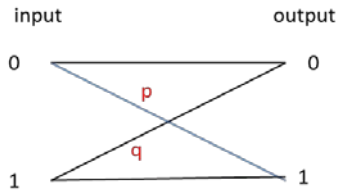
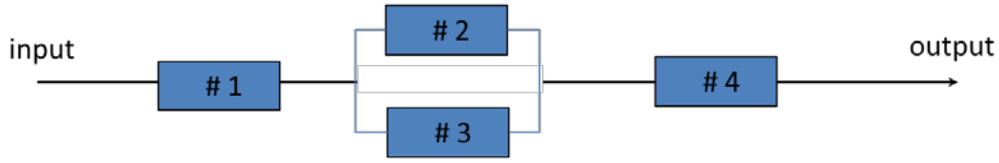
ECE 361: Probability for Engineers HW # 2 due April 16

1. A missile can be launched if two relays A and B both have failed. Probabilities of A and B failing are 0.01 and 0.03 respectively. It is known that B is more likely to fail (probability of 0.06) if A has failed. (i) What is the probability of an accidental missile launch (ii) what is the probability that A will fail if B has failed. (iii) Are the events "A fails" and "B fails" statistically independent.
2. A non-symmetric binary channel is shown below. The inputs [0, 1] are equiprobable. If you received a "0", what is the probability that it was actually transmitted as a "0".



3. A circuit has 4 components connected as shown. If each component can fail independently with a probability of p, what is the probability that the system does not fail.



4. Companies do not test every component they produce. A company that manufactures microphones takes two microphones from a batch consisting of 50 (in a box). If both microphones work, the whole box is accepted. Otherwise, the whole box is rejected. If there are 45 excellent ones and 5 defective ones in a box, what is the probability that the box is accepted.

5. Five good light bulbs and two defective ones were mixed up unknowingly. To identify the defective ones for disposal, the bulbs are tested one one by one. (a) If both defective ones are identified in the testing of the second one, what is the probability of such a 'lucky' break? (b) If three bulbs had to be tested to get the two defective ones, what is the probability of such an event.

6. Lifespan of male elephants using continuous probability model. The probability that a selected male lives past the age x is $\exp\left(-\frac{x}{46}\right)$.
   (a) What is the probability that a randomly selected male will live past 55 years (b) If it is known a certain male elephant is alive at 55, what is the probability that it might make it to 60.
7. Two numbers are drawn, each in the range {0,1}. What is the probability that their sum is less than 1 while their product is greater than 1/5?
8. Two signals reach the receiver independently, each in an interval of 0- 2 minutes. Jamming occurs if the signals arrive if the time difference between their arrival is less than 0.2 min. What is the probability that jamming will take place?

9. An organization needs 100 computers. It buys 40 from manufacturer A, 35 from B and 25 from C. The manufacturer A certifies that only 2% may be defective while B and C certify that their products are only 1% defective. If a computer is chosen randomly at the orginaztion, what is the probability that it is not defective.
10. In certain parts of the world tuberculosis (a very treatable and curable illness) is present in 25% of the people. If a simple chest X-ray can be used as a diagnostic tool with an accuracy of 99% detection of TB if it is present. Only in 0.5% of the cases, normal people get a positive diagnosis. If a person is selected at random and the diagnosis is positive, what is the probability that the person is actually infected.
11. Professors often attempt to determine if the submissions by the students are genuine or copied off the web sources. The program that performs this task is only 95 % accurate in correctly identifying a genuine submission and 80% accurate in correctly identifying copies. Based on the past statistics, 15% of the student turned in copied work. If a work is identified as a copy by the program, what is the probability that it is indeed a sample of copied work.
12. You are given the following data sets, X represents the data collected from an infra red (IR) receiver when there was no target in front of the transmitter and Y represents the data collected from an IR receiver when a target was present in the field of view of the transmitter. With these data sets, obtain apriori probabilities. Obtain the probabilities of false alarm, miss and error rates when the threshold is set at 3, 5, and 6. We expect the presence of the target to result in higher values of the measurand.

```
X=[1.6978    7.7368    8.7591    1.9001    0.7486    0.2048    6.1228,
1.6926    2.2690   13.2931];

Y=[11.4976,4.5624,17.0159,5.1402,8.9841,8.9526,2.0939,6.1407];
```

13. (Individual assignments) A single column of data is provided. The first 40 values consists of the data collected from the IR receiver when no target is present and the remaining values constiture the data collected when the target is present. After deteremining the threshold (obtain the peaks of the hostograms and take the mid point), obtain the following:
    a priori probabilities
    probablities of miss, false alarm
    sensitivity, specificity
    positive predictive value
    accuracy
    Each student has a file with his/her last name. ONLY USE the FILE with your name.

