DU 3

1.

Vztah stupňů

stupeň čitatele	< stupeň jmenovatele	0
-"-	= -"-	$\frac{a_1}{a_2}$?
-"-	> -"-	$\pm\infty$

$ax^n + bx^{n-1} \dots 2^{n-n}$

2.

Když $f(nb-\varepsilon) > 0$ a $f(nb+\varepsilon) > 0$, tak se značí $+$

Když $f(nb-\varepsilon) < 0$ a $f(nb+\varepsilon) < 0$, tak se značí $-$

Když $f(nb-\varepsilon) > 0$ a $f(nb+\varepsilon) < 0$, nebo

$f(nb-\varepsilon) < 0$ a $f(nb+\varepsilon) > 0$, limita neexistuje.

3.

(a) $\lim\limits_{x\to -1} \frac{x^2-3x+4}{3x^2+1} = \frac{1+3+4}{3+1} = \frac{8}{4} = 2.$

(b) $\lim\limits_{x\to 1} \frac{x^2-3x+2}{2x^2-2} = \frac{(x-1)(x-2)}{2(x^2-1)} = \frac{(x-1)(x-2)}{2(x-1)(x+1)} = \frac{(x-2)}{2(x+1)} = \frac{1-2}{2\cdot 2} = -\frac{1}{4}$

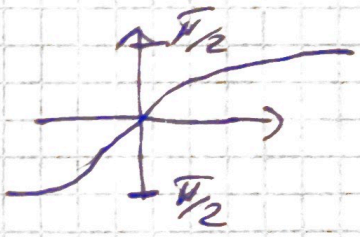
(c) $\lim\limits_{x\to 2} \frac{x^2+x+1}{x^2-3x+2} = \left|\frac{7}{0}\right| =$ neexistuje (oboustranná limita)

$\lim\limits_{x\to 2_+} \frac{x^2+x+1}{x^2-3x+2} = \frac{7}{0_+} = +\infty$

$\lim\limits_{x\to 2_-} \frac{x^2+x+1}{x^2-3x+2} = \frac{7}{0_-} = -\infty$

(d) $\lim\limits_{x\to -\infty} \frac{3x+2}{\sqrt{4x^2-1}+3} = \frac{x(3+2x^{-1})}{|x|\sqrt{4-x^{-2}}+3} = \frac{3x}{2|x|+3} = \frac{x\cdot 3}{|x|(2+x^{-1})} = \frac{-3}{2} = -\frac{3}{2}$

(e) $\lim\limits_{x\to\infty} \frac{\arctan x}{2x+3} = \left|\frac{\frac{\pi}{2}}{\infty}\right| = 0$



(f) $\lim\limits_{x\to -\infty} \frac{2+\sin x}{x-\cos x} = \frac{2}{x} = \left|\frac{2}{-\infty}\right| = 0$ (čitatel omezená, jmenovatel omezená)

(g) $\lim\limits_{x\to -\infty} 3x\sin 3x = |-\infty \cdot \text{om}| =$ neex

(h) $\lim\limits_{x\to\infty} (3x+\cos 2x) = |\infty + \text{omez.}| = \infty$

(i) $\lim\limits_{x\to\infty} \arccos\left(\frac{1-x}{1+x}\right) = (g\circ f)(x) = \pi$

$f: \lim\limits_{x\to\infty} \frac{1-x}{1+x} = \frac{x(x^{-1}-1)}{x(x^{-1}+1)} = \frac{-1}{+1} = -1$

$g: \lim\limits_{x\to -1} \arccos(-1) = \pi$

(j) $\lim_{x \to -\infty} \arcsin\left(\frac{1-x}{1+x}\right) = -\frac{\pi}{2}$ //

$\lim_{x \to -\infty} \frac{1-x}{1+x} = \frac{x(x^{-1}-1)}{x(x^{-1}+1)} = \frac{-1}{1} = -1$

$\lim_{x \to -1} \arcsin(x) = -\frac{\pi}{2}$

4. $f(x) = \frac{\cos 5x}{1-2^x}$ $\quad D(f) = (-\infty; 0) \cup (0; +\infty)$

$\lim_{x \to -\infty} \frac{\cos 5x}{1-2^x} = \left| \frac{\text{omez}}{1 - \frac{1}{2^\infty}} \right| =$ neexistuje //

$\lim_{x \to \infty} \frac{\cos 5x}{1-2^x} = \left| \frac{\text{omez}}{-\infty} \right| = 0$ //

5. Limita v první polovině není definována

6. (a) $(x^5 - 7x^3 + 3x + \pi)' = 5x^4 - 21x^2 + 3$ //

(b) $\left(\frac{1}{x^3} + 7\sqrt[4]{x}\right)' = \left(x^{-3} + 7x^{\frac{1}{4}}\right)' = -3x^{-4} + \frac{7}{4}x^{-\frac{3}{4}} = \frac{-3}{x^4} + \frac{7}{4\sqrt[4]{x^3}}$ //

(c) $(5e^{3x} - 2\sin 5x) = 15e^{3x} - 10\cos 5x$ //

(d) $(\cosh x)' = \sinh x$ //