CS 270 Assignment 4 on Induction

Due Thur. Nov. 30 at 9:00 am.

Instructions: Assignments should be submitted to BBLearn and must be submitted as a pdf file. Handwritten solutions will not be accepted. You may use Microsoft word, LaTeX (designed for mathematical and scientific typesetting) or any word processing program (ideally supporting mathematics) for your homework so long as the output is converted to pdf. You may create your solution with a text editor, you can use _ for subscripts and ^ for superscripts, provided the output is submitted as a pdf file. In this case sums should use the following notation: sum_{i=1}^n a_i. There are 5 problems worth 15 points and one problem worth 25 points and one extra credit problem worth 10 points. Make sure you clearly indicate your base case and inductive hypothesis in your inductive proofs. You should state the reasoning used in the steps of your proof as was done in the examples from class.

An example LaTeX source file is available from the lecture outline on induction in the course website - https://www.cs.drexel.edu/~jjohnson/2017-18/fall/CS270/Lectures/induction.html. There are free distributions of LaTeX for all platforms including a web interface (ShareLaTeX). LaTeX

1) [15 points] Use induction to prove that $\sum_{i=1}^{n} ba_i = b\sum_{i=1}^{n} a_i$ and $\sum_{i=1}^{n}(a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$. You should assume that the associative and commutative laws of addition hold as does the distributive law of multiplication over addition holds; i.e. b(x+y) = bx+by.
2) [15 points] Prove by induction on n that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$. You should compute an example to make sure you understand the statement.

3) [15 points] Prove by induction that a truth table with n>0 variables has 2^n rows.

4) [15 points] Let $x_1,\ldots,x_n$ be binary variables, i.e. they can be either 0 or 1. Prove by induction that parity($x_1,\ldots,x_n$) = $x_1 \oplus\cdots\oplus x_n$, where $\oplus$ is exclusive or. The parity function returns 1 when the number of 1s in the input is odd and 0 when the number of 1s in the input is even. E.G. parity(1,0,1) = 0 and parity(1,1,1) = 1.

5) [15 points] An n-bit Gray code is a 1-1 onto mapping from [0..2^n-1] such that the binary representation of consecutive numbers differ by exactly one bit. For example, [000,001,011,010,110,111,101,100] is a 3-bit Gray code. A binary reflected Gray code is recursively constructed by

   $G_1 = [0,1]$ and $G_n = [0G_{n-1},1G'_{n-1}]$,

   G'_{n-1} reverses the order of G_{n-1}, "," is the concatenation operator for sequences, and bG_{n-1} prepends the bit b in front of each element of G_{n-1}. Note that G'_{n-1} can be obtained by complementing the leading bit of the elements in G_{n-1}. For example, $G_2 = [0G_1,1G'_1]$ = [00,01,11,10], $G_3 = [0G_2,1G'_2]$ = [000,001,011,010,110,111,101,100]. Use induction to prove that this recursive construction produces a Gray code, i.e. it satisfies the properties above.

6) [25 points] Let L be a list of length n>0 with L = (L_1 … L_n). Prove using induction that the ith element of (reverse L) is (n+1-i)th element of L. What is the base case?

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(define (reverse l)
  (if (null? l)
    null
    (append (reverse (rest l)) (cons (first l) null))))
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You may assume the following property of append

- $0 < i \leq$ (length x) $\rightarrow$ the ith element of (append x y)) = ith element of x and (length x) $< i \leq$ (length x) + (length y) $\rightarrow$ ith element of (append x y) =(i – (length x)th element of y.

[Extra credit – 10 points] Let $G_n(i)$ be the function from $[0,\ldots,2^n-1]$ defined by $G_n(i) = i$ ^ (i >> 1) [exclusive or of i and i/2]. For example, $G_2(0) = 0$, $G_2(1) = 1$, $G_2(2) = 3$, $G_2(3) = 2$. Use induction to prove that the sequence $G_n(i)$, $i=0,\ldots,2^n-1$ is a binary-reflected Gray code.