DU 11

1 a) $\int_{-\infty}^{\infty} x^2\,dx = \left[\frac{x^3}{3}\right]_{-\infty}^{\infty} = \lim_{x\to\infty}\frac{x^3}{3} - \lim_{x\to-\infty}\frac{x^3}{3} = \left|+\infty-(-\infty)\right| = +\infty$ //

b) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} tg\,x\,dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} tg\,x\,dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin x}{\cos x} = \left|\begin{array}{l} \cos x = t \\ -\sin x = dt \\ x=\frac{\pi}{2} \rightsquigarrow t\to 0 \\ x=-\frac{\pi}{2} \rightsquigarrow t\to 0 \end{array}\right| =$

$= \int_0^0 t\,dx = 0$ //

c) $\int_3^{\infty} \frac{2x-1}{x^2-x}\,dx = \left[\ln|x^2-x|\right]_3^{\infty} = \lim_{x\to\infty}\ln|x^2-x| - \lim_{x\to 3}\ln|x^2-x| =$

$x\in\mathbb{R}\setminus\{0,1\}$

$= \lim_{x\to\infty}\left(\ln|x| + \ln|x-1|\right) - \lim_{x\to 3}\ln|x^2-x| = \left|\infty - \text{const}\right| = +\infty$

d) $\int_0^{\frac{1}{2}} \frac{1}{x(\ln^2 2x+1)}\,dx = \left|\begin{array}{l} \ln 2x = t \\ \frac{2}{2x} = \frac{1}{x} = dt \end{array}\right| = \int_{-\infty}^{0}\frac{1}{t^2+1} =$

$x=\frac{1}{2} \rightsquigarrow t\to 0$

$x\to 0 \rightsquigarrow t\to-\infty$

$= \left[\operatorname{arctg} t\right]_{-\infty}^{0} = 0 - \lim_{t\to-\infty}\operatorname{arctg} t = \frac{\pi}{2}$ //

e) $\int_0^\infty x^2 e^{-x}\, dx = \left| \begin{matrix} u = x^2 & v' = e^{-x} \\ u' = 2x & v = -e^{-x} \end{matrix} \right| = \left[x^2 (-e^{-x}) \right]_0^\infty +$

$+2\int_0^\infty x (+e^{-x}) = \left| \begin{matrix} u = x & v' = e^{-x} \\ u' = 1 & v = -e^{-x} \end{matrix} \right| = \left[-x^2 e^{-x} \right]_0^\infty +$

$+2\left(\left[-x e^{-x} \right]_0^\infty - \int_0^\infty -e^{-x} \right) = \left[-x^2 e^{-x} \right]_0^\infty + 2\left(\left[-x e^{-x} \right]_0^\infty + \left[e^{-x} \right]_0^\infty \right)$

$= \lim_{x \to \infty} -x^2 e^{-x} - \lim_{x \to 0} -x^2 e^{-x} + 2\Big(\lim_{x \to \infty} -x e^{-x} - \lim_{x \to 0} -x e^{-x} +$

$+ \lim_{x \to \infty} -e^{-x} - \lim_{x \to 0} -e^{-x} \Big) = 0 - 0 + 2(0 - 0 + 0 + 1) = 2$

f) $\int_0^\pi \frac{1}{1 + \cos^2 x}\, dx$

g) –

2. $\int_0^\infty \frac{\sin x}{x^2 + 1} = \left| \begin{matrix} u = \sin x & v' = \frac{1}{x^2+1} \\ u' = \cos x & v = \operatorname{arctg} x \end{matrix} \right| = \left[\sin x \cdot \operatorname{arctg} x \right]_0^\infty - \int \cos x \operatorname{arctg} x$

$= \lim_{x \to \infty} \sin x \operatorname{arctg} x - \lim_{x \to 0} \sin x \cdot \operatorname{arctg} x - \int_0^\infty \cos x \operatorname{arctg} x =$

$= \left| \infty - 0 - \int_0^\infty \cos x \operatorname{arctg} x \right| = \infty$ nekonverguje

3. $f = x^2 + y^2 = r^2$

$y = \sqrt{r^2 - x^2}$

$\cancel{y' = (r^2 - x^2)^{\frac{1}{2}} = \left(e^{\frac{1}{2} \ln (r^2 - x^2)}\right)' = \sqrt{r^2 - x^2} \cdot \left(\frac{r^2 - 2x}{2(r^2 - x^2)} \right)}$

$\cancel{4\int_0^r \sqrt{1 + \left(\frac{r^2 - 2x}{2\sqrt{r^2 - x^2}} \right)^2}\, dx}$

$y' = (r^2 - x^2)^{\frac{1}{2}} = \sqrt{r^2 - x^2} \cdot \left(\frac{-2x}{2(r^2 - x^2)} \right) =$

$= \frac{-x}{\sqrt{r^2 - x^2}}$

$4\int_0^r \sqrt{1 + \frac{x^2}{r^2 - x^2}}\, dx = \int_0^r \sqrt{\frac{x^2 + r^2 - x^2}{r^2 - x^2}}\, dx = \int_0^r \sqrt{\frac{r^2}{r^2 - x^2}}\, dx =$

$$\int_0^r \frac{r}{\sqrt{r^2-x^2}} = r\int_0^r \frac{1}{\sqrt{r^2-x^2}} = r\int_0^r \frac{1}{r\sqrt{1-\left(\frac{x}{r}\right)^2}} =$$

$$= \left| \begin{array}{l} \frac{x}{r} = t \\ \frac{1}{r} = dt \\ x \to r \leadsto t \to 1 \\ x \to 0 \leadsto t \to 0 \end{array} \right| = r\int_0^1 \frac{1}{\sqrt{1-t^2}} = r \cdot [\arcsin t]_0^1 = r \cdot \left(\frac{\pi}{2} - 0\right) =$$

$$= \frac{\pi r}{2}$$

$$P_{kr} = 4 \cdot \frac{\pi r}{2} = 2\pi r$$

4. Objem rotačního elipsu

$$y = \sqrt{\frac{a^2b^2 - x^2b^2}{a^2}} \qquad a < b$$

$$V = \pi \int_a^b f^2(x)\, dx = \pi \left(\int_a^b b^2 - \int_a^b \frac{x^2b^2}{a^2} \right) =$$

$$= \pi b^2 - \pi \int_a^b \frac{x^2 b^2}{a^2} = \pi b^2 - \frac{\pi b^2}{a^2} \int_a^b x^2 = \pi b^2 - \frac{\pi b^2}{a^2} \left[\frac{x^3}{3} \right]_a^b =$$

$$= \pi b^2 - \frac{\pi b^2}{a^2} \left(\frac{b^3}{3} - \frac{a^3}{3} \right)$$

5. Objem a povrch rotačního hyperboloidu $f(x) = \frac{1}{x}$ $[1, +\infty)$

$$V = \pi \int_1^\infty \left(\frac{1}{x}\right)^2 dx = \pi \int_1^\infty \frac{1}{x^2} = \pi \left[-\frac{1}{x} \right]_1^\infty = |0 - (-1)| = 1 \cdot \pi = \pi$$

$$S = 2\pi \int_1^\infty \frac{1}{x} \sqrt{1 + \left(\left(\frac{1}{x}\right)'\right)^2}\, dx = 2\pi \int_0^\infty \frac{1}{x} \sqrt{\frac{1+x^4}{x^4}}$$