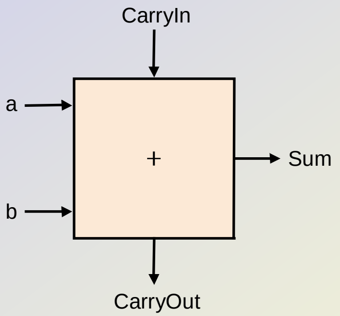
Lab 3 Solution
(CS 270 – Fall 2016)

- Derive a truth table for the output bits (Sum and CarryOut) of a full adder.
- Using the truth table derive a sum of products expression for Sum and CarryOut. Draw a circuit for these expressions.
- Express Sum and CarryOut using the parity and majority functions
  - parity(x_1,…,x_n) = 1 if an odd number of x_is are 1
  - majority(x_1,…,x_n) = 1 if a majority of x_is are 1
- Using properties of Boolean algebra simplify your expressions. Use xor for Sum. Draw the simplified circuits.

Full Adder

- Used to add to binary numbers stored as an array of bits using carry ripple addition



- Three binary inputs
  - a, b and CarryIn
- Two binary outputs
  - Sum and CarryOut such that a + b + CarryIn = 2*CarryOut + Sum

Carry	110
A	101
B	111
A+B =	1100

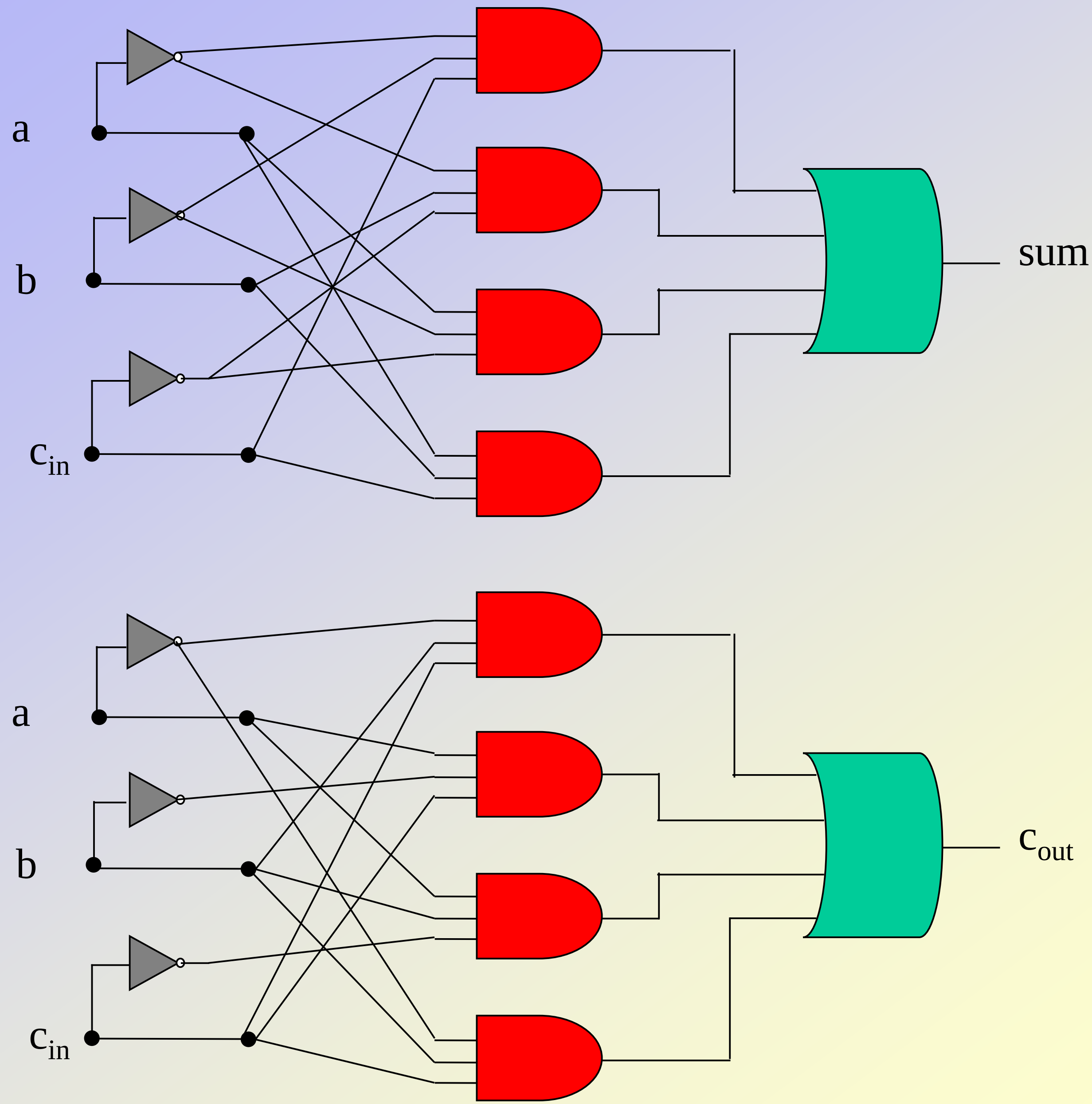
Solution

❖ Derive a truth table for the output bits (Sum and CarryOut) of a full adder.

