DU 5.

1.

$f(x) = e^{2x} \cos x$

$f'(x) = 2e^{2x} \cos x + e^{2x}(-\sin x) = e^{2x}(2\cos x - \sin x)$

$f''(x) = 2e^{2x}(2\cos x - \sin x) + e^{2x}(-2\sin x - \cos x) = e^{2x}(4\cos x - 2\sin x -$
$- 2\sin x - \cos x) = e^{2x}(3\cos x - 4\sin x)$

$f'''(x) = 2e^{2x}(3\cos x - 4\sin x) + e^{2x}(-3\sin x - 4\cos x) =$
$= e^{2x}(6\cos x - 8\sin x - 3\sin x - 4\cos x) = e^{2x}(2\cos x - 11\sin x)$

$$T_3^{f,a} = e^{2a}\cos a + \frac{e^{2a}(2\cos a - \sin a)}{1}(x-a) + \frac{e^{2a}(3\cos a - 4\sin a)}{2}(x-a)^2 +$$
$$+ \frac{e^{2a}(2\cos a - 11\sin a)}{6}(x-a)^3.$$

$$a = 0: \quad T_3^{f,0} = 1 + 2x + 1{,}5x^2 + \frac{2x^3}{6} = 1 + 2x + 1{,}5x^2 + \frac{x^3}{3}$$

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3.

a) $\lim\limits_{x\to 3} \dfrac{\ln(x^2-8)}{x^2-3x} = \left|\dfrac{0}{0}\right|$

$\xrightarrow{l'H} \lim\limits_{x\to 3} \dfrac{\frac{2x}{x^2-8}}{2x-3} = \dfrac{2x}{(x^2-8)(2x-3)} = \left|\dfrac{6}{3}\right| = 2$

b) $\lim\limits_{x\to 0} \dfrac{\sin ax}{\sin bx} = \left|\dfrac{0}{0}\right|$

$\xrightarrow{l'H} \lim\limits_{x\to 0} \dfrac{\cos ax}{\cos bx} = \lim\limits_{x\to 0} \dfrac{\cos 0}{\cos 0} = 1$

c) $\lim\limits_{x\to 0_+} \frac{\ln \sin ax}{\ln \sin bx} = \left|\frac{\ln 0}{\ln 0}\right|$

$\xrightarrow{L'H} \lim\limits_{x\to 0_+} \frac{\frac{\cos ax}{\sin ax}}{\frac{\cos bx}{\sin bx}} = \frac{\cos ax \cdot \sin bx}{\sin ax \cos bx} = \left|\frac{0}{0}\right|$

$\xrightarrow{L'H} \lim\limits_{x\to 0_+} \frac{-\sin ax \sin bx + \cos ax \cos bx}{\cos ax \cos bx + \sin ax \sin bx}$

$= \frac{0+1}{1+0} = 1$

d) $\lim\limits_{x\to\infty} \frac{2x+\sin x}{x+1} = \left|\frac{\infty + \text{omez}}{\infty}\right|$

$\xrightarrow{L'H} \lim\limits_{x\to\infty} \frac{2+\cos x}{1} = 2$

e) $\lim\limits_{x\to 0} x \operatorname{cotg} x = |0 \cdot ND| = \frac{x}{\operatorname{tg} x}$

~~$\xrightarrow{L'H} \lim\limits_{x\to 0} \operatorname{cotg} x + \frac{x(-1)}{\sin^2 x} = \left|ND + \frac{0}{0}\right|$~~

~~$\to \lim\limits_{x\to 0} \frac{-1}{\sin^2 x} + \frac{+1}{}$~~

~~L'H~~

$\xrightarrow{L'H} \lim\limits_{x\to 0} \frac{1}{\frac{1}{\cos^2 x}} = \cos^2 x = 1$

f) $\lim\limits_{x\to 0_+} x^x = |0^0| = e^{x \ln x} = e^{\frac{\ln x}{\frac{1}{x}}}$

$\xrightarrow{L'H} \lim\limits_{x\to 0_+} e^{\frac{\frac{1}{x}}{-x^{-2}}} = e^{\frac{\frac{1}{x}}{\frac{-1}{x^2}}} = e^{-\frac{x^2}{x}} = e^{x} = |e^0| = 1$

g) $\lim\limits_{x\to\infty} \sqrt[x]{x} = x^{\frac{1}{x}} = e^{\frac{1}{x}\ln x} = e^{\frac{\ln x}{x}}$

