

CS 270 Lab 8 (Recursion and Induction)

Week 8 – Nov. 13 – Nov. 17, 2017.

Name 1: _____

Drexel Username 1: _____

Name 2: _____

Drexel Username 2: _____

Name 3: _____

Drexel Username 3: _____

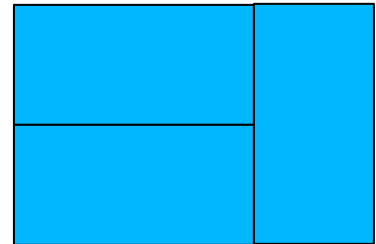
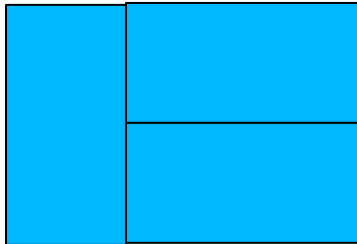
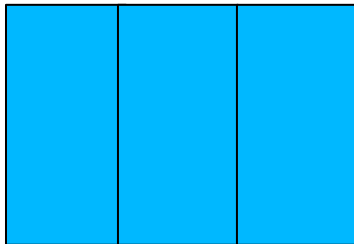
Instructions: For this exercise you are encouraged to work in groups of two or three so that you can discuss the problems, help each other when you get stuck and check your partners work. There are four problems that ask you to form conjectures and to prove your conjectures using induction. The last problem requires a recursive construction and inductive proofs based on the construction.

1. Create a table of values for n^2 and 2^n and verify that eventually $n^2 < 2^n$. Use induction to prove $n^2 < 2^n$, $\forall n \geq 5$.

2. Create a table of values for $n!$ and 2^n and verify that eventually $2^n < n!$. Use induction to prove $2^n < n!$, $\forall n \geq 4$.

3. Use induction to prove that any postage amount greater than 11 can be created using only 4 and 5 cent stamps. First determine which values upto 15 can be done with 4 and 5 cent stamps. How many base cases do you need? What if instead you use 4 and 6 cent stamps?

4. Given a set of 1×2 dominoes, how many different ways are there to tile and $2 \times n$ rectangle placing a single domino vertically (2×1) or stacking two dominoes horizontally? For example, for $n=3$, there are 3 ways.



Think recursively: What are the base cases? How do you solve the $2 \times n$ using solutions to smaller problems?

- Derive a recurrence relation for $D(n)$ the number of tilings of a $2 \times n$ rectangle.
- Produce a table of values of $D(n)$ and compare to 2^n and $(3/2)^n$
- Formulate a conjecture and use induction to prove your result.