# DU 1

1. a) $x^2+6x=-16$

$x^2+6x+16=0$

$D=36-4(16)=-28$

$x_{1,2}=\frac{-6\pm\sqrt{-28}}{2}=-3\pm\sqrt{7}i$

Odp: $x_{1,2}=-3\pm\sqrt{7}i$

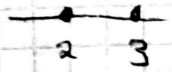
b) $x^2-5x\leq -6$

$x^2-5x+6\leq 0$

$D=25-4(6)=1$

$x_1=\frac{5+1}{2}=3$

$x_2=\frac{4}{2}=2$




Odp: $x\in[2;3]$

c) $\frac{x^3-2x^2-3x}{x+1} \geq 0$

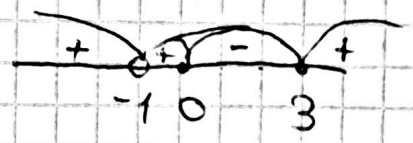
$\frac{x(x^2-2x-3)}{x+1} = 0$

$x^2-2x-3=0$

$D = 4-4(-3) = 4^2$

$x_1 = \frac{2+4}{2} = 3 \quad x_3 = 0$

$x_2 = \frac{2-4}{2} = -1 \quad x_4 : x \neq -1$




Odp: $(-\infty; -1) \cup (-1; 0] \cup [3; +\infty)$

2. $x^2+6x+9+4 = (x+3)^2+4$

Odp: $(x+3)^2+4$

3.

a) $2\mathbb{Z} = \{2 \cdot k \mid k \in \mathbb{Z}\}$

Odp: $\max 2\mathbb{Z}$ : nieex. $\quad \sup 2\mathbb{Z} : +\infty$

$\min 2\mathbb{Z}$ : nieex $\quad \inf 2\mathbb{Z} : -\infty$

b) $M = \{r \in \mathbb{R} \mid r^2 \leq 2 \ \& \ |r| = r\} \cap \mathbb{Q}$

$\begin{cases} r^2 \leq 2 \\ |r| = r \end{cases} \Rightarrow \begin{cases} r \in [-\sqrt{2}; \sqrt{2}] \\ r \geq 0 \end{cases} \Rightarrow \{ r \in [0; \sqrt{2}]$

$M = [0; \sqrt{2}] \cap \mathbb{Q} = [0; \sqrt{2})$

Odp: $\max M$ : nieex $\quad \sup M : \sqrt{2}$

$\min M : 0 \quad \inf M : 0$

c) $N = \{\frac{n+1}{n+2} \mid n \in \mathbb{N}\}$

Odp: $\max N$ : nieex $\quad \sup N : +\infty$

$\min N$ : nieex $\quad \inf N : 0$

4. $\sqrt{7} \in \mathbb{I}$ sporem (předpoklad $\sqrt{7}$ je racionální) spor

$\sqrt{7} = \frac{a}{b}$, $a, b$ jsou nesoudělná, $a, b \in \mathbb{N}$

$7b^2 = a^2$ dělitelné 7, $a$ je dělitelné 7 $a = 7c$, $c \in \mathbb{N}$

$7b^2 = (7c)^2$

$7b^2 = 49c^2$

$b^2 = 7c^2$ dělitelné 7, $b$ je dělitelné 7

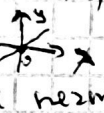
$a, b$ soudělná

□

5. $f$ je rostoucí funkce

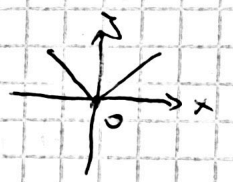
a) $-3f$ je klesající funkce (změna znaménka)

b) $5f$ je rostoucí funkce (znaménko se nezměnilo)



