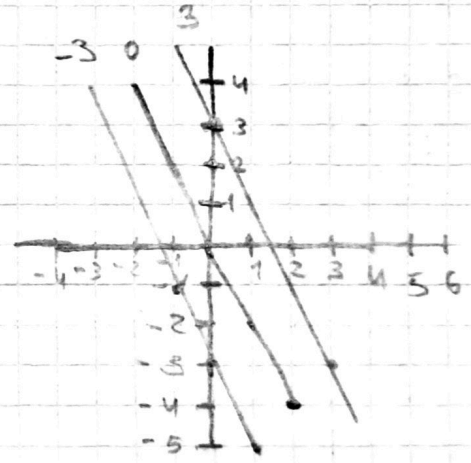
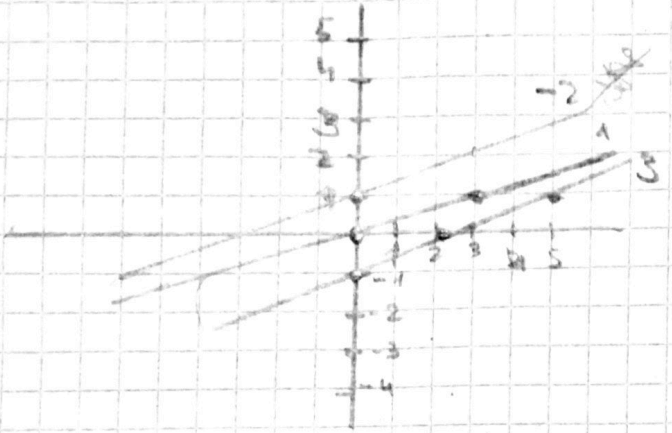
CVI07

