CVI 12

15.2

$$\max\left\{\sum_{i=1}^{n} c_i x_i \;\middle|\; -1 \le x_i \le 1\right\}$$

$$\max\left\{c^T x \;\middle|\; -1 \le x_i \le 1\right\}$$

a) $c_i \ge 0 \to x_i = 1$

$c_i < 0 \to x_i = -1$ $\Big\} \Rightarrow \sum_{i=1}^{n} |c_i|$

b) $\max c^T x$

z. p. $x \ge -1$

$x \le 1$

$\min +y_1 - y_2$

z. p. $y_1 \ge 0$

$y_2 \le 0$

Nejoptimalnější řešení bude, když

$+y_{1i} - y_{2i} = |c_i^T|$

$\downarrow$

$$c_i^T \geq 0 \rightarrow \quad y_{1i} = c_i^T \qquad (x = 1)$$
$$y_{2i} = 0$$

$$c_i^T \leq 0 \rightarrow \quad y_{1i} = 0 \qquad (x = -1)$$
$$y_{2i} = c_i^T$$

c)

$x_i = -1$ nebo $y_{1i} = 0$

$x_i = 1$ nebo $y_{1i} = 0$

d) $(c_1, c_2, c_3) = (-2, 3, 4)$

$x = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ $\quad c^T x = 2+3+4 = 9$ $\qquad y_1 = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$ $\quad y_2 = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$

$y_1 - y_2 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

~~5.3~~

~~a) min $2x_1 - 3x_3 + x_4$~~

~~z.p. $x_1 - x_2 - x_3 \geq 0$~~

~~$-x_1 + 2y_2 - 3x_3 \leq 5$~~

5.3

$$\begin{aligned}
\min \quad & 2x_1 - 3x_3 + x_4 \\
\text{z.p.} \quad & x_1 - x_2 - x_3 \geq 0 \\
& -x_1 + 2x_2 - 3x_3 \leq 5 \\
& 2x_1 - x_2 - x_3 + 2x_4 = 6 \\
& x_1, x_2, x_3, x_4 \geq 0
\end{aligned}$$

$$\begin{aligned}
\max \quad & +5y_2 + 6y_3 \\
\text{z.p.} \quad & y_1 \geq 0 \\
& y_2 \leq 0 \\
& y_3 \in \mathbb{R} \\
& y_1 - y_2 + 2y_3 \leq 2 \\
& -y_1 + 2y_2 - y_3 \leq 0
\end{aligned}$$

$$\rightarrow -y_1 - 3y_2 - y_3 \leq -3$$
$$2y_3 \leq 1$$

b) $\min\{ \max_{i=1}^{n} |a_i - x| \mid x \in \mathbb{R}\}$

$\cancel{\max}$ max min $\cancel{z}$

$\min \quad z$

z.p. $\quad a_1 - x - z \leq 0$

$-(a_1 - x) - z \leq 0$

$\vdots$

$a_n - x - z \leq 0$

$-(a_n - x) - z \leq 0$

$x \in \mathbb{R}$

$\min \quad z_i$

z.p $\quad -x - z_i \leq -a_1$

$x - z_i \leq a_1$

$\vdots$

$-x - z_i \leq -a_n$

$x - z_i \leq a_n$

$\max \ (y_{1i} - y_{2i}) \cdot a_i$

$y_1 \leq 0$

$y_2 \leq 0$

$y_1 + y_2 = 1$

$y_1 - y_2 = 0$

g) ii) $\min_{x \in \mathbb{R}^n} \max_{i=1}^{m} (a_i^T x + b_i)$

$\min_{x \in \mathbb{R}^n} \max\{ a_1^T x + b_1, \ldots, a_m^T x + b_m \}$

$\min \ z$

$a_1^T x + b_1 \overset{-z}{\leq} 0$

$\vdots$

$a_m^T x + b_m \overset{-z}{\leq} 0$

$=$

$\min \ z$

$a_1^T x \overset{-z}{\leq} -b_1$

$\vdots$

$a_m^T x \overset{-z}{\leq} -b_m$

$=$

min z

$a^T x \leq b$

$\min \ z_i$

$a^T x - z_i \leq b$

$z_i \in \mathbb{R}$

$x \in \mathbb{R}$

$\max \quad y_i \cdot a_i + b_i$

$y \leq 0$

$a y = 1$