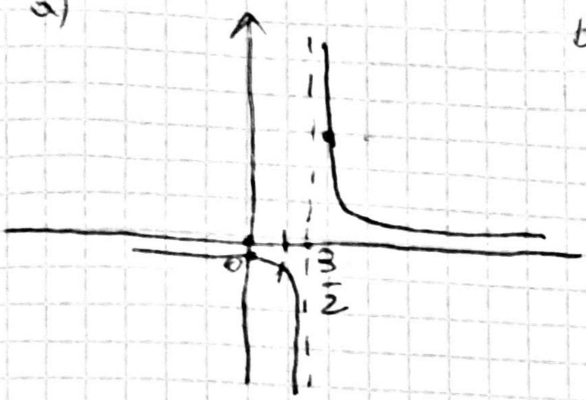
$\xrightarrow{L'H} e^{\lim\limits_{x\to\infty} \frac{\frac{1}{x}}{1}} = e^{\lim\limits_{x\to\infty} \frac{1}{x}} = e^0 = 1$

h) $\lim\limits_{x\to 1} x^{\frac{1}{1-x}} =$ ~~$|1^{\infty}|$~~ $e^{\frac{1}{1-x}\ln x} = \left|e^{\infty \cdot 0}\right|$

$\xrightarrow{L'H} e^{\lim\limits_{x\to 1} \frac{\frac{1}{x}}{-1}} = e^{\lim\limits_{x\to 1} \frac{1}{-x}} = e^{-1} = \frac{1}{e}$

4. $f(x)=\frac{x+1}{2x-3}$

a)



b) $f'(x)=\frac{(x+1)'(2x-3)-(x+1)(2x-3)'}{(2x-3)^2}=$

$=\frac{2x-3-(x+1)2}{4x^2-6x+9}=\frac{-3-2}{4x^2-6x+9}=$

$=\frac{-5}{(2x-3)^2}$

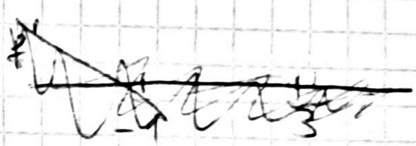
f': $-$ | $\frac{3}{2}$ | $-$

klesajici ~~f~~: $\left(-\infty;\frac{3}{2}\right),\left(\frac{3}{2};+\infty\right)$

5.

$g(x)=(x+1)\cdot|x-3|+1$

$g(x)=\begin{cases}(x+1)\cdot(x-3)+1, & x\geq 3\\ (x+1)\cdot(-(x-3))+1, & x<3\end{cases}$



$(x+1)(x-3)+1=0, \quad x\geq 3$

$x^2-3x+x-3+1=0$

$x^2-2x-2=0$

$D=4-4(-2)=12$

$x_1=\frac{-2+\sqrt{4\cdot3}}{2}=\frac{-2+2\sqrt{3}}{2}=$

$=-1+\sqrt{3}<3\to\otimes$

$x_2=\frac{-2-2\sqrt{3}}{2}=-1-\sqrt{3}<3\to\otimes$

$-(x+1)(x-3)+1=0, \quad x<3$

$-(x^2-2x-3)+1=0$

$-x^2+2x+3+1=0$

$-x^2+2x+4=0$

$D=4-4(-1)(4)=20$

$x_1=\frac{-2+\sqrt{20}}{-2}=\frac{-2+2\sqrt{5}}{-2}=+1-\sqrt{5}$ ✓

$x_2=\frac{-2-2\sqrt{5}}{-2}=+1+\underbrace{\sqrt{5}}_{>2}$ >3 $\otimes$

a)

$g'(x)=\begin{cases}2x-2, & x\geq 3\\ -2x+2, & x\leq 3\end{cases}$

$+$ | 1 | $-$ | 3 | $+$

rostouci: $(-\infty;1]$, $[3;+\infty)$

klesajici $(1;3)$

b) $\lim_{x\to3^+}f'(x)=2x-2=4$

$\lim_{x\to3^-}f'(x)=-2x+2=-4$