ECE 361: Probability for Engineers HW # 6 due May 14

1. In a digital communication system, 1's transmitted 55% of the time and -1's transmitted during the rest of the time. If noise is present in the channel (Gaussian noise N(0,1/25)), and the threshold is set to 0, what is the probability of error?
2. X and Y are independent and identically distributed exponential random variables, each with a parameter 5. Obtain the mean of and variance of Z=4X+3Y.
3. X and Y are independent and identically distributed Rayleigh variables, each b = 5. What is the probability that X>2Y?
4. If X is Rayleigh distributed (amplitude) with an average power of 8 units (average power is the second moment), obtain the pdf and CDF of the power conditioned on being always greater than 8 units.
5. Consider an experiment in which a die is rolled. Depending on the outcome of the roll of the die, a sample of an exponential random variable Y is chosen such that its mean is the outcome seen. Obtain the density of Y.
6. X and Y are lifetimes of two computers in an office. X and Y are independent and identically distributed exponential variables with mean lifetimes of 5 years. What is the probability that both computers will become inoperable within 4 years?
7. If X and Y are each uniform in [-1,1], obtain the pdf of Z=X-Y graphically.
8. The joint pdf of X and Y is

$$f(x,y)=\begin{cases}\frac{1}{8}, & 0<x<2, 0<y<4\\ 0, elsewhere\end{cases}$$

   a. What is the $P\left[X+Y<\frac{3}{2}\right]$?

   b. What is the $P\left[Y<4X\right]$?

9. X and Y are independent and identically distributed Rayleigh random variables, each with a parameter b. Obtain the mean and variance of W=X-Y and Z=X+Y.
10. You are given a data set consisting of 200 entries. Determine the best fit using chi square testing. Check for Nakagami, gamma, Weibull if the data set is completely positive. If data set contains negative values, test for normal or Laplacian. Laplacian is not a built in pdf in Matlab. To get the parameters of the Laplacian, simply find the mean and variance as indicated in HW#4. [You will see that a file named `HW4_data_shankar_Spring`. You will see your name (last name only) at the top of the column. You are required to use the data in that column].

Sample Result

Summary of χ^2 tests

	degF.	χ^2 stat	h	REJECT-YES/NO
Normal distribution	6	4.91	0	NO
Laplace distribution	6	20.16	1	YES

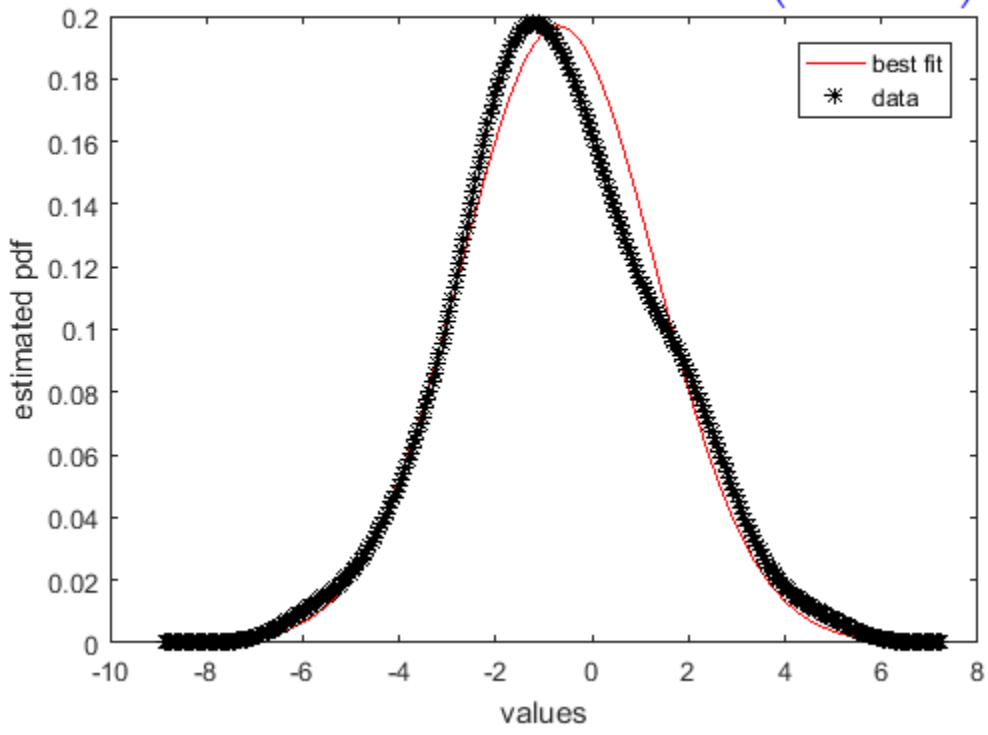
data set contains -ve values
cannot be gamma, Nakagami, Weibull

best fit: Normal pdf $f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$

μ = -0.70345 σ = 2.0267

p m shankar

best fit: Normal pdf $f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$



Example when data set contains only positive values

Summary of χ^2 tests

	degF.	χ^2 stat	h	REJECT-YES/NO
Weibull distribution	5	4.65	0	NO
Nakagami distribution	5	4.53	0	NO
gamma distribution	5	8.74	0	NO

data set is completely positive; cannot be Gaussian, Laplacian

best fit: Nakagami pdf $f_X(x) = 2\left(\frac{m}{\Omega}\right)^m \frac{x^{2m-1}}{\Gamma(m)} exp\left(-\frac{m}{\Omega}x^2\right) U(x)$

m = 0.97161 Ω = 7.3576

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best fit: Nakagami pdf $f_X(x) = 2\left(\frac{m}{\Omega}\right)^m \frac{x^{2m-1}}{\Gamma(m)} exp\left(-\frac{m}{\Omega}x^2\right) U(x)$