ECE 361: Probability for Engineers HW # 4 April 30

1. The voltage measured has been observed to follow a Rayleigh density. If the probability that the measured voltage exceeds 5 V is 0.3, what is the mean voltage? If a load of 2 Ohms exists, obtain the pdf of the power dissipated across the load.
2. Radar return amplitude X is modeled using the following density. $f(x)=\frac{1}{4}e^{-\frac{x}{4}}, \qquad x>0$
   Obtain the pdf of $Y=1/X^2$ using (a) the derivative of Y and (b) using the CDF of X.
3. A string has a length of 5 units. A point is chosen on the string randomly and the string is cut at a length X (to that point), what is the probability that this segment is at least 1.5 times longer than the remaining piece?
4. X is uniform in the range [0,1]. Obtain the pdf of $Y=-3\log(X)$.
5. A random variable X has the following property.
$$P(X>x)=\left(\frac{3}{x}\right)^{\alpha}, x>x_0 \quad \alpha>0$$
   a. What is the CDF?
   b. What is the pdf
6. If $g(X)=2F_X(x)$, where X is a continuous random variable, (1) what is the mean of g(X) (2) what is the mean of g(X)-1. What is the pdf of Y=2g(X), if X is a Gaussian random variable.
7. X is uniform in [-2,2] and Y is uniform in [2,4]. For non-zero real values of p and q, define
$$Z = pX^2 + qY^2$$
Obtain a relationship between p and q such that E(Z)=0.
8. X is uniform in [-2,6]. Obtain the pdf and CDF of $Y=1/X^2$. What is the mean of Y?
9. The lifetimes of computers is modeled in terms of a Rayleigh density. Three manufacturers A, B, C are available. Their computers have average lifetimes of 5, 6 and 7 years respectively. A company orders 20 computers from A, 30 from B and 50 from C. What is the probability that a computer randomly examined is operating beyond 6 years?
10. An organization purchases 10 computers from a manufacturer with lifetimes modeled as exponential variables (mean of 5 years). What is the probability that at least 6 computers are functioning beyond 6 years?
11. IN medical imaging, the amplitude of the image displayed is modeled as a Rayleigh random variable with an average brightness (intensity or power) of 4 units when dark suspicious regions in the image and average brightness 512 units when there are no dark regions in the image. If a threshold is set at 16 units of brightness, what is the probability that (a) a suspicious region is misidentified as a normal one (b) what is the probability that a normal region is considered to be a suspicious one. The probability in part (a) is identified as the miss rate and the probability in part (b) is considered as the false alarm rate.
12. You are given a data set consisting of 200 entries. Check if the data fits a Nakagami, gamma, Weibull if the data set is completely positive. If data set contains negative values, test if it normal or Laplacian. Laplacian is not a built in pdf in Matlab. The Laplacian has the following properties

$$f_X(x)=\frac{1}{2b}\exp\left(-\frac{|x-a|}{b}\right) \qquad F_X(x)=\begin{cases}\frac{1}{2}\exp\left(\frac{x-a}{b}\right), x<a \\ 1-\frac{1}{2}\exp\left(-\frac{x-a}{b}\right), x>a\end{cases}$$

$$E(X) = a \qquad \text{var}(X) = 2b^2$$

To get the parameters of the Laplacian, simply find the mean and variance as indicated above. Once you have the mean, you may substitute the values of a and b in f(x) to get the pdf.

For the other densities, the Matlab command is

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pd = fitdist(data,distname)
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Using the estimated parameters, you may get the theoretical density using the Matlab command

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pd = pdf(distname,x,parameters)
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Using the estimated parameters, plot the estimated densities. Plot the data pdf using ksdensity and in each case, estimate the MSE. Determine the best fit based on the lowest value of MSE.

$$MSE = \frac{1}{n}\sum_{k=1}^{n}\left[datapdf\,(k) - fittedpdf\,(k)\right]^2$$

[You will see that a file named HW4_data_shankar_Spring. You will see your name (last name only) at the top of the column. You are required to use the data in that column].

