \begin{cases} 4x^3=4y^3 \\ \lambda=-4x^3 \\ \lambda=-4z^3 \\ x+y+z-1=0 \end{cases} \quad \begin{cases} x=y \\ x=z \\ x+y+z-1=0 \end{cases}$$

$3x=1$ $\quad A\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

$x=\frac{1}{3}$

$y=\frac{1}{3}$

$z=\frac{1}{3}$

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Vnitřek ?

$$\begin{cases} 4x^3=0 \\ 4y^3=0 \\ 4z^3=0 \end{cases} \quad \{x=y=z=0 \qquad (0,0,0)\notin M$$

$f(A) = \frac{1}{3^4}+\frac{1}{3^4}+\frac{1}{3^4} = \frac{3}{3^4} = \frac{1}{3^3} = \frac{1}{27}$ – abs max

Odpověď: $\frac{1}{27}$

$\boxed{3.}$ $F = (2xz+\sin y,\ x\cos y,\ x^2)$

a) rot F:

$$\begin{vmatrix} e_1 & e_2 & e_3 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xz+\sin y & x\cos y & x^2 \end{vmatrix}$$

$$\begin{matrix} e_1 & e_2 & e_3 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{matrix}$$

$0+e_3\cos y + e_2(2x) - (\cos y)e_3 -$

$-0-(2x)e_2$

$e_1\cdot 0 + e_2\cdot 0 + e_3\cdot 0$

$(0,0,0)$

rot F = (0,0,0) → vektorové pole je konzervativní

b) $$\begin{cases} \frac{\partial f}{\partial x} = 2xz + \sin y \\ \frac{\partial f}{\partial y} = x\cos y \\ \frac{\partial f}{\partial z} = x^2 \end{cases}$$

$$f(x,y,z) = x^2 z + x\sin y + C(y,z)$$

$$\frac{\partial f}{\partial y} = x\cos y \ ;$$

$$0 + x\cos y + \frac{\partial C(y,z)}{\partial y} = x\cos y$$

$$\frac{\partial C(y,z)}{\partial y} = 2\cancel{x}\cos y \quad 0$$

$$C(y,z) = 2x\sin y + C(z)$$

$$\frac{\partial f}{\partial z} = x^2 \ ;$$

$$0 + \frac{\partial C(z)}{\partial z} = x^2$$

$$\frac{\partial C(z)}{\partial z} = x^2$$

$$C(z) = zx^2 + C$$

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$$f(x,y,z) = x^2 z + x\sin y + 2x\sin y + zx^2 + C$$

$$f(x,y,z) = x^2 z + 3x\sin y + \cancel{zx^2} + C \quad \text{je potenciálem } F$$

4. $$\int_0^8 \int_{y^{\frac{1}{3}}}^2 e^{x^4}\,dx\,dy = \int_0^8 \left[4x^3 e^{x^4}\right]_{y^{\frac{1}{3}}}^2 dy = \int_0^8 32e^{16} - 4ye^{y^{\frac{4}{3}}}\,dy =$$

$(e^{x^4})' = 4x^3 e^{x^4}$

$$= \left| \begin{matrix} u = y^{\frac{4}{3}} \\ du = \frac{4}{3}y^{\frac{1}{3}} \end{matrix} \right| = \int_0^8 32e^{16} - \frac{3}{4}\int_0^{16} 4u^{\frac{2}{3}} e^u\,du =$$

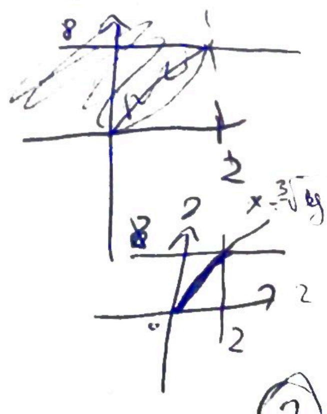
$$= 32\cdot 8\cdot e^{16} - \left[u^4 -\right.$$

~~Záměna~~ z

$$\int_0^2 \int_0^{x^3} e^{x^4}\,dy\,dx =$$

$x = \sqrt[3]{y}$

$x^3 = y$



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$$= \int_0^2 \left[e^{x^4} y\right]_{x^3}^{8} dx = \int_0^2 8e^{x^4} - e^{x^4}x^3\,dx.$$

Jauheni Zviazdou 3

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E

1. a) $\lim_{(x,y)\to(0,0)} \frac{x^3+x^2y^2}{x^2+y^2}$

· Vyjadříme x pomocí y : $x = ky$

$\lim_{(x,y)\to(ky,0)} \frac{x^3+x^2y^2}{x^2+y^2} = \lim_{(x,y)\to(ky)} \frac{k^3y^3+0}{k^2y^2+0} = \lim_{(x,y)} \frac{k^3y^3}{k^2y^2} = ky = 0$

