DU 9.

1. a) $$\frac{3x^2+2x-9}{x^2+x-2} = \frac{3(x^2+x-2)-x-3}{x^2+x-2} = \frac{3(x^2+x-2)}{x^2+x-2} - \frac{x+3}{x^2+x-2} =$$

$$= 3 - \frac{x+3}{(x-1)(x+2)} = 3 - \frac{A}{(x-1)} - \frac{B}{(x+2)}$$

$$A: \frac{x+3}{\cancel{(x-1)}(x+2)} = \frac{1+3}{1+2} = \frac{4}{3}$$

$$B: \frac{x+3}{(x-1)\cancel{(x+2)}} = \frac{-2+3}{-2-1} = \frac{1}{-3}$$

$$\frac{3x^2+2x-9}{x^2+x-2} = 3 - \frac{4}{3(x-1)} + \frac{1}{3(x+2)}$$

b) $$\frac{1}{(x+1)(x+2)^2(x+3)^3} = \frac{A}{(x+1)} + \frac{B}{(x+2)^2} + \frac{C}{(x+3)^3} + \frac{D}{(x+2)} + \frac{E}{(x+3)^2} + \frac{F}{(x+3)}$$

$$A: \frac{1}{\cancel{(x+1)}(x+2)^2(x+3)^3} = \frac{1}{(-1+3)^3} = \frac{1}{8}$$

$$B: \frac{1}{(x+1)\cancel{(x+2)^2}(x+3)^3} = \frac{1}{(-2+1)(-2+3)^3} = -1$$

$$C: \frac{1}{(x+1)(x+2)^2\cancel{(x+3)^3}} = \frac{1}{(-3+1)(-3+2)^2} = -\frac{1}{2}$$

$$1 = A(x+2)^2(x+3)^3 + B(x+1)(x+3)^3 + C(x+1)(x+2)^2 + D(x+1)(x+2)(x+3)^3 + E(x+1)(x+2)^2(x+3) + F(x+1)(x+2)^2 \cdot (x+3)^2$$

$$\begin{aligned}1 = {} & A(x^5+13x^4+67x^3+171x^2+216x+108)+\\ & B(x^4+10x^3+36x^2+54x+27)+\\ & C(x^3+5x^2+8x+4)+\\ & D(x^5+12x^4+56x^3+126x^2+135x+54)+\\ & E(x^4+8x^3+23x^2+28x+12)+\\ & F(x^5+11x^4+47x^3+97x^2+96x+36)\end{aligned}$$

x^5: $A+D+F=0$

x^4: $13A+B+12D+E+11F=0$

x^3: $67A+10B+C+56D+8E+47F=0$

x^2: $171A+36B+5C+126D+23E+97F=0$

x: $216A+54B+8C+135D+28E+96F=0$

x^0: $108A+27B+4C+54D+12E+36F=1$

c) $\dfrac{3x^2+x+2}{(x+1)(x^2+x+2)} = \dfrac{3(x^2+x+2)-2x-4}{(x+1)(x^2+x+2)} = \cancel{\dfrac{3}{x+1} - \dfrac{2x+4}{(x+1)(x^2+x+2)}} =$

$$= \frac{A}{(x+1)} + \frac{Bx+C}{x^2+x+2}$$

$A: \dfrac{3x^2+x+2}{\cancel{(x+1)}(x^2+x+2)} = \dfrac{3-1+2}{1-1+2} = \dfrac{4}{2} = 2$

$3x^2+x+2 = A(x^2+x+2) + (Bx+C)(x+1)$

$3x^2+x+2 = (A+B)x^2 + (A+B+C)x + (2A+C)$

$$\left.\begin{aligned} A+B&=3 \\ A+B+C&=1 \\ 2A+C&=2 \end{aligned}\right\} \quad \begin{aligned} A&=2 \\ B&=1 \\ C&=-2 \end{aligned}$$

$$\frac{3x^2+x+2}{(x+1)(x^2+x+2)} = \frac{2}{(x+1)} + \frac{x-2}{x^2+x+2}$$

d) $$\frac{x^3-x^2+4x+1}{(x^2+2)^2} = \frac{Ax+B}{(x^2+2)^2} + \frac{Cx+D}{(x^2+2)}$$

$$x^3-x^2+4x+1 = Ax+B+(Cx+D)(x^2+2) = Ax+B+Cx^3+2Cx+$$
$$+Dx^2+2D = Cx^3+Dx^2+(A+2C)x+(B+2D)$$

$C=1$; $D=-1$; $A=2$; $B=3$

$$\frac{x^3-x^2+4x+1}{(x^2+2)^2} = \frac{2x+3}{(x^2+2)^2} + \frac{x-1}{(x^2+2)}$$

2\. a) $$\int\frac{3x^2+2x-9}{x^2+x-2} = \int 3dx - \int\frac{4}{3(x-1)}dx + \int\frac{1}{3(x+2)}dx =$$
$$= 3\int dx - \frac{4}{3}\int\frac{1}{x-1} + \frac{1}{3}\int\frac{1}{x+2} = 3x - \frac{4}{3}\ln|x-1| + \frac{1}{3}\ln|x+2|$$
$$+C, \quad x\neq 1, \; x\neq -2$$