Sample results in each case (data only positive and data set containing –ve values) is shown below: You do not have to display the data. You may also display the summary as a separate text file.

If data set contains negative values

data (Shankar)

Data set contains -ve values

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$ normal pdf

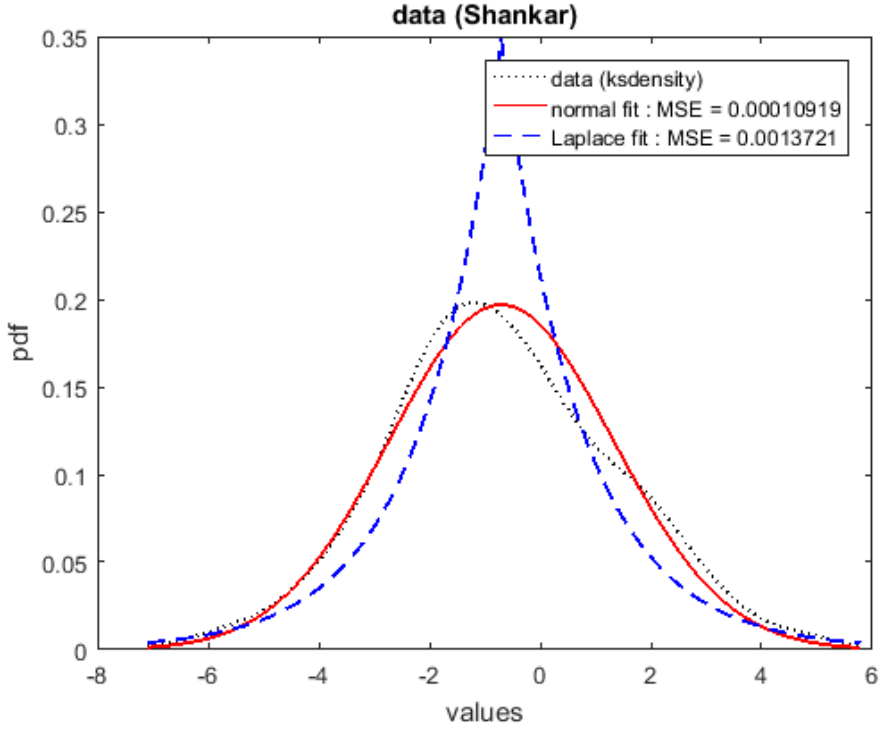
μ = -0.70345 σ = 2.0267 normal fit : MSE = 0.00010919

$$f_X(x) = \frac{1}{2b} exp\left(-\frac{|x-a|}{b}\right)$$ Laplacian pdf

a = -0.70345 b = 1.4331 Laplace fit : MSE = 0.0013721

best fit: normal

p m shankar



Data set only contains positive values

data (shankar)

Data set is completely positive

$$f_X(x) = \left(\frac{b}{a}\right)\frac{x}{a}^{b-1} exp\left(-\frac{x}{a}^{b}\right) U(x)$$ Weibull pdf

a = 2.706 b = 2.706 Weibull fit : MSE = 0.00016626

$$f_X(x) = 2\left(\frac{m}{\Omega}\right)^{m} \frac{x^{2m-1}}{\Gamma(m)} exp\left(-\frac{m}{\Omega}x^{2}\right) U(x)$$ Nakagami pdf

m = 0.97161 Ω = 7.3576 Nakagami fit : MSE = 0.00016093

$$f_X(x) = \frac{1}{b^{a}\Gamma(a)}\frac{x}{a}^{-1} exp\left(-\frac{x}{b}\right) U(x)$$ gamma pdf

a = 2.999 b = 0.80044 gamma fit : MSE = 0.00056078

best fit: Nakagami

p m shankar