DU 8.

1 a) $\int \left(\frac{1-x}{x}\right)^2 dx = \int \left(\frac{1}{x}-1\right)^2 dx = \int \left(\frac{1}{x^2}-\frac{2}{x}+1\right) dx =$

$= \int x^{-2} dx - \int \frac{2}{x} dx + \int 1 \, dx = -x^{-1} - 2\ln|x| + x + c, \quad x \in R \setminus \{0\}$

b) $\int 2^x \, dx = \int e^{x \ln 2} dx = \frac{e^{x \ln 2}}{\ln 2} = \frac{2^x}{\ln 2} + c, \quad x \in R$

c) $\int (2^x + 3^x)^2 dx = \int (2^{2x} + 2^{x+1} 3^x + 3^{2x}) \, dx = \int e^{2x \ln 2} dx + \int e^{2x \ln 3} +$

$+ \int e^{(x+1)\ln 2 + x \ln 3} = \frac{2^{2x}}{2\ln 2} + \frac{3^{2x}}{2\ln 3} + \frac{2^{x+1} 3^x}{\ln 2 + \ln 3} =$

$= \frac{2^{2x}}{2\ln 2} + \frac{2 \cdot 6^x}{\ln 6} + \frac{9^x}{2\ln 3} =$

$= \frac{2^{2x}}{2\ln 2} + \frac{2 \cdot 6^x}{\ln 6} + \frac{9^x}{2\ln 3} + C, \; x \in R$

d) $\int \frac{1+x}{1-x}\,dx = \cancel{\int \frac{1-x+2x}{1-x} = \int 1\,dx + \int \frac{2x}{1-x}} = \int$

$= \int \frac{(x-1)+2}{1-x} = -\int 1\,dx + \int \frac{2}{1-x}\,dx = -\int 1\,dx + 2\int \frac{1}{1-x}\,dx =$

$= -x - 2\ln|1-x| + c, \quad x \in \mathbb{R} \setminus \{1\}$

2. a) $\int (x^2+x)\sin 2x\,dx = \left| \begin{matrix} x^2+x = u & v' = \sin 2x \\ u' = 2x+1 & v = \frac{-\cos 2x}{2} \end{matrix} \right| = \frac{(x^2+x)\cdot(-\cos 2x)}{2} +$

$+ \int (2x+1)\left(\frac{-\cos 2x}{2}\right) = \cancel{\begin{matrix} u = 2x+1 \\ u' = 2 \end{matrix}}$

$= \frac{(x^2+x)(-\cos 2x)}{2} - \frac{1}{2}\int (2x+1)(\cos 2x) = \left| \begin{matrix} u = 2x+1 & v' = \cos 2x \\ u' = 2 & v = \frac{\sin 2x}{2} \end{matrix} \right| =$

$= \frac{(x^2+x)(-\cos 2x)}{2} - \frac{1}{2}\left(\left(\frac{(2x+1)\cdot\sin 2x}{2}\right) + \int \frac{2\sin 2x}{2}\right) =$

$= \frac{(x^2+x)(-\cos 2x)}{2} - \frac{1}{2}\left(\frac{(2x+1)\cdot\sin 2x}{2} + \int \sin 2x\right) =$

$= \frac{(x^2+x)(-\cos 2x)}{2} - \frac{1}{2}\left(\frac{(2x+1)\cdot\sin 2x}{2} - \frac{\cos 2x}{2}\right) =$

$\frac{(x^2+x)(-\cos 2x)}{2} - \left(\frac{(2x+1)\sin 2x}{4} - \frac{\cos 2x}{4}\right) =$

$= \frac{-x^2\cos 2x - x\cos 2x - x\sin 2x}{2} - \frac{\sin 2x + \cos 2x}{4} + c, \quad x \in \mathbb{R}$

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b) $\int (3x^2-x)\ln 4x\,dx = \cancel{\left| \begin{matrix} u = 3x^2-x & v' = \\ u' = 6x-1 & \end{matrix} \right|} \; \cancel{\left| \begin{matrix} u = \ln 4x & v' = 3x^2-x \\ u' = \frac{4}{4x} = \frac{1}{x} & \end{matrix} \right|}$