a	b	CarryIn	Sum	CarryOut
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$s = \bar{a}\bar{b}c_i + \bar{a}b\overline{c_i} + a\bar{b}\overline{c_i} + abc_i$$
$$c_o = \bar{a}bc_i + a\bar{b}c_i + ab\overline{c_i} + abc_i$$

Note that s = parity(a,b,c_i), c_o= majority(a,b,c_i)


a
b
c_{in}
sum
a
b
c_{in}
c_{out}

Solution

- Simplification of carry out

$$c_o = \bar{a}bc_i + a\bar{b}c_i + ab\overline{c_i} + abc_i$$
$$c_o = bc_i(\bar{a} + a) + ac_i\left(\bar{b} + b\right) + ab(\overline{c_i} + c_i)$$
$$c_o = bc_i1 + ac_i1 + ab1$$
$$c_o = bc_i + ac_i + ab$$

- Note the simplified expression has 5 operations. It can be simplified further (4 operations)

$$c_o = (b + a)c_i + ab$$

Solution

- Simplification of sum

$$s = \bar{a}\bar{b}c_i + \bar{a}b\overline{c_i} + a\bar{b}\overline{c_i} + abc_i$$
$$s = (\bar{a}b + a\bar{b})\overline{c_i} + (ab + \bar{a}\bar{b})c_i$$
$$s = (a \oplus b)\overline{c_i} + \overline{(a \oplus b)}c_i$$
$$s = (a \oplus b \oplus c_i)$$

- That $\left(ab + \bar{a}\bar{b}\right) = \overline{(a \oplus b)}$ easily follows from the truth table. It can also be derived using Boolean algebra (next slide)

Solution

$$\overline{(a \oplus b)} = \overline{(\bar{a}b + a\bar{b})}$$

$$= \overline{\bar{a}b} \cdot \overline{a\bar{b}}$$

$$= (\bar{\bar{a}} + \bar{b}) \cdot (\bar{a} + \bar{\bar{b}})$$

$$= (a + \bar{b}) \cdot (\bar{a} + b)$$

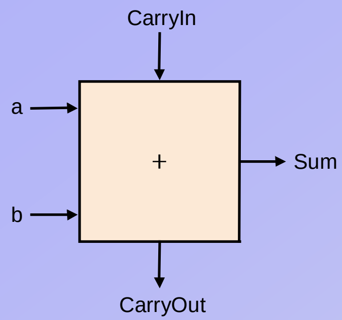
$$= a(\bar{a} + b) + \bar{b}(\bar{a} + b)$$

$$= a\bar{a} + ab + \bar{b}\bar{a} + \bar{b}b$$

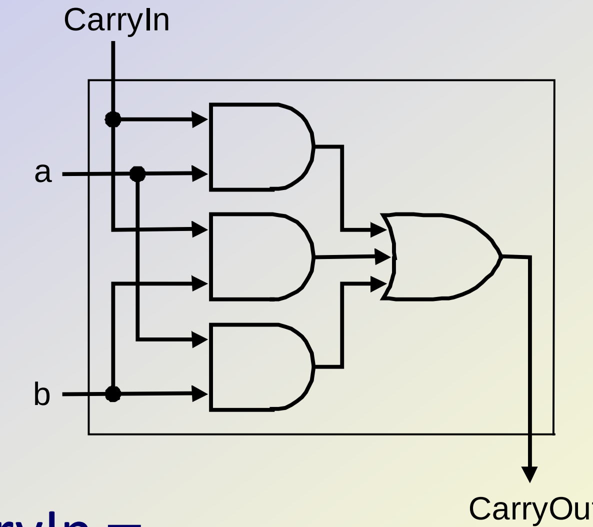
$$= 0 + ab + \bar{b}\bar{a} + 0$$

$$= ab + \bar{b}\bar{a}$$

$$= ab + \bar{a}\bar{b}$$



Full Adder



- Sum = parity(a,b,CarryIn)
  - $a \oplus b \oplus c + a \bullet b \bullet c \equiv a \oplus b \oplus c$
- CarryOut = majority(a,b,CarryIn)
  - $b \bullet CarryIn + a \bullet CarryIn + a \bullet b + a \bullet b \bullet CarryIn \equiv$
  - $b \bullet CarryIn + a \bullet CarryIn + a \bullet b$

a	b	CarryIn	Sum	CarryOut
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1