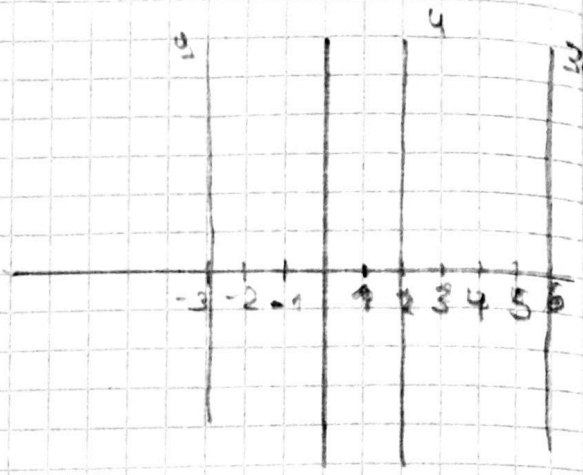
8.1 a) $f(x;y) = 2x + y$



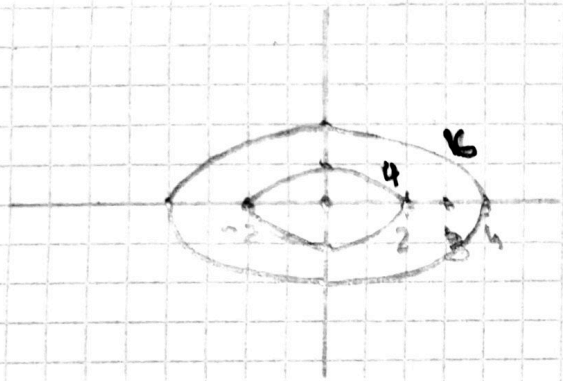
b) $f(x,y) = x - 3y + 1$



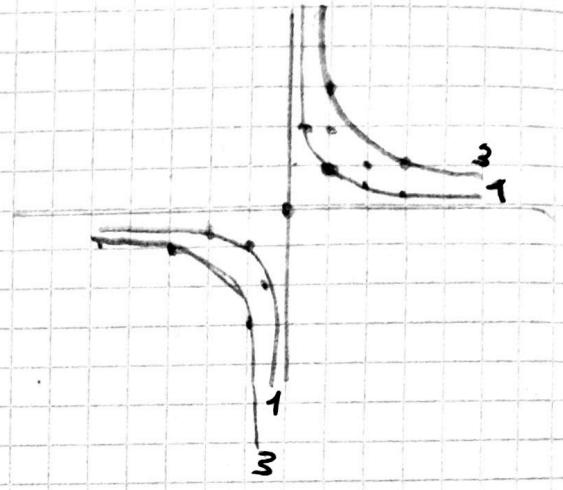
c) $f(x,y) = x^2$



d) $f(x,y) = x^2 + 4y^2$



f) $f(x,y) = xy$



8.3

d) $f(x,y) = \ln(1+xy)$

$f'(x,y) = \left(\frac{y}{1+xy}, \frac{x}{1+xy}\right)$

$x_0 = 1$

$y_0 = 2$ $\quad f'(1,2) = \left(\frac{2}{3}, \frac{1}{3}\right)$

g) $H = \begin{bmatrix} \frac{-y^2}{(1+xy)^2} & \frac{1+xy-xy}{(1+xy)^2} \\ \frac{1}{(1+xy)^2} & \frac{-x^2}{(1+xy)^2} \end{bmatrix}$ $\quad H(x_0,y_0) = \begin{bmatrix} \frac{-4}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{-1}{9} \end{bmatrix}$

8.10

$h(d,s) = 2s^2 + 3sd - d^2 + 5$

a) $\text{grad } h = (3s - 2d, 4s + 3d)$

$\frac{(5,1)}{\|(5,1)\|} = \frac{(5,1)}{\sqrt{26}} = (5/\sqrt{26}, 1/\sqrt{26})$

8.13 $f(x,y) = 6xy^2 - 2x^3 - 3y^3 \quad (x_0, y_0) = (1, -2)$

$T^0_{(x_0,y_0)} = f(x_0,y_0) = 46$

$$T^1_{(x_0,y_0)} = f(x_0,y_0) + f'(x_0,y_0)\begin{bmatrix} x-x_0 \\ y-y_0 \end{bmatrix} =$$

$$= 46 + \begin{bmatrix} 6y^2 - 6x^2 & 12xy - 3y^2 \end{bmatrix}_{x_0,y_0} \begin{bmatrix} x-x_0 \\ y-y_0 \end{bmatrix} =$$

$$= 46 + \begin{bmatrix} 18 & -60 \end{bmatrix} \begin{bmatrix} x-1 \\ y+2 \end{bmatrix} = 46 + 18x - 18 - 60y - 120$$

$$= 18x - 60y - 92$$

$$T^2_{(x_0,y_0)} = T^0_{(x_0,y_0)} + T^1_{(x_0,y_0)} + \frac{1}{2}\begin{bmatrix} x-1 \\ y+2 \end{bmatrix}^T f''(x_0,y_0) \begin{bmatrix} x-1 \\ y+2 \end{bmatrix}$$

$$= 18x - 60y - 46 + \frac{1}{2}\begin{bmatrix} x-1 \\ y+2 \end{bmatrix}^T \cdot \begin{bmatrix} -12x & 12y \\ 12y & 12x - 18y \end{bmatrix}_{x_0,y_0} \begin{bmatrix} x-1 \\ y+2 \end{bmatrix}$$

$$= 18x - 60y - 46 + \frac{1}{2}\begin{bmatrix} x-1 \\ y+2 \end{bmatrix}^T \begin{bmatrix} -12 & -24 \\ -24 & 48 \end{bmatrix} \begin{bmatrix} x-1 \\ y+2 \end{bmatrix} =$$

$$= 18x - 60y - 46 + \frac{1}{2} \begin{bmatrix} -12x + 12 - 24y - 48 \\ -24x + 24 + 48y + 96 \end{bmatrix}^T \begin{bmatrix} x-1 \\ y+2 \end{bmatrix}$$

$$= 18x - 60y - 46 + \begin{bmatrix} -6x - 12y - 18 \\ -12x + 24y + 60 \end{bmatrix}^T \begin{bmatrix} x-1 \\ y+2 \end{bmatrix} =$$

$$= 18x - 60y - 46 - 6x^2 - 12yx - 18x + 6x + 12y + 18 - 12xy +$$
$$+ 24y^2 + 60y - 24x + 48y + 120 =$$
$$= -6x^2 - 24xy + 24y^2 - 18x + 60y + 46$$