· Vyjadříme y pomocí x : $y = kx$

$\lim_{(x,y)\to(0,kx)} \frac{x^3+x^2y^2}{x^2+y^2} = \frac{0+0}{0+k^2x^2} = 0$

limita se rovná 0

nedostatečný důvod

Odpověď : 0

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b) $\lim_{(x,y)\to(0,0)} \frac{\sin(x)+y}{|x|+|y|}$

· Vyjadříme x pomocí y : $x = ky$

$\lim_{(x,y)\to(ky,0)} \frac{\sin(x)+y}{|x|+|y|} = \lim_{(x,y)\to(ky,0)} \frac{\sin(ky)+0}{0+|ky|} = \frac{\sin(ky)}{|ky|}$?

?

· Vyjadříme y pomocí x : $y = kx$

$\lim_{(x,y)\to(0,kx)} \frac{\sin(x)+y}{|x|+|y|} = \lim_{(x,y)\to(0,kx)} \frac{0+kx}{|kx|+0}$? = ?

nerovnají

limita neex

Odpověď: limita neexistuje

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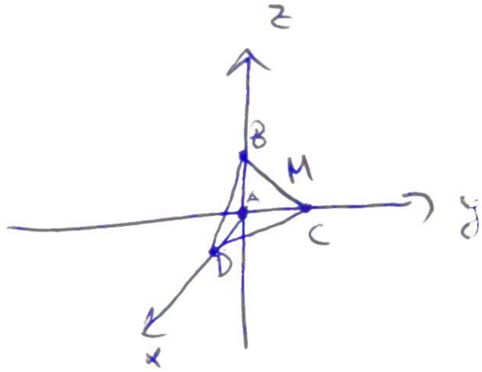
$\boxed{5.}$ $\iint_{(S)} F \cdot dS \qquad F = (3xy,\ y^2,\ -x^2y^4)$

A = (0,0,0)
B = (0,0,1)
C = (0,1,0)
D = (1,0,0)

(4)

$$\iint_{(S)} F\,dS \ \overset{\text{Gaussova věta}}{=\!=\!=}\ \iiint_M \operatorname{div} F\, dM$$



$$\operatorname{div} F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

$$\operatorname{div} F = 3y + 2y + 0 = 5y \qquad (0,5,0)$$

$$\int_0^1 \int_0^{1-z\ \cancel{z+1}} \int_0^{1-x-z\ \cancel{x+1}} 5y \; dy\,dx\,dz =$$

Vnější orientace

$$= \frac{5}{2}\int_0^1 \int_0^{z+1} [y^2]_0^{x+1}\, dx\,dz = \frac{5}{2}\int_0^1 \int_0^{z+1} x^2 + 2x + 1\, dx\,dz =$$

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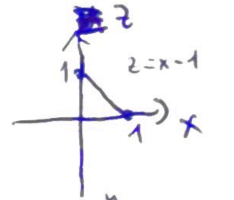

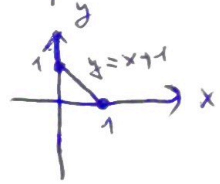

$$= \frac{5}{2}\int_0^1 \left[\frac{x^3}{3} + x^2 + x\right]_0^{z+1} dz = \frac{5}{2}\int_0^1 \frac{z^3 + 3z^2 + 3z + 1}{3} +$$

$$+ z^2 + 2z + 1 + z + 1\ dz = \frac{5}{6}\int_0^1 z^3 + 3z^2 + 3z + 1\, dz +$$

$$+ \frac{5}{2}\int_0^1 z^2 + 3z + 2\, dz = \frac{5}{6}\left[\frac{z^4}{4} + z^3 + \frac{3z^2}{2} + z\right]_0^1 + \frac{5}{2}\left[\frac{z^3}{3} + \frac{3z^2}{2} + 2z\right]_0^1$$

$$= \frac{5}{6}\underbrace{\left(\frac{1}{4} + 1 + \frac{3}{2} + 1\right)}_{\frac{1+4+6+4}{4} = \frac{15}{4}} + \frac{5}{2}\underbrace{\left(\frac{1}{3} + \frac{3}{2} + 2\right)}_{\frac{2}{6} + \frac{9}{6} + \frac{12}{6} = \frac{23}{6}} = \frac{5 \cdot 15}{6 \cdot 4} + \frac{5 \cdot 23}{2 \cdot 6} =$$

$$= \frac{5 \cdot 5}{8} + \frac{5 \cdot 23}{12} = \frac{25}{8} + \frac{115}{12} = \frac{3 \cdot 25 + 230}{24} = \frac{305}{24}$$