ECE 361: Probability for Engineers HW # 5 May 7

1. The power received on your cellphone is exponentially distributed with a mean of -85 dBm. Note that exponential distribution is defined for power defined in units such as Watts, milliW, nW or pW. Outage of service occurs when power falls below P_{thr}. What is the outage probability if P_{thr}=-100 dBm?
2. X is N(3,4). Obtain the following probabilities using the Normal Probability Table. In each case, express the Z_{score} to be used. (a) $|X| > 5$ (b) $|X - 4| < 3$
3. X is a Rayleigh distributed random variable with parameter 4. An electronic clipper operates on X as follows:

   $$Y = \begin{cases} 3, & X \le 3 \\ X, & X > 3 \end{cases}$$

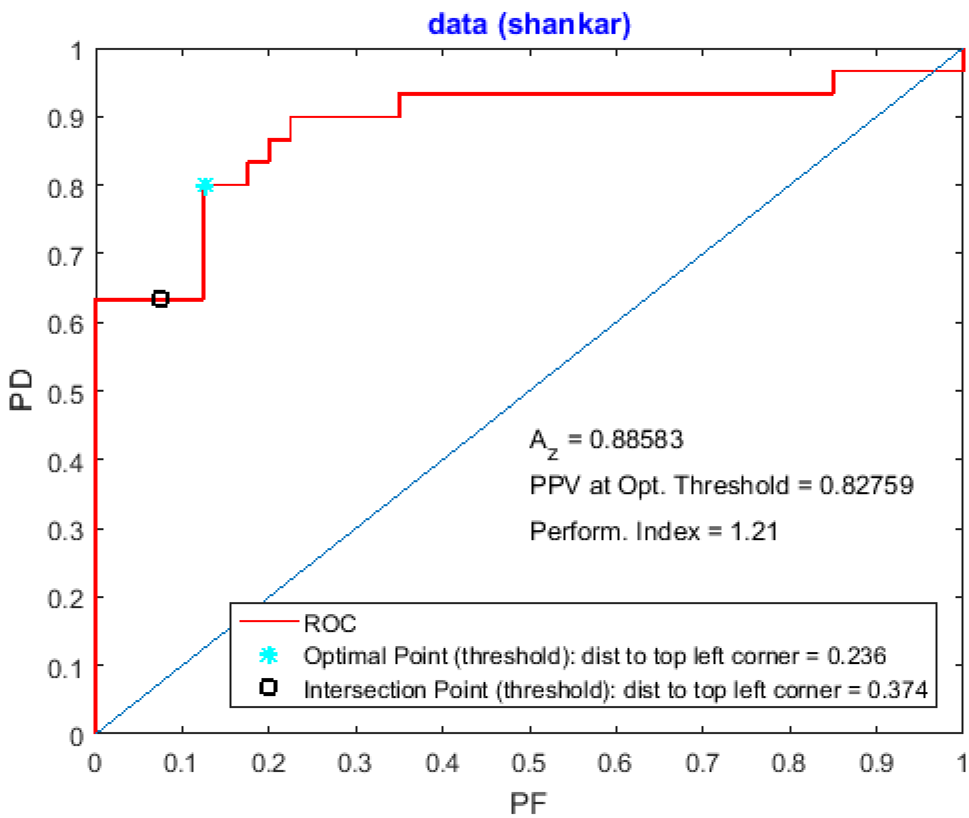
   Obtain the pdf and CDF of Y.
4. X is exponentially distributed density of the power expressed in mW with an average of 5 mW. Obtain the pdf of the power expressed in dBm units.
5. The power in a wireless system X is gamma distributed with a pdf G(a,b). If the power is scaled such that $Y=X^k$, k>1, obtain an expression for the pdf of Y.
6. Power received at the radar receiver can be described in terms of an exponential random variable. In the absence of any target, the power received is an exponential variable with a mean of 1 mW. When target is present, power received is also an exponential variable with a mean of 16 mW. If the threshold for decision is set to 4 mW, obtain the probabilities of False Alarm and Miss. If the chances of the target being present are only 10%, what is the probability of error in this decision making process?
7. X is Gaussian, N(4,9). The noise modeled using this Gaussian is passed through an A/D converter such that any voltage less than -10 becomes -10 and any voltage greater than 10 becomes 10. The values between -10 and 10 are equally divided into 5 bins. Obtain the pdf of the digitized output.
8. X is Gaussian, N(0,16). Obtain the pdf of Y=[X|X>4].
9. X is Gaussian, N(3,16). Obtain the pdf of Y=[X|-4<X<4].
10. Data collected from a machine vision lab to see the efficiency of an object recognition system is given (HW5_data_shankar). The IR data set consist of a column of samples (one for each of you as indicated by your last name), the first 40 values are the responses when there is no target in the field of view of the robot with remaining 30 values are the responses when there is target in field of view. As shown in the lecture, obtain the receiver operating characteristic curve for the system, determine the area under the ROC curve, optimal operational point, the positive predictive value corresponding to the threshold at the optimal operational point. A sample result is shown on the next page.

