10.1

a) $\{2,3,-1\}$ – Hessova matice je indef – nemá lok extrém v $(2,1,5)$

b) $\{2,3,0\}$ – Hessova matice je pos. semidef – nabývá min

c) $\{2,1,1\}$ – Hessova matice je pos. def – ~~nabývá~~ má ostré lok min.

10.2

d) $f(x,y) = 3x - x^3 - 3xy^2$

$f'(x,y) = (3 - 3x^2 - 3y^2, -6xy)$

$f'(x,y) = (0,0) \Rightarrow (x,y) = (0,1), (0,-1), (1,0), (-1,0)$

$f''(x,y) = \begin{pmatrix} -6x & -6y \\ -6y & -6x \end{pmatrix}$

$f''(0,1) = \begin{pmatrix} 0 & -6 \\ -6 & 0 \end{pmatrix}$ indef – sedlový bod

$f''(0,-1) = \begin{pmatrix} 0 & 6 \\ 6 & 0 \end{pmatrix}$ indef – sedlový bod

$f''(1,0) = \begin{pmatrix} -6 & 0 \\ 0 & -6 \end{pmatrix}$ neg def $\rightarrow$ $(1,0)$ je ostré lok max

$f''(-1,0) = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$ pos def $\rightarrow$ $(-1,0)$ je ostré lok min

e) $f(x,y) = 6xy^2 - 2x^3 - 3y^4$

$f'(x,y) = (6y^2 - 6x^2, 12xy - 12y^3)$

$f''(x,y) = \begin{pmatrix} 12x & 12y \\ 12y & 12x - 36y^2 \end{pmatrix}$

$f'(x,y) = (0,0) \Rightarrow (x,y) = (0,0), (1,1), (1,-1)$

$f''(0,0) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ pos semidef → lok min

$f''(1,1) = \begin{pmatrix} 12 & 12 \\ 12 & -24 \end{pmatrix}$ neg def → ostré lok max

$f''(1,-1) = \begin{pmatrix} 12 & -12 \\ -12 & -24 \end{pmatrix}$ neg def → ostré lok max

10.3 $f(x) = a^T x - \sum_{i=1}^{n} x_i \log x_i \qquad \mathbb{R}^n \to \mathbb{R}$

$f(x) = a^T x - x^T \log x$

~~$\frac{1}{x_i} \cdot x_i = \log x_i$~~

$f'(x) = a^T - \log x - 1$

$f'(x) = (0 \dots 0) \qquad \log x = a - 1$

$x = (e^{a_1 - 1}, e^{a_2 - 1}, \dots, e^{a_n - 1})$

10.5

$\sin x = \frac{1}{2}x$

$f(x) = \frac{1}{2}x - \sin x \qquad x = 0$

$f'(x) = \frac{1}{2} - \cos x \qquad x = -1.8954942670339 81$ (Newtonova metoda)

$x = 1.8954942670339 81$ (Newtonova metoda s jinou začátečí hodnotou)

10.6 $f(x,y) = x^2 y + \sin(y^2 - 2x)$

$(x_0, y_0) = (1,1)$

$g(x,y) = (2x + \cos(y^2 - 2x) \cdot (-2), -1 + \cos(y^2 - 2x) \cdot 2y)$

$x = 0,68074537238326 06$

$y = 0,73489017897436 7$