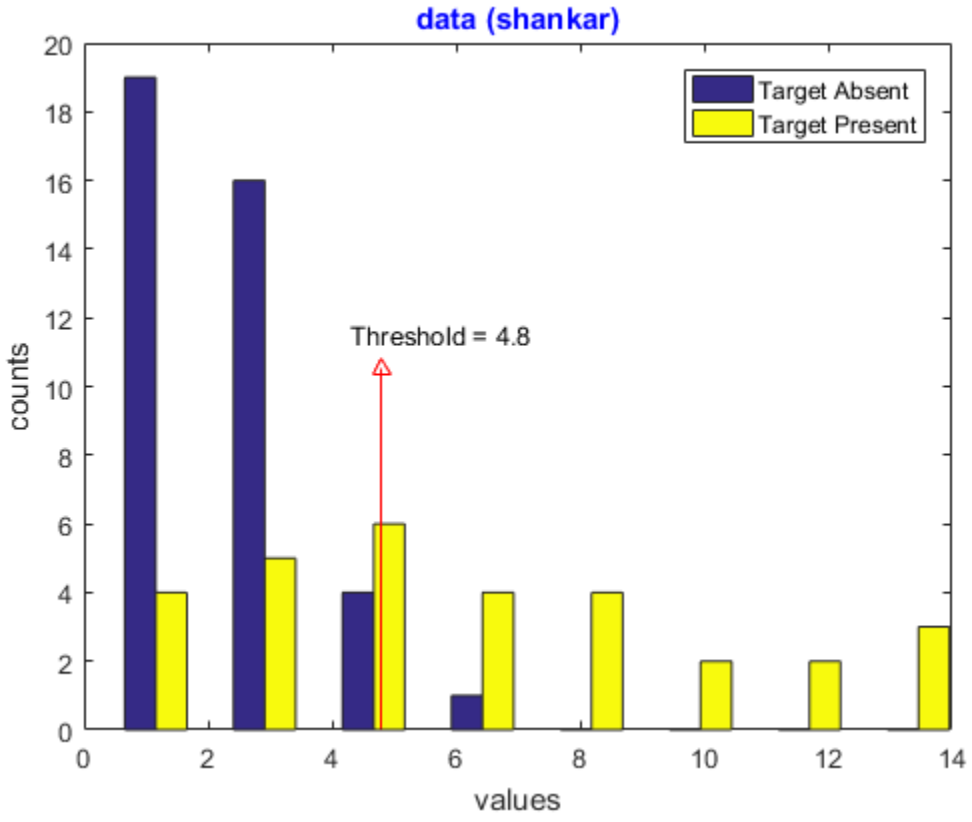
    Repeat the analysis by obtaining a continous density plot using ksdensity(.) in Matlab and using the point of intersection of the two densities as the threshold.
    In addition to the parameters listed above, for this case, obtain the confusion matrix also.

    Sample results are displayed.

data (shankar)

Target Absent					Target Present				
1.3327	2.8137	1.4092	0.7188	1.967	2.4904	12.4877	4.1481	8.5447	11.4267
1.7006	5.1574	1.5566	1.064	1.8101	14.3372	0.2788	13.5726	5.0231	2.4758
1.2989	3.1482	2.7676	1.1658	2.6732	6.1244	5.2099	8.8066	2.2599	9.9207
3.3691	1.3687	1.6516	1.2788	6.2759	4.675	4.3574	1.2098	3.4559	5.868
2.0095	1.0061	2.0891	2.6408	3.0304	5.0417	7.4526	10.4369	2.4114	7.5045
2.9398	2.493	1.3638	2.8123	1.8705	7.0243	6.1976	1.5231	1.4325	13.92
2.7457	2.51	2.0927	3.0023	3.8574					
1.8722	4.9084	3.237	1.3146	4.6582					

Not needed



Sorted and Partitioned Data : Threshold at 4.8

Target Absent					Target Present				
6.2759	3.0304	2.6408	1.8705	1.3327	14.3372	9.9207	6.1976	4.675	2.4114
5.1574	3.0023	2.51	1.8101	1.3146	13.92	8.8066	6.1244	4.3574	2.2599
4.9084	2.9398	2.493	1.7006	1.2989	13.5726	8.5447	5.868	4.1481	1.5231
4.6582	2.8137	2.0927	1.6516	1.2788	12.4877	7.5045	5.2099	3.4559	1.4325
3.8574	2.8123	2.0891	1.5566	1.1658	11.4267	7.4526	5.0417	2.4904	1.2098
3.3691	2.7676	2.0095	1.4092	1.064	10.4369	7.0243	5.0231	2.4758	0.2788
3.237	2.7457	1.967	1.3687	1.0061					
3.1482	2.6732	1.8722	1.3638	0.7188					

