CVIO3

4.1 $x = (1,2,3) \quad y = (-1,0,1)$

a) $d = \sqrt{1^2+2^2+3^2} = \sqrt{1+4+9} = \sqrt{14}$

b) $d(x,y) = \|x-y\| = \|(1,2,3)-(-1,0,1)\| = \|(2,2,2)\| = \sqrt{4+4+4} = \sqrt{12}$

c) $\cos\varphi = \frac{-1+0+3}{\sqrt{1+4+9}\cdot\sqrt{1+0+1}} = \frac{2}{\sqrt{14}\cdot\sqrt{2}} = \frac{\sqrt{6}}{6}$

$\varphi = \arccos\frac{\sqrt{6}}{6}$

4.2 Pro takové x, y, které jsou stranami trojúhelníku.

4.3 $X = \text{span}\{(0,1,1),(1,2,3)\}$

$X^\perp = \{y \in \mathbb{R}^n \mid y \perp X\}$

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$-y = -1$

$X^\perp = \text{span}\left\{\begin{pmatrix}1\\1\\-1\end{pmatrix}\right\}$

4.5

a) $\|x\| = \|y\| \longrightarrow (x+y) \perp (x-y)$

$x^Tx = y^Ty$

$x^Tx - y^Ty = 0$

$x^Tx - x^Ty + y^Tx - y^Ty = 0 \qquad (x^Ty = y^Tx$, skalární součin)

$(x^T+y^T)\cdot(x-y) = 0$

$(x+y)^T \cdot (x-y) = 0$

$(x+y) \perp (x-y)$ ✓

b) $x \perp y \longrightarrow \|x\|^2 + \|y\|^2 = \|x-y\|^2$

$x^Ty = 0 = y^Tx$

$\|x\|^2 = \|x\|^2 \quad / + \|y\|^2$

$\|x\|^2 + \|y\|^2 = \|x\|^2 + \|y^*\|^2$

$(x^Tx)^2 + (y^Ty)^2 = (x^Tx)^2 + (y^Ty)^2$

$(x^Tx)^2 + (y^Ty)^2 = (x^Tx)^2 + (y^Ty)^2 + xx^Tyy^T + yy^Txx^T$

$xx^Tyy^T = x(x^Ty)y^T = 0$ (x^Ty = 0)

$yy^Txx^T = y(y^Tx)x^T = 0$ (y^Tx = 0)

$(x^Tx)^2 + (y^Ty)^2 = (x^Tx)^2 + (y^Ty) + (xx^Tyy^T)^T + (yy^Txx^T)^T$

$(x^Tx)^2 + (y^Ty)^2 = (x^Tx + y^Ty)^2$

$x^Ty = 0$

$y^Tx = 0$

$(x^Tx)^2 + (y^Ty)^2 = (x^Tx - x^Ty - y^Tx + y^Ty)^2$

$\|x\|^2 + \|y\|^2 = \left((x^T - y^T)(x-y)\right)^2$

$\|x\|^2 + \|y\|^2 = \left((x-y)^T(x-y)\right)^2$

$\|x\|^2 + \|y\|^2 = \|x-y\|^2$ ✓

4.13 $x^Ty = 0$ $\quad \text{span}\left\{\begin{pmatrix}0\\1\\1\end{pmatrix}, \begin{pmatrix}1\\2\\3\end{pmatrix}\right\}$

$\vec{u}_1 \quad \vec{u}_2$

$x = \vec{u}_1 = (0\ 1\ 1)$

$y = \vec{u}_2 - \frac{\vec{u}_2 \cdot \vec{u}_1}{\|u_1\|^2} \cdot \vec{u}_1 = (1,2,3) - \frac{(0,1,1)\cdot(1,2,3)}{\left(\sqrt{0+1^2+1^2}\right)^2} \cdot (0,1,1) =$

$= (1,2,3) - \frac{5}{2} \cdot (0,1,1) = (1, -0.5, 0.5)$

$\text{span}\left\{\begin{pmatrix}0\\1\\1\end{pmatrix}, \begin{pmatrix}1\\-0.5\\0.5\end{pmatrix}\right\}$

$x \quad y$ //