$= \left| \begin{matrix} u = \ln 4x & v' = (3x^2-x) \\ u' = \frac{1}{x} & v = x^3 - \frac{x^2}{2} \end{matrix} \right| = \ln 4x\left(x^3 - \frac{x^2}{2}\right) - \int \frac{1}{x}\left(x^3 - \frac{x^2}{2}\right) =$

$= \ln 4x\left(x^3 - \frac{x^2}{2}\right) - \int\left(x^2 - \frac{x}{2}\right) = \ln 4x\left(x^3 - \frac{x^2}{2}\right) - \frac{x^3}{3} + \frac{x^2}{4} + c,$

$x \in (0; +\infty)$

c) $\int x\sin x\,dx = \left| \begin{matrix} u = x & v' = \sin x \\ u' = 1 & v = -\cos x \end{matrix} \right| = -x\cos x - \int -\cos x =$

$= -x\cos x + \int \cos x = -x\cos x + \sin x + C, \quad x \in \mathbb{R}$

d) $\int \cos^2 x \, dx = \left| \begin{matrix} u = \cos^2 x & v' = 1 \\ u' = 2\sin 2x & v = x \end{matrix} \right| = x\cos^2 x + \int 2x \sin 2x$

$= x\cos^2 x + 2\int x \sin 2x = \left| \begin{matrix} u = x & v' = \sin 2x \\ u' = 1 & v = -\frac{\cos 2x}{2} \end{matrix} \right| =$

$\cancel{= x\cos^2 x + 2(x(-\frac{\cos^2 x}{2}) + \int \cos x) = x\cos^2 x + -2\cos x =}$

$\cancel{= x\cos^2 x - 2x\cos x + 2\sin x + c, \ x \in \mathbb{R}}$

$= x\cos^2 x + 2\left(x\left(-\frac{\cos 2x}{2}\right) + 2\int \frac{\cos 2x}{2}\right) =$

$= x\cos^2 x + x(-\cos 2x) + 2\int \cos 2x =$

$= x\cos^2 x - x\cos 2x + \frac{\sin 2x}{2} + c, \quad x \in \mathbb{R}$

3.

a) $\int \sqrt[6]{2-3x} \, dx = \left| \begin{matrix} 2-3x = t \\ -3dx = dt \\ dx = \frac{dt}{-3} \end{matrix} \right| = -\frac{1}{3}\int t^{\frac{1}{6}} \, dt =$

$= -\frac{1}{3} t^{\frac{7}{6}} \cdot \frac{6}{7} = -\frac{2\sqrt[6]{(2-3x)^7}}{7} = -\frac{2(2-3x)\sqrt[6]{2-3x}}{7} + c, \quad x \in \mathbb{R}$

b) $\int \frac{1}{x}\sqrt{\ln x} = \left| \begin{matrix} t = \ln x \\ \frac{1}{x}dx = dt \\ dx = x\,dt \end{matrix} \right| = \int \sqrt{t} \, dt = \int t^{\frac{1}{2}} \, dt = t^{\frac{3}{2}} \cdot \frac{2}{3} =$

$= \frac{2\ln x\sqrt{\ln x}}{3} + c, \quad x \in [1, +\infty)$

c) $\int \sin x \cdot \cos^3 x \, dx = \left| \begin{matrix} \cos x = t \\ -\sin x \, dx = dt \\ dx = \frac{dt}{-\sin x} \end{matrix} \right| = -\int t^3 =$

$= \frac{t^4}{4} = \frac{-\cos^4 x}{4} + c, \quad x \in \mathbb{R}$

d) $\int \operatorname{ctg} 3x \, dx = \int \frac{\cos 3x}{\sin 3x} dx = \left| \begin{matrix} \sin 3x = t \\ 3\cos 3x \, dx = dt \\ dx = \frac{dt}{3\cos 3x} \end{matrix} \right| =$

$= \int \frac{1}{3t} \, dt = \frac{1}{3}\int t^{-1} \, dt = \frac{1}{3}\ln|t| = \frac{1}{3}\ln|\sin 3x| + c$

$x \in \mathbb{R}.$