Number of samples above threshold = 3

False Alarm rate : 3 in 40

Number of samples above threshold = 18

Miss rate : 12 in 30

Not needed

(c) P. M. Shankar

Sorted and Partitioned Data : Threshold at 4.8

Sample size (Target Absent) = 40

Sample size (Target Present) = 30

a priori probability : Target Absent = 0.57143

a priori probability : Target Present = 0.42857

Threshold value = 4.8

Probability of correct TARGET detection (sensitivity) = 0.6

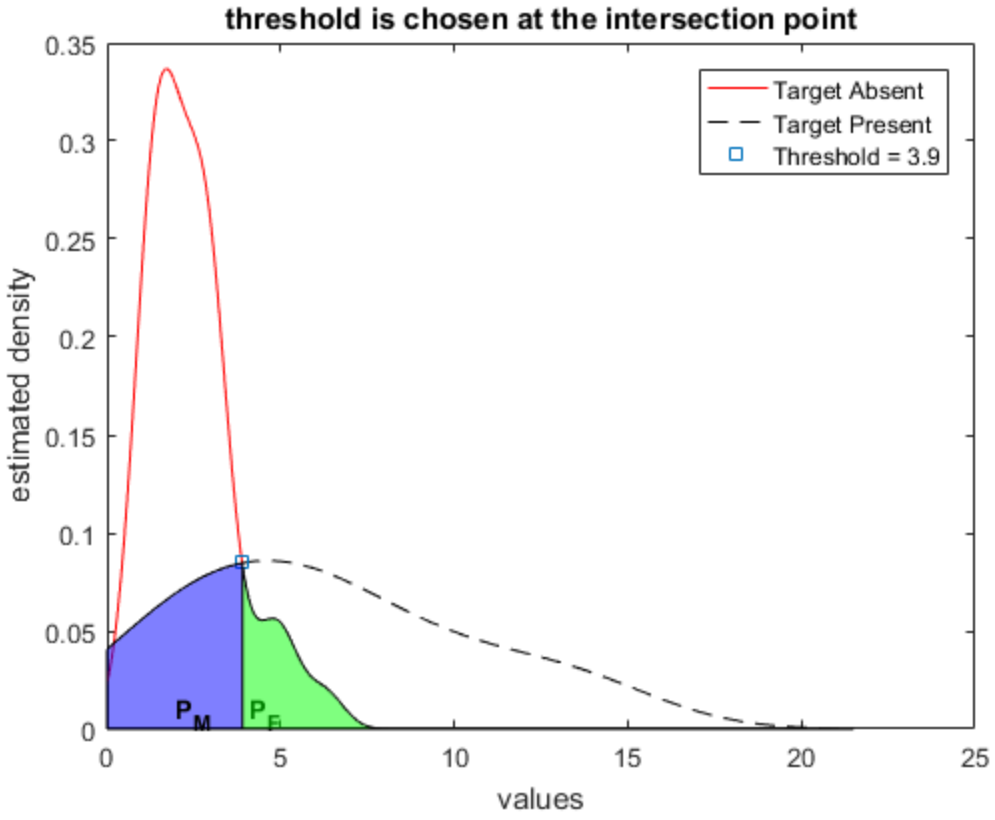
Probability of Miss = 1- sensitivity = 0.4

Probability of correct NO TARGET detection (specificity) = 0.925

Probability of False Alarm = 1- specificity = 0.075

Positive Predictive Value (PPV) = a posteriori probability = 0.85714

Overall Accuracy = 0.78571



Sorted and Partitioned Data : Threshold at 3.9
Threshold is chosen at the intersection point

Target Absent					Target Present				
6.2759	3.0304	2.6408	1.8705	1.3327	14.3372	9.9207	6.1976	4.675	2.4114
5.1574	3.0023	2.51	1.8101	1.3146	13.92	8.8066	6.1244	4.3574	2.2599
4.9084	2.9398	2.493	1.7006	1.2989	13.5726	8.5447	5.868	4.1481	1.5231
4.6582	2.8137	2.0927	1.6516	1.2788	12.4877	7.5045	5.2099	3.4559	1.4325
3.8574	2.8123	2.0891	1.5566	1.1658	11.4267	7.4526	5.0417	2.4904	1.2098
3.3691	2.7676	2.0095	1.4092	1.064	10.4369	7.0243	5.0231	2.4758	0.2788
3.237	2.7457	1.967	1.3687	1.0061					
3.1482	2.6732	1.8722	1.3638	0.7188					

Number of samples above threshold = 4

False Alarm rate : 4 in 40

Number of samples above threshold = 21

Miss rate : 9 in 30

not needed

Summary of the analysis
shankar

Sample size (Target Absent) = 40

Sample size (Target Present) = 30

a priori probability : Target Absent = 0.57143

a priori probability : Target Present = 0.42857

Threshold value = 3.9

Probability of correct TARGET detection (sensitivity) = 0.7

Probability of Miss = 1- sensitivity = 0.3

Probability of correct NO TARGET detection (specificity) = 0.9

Probability of False Alarm = 1- specificity = 0.1

Positive Predictive Value (PPV) = a posteriori probability = 0.84

Overall Accuracy = 0.81429

Confusion Matrix (Threshold Value = 3.9)

Data Collected	Target Detected	Target Not Detected	Total Counts
Target Absent	4	36	40
Target Present	21	9	30
Total Counts	25	45	70

Errors circled

$P_F = \frac{1}{10}$ $P_M = \frac{3}{10}$ $\mathrm{PPV} = \frac{21}{25}$ $\mathrm{err} = \frac{13}{70}$ $\mathrm{acc} = \frac{57}{70}$

Summary of the analysis
shankar

p m shankar