3\. a) $$\int\frac{3x+6}{2x^2+8x+9}dx =$$

$$\int\frac{3x+6}{2x^2+8x+9} = \int\frac{6x+8}{2(2x^2+8x+9)} + \int\frac{6-12}{2x^2+8x+9} =$$

$$\# \int\frac{3x+8}{2x^2+8x+9}$$

a) $\int \frac{3x+6}{2x^2+8x+9}\,dx = \int \frac{3(x+2)}{2x^2+8x+9} = \left| \begin{array}{l} t=2x^2+8x+9 \\ dt = 4x+8 \\ dt = 4(x+2) \\ \frac{3}{4}dt = 3(x+2) \end{array} \right| = \int \frac{3}{4}\cdot\frac{dt}{t}$

$= \frac{3}{4}\cdot\int\frac{dt}{t} = \frac{3}{4}\ln|2x^2+8x+9| + C, \quad x\in\mathbb{R}$

b) $\int \frac{x}{(x^2+1)^3}\,dx$

$\cancel{\frac{x}{(x^2+1)^3} = \frac{Ax+B}{(x^2+1)^3} + \frac{Cx+D}{(x^2+1)^2} + \frac{Ex+F}{(x^2+1)}}$

$\cancel{Ax+B}$

(x^4+2x^2+1)

$\cancel{x = Ax+B+(Cx+D)(x^2+1)+(Ex+F)(x^2+1)^2}$

$\cancel{x = Ax+B+Cx^3+Cx+Dx^2+D+Ex^5+2Ex^3+Ex+Fx^4+2Fx^2+F}$

$\int \frac{x}{(x^2+1)^3}\,dx = \left| \begin{array}{l} t=x^2+1 \\ dt = 2x \\ \frac{dt}{2} = x \end{array} \right| = \int \frac{dt}{2t^3} = \frac{1}{2}\int\frac{dt}{t^3} = \frac{1}{2}\int t^{-3} =$

$= \frac{1}{2}\cdot\frac{t^{-2}}{-2} = -\frac{1}{4(x^2+1)^2} + C, \quad x\in\mathbb{R}$

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c) $\int \frac{4x+2}{(x^2+x+3)^2}\,dx = \left| \begin{array}{l} t=x^2+x+3 \\ dt = 2x+1 \\ 2dt = 4x+2 \end{array} \right| = \int\frac{4\,dt}{t^2} = 4\int\frac{dt}{t^2} =$

$= 4\int t^{-2}\,dt = -4t^{-1} = -4(x^2+x+3)^{-1} = -\frac{4}{(x^2+x+3)} + C$

$x\in\mathbb{R}.$

d) $\int \frac{1}{x^2-2x+10}\,dx = \int\frac{1}{(x-1)^2+9} = \int \frac{1}{9\left(\frac{(x-1)^2}{9}+1\right)} =$

$= \int \frac{1}{9}\,\frac{1}{\frac{(x-1)^2}{9}+1} = \frac{1}{9}\int\frac{1}{\left(\frac{x-1}{3}\right)^2+1} =$

$$= \frac{1}{9}\int \frac{1}{\left(\frac{x-1}{3}\right)^2+1} = \left| \begin{array}{l} t = \left(\frac{x-1}{3}\right) \\ dt = \frac{3}{9} = \frac{1}{3} \\ 3dt = 1 \end{array} \right| = \frac{1}{9}\int \frac{3\,dt}{t^2+1} = \frac{1}{3}\int \frac{1\cdot dt}{t^2+1} =$$

$$= \frac{\operatorname{arctg}\left(\frac{x-1}{3}\right)}{3}, \quad x \in \mathbb{R}$$

e) $$\int \frac{5x+1}{x^2-2x+5}\,dx = \left| \begin{array}{l} \cancel{t = x^2-2x+5} \\ \cancel{dt = 2x-2} \end{array} \right.$$

$$= \int \frac{5x-5}{x^2-2x+5}\,dx + \int \frac{6}{x^2-2x+5} = 5\int \frac{x-1}{x^2-2x+5}\,dx + 6\int \frac{1}{x^2-2x+5}$$

$$= \left| \begin{array}{l} t = x^2-2x+5 \\ dt = 2x-2 \\ \frac{dt}{2} = x-1 \end{array} \right| = \frac{5}{2}\int \frac{1}{t}\,dx + 6\int \frac{1}{x^2-2x+5} = \frac{5}{2}\ln|x^2-2x+5| +$$

$$+ 6\int \frac{1}{x^2-2x+5} = 2{,}5\ln|x^2-2x+5| + 6\int \frac{1}{(x-1)^2+4} =$$

$$= 2{,}5\ln|x^2-2x+5| + \frac{6}{4}\int \frac{1}{\left(\frac{x-1}{2}\right)^2+1} = \left| \begin{array}{l} t = \frac{x-1}{2} \\ dt = \frac{2}{4} \\ \frac{4}{2}dt = 1 \end{array} \right| =$$

$$= 2{,}5\ln|x^2-2x+5| + \frac{6\cdot 4}{4\cdot 2}\int \frac{1\,dt}{t^2+1} = 2{,}5\ln|x^2-2x+5| + 3\cdot$$

$$\cdot \operatorname{arctg}\left(\frac{x-1}{2}\right) =$$

$$= 2{,}5\ln|x^2-2x+5| + 3\operatorname{arctg}\left(\frac{x-1}{2}\right) + c, \quad x \in \mathbb{R}$$