CVIO8

11.1 d) $f(x,y) = x^2 y$ $\quad$ $X = \{(x,y) \in \mathbb{R}^2 \mid g(x,y) = x^2 + y^2 - 1 = 0\}$

$x^2 + y^2 - 1 = 0 \quad ; \quad x^2 = 1 - y^2$

$f(x,y) = y(1-y^2) = y - y^3$

$\cancel{f'(x,y)}$ $f'(y) = 1 - 3y^2$

$f'(y) = 0 \quad , \quad y = \pm \frac{\sqrt{3}}{3} \quad , \quad x = \pm\sqrt{\frac{2}{3}}$

$\left(\frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{3}}{3}\right), \left(-\frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{3}}{3}\right)$

$\left(-\frac{\sqrt{2}}{\sqrt{3}}, -\frac{\sqrt{3}}{3}\right), \left(+\frac{\sqrt{2}}{\sqrt{3}}, -\frac{\sqrt{3}}{3}\right)$

+ krajní body

$(0,1)$, $(0,-1)$

11.4 c) $x^2y=1$

$\min\{\|(x,y)\| \mid x^2y=1\}$

$\min\{\sqrt{x^2+y^2} \mid x^2y=1\}$ převedeme na úlohu se stejnými

kořeny: $\min\{x^2+y^2 \mid x^2y=1\}$

$x^2=\frac{1}{y}$

$f(y)=\frac{1}{y}+y^2=\frac{1+y^3}{y}$

$f'(y)=\frac{-1}{y^2}+2y=2y-\frac{1}{y^2}=\frac{2y^3-1}{y^2}$

$f'(y)=0$, $2y^3-1=0$

$y^3=\frac{1}{2}$, $y=\frac{1}{\sqrt[3]{2}}$ $x=\pm\frac{1}{\sqrt[6]{2}}$

11.8 $f(x,y)=x$ $\quad x^3=y^2$ $\quad \min=(0,0)$

Dk. $x^3=y^2$

$y^2\geq 0 \Rightarrow x\geq 0.$

~~Ať~~ $f(x,y)$ nabývá min iff $x=0$

$L=x+\lambda(x^3-y^2)$

$L'=(1+3\lambda x^2, -2\lambda, x^3-y^2)$

$$\begin{cases} 1+3\lambda x^2=0 & (1) \\ -2\lambda=0 & (2) \\ x^3-y^3=0 & (3) \end{cases}$$

Podle (2), $\lambda=0$. Dosazením $\lambda=0$ do rovnice (1) dostaneme $1=0 \Rightarrow$ soustava nemá řešení.

11.11 a) i. ?

11.16 $\min\{x^T x \mid a^T x = 1\}$ máme minimalizovat n-rozměrný elipsoid?

$$L = x^T x + \lambda(a^T x - 1)$$

$$\begin{cases} 2x^T + \lambda a^T = 0 \\ a^T x - 1 = 0 \end{cases} \quad \begin{cases} 2x^T + \lambda a^T = 0 \\ x^T a = 1 \end{cases} \quad \begin{cases} x^T = \dfrac{-\lambda a^T}{2} \\ x^T a = 1 \end{cases} \quad \begin{cases} x^T = -\dfrac{\lambda a^T}{2} \\ \dfrac{-\lambda a^T a}{2} = 1 \end{cases}$$

$$\begin{cases} x^T = -\dfrac{\lambda a^T}{2} \\ \lambda = \dfrac{-2}{a^T a} \end{cases} \quad x^T = \frac{-2a^T}{2a^T a} = \frac{a^T}{a^T a}; \quad x = \frac{a}{a^T a} = \frac{a}{\|a\|^2}$$

Nabývá min iff $x = \dfrac{a}{a^T a}$