CS 270 Lab 6 (Boolean Algebra and Natural Deduction)

Week 6 - Oct. 30 – Nov. 3, 2017.

Name 1:____

Drexel Username 1:____

Name 2:____

Drexel Username 2:____

Name 3:____

Drexel Username 3:____

Grading:

Question 1 (20pts) ____

Question 2 (20pts) ____

Question 3 (20pts) ____

Question 4 (20pts) ____

Question 5 (20pts) ____

Instructions: For this exercise you are encouraged to work in groups of two or three so that you can discuss the problems, help each other when you get stuck and check your partner's work. There are five problems relating to Boolean Algebra and Natural Deduction. The objective of the lab is to get proficient with direct and indirect proofs in natural deduction and to present your proofs formally so that they can be verified by a proof checker. You will use the proof checker at http://proofs.openlogicproject.org The problems relate to the laws of Boolean algebra and the lab will help reinforce that material.

All questions must be done using the proof checker at http://proofs.openlogicproject.org The proof checker can be run from your browser, however, you cannot save your proofs. So After successfully completing a proof you should document it by either using screen capture or printing to pdf.

Before beginning the lab you should review the proof of the distributive law in the lecture slides on deduction and the proofs of double negation and De Morgan's law in the lecture slides on indirect proofs. You should also familiarize yourself with the proof checker. Note that the proof checker has all of the natural deduction rules discussed in class, however, a few of the rules have different names. In particular, bottom elimination ($\perp$E) is called (X), bottom introduction ($\perp$I) is called negation elimination ($\neg$E), and negation elimination ($\neg$E) is called (IP) for indirect proof. Questions 1 and 3 are easier and serve as warm up exercises to familiarize yourself with the proof checker.

The questions are organized into two parts. The first part contains direct proofs and the second part contains indirect proofs (proof by contradiction). In the following the $\therefore$ symbol separates the premises from the conclusion in an argument. Thus when asked to prove $\mathrm{A} \therefore \mathrm{B}$, you are to provide a proof that starting with the premises A derives the conclusion B.

Part I (Direct Proofs)

1) Commutativity of $\wedge$ and $\vee$.
   Prove $A \wedge B \therefore B \wedge A$
   Prove $A \vee B \therefore B \vee A$

2) Distributivity of $\vee$ over $\wedge$.
   Prove $A \vee (B \wedge C) \therefore (A \vee B) \wedge (A \vee C)$
   Prove $(A \vee B) \wedge (A \vee C) \therefore A \vee (B \wedge C)$

Part II (Indirect Proofs)

3) Double Negation Law.
   Prove $\neg\neg A \therefore A$
   Prove $A \therefore \neg\neg A$

4) Definition of the Conditional.
   Prove $A \rightarrow B \therefore \neg A \vee B$
   Prove $\neg A \vee B \therefore A \rightarrow B$

5) DeMorgan's Laws.
   Prove $\neg A \vee \neg B \therefore \neg(A \wedge B)$
   Prove $\neg(A \wedge B) \therefore \neg A \vee \neg B$