1. Test whether the following expression is a valid CDF. The range of validity of the variable is 0 to ∞. If it is a valid CDF, obtain the corresponding pdf.

$$h(x) = \frac{1}{2}\left[1 - \exp\left(-\frac{x}{5}\right)\right] + \frac{1}{4}\left[1 - \exp\left(-\frac{x}{8}\right)\right] + \frac{1}{4}\left[1 - \exp\left(-\frac{x}{15}\right)\right]$$

2. For the function h(x) below to be a valid CDF, what must be the range of the variable? What is the pdf?

$$h(x) = x^2, \quad a < x < b, \quad a \geq 0, \quad b > 0$$

3. A function h(x) is given as $h(x) = \frac{1}{\sqrt{x}}, \quad 0 < x < a, \quad a > 0$. What value of a will make h(x) a valid pdf? What is the CDF?

4. A number X is picked randomly between 0 and 4.
   What is the probability that X>3?
   What is the probability that X>2 given that X has been observed to be always been larger than 1
   What is the probability that the first decimal place of X is 0.4?

5. For the following triangular pdf, obtain the CDF and P(|X|<1)

$$f_x(x) = \frac{(2 - |x|)}{4}, \quad |x| < 2$$

6. A dart is thrown at a dartboard with concentric circles. If the radius of the circle is a Rayleigh variable with a parameter 0.5, what is the probability that the dart thrown lands in a disc of thickness 0.2 units with the center at 1 unit.

7. If X has the following pdf, obtain the value of the constant A and the CDF. What is the prob{X<pi/4}.

$$f_X(x) = A\sin(x), 0 < x < \pi$$

8. X is uniform in [0,π]. What the probability that sin(X) lies between 0.2 and 0.3.

9. The probability density function of (lifetime of an electronic component in years) X is

$$f_X(x) = A x \exp\left(-\frac{x^2}{32}\right) U(x)$$

   (a) What value of A will make this a valid pdf?
   (b) What is the probability that it will fail within 6 years, given that normally these units tend to fail within 4 to7 years?
   (c) What is P[|X-3|<6]?

(d) If the unit is known to fail within 6-8 years, what is the probability that it fail within 7 years.

10. A call center receives telephone calls with an average duration of 3 minutes. What is the probability that the next call received will exceed the average duration given that calls generally last longer than 2 minutes. (Call duration is exponentially distributed).

11. In a test with 10 Yes/No questions, the probability of being correct is 0.35. If the number of Y's is modeled as discrete random variable X, obtain expressions for the CDF and pdf of this random variable. What are the mean and variance of X?

12. A random variable has the following density. Determine if it a valid density and if it is a valid density, obtain expression for the CDF

