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```
(define (is-simplified? expr)
  (if (constant? expr)
      #t
      (and (noconstant-arith? expr) (nozeros? expr) (nomult1? expr))
  )
)
```

$(\text{plus-simp } 2 \ 5) = (+ \ 2 \ 5) = 7$
 $(\text{eval } (\text{plus-simp } a \ b) \ \text{env}) = (\text{eval } (+ \ a \ b) \ \text{env})$
case 1:
let c1 be a constant int
let c2 be a constant int
let $(+ \ c1 \ c2) = c3$
 $(\text{eval } (\text{plus-simp } c1 \ c2) \ \text{env})$
by def:
 $= (\text{eval } (+ \ c1 \ c2) \ \text{env})$
 $= c3$
 $(\text{eval } (+ \ c1 \ c2) \ \text{env})$
 $= c3$

let c2 be any value
then $(\text{eval } c2 \ \text{env}) = c2$
and $(+ \ c2 \ 0) = c2$
 $(\text{eval } (\text{plus-simp } 0 \ c2) \ \text{env})$
 $= (\text{eval } c2 \ \text{env})$
 $= c2$
 $(\text{eval } (+ \ 0 \ c2) \ \text{env})$
 $= c2$

Question 2:

$(\text{arith-eval } (\text{arith-simp } \text{expr}) \ \text{env}) = (\text{arith-eval } \text{expr} \ \text{env})$
 $\text{expr} = (* \ E1 \ E2)$

```
(arith-simp expr)
= (arith-simp (* E1 E2))
= (mult? (* E1 E2))
= (let ([simpexpr1 (arith-simp (op1 expr))] [simpexpr2 (arith-simp (op2 expr))])
    (mult-simp simpexpr1 simpexpr2))
= (mult-simp (arith-simp (op1 expr)) (arith-simp (op2 expr)))
= (multi-simp (arith-simp E1) (arith-simp E2))
= (multi-simp (constant? E1) (constant? E2))
= (multi-simp E1 E2)
= (make-mult expr1 expr2)
```

= (* E1 E2)

Question 3:

Prove that (is-simplified? (arith-simp expr)) = #t

1. Prove k=0, 1 case is true

assume expr is constant:

```
(arith-simp expr)
= (arith-simp constant)
= (constant? constant)
= constant
(is-simplified? constant)
= (constant? constant) = #t
```

assume expr is variable:

```
(arith-simp expr)
= (arith-simp variable)
= (variable? variable)
= variable
(is-simplified? variable)
= (constant? variable) = #f
= (and (noconstant-arith? expr) (nozeros? expr) (nomult1? expr)))
= (and (variable? expr) (variable? expr) (variable? expr))
= (and #t #t #t) = #t
```

2. Assume k=n case is true

expr is (+ E1 E2):

```
(arith-simp expr)
= (plus? expr) => #t
= (let ([simpexpr1 (arith-simp (op1 expr))] [simpexpr2 (arith-simp (op2 expr))])
    (plus-simp simpexpr1 simpexpr2))
= (plus-simp simpexpr1 simpexpr2)
= (make-plus expr1 expr2)
= (+ E1 E2) = E3
```

```
= (is-simplified? expr)
= (is-simplified E3)
= (constant? expr) = #t
```

expr is (* E1 E2):

```
(arith-simp expr)
form previous steps
= expr = E3
```

```
= (is-simplified? expr)
= (constant? expr) = #t
```

3. Prove $k=n+1$ case is true

expr is $(+ (+ E1 E2) E3)$:

```
(arith-simp expr)
= (arith-simp (+ (+ E1 E2) E3))
= (plus? expr)
= (let ([simpexpr1 (arith-simp (op1 expr))] [simpexpr2 (arith-simp (op2 expr))])
      (plus-simp simpexpr1 simpexpr2))
= (plus-simp (arith-simp (+ E1 E2)) (arith-simp E3))
from previous
= (plus-simp (arith-simp E3) (arith-simp E3))
= E6 = expr
= (is-simplified? expr)
= (constant? expr) = #t
```

expr is $(* (+ E1 E2) E3)$:

```
(arith-simp expr)
= (arith-simp (* (+ E1 E2) E3))
= (mult? expr)
= (let ([simpexpr1 (arith-simp (op1 expr))] [simpexpr2 (arith-simp (op2 expr))])
      (mult-simp simpexpr1 simpexpr2))
= (mult-simp (arith-simp (+E1 E2)) (arith-simp E3))
from preious
= (mult-simp E3 E3)
= E9 = expr
= (is-simplified? expr)
= (constant? expr) = #t
```