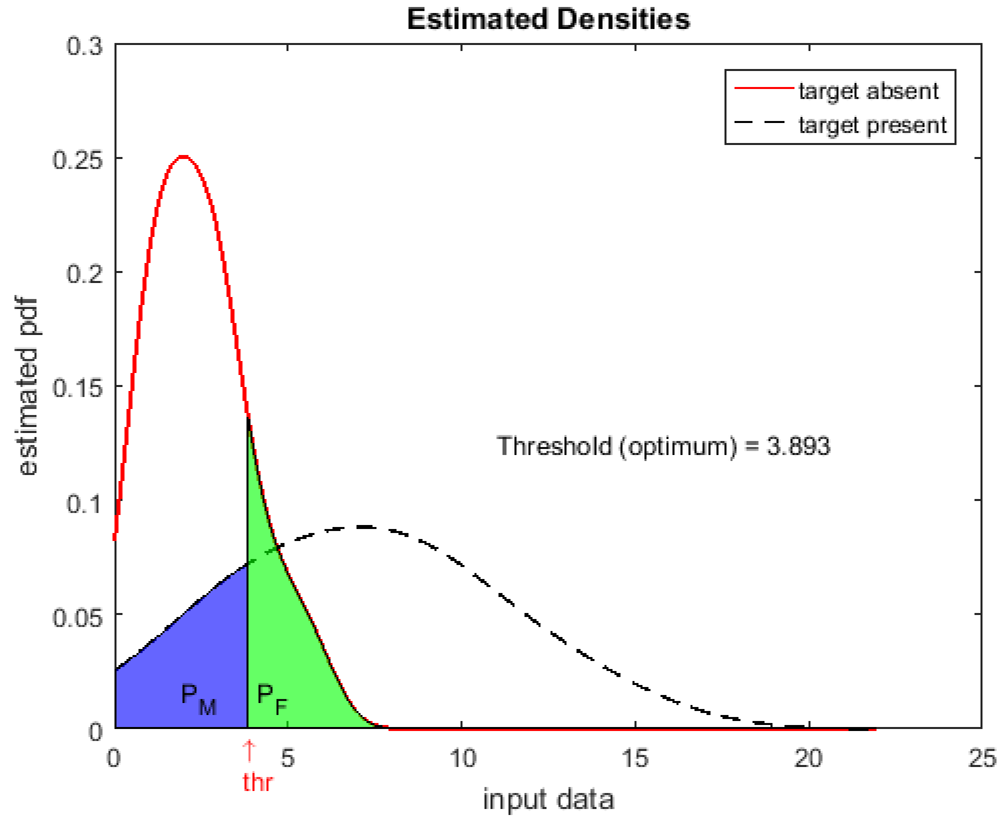
data (shankar)

Target Absent					Target Present				
3.893	0.741	1.18	0.758	0.526	8.056	4.448	14.66	8.952	8.061
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	7.037	10.126
[illegible]								[illegible]	2.83
[illegible]								[illegible]	9.804
[illegible]								[illegible]	6.994
[illegible]								[illegible]	6.768
2.396	3.569	2.26	1.191	5.805					
1.889	3.308	1.333	2.048	2.849					

NOT NEEDED

(c) P. M. Shankar





Confusion Matrix : Threshold (optimum) = 3.893

Data Collected	Target Detected	Target Not Detected	Total Counts
Target Absent	5	35	40
Target Present	24	6	30
Total Counts	29	41	70

Errors circled

dist to top left corner of the ROC curve = 0.236

Transition Matrix: Threshold (optimum) = 3.893

$$T_X = \begin{bmatrix} P(NotDetected|Absent) & P(NotDetected|Present) \\ P(Detected|Absent) & P(Detected|Present) \end{bmatrix}$$

$$T_X = \begin{bmatrix} 1-P_F & P_M \\ P_F & 1-P_M \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{7}{8} & \frac{1}{5} \\ \frac{1}{8} & \frac{4}{5} \end{bmatrix} \Rightarrow \begin{bmatrix} 0.875 & 0.2 \\ 0.125 & 0.8 \end{bmatrix}$$

$P_F = \frac{1}{8}$ = 0.125 $\quad P_M = \frac{1}{5}$ = 0.2 $\quad \mathrm{PPV} = \frac{24}{29}$ = 0.82759

p m shankar