$$f(x) = 0.2\delta(x-2) + 0.1\delta(x+2) + 0.3\delta(x+4) + 0.4\delta(x-3)$$

13. The density of X is (1/4)*exp(-x/4)U(x).

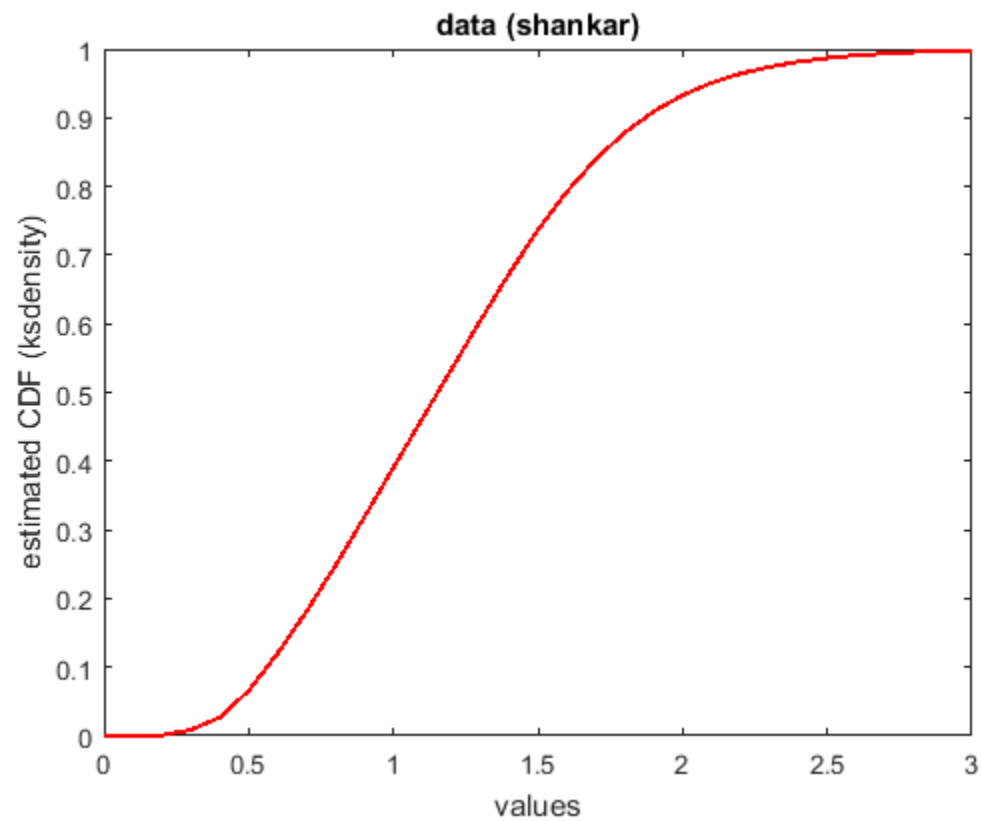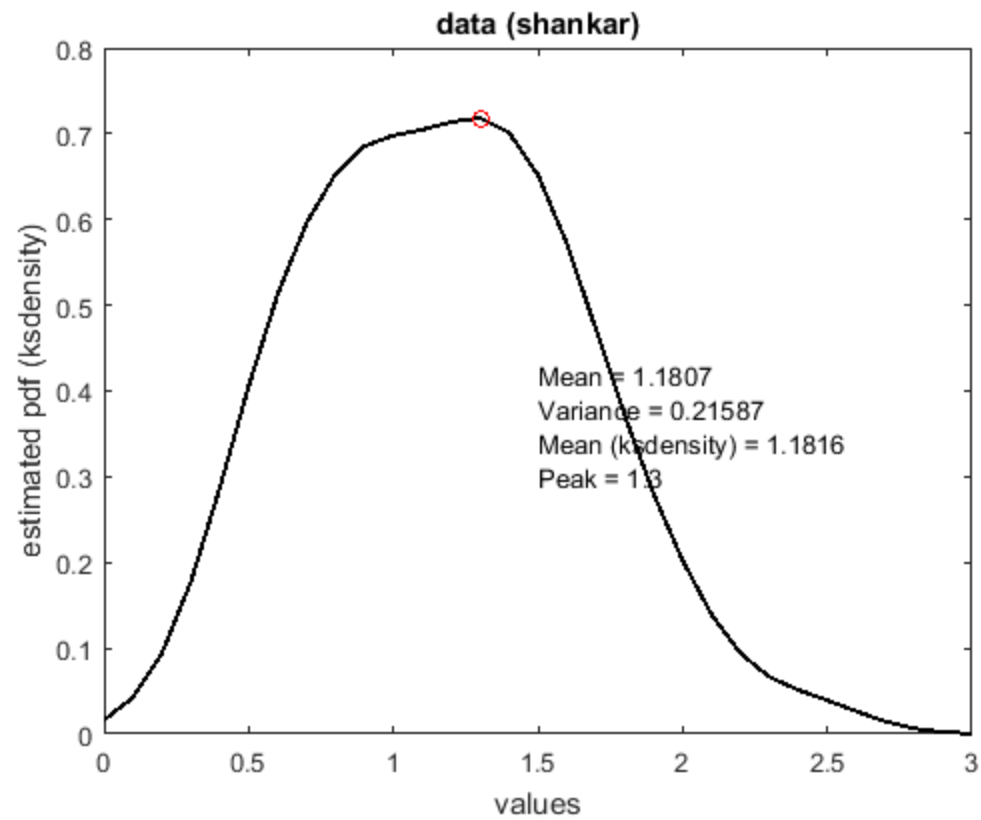What is (1) $P(X^2 > 2)$ (2) $P(\sqrt{X} < 2)$ (3) $P(X^2 = 3)$
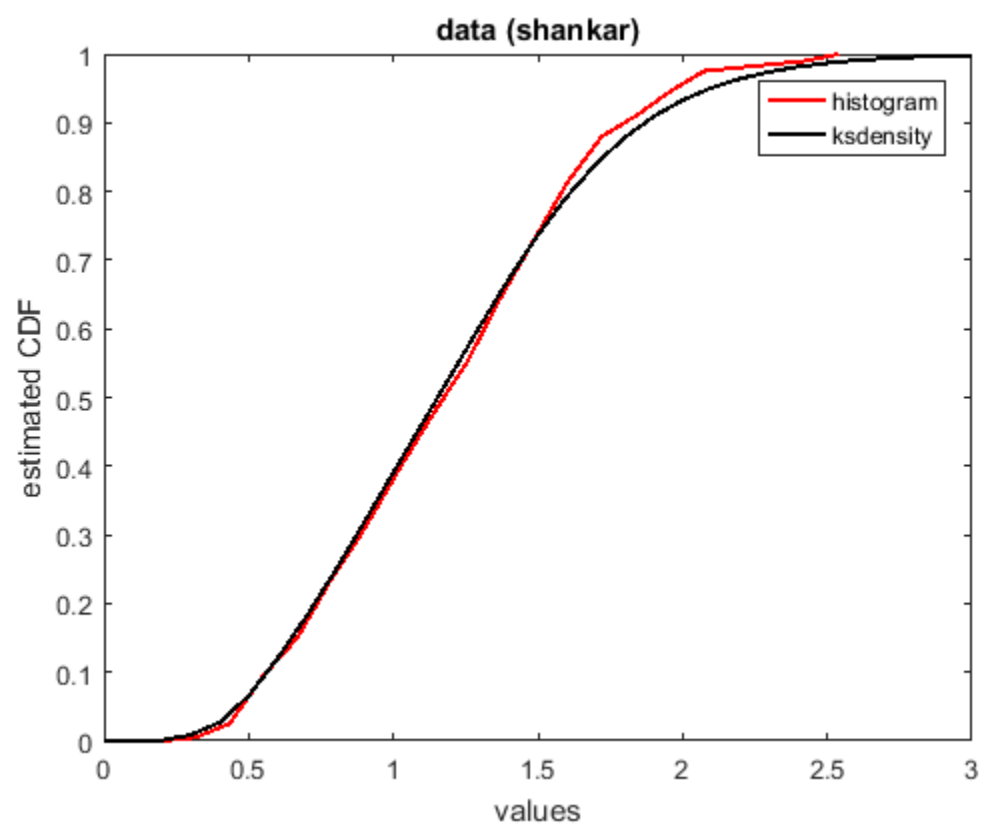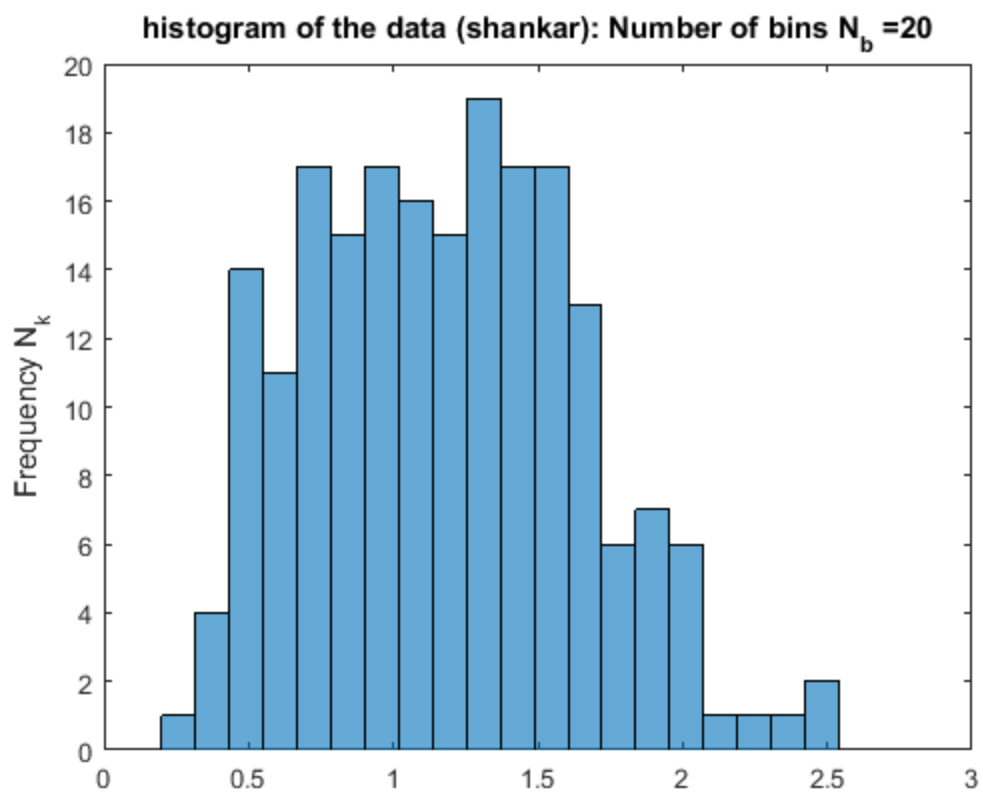
14. You are given a data set collected with 200 entries. Estimate the pdf, CDF, mean and variance from the data. Also determine the location of the peak of the density. Verify the mean from the calculated density using the definition of the mean. Verify your CDF estimate by obtaining the histogram of the data.

There is a file named `HW3_data_shankar_Spring`. The column headings have the last names of the students. Choose the set meant for you.

Sample results are shown. You must produce all the displays containing the information shown.

[Hint: Use **ksdensity**(.) in Matlab. It can generate the estimated pdf and the CDF for any given data. Most of what is needed with this HW problem can be undertaken using the Matlab published document posted in week # 1 under the Supplementary Materials Tab.]

data (shankar)

Mean = 1.1807
Variance = 0.21587
Mean (ksdensity) = 1.1816
Peak = 1.3

data (shankar)

histogram of the data (shankar): Number of bins N_b =20

data (shankar)

P. M. Shankar, April 2, 2018