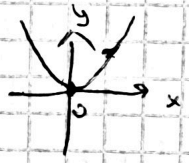
c) $|f|$ klesá na úseku $(-\infty; 0]$

roste na úseku $[0; +\infty)$

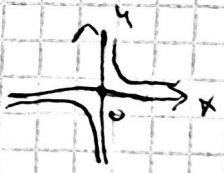


d) $f^2$ klesá na úseku $(-\infty; 0]$

roste na úseku $[0; +\infty)$



e) $\frac{1}{2f}$ je klesající funkce na úseku $(-\infty; 0) \cup (0; +\infty)$



6. a) $f(x) = \frac{2-x}{x^2-11}$

$x^2 - 11 \neq 0$

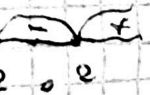
$x \neq \pm\sqrt{11}$

Odp: $D(f) = \mathbb{R} - \{\pm\sqrt{11}\}$

b) $g(x) = \sqrt{x^2-4} + \sqrt{3x^2+7} + \frac{1}{7}\sqrt{2-x}$

$$\begin{cases} x^2 - 4 \geq 0 \\ 3x^2 + 7 \geq 0 \\ 2 - x \geq 0 \end{cases} \quad \begin{cases} x^2 - 4 \geq 0 \\ 2 - x \geq 0 \end{cases} \quad \begin{cases} (x-2)(x+2) \geq 0 \\ 2 - x \geq 0 \end{cases}$$

$$\begin{cases} (x-2)(x+2) \geq 0 \\ x \leq 2 \end{cases} \quad ;$$



$$\begin{cases} x \leq -2 \\ x \geq 2 \\ x \leq 2 \end{cases} \quad \begin{cases} x = 2 \end{cases}$$

Odp: $D(g) = \{2\}$

c) $h(x) = \log(1 - \log(x^2 - 5x + 16))$

$\begin{cases} 1 - \log(x^2 - 5x + 16) > 0 \\ x^2 - 5x + 16 > 0 \end{cases}$ $\begin{cases} \log 10 > \log x^2 - 5x + 16 \end{cases}$

$x^2 - 5x + 16 > 0$

$D = 25 - 4(16)$, nemá řeš v $\mathbb{R}$

$x^2 - 5x + 16$ vždy větší než 0

$x^2 - 5x + 16 < 10$

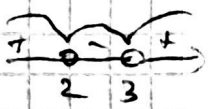
$x^2 - 5x + 6 < 0$

$D = 25 - 4(6) = 1$

$x_1 = \frac{5+1}{2} = 3$

$x_2 = \frac{5-1}{2} = 2$

$(x-3)(x-2) < 0$




$x \in (2; 3)$

Odp: $D(h) = (2; 3)$

d) $j(x) = \arccos\sqrt{x + \frac{1}{2}}$

$-1 \leq \sqrt{x + \frac{1}{2}} \leq 1$

$x + \frac{1}{2} \leq 1 \; ; \; x + \frac{1}{2} \geq 0$

$x \leq \frac{1}{2} \; ; \; x \geq -\frac{1}{2}$

$x \in \left[-\frac{1}{2}; \frac{1}{2}\right]$

Odp: $D(j) = \left[-\frac{1}{2}; \frac{1}{2}\right]$

e) $k(x) = \ln|\cos x|$

$|\cos x| > 0$

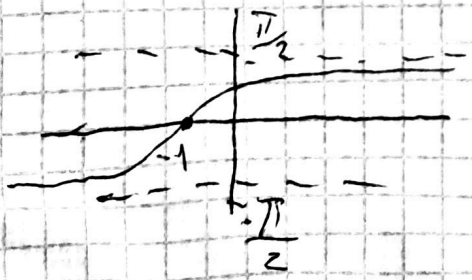
$\cos x \neq 0$

$x \neq \frac{\pi}{2} + \pi k, \; k \in \mathbb{Z}$

Odp: $D(k) = \mathbb{R} - \left\{k \in \mathbb{Z} \mid \frac{\pi}{2} + \pi k\right\}$

f) $l(x) = (\arctan(x+1))^{-\frac{1}{x}} = e^{-\frac{1}{x}\ln\arctan(x+1)}$

$\begin{cases} x \neq 0 \\ \arctan(x+1) > 0 \end{cases}$ $\begin{cases} x \neq 0 \\ x > -1 \end{cases}$




Odp: $D(l) = (-1; +\infty)$